On the A_α-spectral radius of connected graphs[*]

Abdollah Alhevaz [ID], Maryam Baghipur [ID]

Faculty of Mathematical Sciences, Shahrood University of Technology,
P.O. Box: 316-3619995161, Shahrood, Iran

Hilal Ahmad Ganie [ID]

Department of School Education, JK Govt. Kashmir, India

Kinkar Chandra Das [†] [ID]

Department of Mathematics, Sungkyunkwan University, Suwon 16419, Republic of Korea

Abstract

For a simple graph G, the generalized adjacency matrix $A_\alpha(G)$ is defined as $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G), \alpha \in [0,1]$, where $A(G)$ is the adjacency matrix and $D(G)$ is the diagonal matrix of the vertex degrees. It is clear that $A_0(G) = A(G)$ and $2A_{\frac{1}{2}}(G) = Q(G)$ implying that the matrix $A_\alpha(G)$ is a generalization of the adjacency matrix and the signless Laplacian matrix. In this paper, we obtain some new upper and lower bounds for the generalized adjacency spectral radius $\lambda(A_\alpha(G))$, in terms of vertex degrees, average vertex 2-degrees, the order, the size, etc. The extremal graphs attaining these bounds are characterized. We will show that our bounds are better than some of the already known bounds for some classes of graphs. We derive a general upper bound for $\lambda(A_\alpha(G))$, in terms of vertex degrees and positive real numbers b_i. As application, we obtain some new upper bounds for $\lambda(A_\alpha(G))$. Further, we obtain some relations between clique number $\omega(G)$, independence number $\gamma(G)$ and the generalized adjacency eigenvalues of a graph G.

Keywords: Adjacency matrix, signless Laplacian matrix, generalized adjacency matrix, spectral radius, degree sequence, clique number, independence number.

Math. Subj. Class. (2020): Primary: 05C50, 05C12; Secondary: 15A18.

A_α-spektralni polmer povezanih grafov[*]

Abdollah Alhevaz ,   Maryam Baghipur

Faculty of Mathematical Sciences, Shahrood University of Technology,
P.O. Box: 316-3619995161, Shahrood, Iran

Hilal Ahmad Ganie

Department of School Education, JK Govt. Kashmir, India

Kinkar Chandra Das [†]

Department of Mathematics, Sungkyunkwan University, Suwon 16419, Republic of Korea

Povzetek

Naj bo G enostaven graf; potem je posplošena matrika sosednosti $A_\alpha(G)$ definirana kot $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G), \alpha \in [0,1]$, kjer je $A(G)$ matrika sosednosti in $D(G)$ diagonalna matrika stopenj vozlišč. Jasno je, da $A_0(G) = A(G)$ in $2A_{\frac{1}{2}}(G) = Q(G)$, kar pomeni, da je matrika $A_\alpha(G)$ posplošitev matrike sosednosti in nepredznačene Laplaceove matrike. V tem članku izpeljemo nekaj novih zgornjih in spodnjih mej za spektralni polmer $\lambda(A_\alpha(G))$ posplošene matrike sosednosti glede na stopnje vozlišč, povprečne 2-stopnje vozlišč, red, velikost, itd. Karakteriziramo ekstremne grafe, ki te meje dosegajo. Pokažemo, da so naše meje boljše od nekaterih že znanih mej za nekatere razrede grafov. Izpeljemo splošno zgornjo mejo za $\lambda(A_\alpha(G))$ glede na stopnje vozlišč in pozitivna realna števila b_i. Z uporabo teh rezultatov dobimo za $\lambda(A_\alpha(G))$ nekaj novih zgornjih mej. Izpeljemo tudi nekaj relacij med številom klik $\omega(G)$, številom neodvisnosti $\gamma(G)$ in lastnimi vrednostmi grafa G za pripadajočo posplošeno matriko sosednosti.

Ključne besede: Matrika sosednosti, nepredznačena Laplaceova matrika, posplošena matrika sosednosti, spektralni polmer, zaporedje stopenj, število klik, število neodvisnosti.

Math. Subj. Class. (2020): Primarna: 05C50, 05C12; sekundarna: 15A18.

E-poštni naslovi: a.alhevaz@shahroodut.ac.ir (Abdollah Alhevaz), maryamb8989@gmail.com (Maryam Baghipur), hilahmad1119kt@gmail.com (Hilal Ahmad Ganie), kinkardas2003@gmail.com (Kinkar Chandra Das)