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Evolution of Human Signed Networks

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Abstract

Two strands of empirical research were based on Heider's work. One was experimental and remained true to idea that mental affect processes operating within human actors are central while the other jumped to the level of group dynamics and relegated the mental affect processes to the status of background phenomena. By the 1980s, both strands had petered out with little cumulated knowledge. Recently, the 'group-level' strand of research has received renewed attention. Much of the recent work, while vigorous, has fallen short precisely because it ignored some of the foundational ideas of Heider. Given this diagnosis, I suggest a new research agenda for structural balance theory that integrates the dynamics of mental affect processes with group-level dynamics.

1 Introduction

Heider (1946, 1958) has been credited with the first general statement of 'structural balance theory' (Taylor, 1970). Two strands of empirical research have been based on these foundations. One is experimental and remained true to idea that mental affect processes operating within the minds of human actors are central to balance theory. By a 'mental affect process' I mean thoughts about, and perceptions of, affect ties between human actors. The other strand jumped to the level of group dynamics and relegated mental affect processes to the status of background phenomena (while assuming they had some unknown relevance.) Both strands seem to have petered out with little cumulated knowledge[2] (Opp, 1984). Recently, the 'group-level' strand of research has received renewed attention (starting with Doreian and Mrvar (1996)) and my concern is focused on this strand. The recent work has raised more questions than it has answered and, while interesting, the recent results point to a need for a serious reconsideration of the foundational ideas of Heider and the integration of the two strands of research.

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[2] However, see Davis (1979) for a more positive assessment.

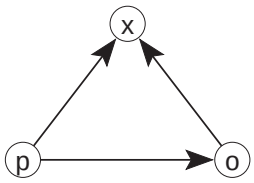

Figure 1: A single unit formation triple, *pox*.

2 Structural balance theory

2.1 Heider's starting points

I start with the first sentence of Heider's 1946 article: "Attitudes towards persons and causal unit formations *influence each other* (1946: 107, emphasis added) and wonder why we seem to have missed this in our zeal to get to the analysis of group-level phenomena. Central to the idea of structural balance is the *pox*-triad where p and o are actors and x is an impersonal entity such as "a situation, event, idea or thing". In this triad, $p \rightarrow o$ is a relation from p to o, $p \rightarrow x$ is a relation from p to x and $o \rightarrow x$ is p's perception of the tie from o to x. One generic type of relation considered by Heider is the 'unit-formation' relation, U, examples of which include proximity (p is close to x), causes (p causes x to occur), membership (p is a member of x) and belonging (p belongs to x). The negation of U is $\neg U$ and denotes 'not proximate to', 'does not cause', 'is not a member' and 'does not belong to' depending on the meaning of U. Figure 1 shows a triple for some impersonal entity, x. If x is a category, then $o \rightarrow x$ represents o belongs to the category x, that is oUx. The tie $p \rightarrow x$ represents the attitude of p towards x and $p \rightarrow o$ is p's tie to o (often contingent on p knowing - or having a perception of - oUx).

The second relation, denoted by L, is signed affect such as like-dislike, love-hate, and respect-disrespect with $\neg L$ denoting the negation of L. For convenience, the positive part is labeled P while $\neg L$ is labeled N. The ties of L in the signed triple can be positive (P), negative (N) or null. Ignoring null ties for the moment, pPo denotes a positive tie from p to o while pNo denotes a negative tie from p to o. The *pox*-triple of Figure 1 becomes the *poq*-triple of Figure 2 when x (an impersonal object) is replaced by q (a third person). The entity $o \rightarrow q$ is p's perception of the tie from o to q. This last sentence is crucial because in the rush towards group-level dynamics in the application of balance theory, many balance theorists, me included, simply assumed that $o \rightarrow q$ was the tie from o to q rather than p's perception of that tie. Table 1 shows the eight possible signed triples (when null ties are excluded). The *pox* and *poq* triples have the same logical structure.

For 'belonging', one potent unit-formation ties is defined by race. Suppose p is a white actor, o is a black actor, and x is a category defined as 'blacks'. The tie $o \rightarrow x$ is a unit formation relation when p recognizes o as belonging to x (oUx or 'o is black'). Next, $p \rightarrow x$ denotes p's attitude to x (like, dislike, or neutral) and $p \rightarrow o$ is p's affect tie to o. Intuitively, it would seem that if p is favorably inclined towards x and sees o as belonging to x, then p would like o. (This would

Table 1: Eight signed triples.

Balanced			Imbalanced		
$p \rightarrow o$	$p \rightarrow x$	$o \rightarrow x$	$p \rightarrow o$	$p \rightarrow x$	$o \rightarrow x$
$p \rightarrow o$	$p \rightarrow q$	$o \rightarrow q$	$p \rightarrow o$	$p \rightarrow q$	$o \rightarrow q$
P	P	P	P	P	N
P	N	N	P	N	P
N	P	N	N	P	P
N	N	P	N	N	N

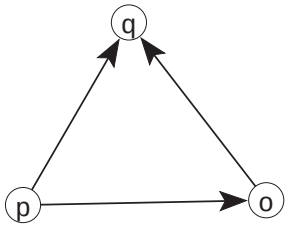

Figure 2: A single affect relational triple, *poq*.

be an example of the triple PPP located among the balanced triples on the left in Table 1.) Alternatively if p does not like x, sees o as belonging to x, it seems reasonable[3] that p will dislike o. (This would an example of the imbalanced triple NNP on the right in Table 1.) Combinations of three ties from $\{L, \neg L, U, \neg U\}$ make up the triples defined by Heider where both L and U are labeled P with $\neg L$ and $\neg U$ labeled by N. These triples differ in crucial ways as shown in Table 1. On the left are the so-called balanced triples while the imbalanced triples are on the right with P and N denoting the signs of the ties in the triples. The structure of the triples with x and the triples with q are the same in Table 1.

2.2 Heiderian mechanisms

In Heider's approach, actors have mental images defined by affect and unit-formation ties. A triple containing an even number of negative ties is defined as balanced while a triple with an odd number of negative ties is imbalanced. The presence of imbalanced triples creates 'tension' for an actor and, with it, psychological discomfort. Accordingly, the actor seeks to reduce the tension by moving an imbalanced triple to a balanced triple. Suppose, for example, p dislikes blacks (pNx), recognizes that o is a black (oUx) and p likes o (pPo). According to Heider, this is imbalanced. Actor p has three options for achieving balance. One, and perhaps the least likely, is to decide that o is not black and so create $o\neg Ux$. Another is to decide that the aversion to blacks is stronger that the liking of o, as an individual, and thus create pNo. The third option to change to pLx given pLo. All three changes, by themselves, lead to

[3]Of the two examples, this one has more force in the sense that dislike of x is more potent for creating pNo (or $p\neg Lo$) if oUx, than pLo operating to generate pLx.

a balanced triple even though the triples differ from each other[4]. Life gets a little more complicated for p if the actor is located in multiple triples and changes towards balance in one triple creates imbalance in another triple. There are 'forces' towards balance that operate through time, for all triples, that impel each actor towards being located only in triples that are balanced . The explicit statement of balance theory is that actors move 'towards balance' while there is an implicit presumption that actors do achieve balance after sufficient time.

2.3 Generalizing structural balance

Cartwright and Harary (1956) generalized Heider's ideas in a way that had far reaching technical and substantive implications. (See also Harary (1959).) First, they moved the empirical focus from mental affect processes to group-level dynamics and observable relations between actors. Second, they also merged the ideas of affect ties and unit formation ties by considering only 'signed ties'. The third change of perspective came when they considered cycles (and, in principle, semi-cycles) of ties and so moved beyond triples. The sign of a triple was defined as the product of the signs in the triple and the idea was extended so that the sign of a cycle (or semi-cycle) was the product of the signs in the cycle (or semi-cycle). A cycle is balanced if the product of signs in the cycle is positive (there is an even number of negative ties in the cycle). Similarly, a cycle with an odd number of negative ties is imbalanced. Instead of dealing with a set of triples, the focus was on networks (or graphs) of signed ties and a signed network is balanced if all of its cycles are balanced.

Cartwright and Harary (1956) proved a remarkable theorem that, informally, states a balanced signed network is balanced if and only if the set of vertices (representing actors) can be partitioned into two subsets, later called plus-sets, such that all positive ties were between actors within plus-sets and all negative ties were between actors from different plus sets[5]. This (first) 'structure theorem' is an elegant way of linking the mental affect processes and the overall (macro) structure of the group. The downside is that the mental affect processes either disappear entirely or are assumed to operate with all actors having an accurate and complete image of the signed networks within which they were located[6]. If there was movement towards balance, this meant that the number of negative cycles would diminish and the group structure moved towards balance in the form of two mutually hostile subgroups, each having internal positive ties.

Assessing movement towards 'balance' requires a measure of imbalance. Much of the early work used the signs of cycles to construct measures of imbalance in terms of the proportion of cycles that were imbalanced. This was complicated by having cycles with different lengths and the need to introduce weighting schemes according to cycle lengths. Using cycles to define imbalance was not that productive

[4]Of course, there are other options for p. Given pLo, the p might simply suspend judgment regarding race and make pLx a null tie and so reach 'vacuous balance'.

[5]In the case of a network having only positive ties, one of the plus-sets is empty.

[6]The work of Freeman *et al.* (1987), albeit for unsigned networks, suggests that this is a risky assumption.

as the calculation of a cycle-based measure of imbalance is a computationally difficult problem for any network, one that was solved in a general fashion by Hummon and Fararo (1995) only in 1995. A more useful measure of imbalance is the smallest number of lines whose reversal of sign leads to a balanced network. An alternative definition of imbalance is the smallest number of lines whose removal leads to a balanced network. The line (sign) reversal index and the line deletion index are identical. See Harary *et al.* (1965).

The idea of groups breaking into two mutually hostile subgroups is appealing. Davis (1967) observing that groups often break into more than two mutually hostile subgroups, explored the balance theoretic ways in which this partitioning could occur. He suggested considering the all negative triple as balanced. Davis defined a cycle as imbalanced if it had exactly one negative tie and balanced otherwise. A network is balanced if all of its cycles are balanced (in this modified sense). He proved a second structure theorem stating that a network is balanced if and only if the vertices can be partitioned into two or more plus-sets where all positive ties are within plus-sets and all negative ties are between actors in different plus-sets. In general, we talk of k-balance where k is the number of plus-sets. For $k = 2$ we have the original definition of structural balance and for $k > 2$ we have the generalized form of balance theory.

2.4 A partitioning algorithm for signed networks

While 'movement towards balance' requires a measure of imbalance whose through time trajectory can be examined, the specific values of the measure are of secondary interest in terms of the structure of the empirical network. Doreian and Mrvar (1996) proposed the use of an algorithm (based on the two structure theorems) that, in addition to measuring imbalance, also provides a description of the partition structure(s) of the group. More formally, they sought to:

> Determine the clustering(s) $\mathbf{C}^*$ for which
>
> $$P(\mathbf{C}^*) = \min_{\mathbf{C} \in \Phi} P(\mathbf{C})$$
>
> where $\mathbf{C}$ is a clustering of the vertices of the network and Φ is the set of all possible clusterings and $P : \Phi \rightarrow \mathbb{R}$ is a *criterion function*.

The criterion function is constructed from inconsistencies with a balanced structure. These inconsistencies take one of two forms: They are either negative ties within a plus-set or positive ties between pairs of plus-sets. Letting $\mathcal{N}$ be the total number of negative ties within plus-sets and $\mathcal{P}$ be the total number of positive ties between plus-sets, the criterion function:

$$P(\mathbf{C}) = \mathcal{N} + \mathcal{P}$$

In this formulation, the two types of inconsistencies are treated as being equally important: the criterion function is simply the count of all inconsistencies regardless of their type[7].

[7] A slightly more general criterion function is:

The criterion function is then minimized by using a relocation algorithm. The procedure provides sets of empirical partitioned structures[8] that are as close to an exactly balanced partition as is possible. In addition, for each such partition, their method provides a list of ties that are inconsistent with an exact balanced partition.

2.5 The fundamental structural balance hypothesis

The fundamental structural balance hypothesis (FSBH) is easy to state: Human signed networks move towards balance through time. In order to examine this hypothesis, longitudinal data are need for a signed network. One of the few such data sets is provided by Sampson (1968) and Doreian and Mrvar (1996) used these data to show that the line index of imbalance did drop through three time points during a period of increased polarization and conflict for the like-dislike relation of a group of trainee monks in a monastery. These trainees were partitioned into the same set of three well-defined subgroups (plus-sets) at each time point considered. Moreover, the measure of imbalance did decline through time. The identities and composition of the three subgroups were supported strongly by the ethnographic narrative of Sampson (1968).

Another legendary longitudinal data set is provided by Newcomb (1961) as reported by Nordlie (1958) for fifteen time points. Doreian *et al.* (1996) used these data to examine the through-time trajectories of reciprocity, transitivity and imbalance. Reciprocity was significantly greater than chance at the first of 15 time points. It fluctuated through the full time period but did not depart much from its initial value. The amount of transitivity was not significantly different from zero at the first time point. However, it increased throughout the first eight time points and then remained essentially stable thereafter. The amount of transitivity became significantly different from zero at the third time point. The amount of structural imbalance declined overall through the whole time (although the decline was not monotonic[9]). Having three coupled processes with different time scales provides an intriguing description, leaving open the task of modeling these coupled processes.

Doreian and Krackhardt (2001) returned to the Newcomb data intent upon a closer look at the temporal balance theoretic phenomena. They focused on the eight types of triples in Table 1 and rephrased the FSBH in terms of two hypotheses: (1) the number of balanced triples will increase through time and (2) the number of imbalanced triples will decrease through time. They defined a 'pre-transitive' condition defined by the ties $p \rightarrow o$ and $o \rightarrow q$ and examined the extent to which the third tie $p \rightarrow q$ was completed in a way that was consistent with the FSBH.

$$P(\mathbf{C}) = \alpha\mathcal{N} + (1-\alpha)\mathcal{P}$$

where $0 \leq \alpha \leq 1$. With $\alpha = 0.5$, the two inconsistencies are equally weighted. For $0 \leq \alpha < 0.5$, positive errors (lines between plus-sets) are more important and for $0.5 < \alpha \leq 1$, the negative errors (lines within plus-sets) are considered as more consequential.

[8]There can be multiple equally well-fitting partitions of a signed network for a given value of k or for a set of values for k.

[9]The lowess smoothed trajectory was monotonic in its decline through time for $k > 2$.

Table 2: Pre-transitive conditions, balance and evidence.

Triple	Tie 1 $p \to o$	Tie 2 $o \to q$	Tie 3 $p \to q$	Balance	Supports Balance
PPP	+	+	+	yes	supports
PPN	+	+	-	no	supports
PNP	+	-	+	no	supports
PNN	+	-	-	yes	supports
NPP	-	+	+	no	contradicts
NPN	-	+	-	yes	contradicts
NNP	-	-	+	yes	contradicts
NNN	-	-	-	yes/no	(supports)

Their results were mixed and are summarized in Table 2 (which is taken from their paper). The frequency of the balanced triples PPP and PNN did increase through time, results that support the FSBH. However, the frequency of the balanced triples NPN and NNP *decreased* through time and their trajectories contradict the FSBH. The imbalanced triples PPN and PNP did decrease in frequency through time and provide additional support for the FSBH. However, the number of imbalanced NPP triples *increased* through time, again contradicting the FSBH. The number of NNN triples increased through time. For balance theory with $k = 2$ this is more contradictory evidence regarding the FSBH but for $k > 2$, these increases provide support for the FSBH. (Hence the use of parentheses in the last line of Table 2.)

While these results are disconcerting for structural balance theory, perhaps Doreian and Krackhardt's (2001) exploration of alternative mechanisms is even more disconcerting. In looking at the trajectories *not* supporting balance theory, they all shared the feature of $p \to o$ is negative. Put differently, if pPo holds, then balance mechanisms operate but if pNo holds, then balance mechanisms do not operate, a result anticipated by Newcomb (1968). Note that the status of the NNN triple is ambiguous as it can be viewed as imbalanced (for $k = 2$) and balanced (for $k > 2$).

Even more disconcerting is another consistency reported in Table 2 when viewed in terms of triples whose trajectories increase in frequency through time. Such trajectories share the feature of having the last two ties either PP or NN. Put differently, increasing frequency comes when $p \to q$ and $o \to q$ have the same sign. Regardless of the state of $p \to o$, the triple increases in frequency when p and o agree in their (like or dislike) ties to q. This suggests that the attributes of q matter - a non-structural feature! Doreian and Krackhardt (2001) speculated that it is possible that differential popularity/unpopularity can be misinterpreted as the operation of structural balance mechanisms. Finally, their results suggested that there may be a form of co-evolution of actors and attributes: actors who were very much disliked at the end of the study period were not disliked much at the beginning of the process. As the structure of the group morphed from three distinct subgroups to a large subgroup with two disliked singletons and a pair of disliked actors in a cluster, the counts of received positive and negative ties changed systematically. At

a minimum, these results suggest that the FSBH has limited utility in understanding structural balance dynamics and that these social balance processes operate in ways that are more complicated than the fundamental structural balance hypothesis suggests. They also point to some of the severe limitations of an approach focusing solely on group-level dynamics. It seems time to bring Heider back into structural balance, an approach taken by Hummon and Doreian (2003).

3 Balance as a pair of multi-level processes

Hummon and Doreian (2003) proposed a new theoretical model in the form of an agent-based simulation model with two coupled processes. One process operates within each actor while the second operates at the group level. Each actor is a pure seeker of structural balance because they make choices to lower the amount of structural imbalance in their image by as much as is possible. The images held by these actors can differ and they make their choices based on the information they have at their decision times. This information is conveyed to the 'group' where a second balance process occurs. This balance process is a 'pure Cartwright and Harary' process while the actor level processes are 'pure Heider' processes. Each type of process informs the other.

When an actor makes a choice regarding one of their ties, this information is transmitted as input to the group-level process and each actor reports their own changes. This means that the group level structure is 'accurate' even though the actors need not perceive the group-level structure accurately. Based on the information for the actors, a group process is simulated by mobilizing the Doreian and Mrvar (1996) algorithm to produce a group-level network with the smallest[10]possible imbalance. The *partition* of the actors for such a model is reported to each group member. Based on the information an actor has in the form of the group-level partition and whatever the actor knows about the signed ties in the group, the actor level process also implements the Doreian and Mrvar (1996) algorithm to generate a structure whose imbalance is a low as possible[11] and does this for each actor.

There are three design variables used in the simulations. They are (1) the amount of contentiousness at the outset (operationalized by the probability of negative ties), (2) the number of actors in the group, and (3) the communication modes by which actors inform the *members* of their group. The communication modes were (1) *simple dyadic* in which only the person to whom a tie is changed is informed of the change, (2) *tell friends* where only actors to whom there are positive ties are told of the change, (3) *tell acquaintances* where other actors to whom there are either positive or negative ties are informed of the change, and (4) *broadcast* where everyone in the group is informed of the change. In addition, a minimum number of ties were specified for actors to preclude actors from disengaging completely from the group as well as a very long run time as a default stopping rule[12].

[10]If there are multiple equally well fitting partitions, one is chosen randomly.

[11]Again, if there are multiple equally well fitting partitions, one is chosen randomly.

[12]Slightly more than 98% of the runs (with 100 runs per set of parameters) were completed before the default was reached.

The outcome variables were (1) the number of acts (actor choices to change ties) needed to reach an equilibrium, (2) the level of group imbalance, (3) the number of actors achieving balance in their cognitive images of their network, and (4) the average number of plus-sets (clusters) in the final group partition. The design variables have complex relations with the outcome variables (and will not be summarized here). However, it is clear that the initial level of contentiousness and the mode of communication are important for balance theoretic dynamics. These dynamics are different in large groups compared to small groups. Finally, there are subtle relations between the number of plus-sets and the types of balance realized by the model. There are also some broad descriptions in the simulation outcomes. One is that some groups reach balance quickly while others can take a long time to reach balance (or do not reach balance). Another is that many *different* partition structures were reached. Additionally, for some groups an equilibrium is reached where the group-level imbalance is non-zero but each actor has a balanced image of the network. (This suggests a plausible reason why some groups do not reach balance even though structural balance processes are operative.)

Hummon and Doreian (2003) had in mind the idea of using the simulations to suggest ways in which data could be collected for empirical studies of balance theory. The long 'behavior streams' (sequences of decisions about ties) generated in the simulations suggest that it will be impossible to collect data in the form generated by the simulations. The results of their study imply that most empirical studies of balance do not have a long enough observation window. To the extent that the multi-level model is an appropriate specification of balance theoretic processes and dynamics, obtaining adequate empirical data becomes a major and more complicated task. This task is complicated further with the realization that while the decisions actors are intended to achieve balance (in their images) their choices can move the group away from balance when the choices of other actors are considered. Both the simulation results of Hummon and Doreian (2003) and the event-sequence arguments of Doreian (2002) suggest strongly the FSBH is, at best, a simplification and, at worst, false in its details if there is movement towards balance at some time points and movements away from balance at other time points. If there is 'movement towards balance' over time it is a net outcome of multiple changes not all of which lower imbalance. At a minimum, long sequences of behavior will be required to study balance processes in a compelling fashion.

In an effort to examine what kinds of data will be required, it seem fruitful to turn to drama and stories for some clues as to how we can proceed to study balance phenomena empirically. One clue takes us right back to the first sentence of Heider's (1946) classic paper.

4 Dramas and stories as a source of ideas

Dramas and stories can be viewed as a source of ideas as well as a context in which story lines detail events that follow in some temporal sequence. As an illustrative example, I consider the 2000 movie *Remember the Titans* with material taken from

Doreian (2003). It depicts a simple 'uplifting' story of a high school football[13] team. Two high schools in Alexandria, Virginia, one white and one black, were forced to merge in 1971 and their football teams were merged. The city was in danger of tearing itself apart over race and the situation was described as 'racially charged'. Adding to the tension, the school board appointed a black head coach (Boone) over a white coach (Yoast) who had expected to be the head coach of the merged programs.

In many ways, the situation was balanced at the outset: blacks versus whites. Boone realized that without the team coming together on race, its season would fail. He set about integrating the team so that black and white players would play together as a team. In short, he created conditions of *imbalance* by forcing black and white players to interact and get to know about each other. This created anger and conflict in the short term. While the movie can be viewed as showing the operation of balance mechanisms, set in motion by the head coach, neither the story nor the mechanisms are that simple.

At the core of the story are Bertier, a star white player, and Julius, a star black player. As the training camp opens, they are mutually and intensely hostile towards each other. As a part of his effort to remake the team, Boone assigns them to room together. The first event depicted in their room was an argument that erupted into a fist fight. Other negative relations between black and white players were depicted. As the story unfolds, the tie between Bertier and Julius is transformed into an intense mutually positive relationship. Other ties change and, by the end of the movie, the team is integrated, cohesive and successful. Balance theory works! However, it does *not* work in the ways suggested by the group-level strand of structural balance theory with its focus on signed ties in the group. The early change in the Bertier-Julius relation is implausible if attention is confined to social relations alone. *Without considering unit formation relations, the balance dynamics are incomprehensible.*

The animosity between Bertier and Julius is driven by race. Early in the movie, Bertier makes it clear that he despises blacks, will not play with them and will not play for a black coach. He even describes Boone as a thief who has stolen Yoast's job. Julius is hostile towards whites and does not expect white players wanting to have him on their team. The unit formation relations are shown in Figure 3. The edge between Bertier and Whites and the edge between Julius and Blacks are the unit formation relations. The negative (dashed) tie from Bertier to Blacks and the negative tie from Julius to Whites depicts the players' sentiments regarding race. The mutual negative tie between Bertier and Julius makes both triples balanced and stable. It is hard to imagine this state of affairs changing.

Bertier's position may be even more entrenched than Figure 3 suggests. This is depicted in Figure 4. For Bertier, there are two unit formation relations that both involve Boone: Boone is black and Boone is a thief. The negative tie from Bertier to Boone means balance in both triples. Bertier makes it clear that he will sit out the season and would not play if Boone is the head coach. In the story line of the movie, Yoast, out of concern for Bertier's future, insists that Bertier play for the team and persuades him to do so.

[13]This is 'football' in the sense of American football and not soccer, otherwise known as football around the globe.

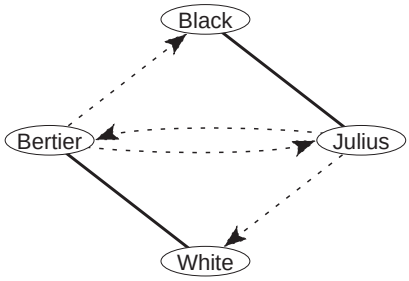

Figure 3: Two unit formation triples for Bertier and Julius.

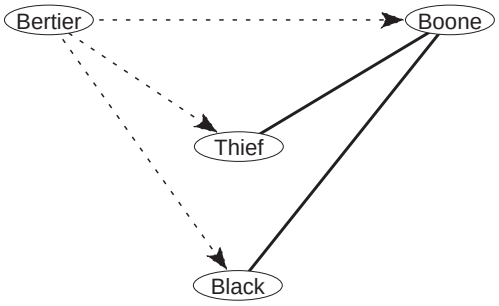

Figure 4: Two unit formation triples for Bertier.

Coach Boone puts additional pressure on players to get to know each other and to play as a team in several ways at the training camp. He demands additional grueling practices sessions until they comply and he demands that every player of one race get to know something about every player of the other race. Bertier is motivated to comply and approaches Julius. Initially, their mutual dislike meant that the session does not go well in terms of Boone's intent. However, they do talk about football but are highly critical of each other's play. They care about football, they want to play well, they want each other to play well and they want to the team to play well. Their joint commitment to football imply the important unit formation relations shown in Figure 5 together with their mutual negative ties. Clearly, the triple is imbalanced and, if Heider is correct, there will be 'tension' and 'forces' towards balance.

The situation is a little more complicated in terms of unit formation relations. Each player is critical of the other's play and thinks the other is not making any positive contributions to the team. This is shown in Figure 6 where all of the (signed) edges are unit formation relations. Julius' unit formation ties are shown in the upper two step path from Julius to Bertier while Bertier's unit formation ties are in the two step path from Bertier to Julius. Bertier thinks negatively about Julius and Julius as a player. Julius reciprocates those sentiments.The two upper triples are balanced given the mutual negative ties between Bertier and Julius. The lower triple, with the positive unit formation ties to football, is imbalanced in both parts. As the training camp continues, each player tries to do better and they come

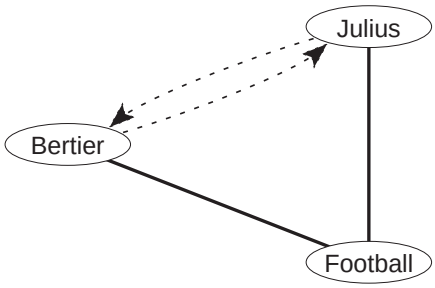

Figure 5: The football formation triple for Bertier and Julius.

to recognize each other's talents and abilities. Playing well together (they both play for the defense) becomes more and more important. Slowly, the negative unit formation relations on figure 6 become positive and this leads to imbalance in *all* of the triples. Clearly, this is not consistent with the FSBH! Given that both players are committed to football and that they reach a positive evaluation of the other as a player, the obvious inconsistency with balance is their mutual negative tie. By the end of the training camp, this is a mutual positive tie, consistent with balance. All of the negative ties of Figure 6 have become positive and the key balance theoretic dynamics involve unit formation relations. If attention is confined solely to the observed affect ties, the changes in the ties between Bertier and Julius are neither plausible nor comprehensible. As Heider wrote "*attitudes towards persons and causal unit formations influence each other* (1946: 107, emphasis added).

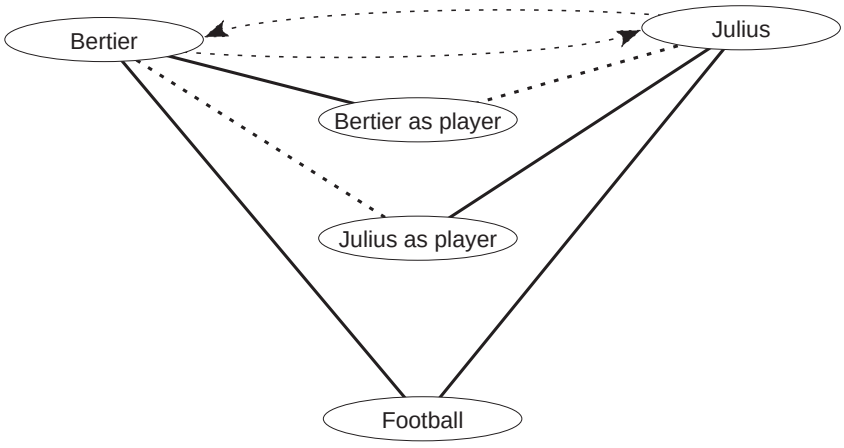

Figure 6: The expanded football formation triple for Bertier and Julius.

A detailed analysis of the balance theoretic dynamics depicted in *Remember the Titans* yields interesting insights into the empirical relevance of balance theory and may provide clues for data collection efforts. The results provided in Doreian (2003) include:

- There is movement towards and away from balance at different points in time;
- Much of the relational dynamics is driven by unit formation relations and concatenations of these ties;
- There is no single generic process operating across the network. (Instead, in dyads, triads and small subgroups, balance dynamics play out separately within a wider network. The outcomes in some of these subgroups have relevance for the dynamics in other subgroups.)
- Networks are embedded in larger networks and the embeddedness changes some of the dynamics. (The team returned from camp as a largely, but not entirely, cohesive team with good relations between pairs of black and white players. Many white adults, especially parents and political officials, and white friends of white players, were not pleased with these developments and brought pressure on some white players to revert to their earlier perceptions and beliefs.)

Every movie has a script with both a narrative and a message. Movies (and novels and plays) are filled with contrived event sequences and outcomes that build to an intended destination or outcome, a description that holds even when a production is 'based on a true story' as was *Titans*. While this appears to create overly 'clean' sequences (in contrast to 'real life'), such narratives have value in sensitizing us to the phenomena we need to observe.

5 An agenda for collecting signed network data

The following agenda items are based primarily on the simulations of Hummon and Doreian (2003) and the above discussion of the movie *Remember the Titans*. While they form only a provisional list, it contains some formidable tasks.

- We need to collect valid and reliable *signed* data for affect ties. This seems most pressing to the extent that many important social relations are intrinsically signed. Obtaining data for only positive ties seems overly restricting and eliminates consideration of balance dynamics.
- To the extent that actors do *not* have complete and accurate images of the signed networks in which they are located, obtaining actor perceptions of their own and other ties seems a useful data collection task.
- Having data on multiple types of signed relations seems important. Together, multiple relations may exhibit dynamics that are more than the simple cumulation of separated dynamics.
- Data on unit formation seem especially useful as is evident in the discussion of *Titans*. Typically, we tend not to collect such data and focus our attention on the signed affect ties between actors. Balance theory may make sense empirically *only* if unit formation relations are considered.

- We need to collect longitudinal data over long time periods. Both the simulation results and the 'movie data' suggest that considerable lengths of time are needed for the balance theoretic mechanisms to operate. This will also help to provide clues concerning the time scales of these phenomena.

- Obtaining information on the contexts or the environments of signed networks appears to be another useful task and will provide additional data for interpreting the operation of balance theoretic mechanisms.

While these are merely suggestions for fruitful data collection actions, they - together with the discussion above - point to a more general substantive agenda.

6 A substantive agenda for signed networks

Questions about the substantive nature of structural balance theory and the operation of balance theoretic dynamics are hindered when we do not have enough data for long enough periods of time. Also, having a substantive agenda will provide additional specification for the data collection tasks listed above. Such an agenda includes the following items.

- Balance is not achieved solely through the changing of ties (sign change or line deletion) and can be reached through the removal of actors from the network[14]. This suggests an alternative measure of imbalance in the form of the smallest number of vertices whose removal creates a balanced network. Unfortunately, computing this for a general network is an NP-hard problem. Of course, we then need to understand the conditions under which actors depart from a network and the conditions under which the sign of ties are changed or ties are deleted.

- It seems necessary to examine the multiple affect mechanisms of structural balance theory to identify the conditions under which they operate (or do not operate).

- Examining further the operation of unit formation mechanisms and the ways they are coupled to the affect processes for actors merits attention, substantively.

- In addition to the balance theoretic - both unit formation and affect - mechanisms, we need to consider how other mechanisms operate. Furthermore, it is necessary to examine closely when these mechanisms are coupled and when they operate as rival mechanisms. Further simulation studies can be useful if we can specify algorithmic rules for coupling multiple processes.

- If different processes have different time scales, we need to determine those scales as well as model their dynamic (and coupled) operation.

[14]The removal of one very prejudiced white player, hitherto a close friend of Bertier - who refused to play in ways that would help black players play well - decreased imbalance.

- Consistent with Stokman and Doreian's (1997) claim that network ties have instrumental value for actors in social networks, it would seem useful to forge a synthesis of structural balance theoretic ideas and principles from rational choice theory.

7 Conclusion and discussion

Heider's (1946) formulation of structural balance theory provided the intellectual foundations for a huge amount of empirical and theoretical research. In retrospect, his initial statement is both simple and complex. The simplicity informed two strands of empirical research, one of which focused on group-level balance theoretic processes. Cartwright and Harary's (1956) generalization of Heider's basic ideas provide a conceptual and analytical framework for exploring and developing balance theory. It informs the simple and fast algorithm of Doreian and Mrvar (1996) for measuring balance and, more importantly, delineating the partition structure of signed networks. However, these developments may have come at too high a cost because of the constraints researchers imposed on themselves when studying group-level phenomena only.

There are two broad problems and each can be traced back to dropping a key item from Heider's statement of balance theory. One came in the form of removing the mental affect processes from balance theory - or assuming that they operated in a way that allowed actors to have complete and accurate images of the signed networks in which they are located. Both the findings of Freeman *et al.* (1987) and the simulations of Hummon and Doreian (2003) suggest strongly that this was a serious mistake. The design of the simulations point to ways in which both actor and group processes can be included and coupled within a model. The second problem came in the form of approaching signed networks as though there was a single signed relation. Dropping the distinction between affect and unit-formation relations was another serious mistake and, as argued above, crippled structural balance theory because unit formation relations play a decisive role in structural balance processes. In short, the group-level strand of research based on Heider's foundation missed not one, but two, essential ideas of Heider.

Overcoming these problems holds the promise of creating a general theory of structural balance. The agendas of substantive and empirical tasks that were outlined above seem necessary steps that must be taken. Of course, the cost will greater complexity both substantively and technically. Although these difficulties are considerable, we may have little choice because the 'simplified version' of structural balance failed. Its failure can spur us on to construct a viable general theory rather than the hollow shell we huddle within.

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