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The Influence Rules of Cutter Parameters on the Contact Characteristics of Tooth Surfaces by Five Cut Processes and the Duplex Helical Method

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The principles of cutter parameters, such as the cutter tilt angle, the blade angle, and the cutter radius, are investigated in the five-cut process and the duplex helical method. The duplex helical method is an advanced and primary manufacturing method of face-milled spiral-bevel and hypoid gears, so it is necessary and urgent to understand its generalized theory. First, the different working principles of the cutter tilt mechanism in the two processing methods are analysed in detail, and the principal idea of the generalized theory for the duplex helical method is fully explored. Next, the influences of the blade angle and the cutter radius on the tooth surface meshing characteristics in the two processing methods are compared and analysed with the numerical analysis method; the results show that the relationships between the cutter parameters and the meshing characteristics are more regular and that the tooth-bending strength is higher in the duplex helical method, which provides a theoretical basis for effectively obtaining the optimal machine settings of spiral-bevel and hypoid gears.
Keywords: duplex helical method, spiral-bevel and hypoid gears, five-cut process, generalized theory, cutter parameter

Highlights
- *The different working principles of the cutter tilt mechanism in the two processing methods are analysed.*
- *The principal idea of the generalized theory for the duplex helical method is fully explored.*
- *The influences of the blade angles and the cutter radius on the meshing characteristics in the two processing methods are compared and analysed.*
- *The research work provides a theoretical basis for obtaining the optimal machine settings of spiral bevel and hypoid gears.*
- *The bending strength of the duplex helical method is significantly higher than that of the five-cut process.*

0 INTRODUCTION

At present, the machining technology of face-milling, spiral-bevel and hypoid gears has two major methods. One method is the traditional face-milling method called the ‘five-cut process’ or the ‘fixed setting process’, which is still widely used in developing countries, such as China [1] to [3]. The other method is called the ‘duplex helical method’ or the ‘completing process’, which has many advantages, such as higher machining efficiency, lower production cost, higher tooth strength, and dry cutting. Therefore, the five-cut process is gradually being replaced by the duplex helical method, and the duplex helical method will become an advanced and primary manufacturing method for face-milling, spiral-bevel and hypoid gears in China. However, there are significant differences and correlations between the generalized theories of the two methods.

The significant differences between the five-cut process and the duplex helical method are: (1) In the five-cut process, the convex and concave tooth surfaces of a pinion are separately cutting with two different head-cutters; the outside head-cutter processes the concave tooth surfaces, and the inside head-cutter cuts the convex tooth surfaces. The duplex helical method can cut both sides of the pinion or gear tooth slots completely, with a spread blade head-cutter, in one operation, from a solid blank. (2) There is a helical motion (does not exist in the five-cut process) in the duplex helical method to control bias on the pinion, which is a relative axial motion between the workpiece and the cutter, along the generating gear axis, during the generating roll. With either method, the cutter represents a tooth of an imaginary gear which rolls with the gear or the pinion being cut. Therefore, its shape and position determine the shape of the tooth surface, and the influences of its parameters on the tooth surface topography play an important role in revealing the generalized theories of the two methods.

The generalized theory of spiral bevel and hypoid gears manufactured with the five-cut process have been comprehensively presented by several gear scientists [4] to [6]. Gleason’s most representative gear scientists establish the spatial geometry of Gleason

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spiral bevel and hypoid gears, analyse the calculation approach of the tooth surface curvature, and introduce the generalized theory of the five-cut process [7] to [9]. The generalized theory and calculation method for the five-cut process are developed by Litvin et al. [10], Fuentes et al. [11], and Litvin and Fuentes [12], which is different from Gleason's technology. To avoid the root profile interference and the tooth edge contact, the blade geometry consists of four sections (tip, toprem, profile, and flankrem), as presented by Fan [13]. The application of the parabolic and Top-Rem profile blade are proposed by Litvin et al.[14] and Fuentes et al. [15] to avoid or reduce the areas of severe contact stresses and increase the endurance of spiral bevel gears. An optimal tooth modification for spiral bevel gears is introduced into the pinion tooth-surface by using a head-cutter with the bicircular profile and the optimal diameter to improve the load distribution and reduce the maximum tooth contact pressure and transmission errors [16] and [17]. Ma et al. [18] investigate the influences of the cutter diameter on the meshing performance of spiral bevel gears, such as the contact area, the contact pressure, the bending stress, the torsional stiffness, and the transmission error, with the finite element analysis methods.

Gleason invented the duplex helical method several decades earlier, but has not clearly published its generalized theory and only shows few formulas and calculating instructions [19]. To the best knowledge of the authors of this paper, little attention has been given to the generalized theory of the duplex helical method until now. The conversion between the specific machine-tool settings of a given hypoid generator and the neutral machine-tool settings of spiral bevel gears manufactured by the duplex helical method is investigated, and the parabolic blade profiles are applied to improve the tooth bearings described in [20]. A mathematical model of the general hypoid machine is proposed by Fong [21], this model includes several common modified motions, such as the modified generating roll ratio, the helical motion, and the cutter tilt, to simulate all face-milled spiral-bevel and hypoid gear-machining methods. A new methodology is presented to determine the basic machine settings of spiral bevel and hypoid gears manufactured by the duplex helical method in [22], and the effect of straight-lined and circular cutting-edge profiles on the meshing and contact of gear pair is analytically investigated to effectively improve meshing performance [23].

In summary, the differences and relations between the theories of the five-cut process and the duplex helical method, the primary idea and many other key scientific problems of the duplex helical method have not yet been solved. Therefore, on the basis of analysing the generalized theory of the five-cut process and the duplex helical method [22], this paper investigates the different working principles of the cutter tilt angle and the blade angle in the two methods and reveals the major idea of the duplex helical method. Furthermore, the influences of the cutter parameters on the contact characteristics of tooth surfaces in the two processing methods are studied, which provides a theoretical reference for the optimization of machining parameters.

1 DIFFERENT WORKING PRINCIPLES OF CUTTER TILT ANGLE

In this paper, the following definitions are made: the convex side of the gear and the concave side of pinion are the drive sides; the other mating surfaces are the coast sides.

In general, the meshing mode for all types of spiral-bevel and hypoid gears is local conjugate contact (or called 'point contact'). To receive the length crowning between the mating flanks of a bevel gearset, the length curvatures (the curvature is inverse to the radius) of the convex flanks have to be larger than the length curvature of the concave flanks, that is, a pair of tooth surfaces with meshing must have a curvature difference. In addition, the normal vector of the two mating flanks at the reference point should be the same, or the two mating flanks should have equal spiral angle and pressure angle at the reference point. The cutter point radius is a good measure for the tooth length curvature in the five-cut process. To obtain a suitable curvature for a pair of the mating tooth surfaces, an inside cutter point radius for the convex flank must be smaller than an outside cutter point radius for the concave flank. The blade angle in the five-cut process is very similar to the pressure angle of the tooth. If there are some small differences, then they are related to the basic setting adjustments which are done in order to fine-tune the rolling performance. With equal pressure angles, the inside blade (IB) radius at the blade reference point (not at the tip) has to be larger than the outside blade (OB) radius at the reference point, as shown in Fig. 1. In this case, the IB and OB for pinion obviously cannot be mounted on one spread blade head-cutter, which cannot be used in the duplex helical method.

In the duplex helical method, the cutter physically requires a smaller inside point radius and a larger OB radius to be able to mount the IB and OB of the pinion on a head-cutter. Comparing the cutter radii in the two methods, it can be seen that this is actually equivalent to result in the correct tooth thickness and a very large amount of the length crowning on both sides of the pinion. When machining spiral bevel and hypoid gears, by changing the blade angle simultaneously with the cutter tilt, the large "natural" length crowning can be reduced to the desirable crowning. This is the primary idea of the generalized theory for the duplex helical method. That is, a suitable curvature radius (the curvature radius is the distance from the blade reference point to the cutter axis, perpendicular to the cutting edge) at the reference point can be changed by adjusting the blade angles and the cutter tilt angles, so that the proper length curvature can be obtained on both sides of the pinion.

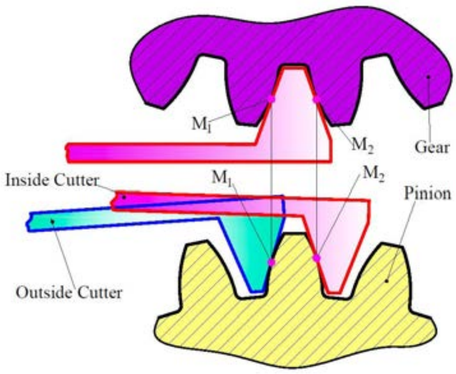

Fig. 1. *Corresponding relationship among cutter radii of gear and pinion*

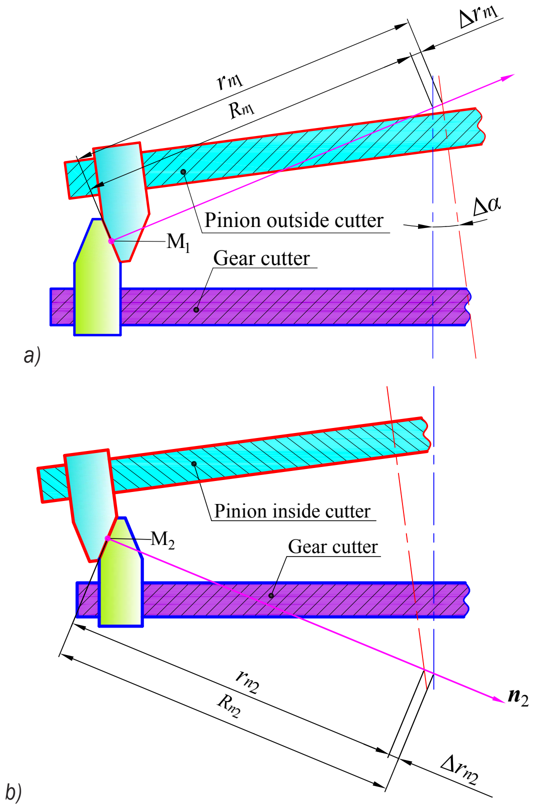

Fig. 2. *Relative position of gear and pinion cutters in five-cut process; a) drive side and b) coast side*

In the following, how the blade angle and the cutter tilt angle affect the length curvature in the five-cut process and the duplex helical method, respectively and how they meet the required meshing conditions will be analysed. In the five-cut process, the blade angles are very similar to the pressure angles of the gear tooth, especially when the gear is cut with the non-generated method (formate). The pressure angle of the gear at the reference point must be equal to the pressure angle of the mating pinion at the reference point to make the pair of tooth surfaces mesh correctly; the relative position of the gear and pinion cutters can be represented in Fig. 2 at the reference point.

In the five-cut process, the gear cutter generally has the same IB and OB angles, or the OB angle is slightly larger than the IB angle, as shown in Fig. 2. However, in the five-cut process or the duplex helical method, the pinion cutter generally has a smaller OB angle and a larger IB angle. When the cutter tilt is used to process the pinion, the curvature radius of the pinion OB r_{n1} is increased by an amount Δr_{n1} relative to that of the gear IB R_{n1} on the dive side, that is, $r_{n1}=R_{n1}+\Delta r_{n1}$. The curvature radius of the pinion IB r_{n2} is reduced by an amount Δr_{n2} relative to that of the gear OB R_{n2} on the coast side, that is, $r_{n2}=R_{n2}+\Delta r_{n2}$. The values of Δr_{n1} and Δr_{n2} are determined by the angle $\Delta\alpha$ caused by the tilt of the cutter. Therefore, the cutter tilt in the five-cut process meets precisely the condition that a pair of meshing flanks have a curvature difference. That is, at the reference point, the curvature radius of the concave side must be larger than that of the convex side. In addition, it should be noted that Fig. 2 is drawn under the same mean cutter radius of the gear and the pinion, and the length curvature crowning is performed in the five-cut process (it is equivalent to counter-crowning the cutter radius), which can further reduce the OB radius and increase the IB radius for the pinion, this will cause the curvature difference required for the correct engagement of the two mating flanks to increase further. The above situation is the reason why the OB radius is smaller than the IB radius of the pinion in the five-cut process.

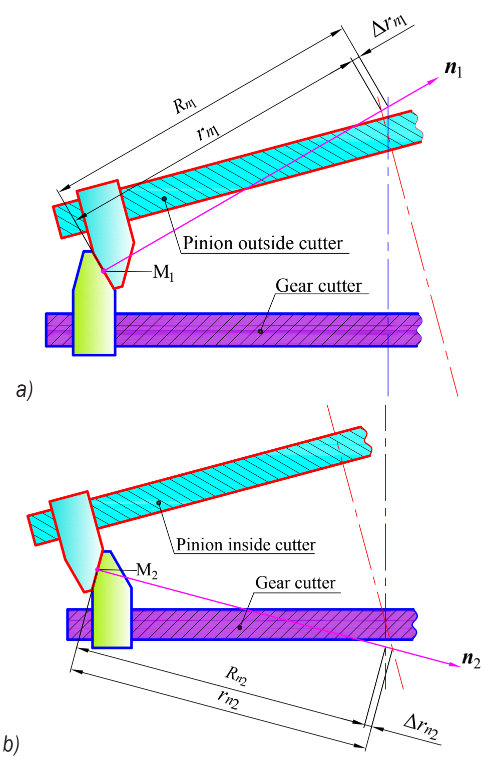

Fig. 3. *Relative position of gear and pinion cutters in duplex helical method; a) drive side and b) coast side*

In the duplex helical method, the spread blade cutter with the smaller IB angle and the larger OB angle are generally chosen to generate the gear and the pinion, and the asymmetry degree between the IB and OB blade angles is larger (the so-called asymmetry degree is the extent to which the IB and OB angles deviate from the mean pressure angle). The reason for this is to control the tooth contact pattern length. A smaller OB angle and a larger IB angle will lengthen the contact pattern, a larger OB angle and a smaller IB angle will shorten the contact pattern. As shown in Fig. 3, when the cutter tilt is used to process the pinion, the curvature radius of the pinion OB r_{n1} is reduced by an amount Δr_{n1} relative to that of the gear IB R_{n1} on the dive side, that is, $r_{n1}=R_{n1}-\Delta r_{n1}$. The curvature radius of the pinion IB r_{n2} is increased by an amount Δr_{n2} relative to that of the gear OB R_{n2} on the coast side, that is, $r_{n2}=R_{n2}+\Delta r_{n2}$. This is equivalent to increasing the concave curvature and reducing the convex curvature for the pinon, and the purpose of effectively reducing the large "natural" length crowning on both sides of the pinion is achieved at the same time, which can satisfy the desirable curvature required for the correct engagement of a gearset.

According to the above analysis, it can be seen that a desirable curvature radius for a pair of mating tooth surfaces can be obtained by the suitable tilt angles and blade angles, and then the different cutter radii required to meet the two processing methods are obtained, which is the essential difference between the five-cut process and the duplex helical method.

2 INFLUENCE RULES OF BLADE ANGLES ON MESHING PERFORMANCE

The selection principle of the blade angles in the five-cut process, and the duplex helical method is significantly different, especially regarding the blade angles of the gear. The tooth contact characteristics are an important indicator of the gearset meshing performance. Therefore, a numerical example of the faced-milled hypoid gearset of the gear ratio 11×45 is considered to investigate the different influences of the blade angle and the cutter radius on meshing performance in the two processing methods. For comparison purposes, the basic design parameters of the hypoid gearset processed by the two methods are as identical as possible, and the HFT (one of the five-cut process, format gear and pinion finished with cutter tilt) and HFDH (one of the duplex helical method, format gear and pinion generated by the duplex helical method) method are used to perform the numerical example, the design parameters are given in Table 1. A computer program is developed to implement the formulation provided above.

Table 1. *Basic design parameters of hypoid gears*

Parameters	Pinion	Gear
Number of teeth	11	45
Module [mm]	-	6
Face width [mm]	44.35	40
Pinion offset [mm]	30	-
Shaft angle [°]	90	-
Mean pressure angle [°]	20	20
Mean spiral angle [°]	48	33.82
Hand of spiral	left	right
Whole depth [mm]	11.89	11.73
Mean cone distance [mm]	125.33	120.98
Outer diameter [mm]	99.68	271.02
Face angle [°]	20.53	74.08
Pitch angle [°]	16.32	73.2
Root angle [°]	15.45	68.87

Based on keeping other machining parameters unchanged (for example, the cutter radius is 114.3 mm), the OB and IB angles of the pinion and gear cutter are changed to investigate the influences of the blade angle on meshing performance in the five-cut process. Fig. 4 shows some of the results of tooth

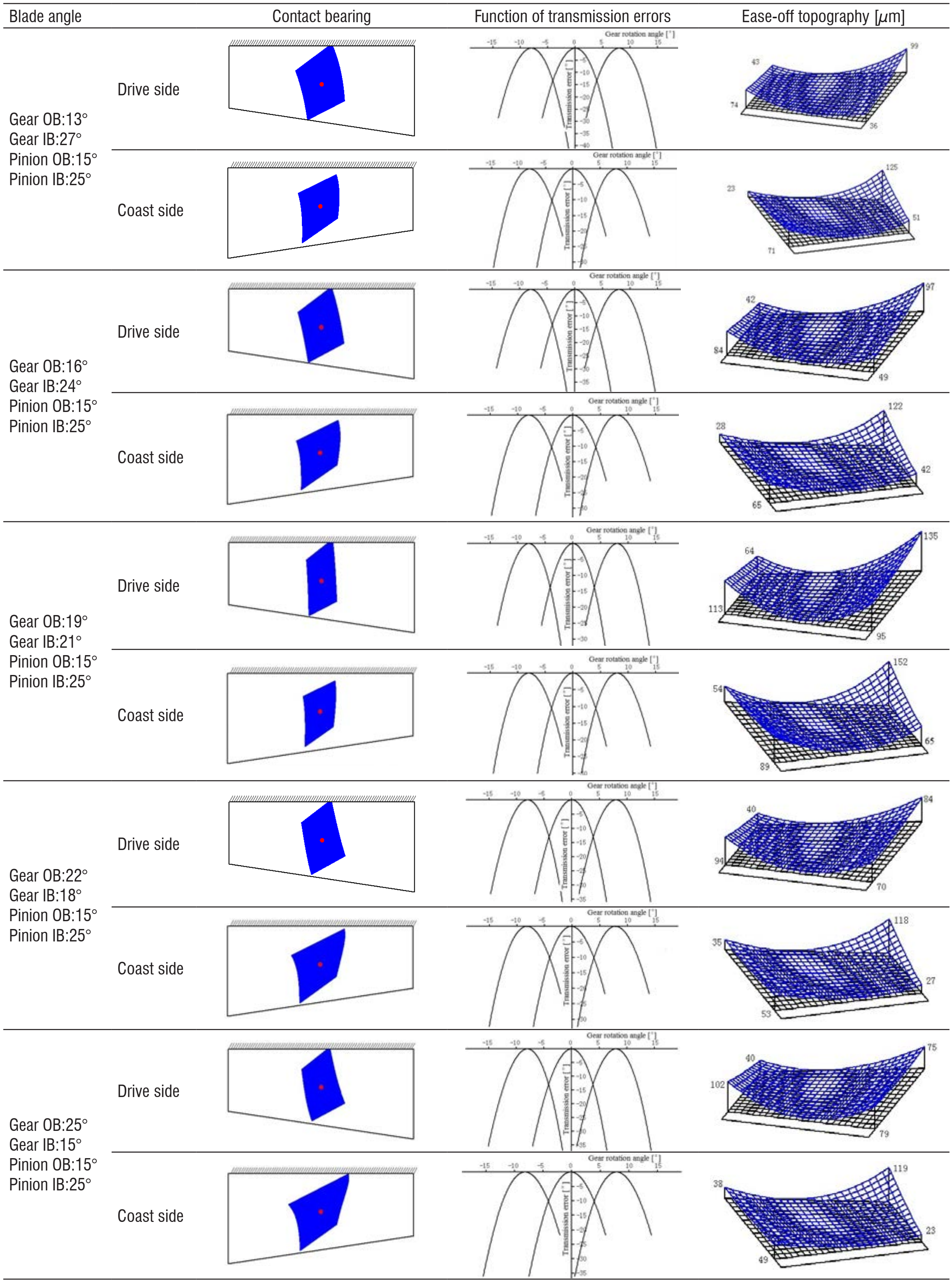

Fig. 4. *TCA results when changing gear blade angles for HFT method*

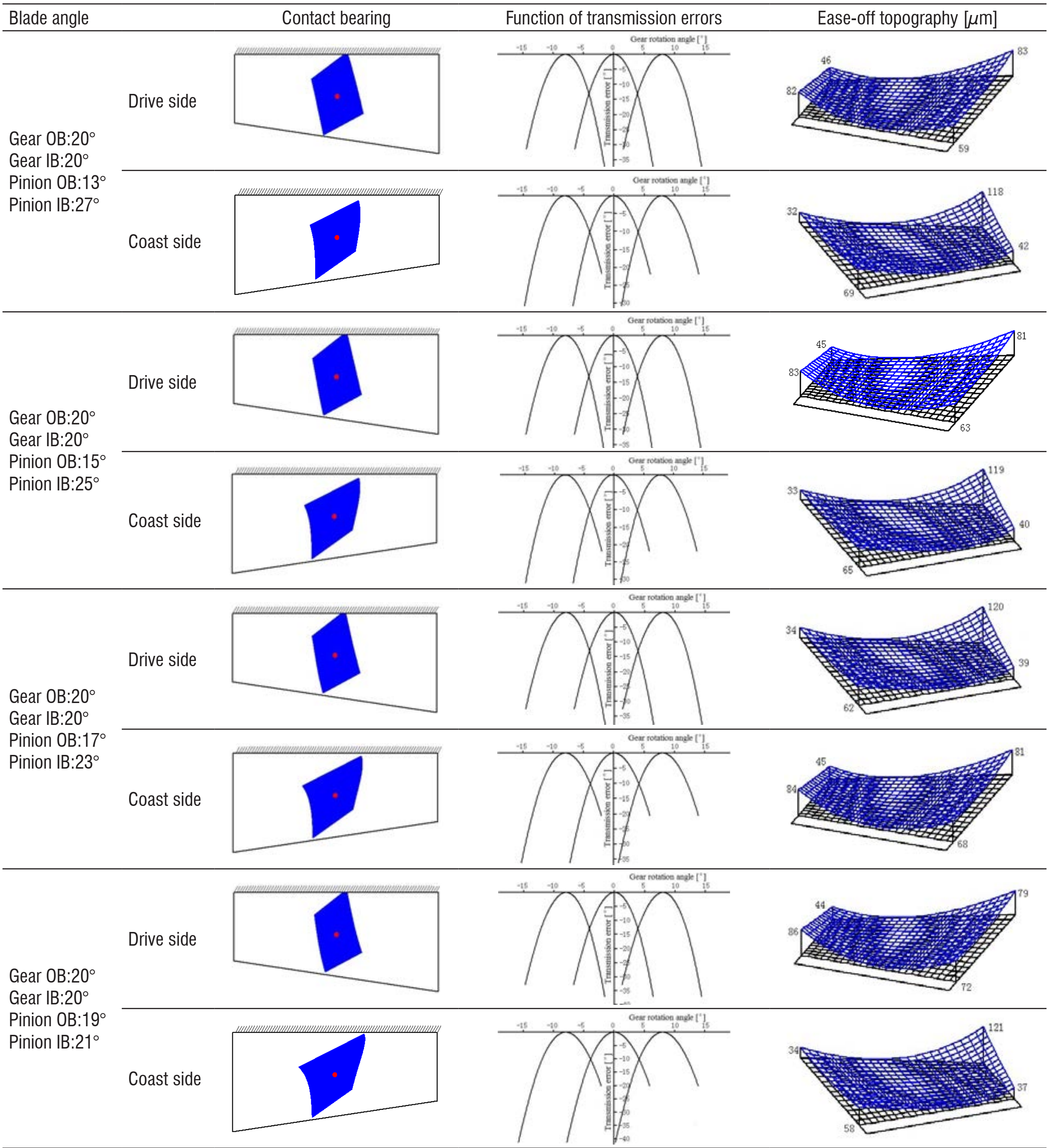

Fig. 5. *TCA results when changing pinion blade angles for HFT method*

surface contact analysis (TCA) when the pinion blade angles remain unchanged, and the gear blade angles change; Fig. 5 shows some of the results of the TCA when the gear blade angles remain unchanged, and the pinion blade angles change. Comparison of results of the TCA in Figs. 4 and 5 have confirmed the following:

(1) As the asymmetry degree of the gear blade angles increases, the length of the contact bearings and the inclination angle of the contact path become larger. The difference of the contact bearing length between the drive side and the coast side becomes significantly larger.
(2) It can be seen from the comparison of different ease-off topographies that the amount of the tooth

surface mismatch increases with the growth of the asymmetry degree for the blade angles.

(3) From the comparison of different transmission error functions, it can be shown that the maximum levels of the transmission error function gradually become larger with the growing of the asymmetry degree for the blade angles, and the maximum levels of the drive side are significantly larger than that of the coast side.

As shown in Figs. 6 and 7, the relationship between the blade angles and the size of the contact bearing is further analysed, it is known that the change of the major axis radius of the contact ellipse has no apparent regularity and the variation tendency of the drive side, and the coast side is different, while the minor axis radius of the contact ellipse has almost no change with the variation of the OB angles for the gear and the unchanged pinion blade angles (OB: 15° and IB: 25°). When the gear blade angles (OB: 20° and IB: 20°) do not change, the major axis radius for the drive side increases, and the coast side decreases substantially with the OB angle of the pinion increasing gradually, while the minor axis radius of the contact ellipse remains approximately unchanged.

The influences of the blade angle on meshing performance in the duplex helical method are investigated with the same analysis method mentioned above, Figs. 8 and 9 show some of the TCA results when the gear and pinion blade angles change respectively, and other head-cutter parameters remain unchanged. Figs. 10 and 11 show the relationship between the contact bearing size and the blade angles. The analysis results obtained are as follows:

(1) Whether with the increase of the OB angles of the gear or the pinion, the length of the contact bearing becomes shorter, which is completely different from the five-cut process and very regular. The slopes of the curves between the drive side and the coast side change only when the OB and IB angles of the gear are 20° (mean pressure angle).
(2) Similar to the five-cut process, the minor axis radius of the contact bearing changes very little, and the minor axis radius of the drive side is larger than that of the coast side.
(3) Compared with the five-cut process, the relationships between the transmission error function, the ease-off topography and the blade angle are more regular. That is, as the OB angle of the gear increases, the maximum level of the transmission error function and the amount of the tooth surface mismatch increase gradually when

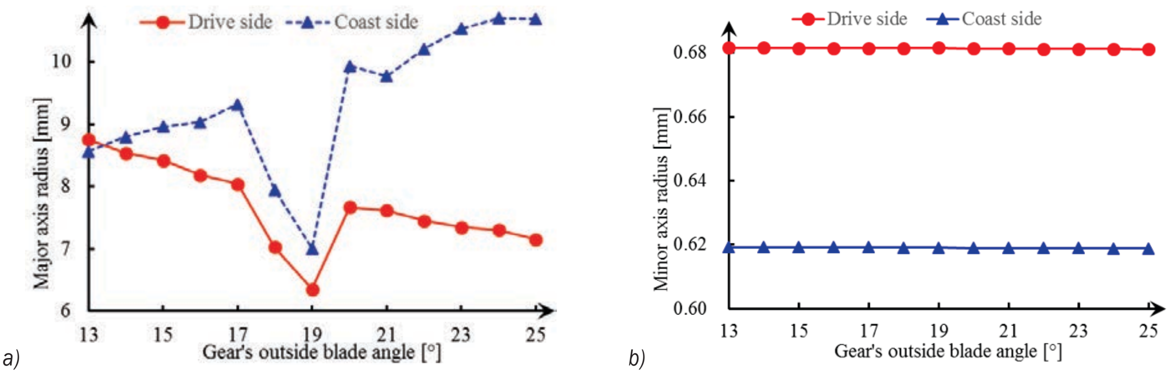

Fig. 6. *Relation between gear blade angles and size of contact ellipse for HFT method; a) major axis radius and b) minor axis radius*

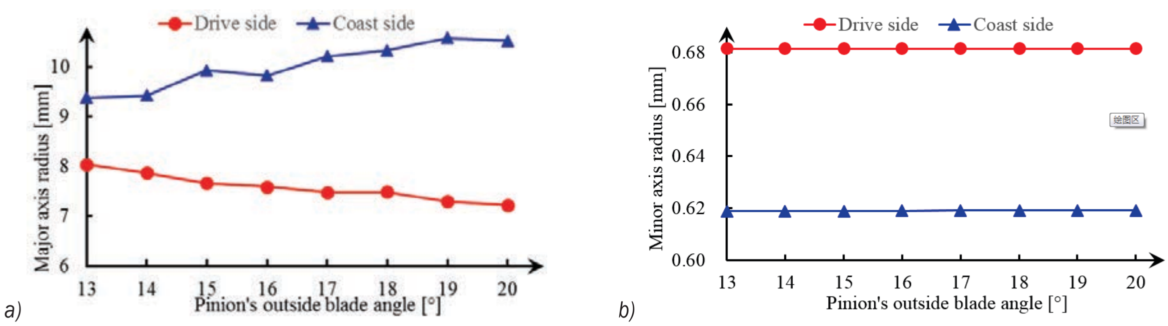

Fig. 7. *Relation between pinion blade angles and size of contact ellipse for HFT method; a) major axis radius and b) minor axis radius*

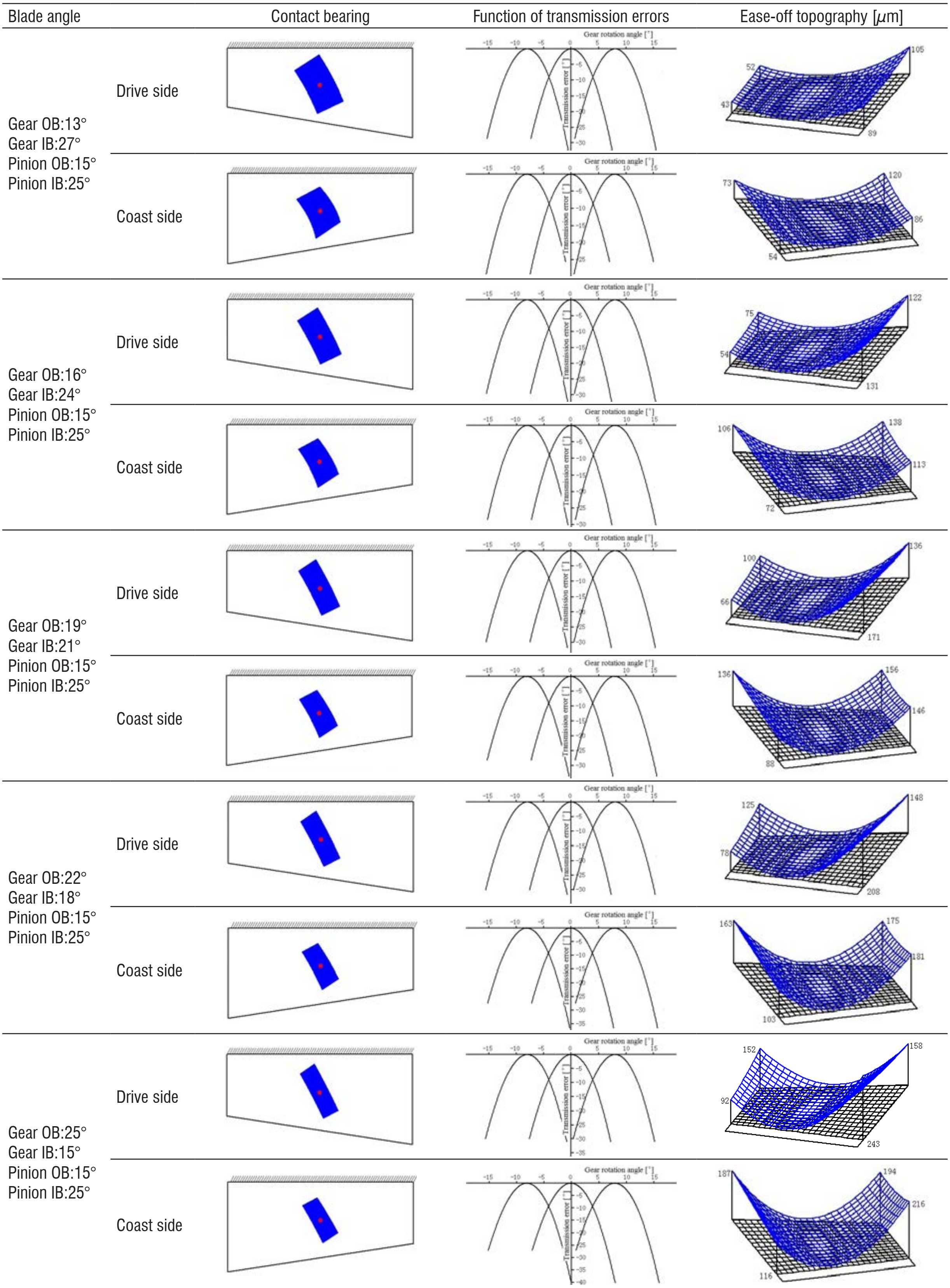

Fig. 8. *TCA results when changing gear blade angles for HFDH method*

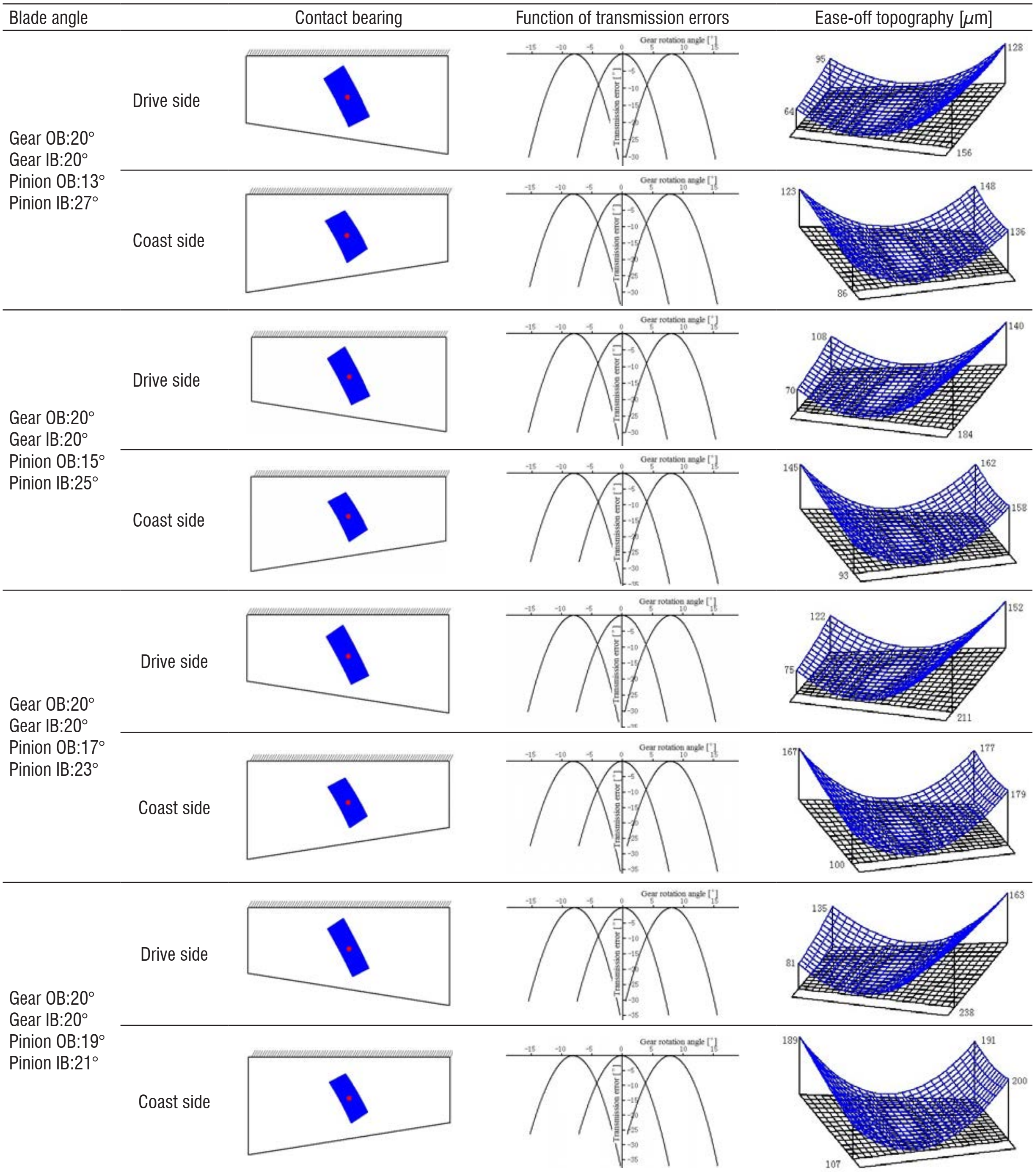

Fig. 9. *TCA results when changing pinion blade angles for HFDH method*

the blade angles of the pinion are constant. The variation trends are similar as above with the increase of the OB angle of the pinion when the blade angles of the gear are unchanged.

Therefore, unlike the five-cut process, the blade angle is an important control parameter for adjusting the size of the contact bearing in the duplex helical method. In general, increasing the asymmetry degree of the IB and OB angles can effectively increase the area of the contact bearing. This is also the reason that the gear and the pinion often can be manufactured by one head-cutter with the same blade angles (for

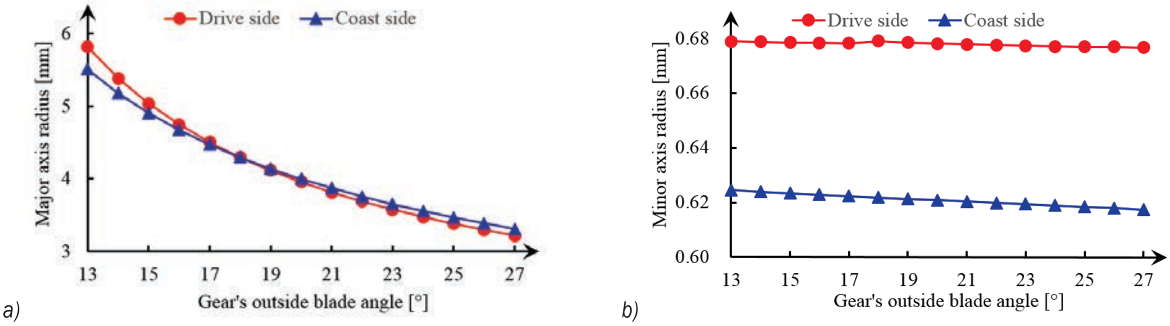

Fig. 10. *Relation between gear blade angles and size of contact ellipse for HFDH method; a) major axis radius and b) minor axis radius*

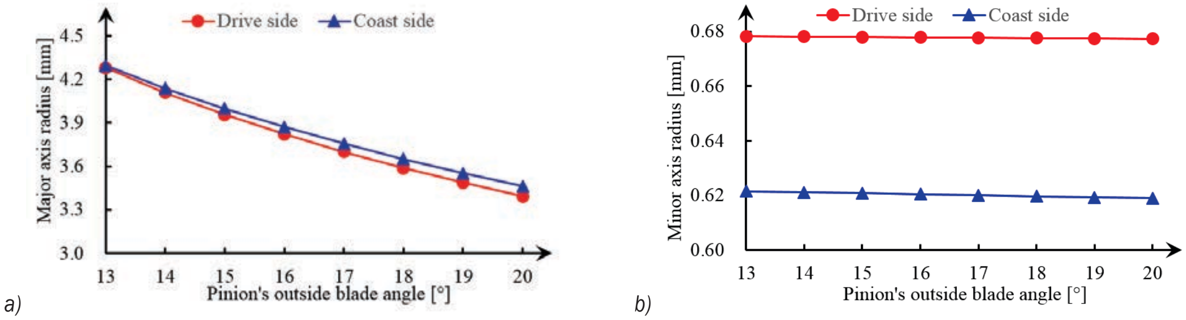

Fig. 11. *Relation between pinion blade angles and size of contact ellipse for HFDH method; a) major axis radius and b) minor axis radius*

example, the OB and IB angle can be 15° and 25° in this numerical example, respectively) and the mean cutter radius.

3 INFLUENCE RULES OF CUTTER RADIUS ON PERFORMANCE

The relationship between the cutter radius and the meshing performance is studied by changing the cutter radius under keeping the blade angles unchanged (Gear OB:20° and IB:20°, Pinion OB:15° and IB:25°) in the two processing methods, respectively. Figs. 12 and 13 show some of the TCA results when the mean cutter radius changes and other head-cutter parameters remain unchanged, Figs. 14 and 15 show the relationship between the contact bearing size and the cutter radius for the five-cut process and the duplex helical method, respectively. The analysis results obtained are as follows:

(1) In the five-cut process, the size, position and direction of the contact bearing and the amount of the tooth surface mismatch have little influence on the cutter radius; only when the cutter radius is less than 114.3 mm does the major axis radius of the contact ellipse become longer with the increase of the cutter radius for the drive side; the coast side is just the opposite, but the minor axis radius of the contact ellipse always changes very little on both surfaces.

(2) In the duplex helical method, the size of the contact bearing increases with the increase of the cutter radius all the time, but the change rates of the drive side and the coast side are different. The amount of the tooth surface mismatch decreases with the increase of the cutter radius.

(3) The influence rules between the maximum level of the transmission error function and the cutter radius are the same in the two processing methods, that is, the maximum level decreases with the increase of the cutter radius, while the coast side is exactly the opposite.

The relationship between the cutter radius and the tooth bending strength is investigated, as shown in Fig. 16; the data in the figure are obtained according to the calculation method of ISO 10300 [24]; the comparison and analysis of the tooth bending strength evolution for the two processing methods are as follows:

(1) Except for a few cutter radii, in general, the bending strength of the gear and the pinion decreases with the increase of the cutter radius.

(2) For the gear, the bending strength in the duplex helical method is slightly higher than that in the five-cut process, but the difference between the two methods is very small, which is because the

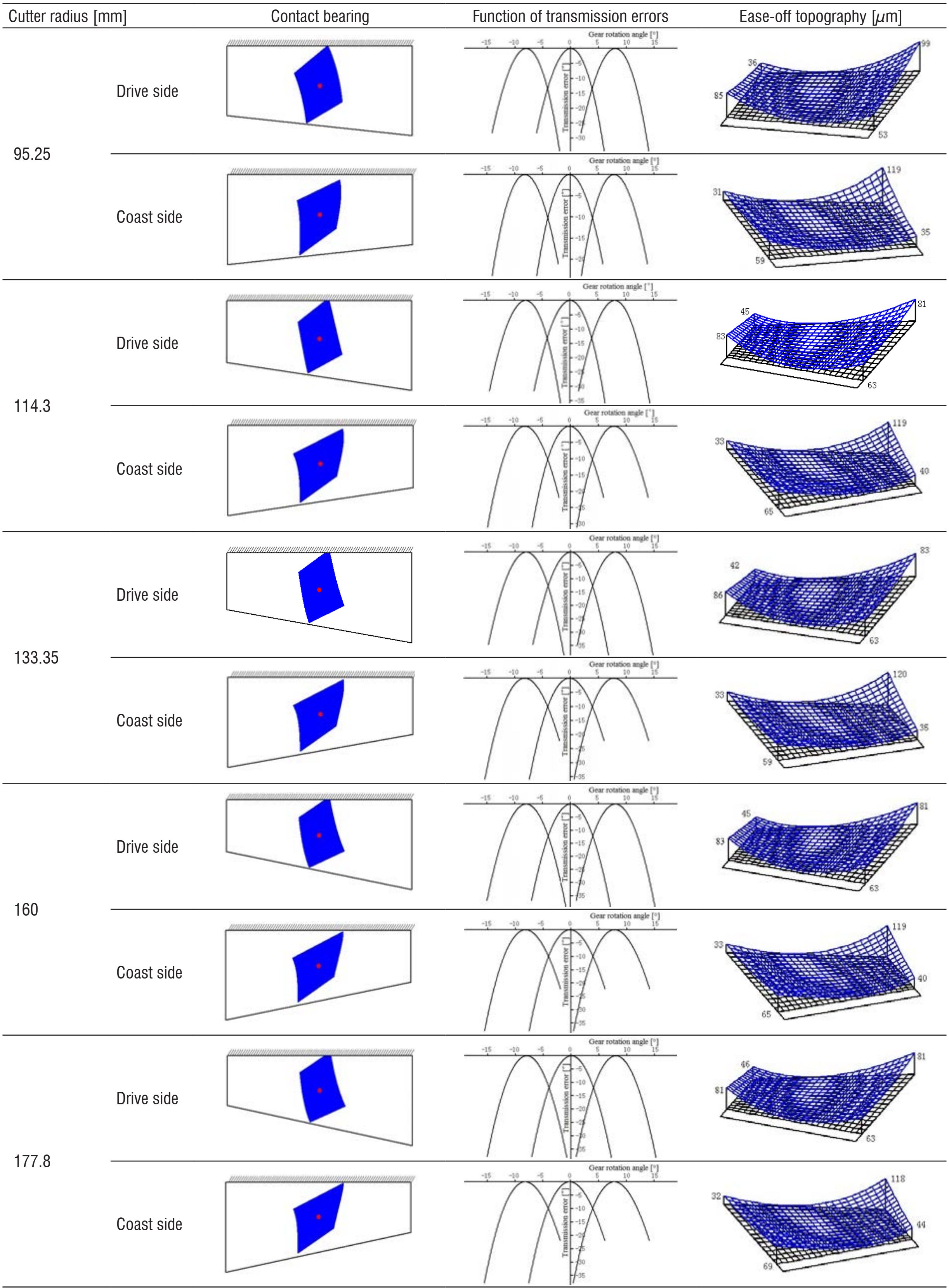

Fig. 12. *TCA results when changing cutter radius for HFT method*

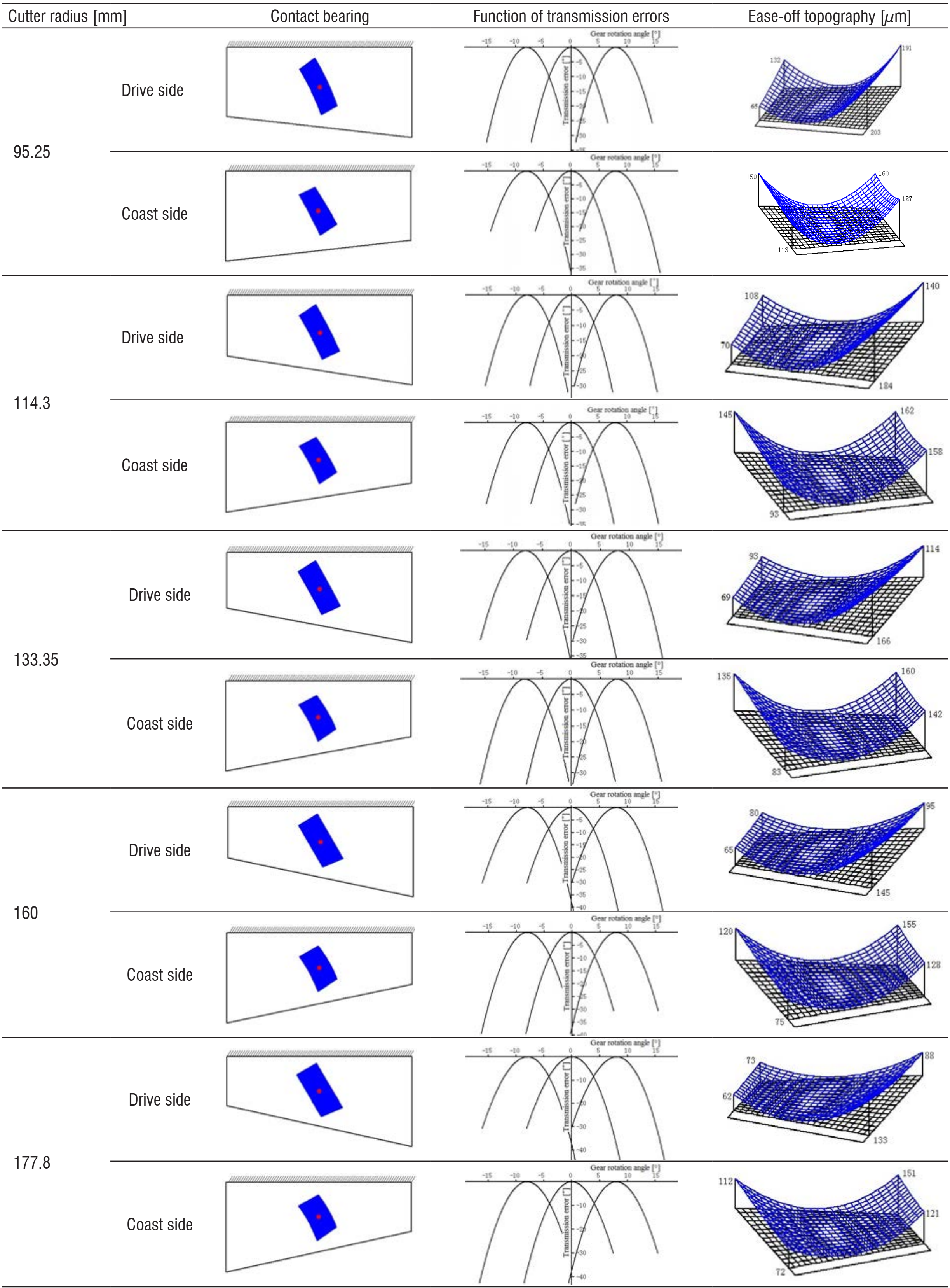

Fig. 13. *TCA results when changing cutter radius for HFDF method*

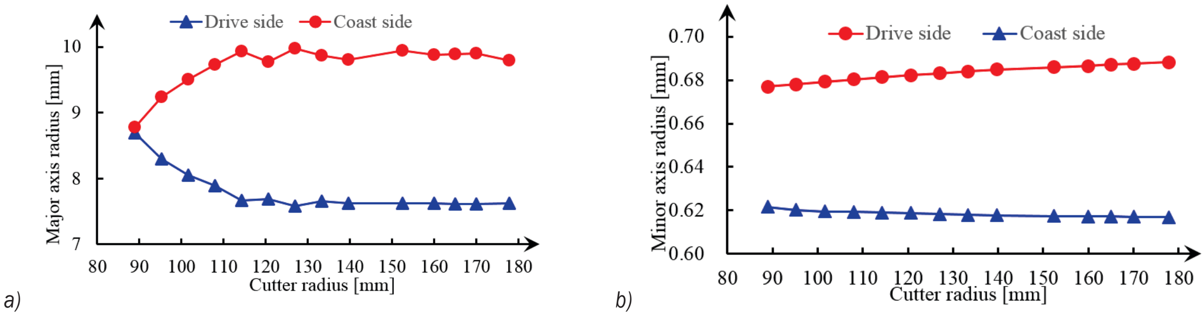

Fig. 14. *Relation between cutter radius and size of contact ellipse for HFT method; a) major axis radius and b) minor axis radius.*

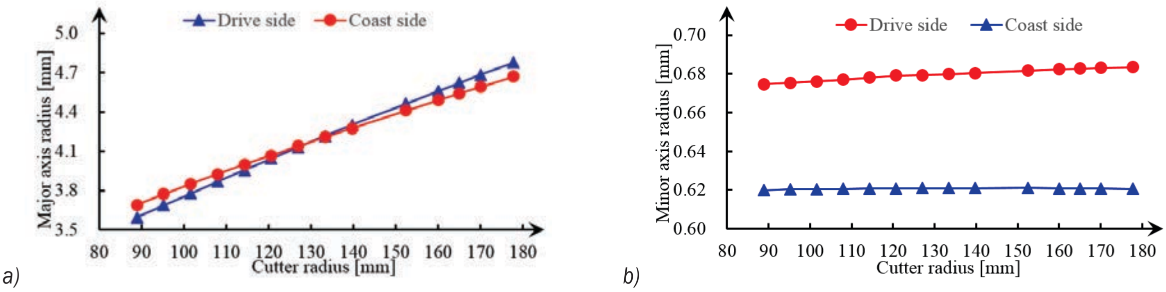

Fig. 15. *Relation between cutter radius and size of contact ellipse for HFDH method; a major axis radius and b) minor axis radius.*

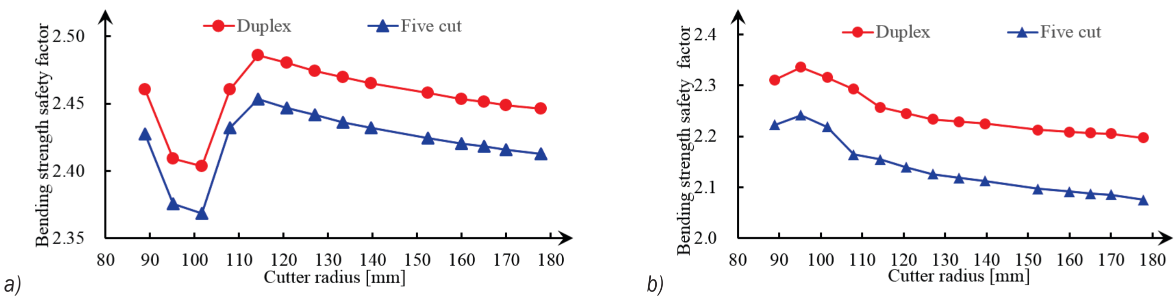

Fig. 16. *Relation between cutter radius and tooth bending strength; a) gear and b) pinion*

gears are all processed by the format in the two processing methods.

(3) For the pinion, the bending strength of the duplex helical method is significantly higher than that of the five-cut process, which is because the slot width has a constant width along lengthwise when the pinion is manufactured with the duplex helical method; the characteristic of the equal slot width ensures that the maximum point width and the maximum cutter edge radius can be used during machining. Therefore, not only the tooth strength can be improved, but also the cutter life can be extended. In addition, both sides of the pinion and gear tooth slots are cut with a single cutter in the duplex helical method; the connection smoothness of the tooth root fillet and the tooth bottom surface is much better than that cut by the five-cut process in actual manufacturing, which also improves the tooth strength of the pinion machined with the duplex helical method.

4 CONCLUSIONS

The following conclusions can be drawn from the performed investigation:

(1) The major idea of the generalized theory for the duplex helical method is revealed by analysing the different working principles of the cutter tilt mechanism in the two processing methods, that is, by changing the OB and IB angles of a spread blade head-cutter simultaneously with the cutter

tilt, the large "natural" length crowning can be reduced, down to the desirable crowning.
(2) Compared with the five-cut process, the influence of the blade angle and the cutter radius on the tooth surface contact characteristics is more regular in the duplex helical method. Therefore, they are often used as important parameters to optimize the meshing performance of spiral bevel and hypoid gears.
(3) In the case of the same design parameters and cutter parameters, the gearset processed by the duplex helical method has higher tooth-bending strength than that processed by the five-cut process; the difference can be up to 2.8 %.
(4) The relationships between the cutter parameters and the contact characteristics of the tooth surface by the five-cut process and the duplex helical method are established and analysed; the research results of this paper provide a theoretical basis for effectively obtaining the optimal machine settings of spiral bevel and hypoid gears.

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