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Analytical Formulae and Applications of Vertical Dynamic Responses for Railway Vehicles

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To effectively improve the estimated level of railway vehicles' vertical dynamic responses and provide a more suitable reference for the selection of its secondary suspension damping parameter, this paper has derived the root mean square values analytical formulae of the car body vertical acceleration, the secondary suspension vertical stroke, and the axle box vertical action force for railway vehicles under the random excitation of the track closer to the actual track characteristics. The correctness of the analytical formulae is verified by testing a real vehicle. Then, according to the analytical formulae derived, an analytical design method of the optimal damping ratio for the secondary suspension system is constructed based on the multi-objective programming and single-objective interval constraint analysis, which can be used to find the best trade-off for conflicting performance indices, such as ride comfort, running smoothness, and running safety, and the influences of the system parameters on the optimal damping ratio are analysed. This research can effectively characterize the vertical vibration response of railway vehicles and provide an effective reference for the initial design of the railway vehicle secondary suspension damping parameter.

Keywords: railway vehicle, vertical dynamic response, model deduction, damping parameter design, optimal compromise

Highlights

- *The analytical formulae of the vertical dynamic response for railway vehicles were derived.*
- *An optimal damping ratio design method for the secondary suspension system was constructed.*
- *The influences of the system parameters on the optimal damping ratio were analysed.*

0 INTRODUCTION

As bogie suspension system parameters are essential for the running stability, safety, and comfort of railway vehicles, the design of their suspension system parameters has become an important part of the bogie system design and has become a key concern for designers [1].

Generally, a bogie suspension system is composed of the elastic element and the damper element; it mainly includes two important parameters to be designed, i.e., the spring static deflection value and the suspension damping parameter value. In order to obtain an accurate and reliable design result for the suspension damping parameter, one widely used method is to use computer simulation technology and vehicle dynamic analysis software to optimize and determine the final value [2] to [4]. This method can simulate the actual operation of railway vehicles well [5] and [6] and can achieve a more accurate design result. However, the simulation analysis requires much time and cannot visually show the one-to-one correspondence between parameters and responses, which is not conducive to the adjustment and optimization of the structural parameters in the initial design stage of suspension systems. Meanwhile, the suspension parameter values before design are often unknown. The complexity of the model makes it complicated to do much work to design the suspension damping parameters, which is not conducive to the designers making a reasonable engineering choice quickly and effectively. In order to solve this problem, the most effective method is to use the analytical method, i.e., through reasonable model simplification, from the theoretical analysis point of view, and then obtain the required design value of the damping parameters [7] to [9]. For this reason, based on the simplified 1/4 vehicle model, using the simplified track irregularity power spectral density (of the form $1/\omega^4$ and $1/\omega^2$) as the input excitation of the system, the analytical formula for calculating the root mean square (RMS) value of the bogie frame vertical acceleration for railway vehicles is derived, and an analytical design method for the primary vertical suspension damping ratio of railway vehicles is given in [1] and [10], respectively. In addition, according to the simplified 1/4 vehicle model (mainly refers to the two-axle bogie railway vehicle), using the simplified track irregularity power spectral density (of the form $1/\omega^4$ and $1/\omega^2$) as the input excitation of the system, the analytical formulae for calculating the RMS values of the car body vertical acceleration, the secondary suspension vertical stroke, and the axle box vertical action force for railway vehicles are derived; an analytical design method for the secondary vertical suspension damping ratio of railway vehicles is given in [11] and [12].

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However, compared with the power spectral density of the form $1/[(\omega^2+v_2\Omega_c^2)\ (\omega_2+v^2\Omega_r^2)]$, which is widely used in the dynamic simulation of railway vehicles, the power spectral density of the form $1/\omega^4$ overestimates and underestimates in the low-frequency and high-frequency parts of the track irregularity, respectively, and the power spectral density of the form $1/\omega^2$ overestimates in both the low-frequency and high-frequency parts of the track irregularity.

Thus, in this paper, using the Germany track vertical irregularity power spectral density, which is closer to the actual track line, as the input excitation of the vehicle system, based on the analytical method, the RMS values analytical formulae of the vertical dynamic response of railway vehicles under the excitation of the form of $1/[(\omega^2+v_2\Omega_c^2)(\omega_2+v^2\Omega_r^2)]$ are deduced, and a more reasonable analytical design method for the secondary vertical suspension damping ratio of railway vehicles is given. As an extension and supplementing [11] and [12], it will provide effective technical support for the engineering selection of the bogie vertical suspension system parameters to deeply explore the vertical dynamic of railway vehicles on actual track lines in the theoretical analysis field.

1 SYSTEM MODEL

1.1 Vertical Vibration Model of Railway Vehicles for Analytical Calculation

Because of the weak coupling between the vertical and lateral dynamic behaviours of railway vehicles, when analysing its dynamic performance, they are often modelled separately and analysed separately, which can simplify the model establishment and facilitate the result analysis [13] on the premise of ensuring sufficient analysis accuracy. Numerous studies have shown that the commonly used vertical dynamic models of railway vehicles mainly include three types [14] to [16]: the 1/4 vehicle model, the single vehicle model, and the multi-marshalling vehicle model; each of the models has its advantages and its application occasions. Here, the 1/4 vehicle model is widely used in the analytical design of the suspension system parameters [10] to [12] and the semi-active and active suspension control research [17] and [18], because of its advantages of easy qualitative understanding of the relationship between the vehicle vertical dynamic characteristics and the structural parameters, simple solving process and easy engineering application. Therefore, in order to construct the analytical calculation model of the vertical dynamic response of railway vehicles and to provide effective guidance for the initial design of the secondary suspension damping parameter, this paper takes the 1/4 railway vehicle model [11] (which mainly contains two bogies per carriage, two wheelsets per bogie) as the model reference and carries out various research work, as shown in Fig. 1.

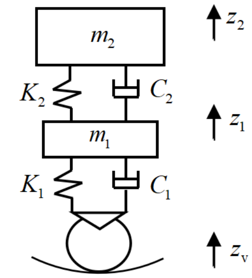

Fig. 1. *1/4 vehicle model*

In Fig. 1, m_1 is the half mass of the bogie frame; m_2 is the quarter mass of the car body; K_1 and K_2 are the vertical equivalent stiffness of the primary suspension and the secondary suspension; C_1 and C_2 are the vertical equivalent damping of the primary suspension and the secondary suspension; z_1 and z_2 are the vertical displacements of the bogie frame and the car body; z_v is the track irregularity.

According to the d'Alembert principle, the vibration differential equations of the 1/4 vehicle model can be obtained.

$$\begin{cases} m_1\ddot{z}_1 + C_1(\dot{z}_1-\dot{z}_v) + C_2(\dot{z}_1-\dot{z}_2) + K_1(z_1-z_v) \\ \qquad + K_2(z_1-z_2) = 0 \\ m_2\ddot{z}_2 + C_2(\dot{z}_2-\dot{z}_1) + K_2(z_2-z_1) = 0 \end{cases}. \quad (1)$$

By using Fourier transform, the transfer functions between $\ddot{z}_2$ and z_v, f_d and z_v, F_d and z_v can be solved respectively according to Eq. (1), as follows:

$$H(\mathrm{j}\omega)_{\ddot{z}_2\sim z_v} = \frac{C_1C_2\omega^4-(C_1K_2+C_2K_1)\mathrm{j}\omega^3-K_1K_2\omega^2}{\left[\begin{aligned}&m_1m_2\omega^4-(C_1m_2+C_2m_1+C_2m_2)\mathrm{j}\omega^3+K_1K_2-\\&(C_1C_2+K_1m_2+K_2m_1+K_2m_2)\omega^2+(C_1K_2+C_2K_1)\mathrm{j}\omega\end{aligned}\right]}, \quad (2)$$

$$H(\mathrm{j}\omega)_{f_d\sim z_v} = \frac{C_1m_2\mathrm{j}\omega^3+K_1m_2\omega^2}{\left[\begin{aligned}&m_1m_2\omega^4-(C_1m_2+C_2m_1+C_2m_2)\mathrm{j}\omega^3+K_1K_2-\\&(C_1C_2+K_1m_2+K_2m_1+K_2m_2)\omega^2+(C_1K_2+C_2K_1)\mathrm{j}\omega\end{aligned}\right]}, \quad (3)$$

$$H(\mathrm{j}\omega)_{F_d\sim z_v} = \frac{\left[\begin{aligned}&-C_1m_1m_2\mathrm{j}\omega^5-(C_1C_2m_1+C_1C_2m_2+K_1m_1m_2)\omega^4+\\&(K_1C_2+C_1K_2)(m_1+m_2)\mathrm{j}\omega^3+K_1K_2(m_1+m_2)\omega^2\end{aligned}\right]}{\left[\begin{aligned}&m_1m_2\omega^4-(C_1m_2+C_2m_1+C_2m_2)\mathrm{j}\omega^3+K_1K_2-\\&(C_1C_2+K_1m_2+K_2m_1+K_2m_2)\omega^2+(C_1K_2+C_2K_1)\mathrm{j}\omega\end{aligned}\right]}, \quad (4)$$

where, f_d is the secondary suspension vertical stroke $f_d=z_2-z_1$; F_d is the axle box vertical action force $F_d = C_1(\dot{z}_1 - \dot{z}_v) + K_1(z_1 - z_v)$.

1.2 Excitation Model of Track Irregularity

As the main excitation source of the vertical vibration of railway vehicles, the random input of the track vertical irregularity is basically a stationary random process along the track. Usually, according to the measured irregularity data of the track, the mathematical statistics is made, and the irregularity is expressed as the power spectral density form by interpolation processing [19]. The research indicates that there are mainly three kinds of analytical expressions for the track irregularity power spectral density commonly used in the research of railway vehicles at present, as shown in Eqs. (5), (6), and (7). The power spectral densities shown in Eqs. (5) and (6) are mainly used in the analytical calculation of railway vehicle dynamics and the semi-active and active suspension control research. The power spectral density shown in Eq. (7) is the closest to the actual track line and is widely used in the dynamic simulation analysis of railway vehicles [19].

$$S_{v1}(\omega) = \frac{A_b(2\pi v)^3}{\omega^4}, \tag{5}$$

where, A_b is the track roughness coefficient, A_b=0.928× 10^{-10} m^{-1}, v is the vehicle speed.

$$S_{v2}(\omega) = \frac{2\pi A_r v}{\omega^2}, \tag{6}$$

where, A_r is the track roughness coefficient, A_r=2.5×10^{-7} m.

$$S_{v3}(\omega) = \frac{2\pi A_v \Omega_c^2 v^3}{\left(\omega^2 + v^2\Omega_c^2\right)\left(\omega^2 + v^2\Omega_r^2\right)}, \tag{7}$$

where, A_v is the track roughness coefficient, A_v= 4.032×10^{-7} m; Ω_c and Ω_r are the truncated spatial frequencies, Ω_c=0.824 6 m^{-1}, Ω_r=0.020 6 m^{-1}.

According to Eqs. (5) to (7), the power spectral density functions of the track irregularity at 300 km/h are expressed in the double logarithmic coordinate system, as shown in Fig. 2. It can be seen from the figure that, the power spectral densities shown in Eqs. (5) and (6) are straight lines with slopes of -4:1 and -2:1, respectively. The power spectral density shown in Eq. (7) can be approximated to a combination of three slopes (0:1, -2:1, -4:1). Compared with Eq. (7), Eq. (5) overestimates and underestimates in the low-frequency and high-frequency parts of the track irregularity, respectively; Eq. (6) overestimates in both the low-frequency and high-frequency parts of the track irregularity, but it is consistent with the actual line in the mid-frequency range.

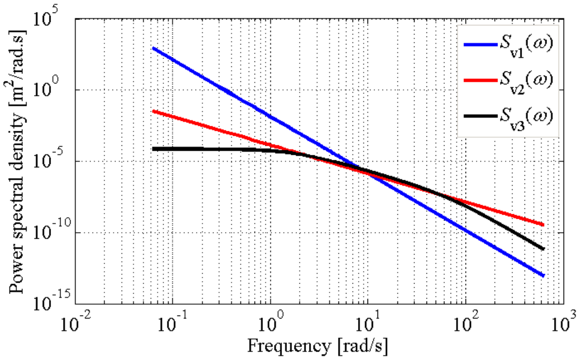

Fig. 2. *Power spectral density curve of track irregularity*

2 ANALYTICAL DESCRIPTION OF THE RAILWAY VEHICLE VERTICAL DYNAMICS RESPONSE

To quickly and effectively characterize the vertical vibration response of railway vehicles in actual operation and to enable designers to make reasonable judgments and choices on the initial design values of the system parameters quickly, using the power spectral density of the track irregularity shown in Eq. (7) as the input excitation source of the railway vehicle, the RMS values analytical formulae of the railway vehicle vertical dynamic response will be solved in this section. Note that the RMS values analytical formulae of the railway vehicle vertical dynamic response under the power spectral densities shown in Eqs. (5) and (6) can be found in references [11] and [12], respectively, which will not be introduced here.

2.1 RMS Values of the railway Vehicle Vertical Vibration Response

In the study of railway vehicle dynamics, the RMS values of the system response are usually used to evaluate the vibration characteristics and isolation effect of the vehicle system [11], [12] and [14]. According to the theory of random vibration, the following equation can be obtained.

$$\sigma_x^2 = \int_{-\infty}^{+\infty} \left| H(\mathrm{j}\omega)_{x\sim z_v} \right|^2 S_v(\omega)\mathrm{d}\omega. \tag{8}$$

Here, x represents the response of the system, $H(\mathrm{j}\omega)_{x\sim z_v}$ is the transfer function, $S_v(\omega)$ is the power spectral density of the system input.

For a linear system, the following relationship exists between its amplitude-frequency characteristics:

$$\left|H(\mathrm{j}\omega)_{x\sim z_{\mathrm{v}}}\right|^2 = H(\mathrm{j}\omega)_{x\sim z_{\mathrm{v}}} H(-\mathrm{j}\omega)_{x\sim z_{\mathrm{v}}}. \tag{9}$$

Therefore, according to Eq. (9), Eqs. (2) to (4) can be expressed as follows

$$\left|H(\mathrm{j}\omega)_{x\sim z_{\mathrm{v}}}\right|^2 = \frac{N(\mathrm{j}\omega)N(-\mathrm{j}\omega)}{D(\mathrm{j}\omega)D(-\mathrm{j}\omega)}, \tag{10}$$

where, N(jω) and D(jω) are the numerator and the denominator of the Eq. (2), Eq. (3) and Eq. (4), respectively.

In addition, the power spectral density function shown in Eq. (7) can be rewritten as follows:

$$S_{\mathrm{v}3}(\omega) = \frac{2\pi A_{\mathrm{v}}\Omega_{\mathrm{c}}^2 v^3}{(v\Omega_{\mathrm{c}}+\mathrm{j}\omega)(v\Omega_{\mathrm{c}}-\mathrm{j}\omega)(v\Omega_{\mathrm{r}}+\mathrm{j}\omega)(v\Omega_{\mathrm{r}}-\mathrm{j}\omega)}, \tag{11}$$

Thus, according to Eqs. (10) and (11), the RMS values of the car body vertical vibration acceleration, the secondary suspension vertical stroke, and the axle box vertical action force can be obtained by using the method of integral solution of the complex variable function [20], respectively.

$$\sigma_{\ddot{z}_2} = \sqrt{\frac{2\pi^2 A_{\mathrm{v}}\Omega_{\mathrm{c}}^2 v^3\left[(a_6a_3^2 - a_4a_3a_5 + a_2a_5^2 - a_1a_6a_5)b_0 + (a_0a_5^2 + a_1a_3a_6 - a_1a_4a_5)b_1 + (a_1^2a_6 + a_0a_3a_5 - a_1a_2a_5)b_2\right]}{\begin{aligned}\big(&a_0^2a_5^3 + a_0a_3^2a_4a_5 - a_0a_3^3a_6 - a_0a_2a_3a_5^2 + 3a_0a_1a_3a_5a_6 - 2a_0a_1a_4a_5^2\\ &+ a_1^3a_6^2 - 2a_1^2a_2a_5a_6 - a_1^2a_3a_4a_6 + a_1^2a_4^2a_5 + a_1a_2^2a_5^2 + a_1a_2a_3^2a_6 - a_1a_2a_3a_4a_5\big)\end{aligned}}}, \tag{12}$$

$$\sigma_{f_{\mathrm{d}}} = \sqrt{\frac{2\pi^2 A_{\mathrm{v}}\Omega_{\mathrm{c}}^2 v^3\left[(a_0a_5^2 - a_1a_4a_5 + a_1a_3a_6)d_0 + a_0(a_1^2a_6 + a_0a_3a_5 - a_1a_2a_5)d_1\right]}{\begin{aligned}\big(&a_0^2a_5^3 + a_0a_3^2a_4a_5 - a_0a_3^3a_6 - a_0a_2a_3a_5^2 + 3a_0a_1a_3a_5a_6 - 2a_0a_1a_4a_5^2\\ &+ a_1^3a_6^2 - 2a_1^2a_2a_5a_6 - a_1^2a_3a_4a_6 + a_1^2a_4^2a_5 + a_1a_2^2a_5^2 + a_1a_2a_3^2a_6 - a_1a_2a_3a_4a_5\big)\end{aligned}}}, \tag{13}$$

$$\sigma_{F_{\mathrm{d}}} = \sqrt{\frac{\begin{aligned}2\pi^2 A_{\mathrm{v}}\Omega_{\mathrm{c}}^2 v^3\big[&(a_1a_3a_4a_6 - a_1^2a_6^2 + 2a_1a_2a_5a_6 - a_1a_4^2a_5 - a_2^2a_5^2 - a_2a_3^2a_6 + a_2a_3a_4a_5 - a_0a_3a_5a_6 + a_0a_4a_5^2)e_0\\ &+ a_0(a_3a_4a_5 - a_2a_5^2 - a_3^2a_6 + a_1a_5a_6)e_1 + a_0(a_1a_4a_5 - a_0a_5^2 - a_1a_3a_6)e_2 + a_0(a_1a_2a_5 - a_1^2a_6 - a_0a_3a_5)e_3\big]\end{aligned}}{\begin{aligned}-a_0\big(&a_0^2a_5^3 + a_0a_3^2a_4a_5 - a_0a_3^3a_6 - a_0a_2a_3a_5^2 + 3a_0a_1a_3a_5a_6 - 2a_0a_1a_4a_5^2\\ &+ a_1^3a_6^2 - 2a_1^2a_2a_5a_6 - a_1^2a_3a_4a_6 + a_1^2a_4^2a_5 + a_1a_2^2a_5^2 + a_1a_2a_3^2a_6 - a_1a_2a_3a_4a_5\big)\end{aligned}}}, \tag{14}$$

where,
$b_0 = C_1^2 C_2^2$,
$b_1 = (C_1K_2 + C_2K_1)^2 - 2C_1C_2K_1K_2$,
$b_2 = K_1^2K_2^2$;
$d_0 = C_1^2m_2^2$,
$d_1 = K_1^2m_2^2$;
$e_0 = C_1^2m_1^2m_2^2$,
$e_1 = [C_1C_2(m_1+m_2)+K_1m_1m_2]^2$
$\quad -2C_1m_1m_2(K_1C_2+K_2C_1)(m_1+m_2)$,
$e_2 = [(K_1C_2+K_2C_1)^2 - 2K_1K_2C_1C_2](m_1+m_2)^2$
$\quad -2K_1^2K_2m_1m_2(m_1+m_2)$,
$e_3 = K_1^2K_2^2(m_1+m_2)^2$;
$a_0 = m_1m_2$,
$a_1 = C_1m_2 + C_2m_1 + C_2m_2 + m_1m_2v(\Omega_{\mathrm{c}}+\Omega_{\mathrm{r}})$,
$a_2 = C_1C_2 + v(\Omega_{\mathrm{c}}+\Omega_{\mathrm{r}})(C_1m_2 + C_2m_1 + C_2m_2)$
$\quad + K_1m_2 + K_2m_1 + K_2m_2 + m_1m_2\Omega_{\mathrm{c}}\Omega_{\mathrm{r}}v^2$,
$a_3 = C_1K_2 + C_2K_1 + v(\Omega_{\mathrm{c}}+\Omega_{\mathrm{r}})$
$\quad \cdot(C_1C_2 + K_1m_2 + K_2m_1 + K_2m_2)$
$\quad + (C_1m_2 + C_2m_1 + C_2m_2)\Omega_{\mathrm{c}}\Omega_{\mathrm{r}}v^2$,
$a_4 = K_1K_2 + v(C_1K_2 + C_2K_1)(\Omega_{\mathrm{c}}+\Omega_{\mathrm{r}})$
$\quad + (C_1C_2 + K_1m_2 + K_2m_1 + K_2m_2)\Omega_{\mathrm{c}}\Omega_{\mathrm{r}}v^2$,
$a_5 = K_1K_2(\Omega_{\mathrm{c}}+\Omega_{\mathrm{r}})(C_1K_2 + C_2K_1)\Omega_{\mathrm{c}}\Omega_{\mathrm{r}}v^3$,
$a_6 = K_1K_2\Omega_{\mathrm{c}}\Omega_{\mathrm{r}}v^2$.

2.2 Test Verification of the Analytical Formulae

In order to verify the correctness of the RMS values analytical formulae for the vertical dynamic response of railway vehicles, a CRH2 EMU, which is widely used in China, is taken as an example to test its vibration, and the result of the vibration test and the analytical calculation is analysed. The vehicle running speed is 200 km/h, the sampling frequency is 500 Hz, and the sampling time length is 120 s. The vehicle parameter values (1/4 vehicle equivalent parameters) of the CRH2 EMU are shown in Table 1.

Table 1. *Equivalent parameters of 1/4 CRH2 EMU*

Parameters	Unit	Values
m_1	kg	1,300
m_2	kg	8,150
K_1	N/m	2,352,000
K_2	N/m	189,140
C_1	N·s/m	39,200
C_2	N·s/m	20,000

Fig. 3 shows a picture of the vehicle vibration test. Table 2 gives the comparisons results of the RMS values of the car body vertical acceleration and the secondary suspension vertical stroke obtained from the test and the analytical calculation.

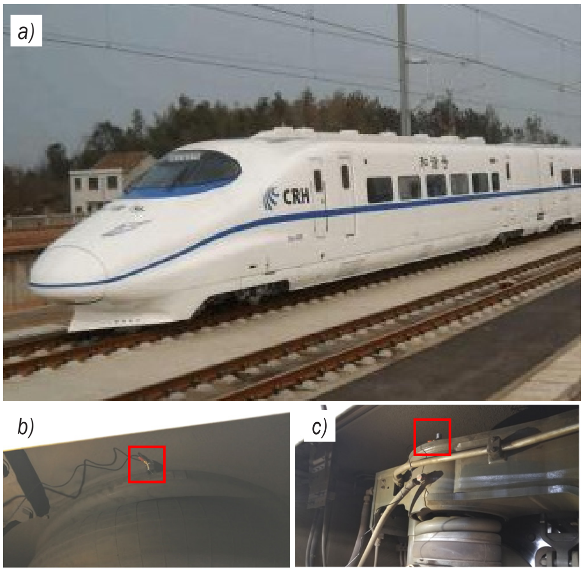

Fig. 3. *Vehicle vibration test; a) test vehicle, b) acceleration sensor installed on the upper end of the secondary suspension, and c) acceleration sensor installed on the lower end of the secondary suspension*

Table 2. *Comparisons of the test and calculation results*

RMS values	Calculation results	Test results	Absolute deviation
$\sigma_{\ddot{z}_2}$ [m/s²]	0.440	0.487	−0.047
σ_{f_d} [m]	0.013	0.012	0.001

As can be seen from Table 2, the analytical results are in good agreement with the actual vehicle test results, and the relative deviations of the RMS values of the car body vertical acceleration and the secondary suspension vertical stroke between the analytical results and the vehicle test results are only 9.65 % and 8.33 %, respectively. This shows that the RMS values analytical formulae for the vertical dynamic response of railway vehicles established are correct and reliable.

3 INFLUENCE OF SECONDARY VERTICAL SUSPENSION ON RAILWAY VEHICLE VERTICAL VIBRATION RESPONSE

In the parameter design of railway vehicles suspension system, in order to make the design result be of more practical value in engineering, the stiffness parameter is usually converted to frequency, and the damping coefficient is usually converted to the damping ratio. Therefore, according to the definition of the frequency and the damping ratio, it is known that the frequency and the damping ratio of the secondary suspension system can be written as [1]:

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{K_2}{m_2}}, \quad \xi_2 = \frac{C_2}{2\sqrt{K_2 m_2}}. \tag{15}$$

According to Eq. (15), substitute it into Eqs. (12) to (14), then the influence of the secondary vertical suspension on the vertical vibration response of the train can be obtained, as shown in Figs. 4 to 6. Here, the vehicle parameter values are shown in Table 1.

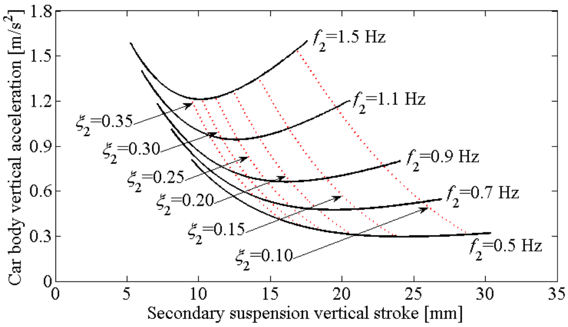

Fig. 4. *Influence of secondary vertical suspension on the car body vertical vibration acceleration and the secondary suspension vertical stroke*

As can be seen from Figs. 4 to 6, under a certain natural frequency f_2, if the damping ratio ξ_2 is too small, the secondary suspension vertical stroke will be too large, which is not conducive to the train operation. With the increase of the damping ratio ξ_2, the secondary suspension vertical stroke decreases gradually, while the car body vertical vibration acceleration and the axle box vertical action force first decrease and then increase, that is, there are minimum extreme points for both. It can be seen that, when the actual frequency value of the high-speed train is adopted, i.e., f_2 = 0.7 Hz to 1.2 Hz [4], selecting an appropriate damping ratio ξ_2 can make the car body vertical vibration acceleration, the secondary suspension vertical stroke, and the axle box vertical action force reach a low compromise effect at the same time.

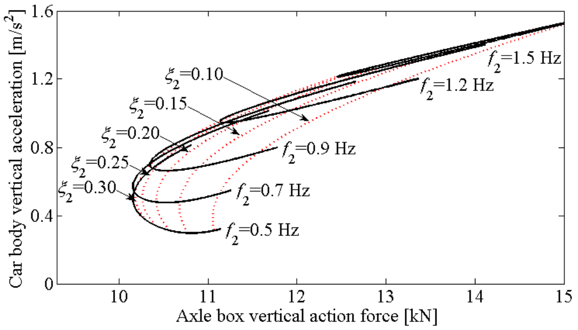

Fig. 5. *Influence of secondary vertical suspension on the car body vertical vibration acceleration and the axle box vertical action force*

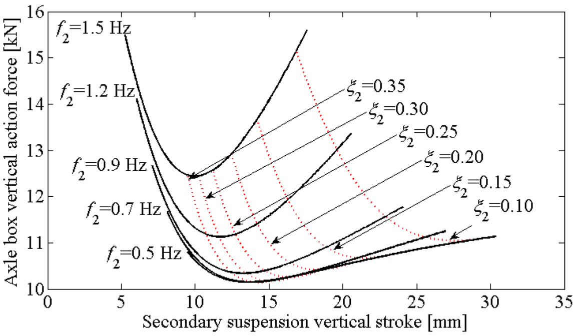

Fig. 6. *Influence of secondary vertical suspension on the secondary suspension vertical stroke and the axle box vertical action force*

4 ANALYTICAL DESIGN OF THE SECONDARY SUSPENSION DAMPING PARAMETER

It can be seen from Section 3 that the secondary vertical suspension damping parameter has a highly significant influence on the car body vertical acceleration, the secondary suspension vertical stroke, and the axle box vertical action force for railway vehicles. Therefore, in order to make the train have good running quality, when designing the secondary suspension damping parameter, the influence of these three indexes should be considered comprehensively. Based on this, a design method of the secondary suspension damping parameter for railway vehicles will be studied in this section by using the established RMS values analytical formulae.

4.1 Damping Ratio Design of the Secondary Suspension System

According to Eq. (15), substituting $C_2 = 2\xi_2\sqrt{K_2 m_2}$ into Eq. (12), solving the partial derivative of the car body vertical acceleration RMS value $\sigma_{\ddot{z}_2}$ with respect to the secondary suspension damping ratio ξ_2, and let $\left(\mathrm{d}\sigma_{\ddot{z}_2}\right)/\left(\mathrm{d}\xi_2\right)=0$, then, the analytical design equation of the optimal damping ratio for the secondary suspension system based on the minimum RMS of the car body vertical acceleration can be established, that is

$$\Lambda_{c0}\xi_2^8+\Lambda_{c1}\xi_2^7+\Lambda_{c2}\xi_2^6+\Lambda_{c3}\xi_2^5+\Lambda_{c4}\xi_2^4 +\Lambda_{c5}\xi_2^3+\Lambda_{c6}\xi_2^2+\Lambda_{c7}\xi_2+\Lambda_{c8}=0, \quad (16)$$

where, Λ_{c0} to Λ_{c8} are the coefficients of the analytical design equation expressed by vehicle parameters and vehicle speed, respectively.

Similarly, if let the partial derivative of the axle box vertical action force RMS value σ_{f_d}, Eq. (14), with respect to the secondary suspension damping ratio ξ_2 equal to zero, the analytical design equation of the optimal damping ratio for the secondary suspension system based on the minimum RMS of the axle box vertical action force can be established.

$$\Lambda_{s0}\xi_2^8+\Lambda_{s1}\xi_2^7+\Lambda_{s2}\xi_2^6+\Lambda_{s3}\xi_2^5+\Lambda_{s4}\xi_2^4 +\Lambda_{s5}\xi_2^3+\Lambda_{s6}\xi_2^2+\Lambda_{s7}\xi_2+\Lambda_{s8}=0, \quad (17)$$

where, Λ_{s0} to Λ_{s8} are the coefficients of the analytical design equation expressed by vehicle parameters and vehicle speed, respectively.

Thus, solving Eqs. (16) and (17) with respect to the positive real root of ξ_2, the optimal damping ratios of the secondary suspension system based on the minimum RMS value of the car body vertical acceleration and the axle box vertical action force can be obtained, respectively, i.e., the design values of ξ_c, and ξ_s.

Furthermore, in order to effectively avoid the probability of the suspension hitting the elastic stop block, according to the relationship between the probability distribution and the standard deviation, the relationship between the RMS value of the vertical stroke and the limit stroke $[f_d]$ of the secondary suspension system can be obtained, that is:

$$3\sigma_{f_d}=[f_d]. \quad (18)$$

Therefore, according to Eq. (13), solving Eq. (18) with respect to the positive real root of ξ_2, the minimum damping ratio of the secondary suspension system, i.e., ξ_b can be obtained based on the maximum

RMS value of the secondary suspension vertical stroke.

It can be seen that the design of the secondary suspension damping parameter is a multi-objective optimization problem. In order to improve a railway vehicle's comprehensive performance and to simplify the design process, this paper transforms the multi-objective optimization problem into a single-objective interval constraint problem by using the linear weighting method. Based on this, the optimal compromise between the minimum RMS value of the car body vertical acceleration and the minimum RMS value of the axle box vertical action force can be obtained. Also, the optimal damping ratio can effectively avoid the suspension impact limit stroke:

$$\xi_2 = \begin{cases} \xi_u = \alpha\xi_c + (1-\alpha)\xi_s, & \xi_b \le \xi_u \\ \xi_b, & \xi_b > \xi_u \end{cases}. \tag{19}$$

Here, α is a weighting factor, and its value can be determined according to the importance of each sub-target, $\alpha \in [0, 1]$. Note that, in order to meet the needs of the engineering and improve the design efficiency, the golden section method [21] can be used to let α=0.618.

4.2 Influence of System Parameters on the Optimal Ration of the Secondary Suspension

To determine the influence of each parameter on the optimal damping ratio of the secondary suspension system, the optimal damping ratios of the secondary

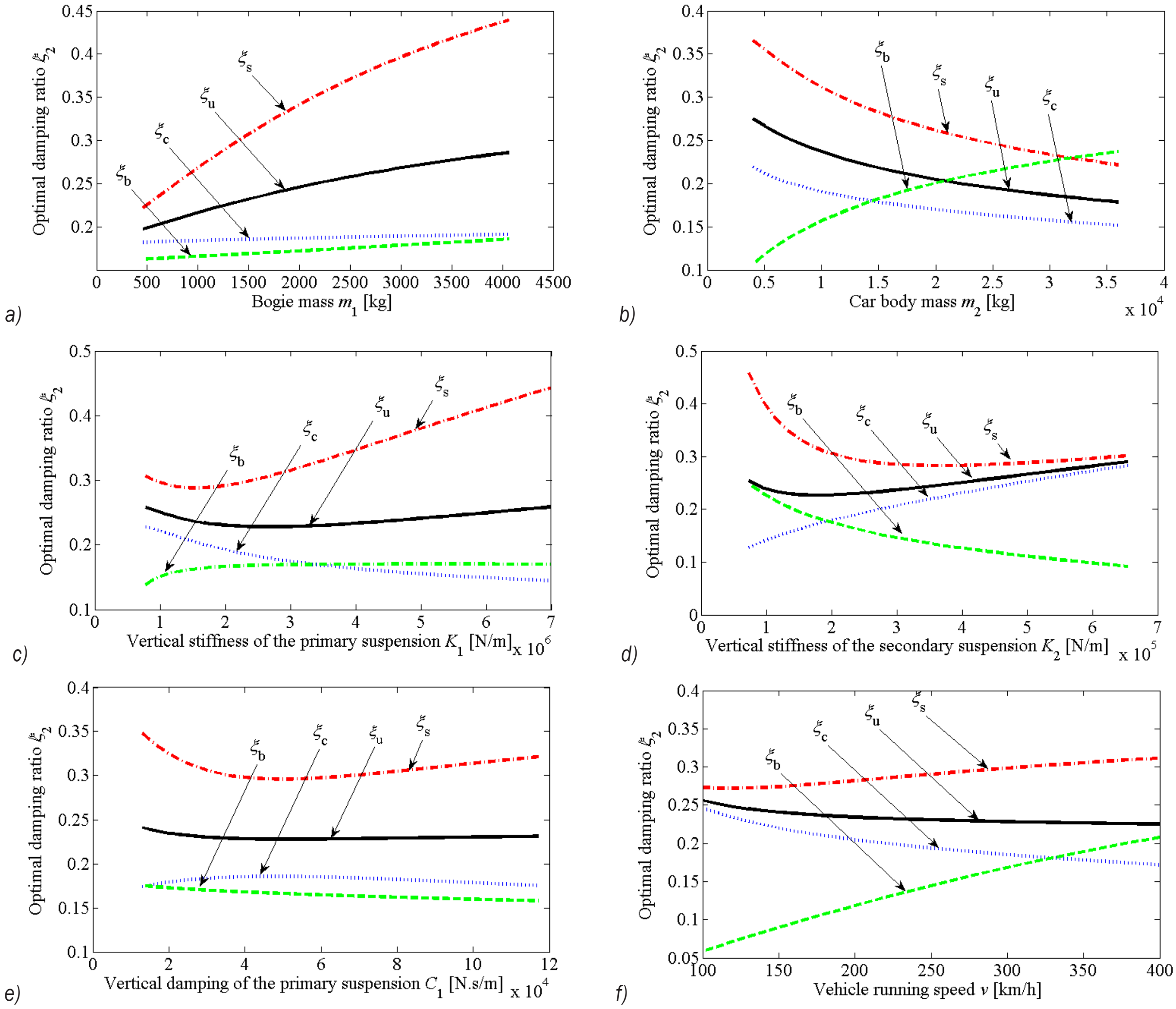

Fig. 7. *Influence of system parameters on the damping ratio of the secondary suspension; a) bogie mass m_1, b) car body mass m_2, c) vertical stiffness of the primary suspension K_1, d) vertical stiffness of the secondary suspension K_2, e) vertical damping of the primary suspension C_1, and f) vehicle running speed v*

suspension system under each parameter are analysed with the example of the railway vehicle shown in Table 1. The curves of the secondary suspension damping ratio with the variation of the system parameters are obtained, as shown in Fig. 7. Here, the weighting factor $\alpha = 0.618$, the vehicle running speed $v = 300$ km/h, and the limit stroke $[f_d] = 60$ mm.

As can be seen from Fig. 7, the optimal damping ratio ξ_c: almost unchanged with the increase of m_1; decreases with the increase of m_2; decreases with the increase of K_1; increases with the increase of K_2; increases first and then decreases with the increase of C_1, but the overall change is not obvious; decreases with the increase of v. The optimal damping ratio ξ_s: increases with the increase of m_1; decreases with the increase of m_2; decreases first and then increases with the increase of K_1; decreases first and then almost unchanged with the increase of K_2; decreases first and then increases with the increase of C_1; increases gradually with the increase of v, but the overall change is not obvious. The minimum damping ratio ξ_b: increases with the increase of m_1; increases with the increase of m_2; first increases with the increase of K_1 and then remains unchanged; decreases with the increase of K_2; almost unchanged with the increase of C_1; increases with the increase of v. The compromised damping ratio ξ_u: increases with the increase of m_1; decreases with the increase of m_2; decreases first and then increases with the increase of K_1; decreases first and then increases with the increase of K_2; almost remains unchanged with the increase of C_1; decreases with the increase of v, but the overall change is small. It can be seen that, among the system parameters, the car body mass m_2 and bogie frame mass m_1 have the greatest influence on the optimal damping ratio of the secondary suspension system, followed by the secondary suspension vertical stiffness K_2, the primary suspension vertical stiffness K_1, the vehicle running speed v, and the primary suspension vertical damping C_1. Therefore, when choosing the damping parameter of the secondary suspension system for railway vehicles, the influence of vehicle running speed should be taken into account in addition to the parameters of the vehicle itself, in which this phenomenon has not been found in previous studies.

4.3 Engineering Design Example

Taking the train shown in Table 1 as an example, the damping ratio of its secondary vertical suspension is designed by using the established analytical design method. The design results of the damping ratio at different vehicle running speeds as shown in Table 3. Here, the secondary suspension vertical limit stroke $[f_d] = 60$ mm.

Table 3. *Design results of the damping ratio*

Damping ratio	Design value			
	$v = 200$ km/h	$v = 250$ km/h	$v = 300$ km/h	$v = 350$ km/h
ξ_c	0.206	0.195	0.187	0.180
ξ_s	0.307	0.321	0.334	0.345
ξ_b	0.107	0.131	0.154	0.175
ξ_u	0.244	0.243	0.243	0.243
ξ_2	0.244	0.243	0.243	0.243

As can be seen from Table 3, the design values of the secondary suspension damping ratio corresponding to the CRH2's operation speed range (200 km/h to 350 km/h) are all around 0.24, which is basically the same. Therefore, in order to take into account the running quality of the vehicle at different running speeds, the damping ratio of the secondary suspension system can be chosen as $\xi_2 = 0.24$. It can be seen that the design result is close to the original vehicle design value 0.25; moreover, it is within the feasible design range (0.2 to 0.4) of the damping ratio of the secondary vertical suspension system given in literature [1], which indicates that the design value of the damping ratio obtained by this method is reliable.

5 CONCLUSIONS

1. According to the 1/4 railway vehicle model, using the Germany track vertical irregularity power spectral density as the input excitation of the vehicle system, the RMS values analytical formulae of the car body vertical acceleration, the secondary suspension vertical stroke, and the axle box vertical action force are derived, and the correctness of the analytical formulae is verified by the real vehicle test. The analytical formulae can more reasonably and accurately estimate the dynamic characteristics of the actual vehicle running on the track.
2. According to the analytical calculation formulae derived, an analytical design method of the optimal damping ratio for the secondary suspension system is proposed based on the multi-objective programming and single-objective interval constraint analysis, which can be used to find the best trade-off for conflicting performance indices such as ride comfort, running smoothness and running safety.
3. The influences of the system parameters on the optimal damping ratio are analysed. It can be seen

that, when choosing the secondary suspension damping parameter, the influence of the vehicle running speed should be taken into account in addition to the parameters of the vehicle itself. This study can provide an effective theoretical reference for the selection of the initial design value of the secondary suspension damping parameter for railway vehicles.

Note that this paper has deduced the analytical formulae of the vertical dynamic responses for railway vehicles and proposed an analytical design method of the damping parameter for its secondary suspension system. Although the research is based on the simplified model of the railway vehicle, it can help to have a qualitative understanding of the phenomena referring to suspension dynamics and enable designers to make reasonable engineering choices quickly and effectively. Moreover, it can greatly simplify the inconvenience of analysis and solution caused by many unknown parameters in the early stage of design.

6 ACKNOWLEDGEMENTS

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