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Future Primary Teachers' Knowledge about the Perimeter and Area of a Rectangle

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POVZETEK – Kakšna je kakovost matematičnega znanja naših učencev in naših študentov – bodočih učiteljev? Ali znajo to znanje uporabiti v realnih situacijah, znajo razložiti pojme in postopke ali pa samo vadijo in avtomatizirajo postopke brez razmišljanja? To so vprašanja, ki se pogosto zastavljajo v kontekstu poučevanja matematike in izobraževanja bodočih učiteljev. Zaradi nenadomestljive vloge osnovnošolskih učiteljev v procesu poučevanja in učenja matematike smo si zastavili sledeči cilj raziskave: preveriti kakovost znanja naših bodočih osnovnošolskih učiteljev o obsegu in ploščini pravokotnika. Učiteljevo znanje o matematičnih vsebinah mora biti predvsem konceptualno in ne proceduralno. Rezultati testiranja so pokazali, da je konceptualno znanje bodočih učiteljev šibkejše od proceduralnega. Zaradi pridobljenih rezultatov in vloge učbenika kot osnovnega učnega vira smo se odločili predstaviti tudi rezultate analize vrst nalog o obsegu in ploščini pravokotnika v učbenikih za osnovno šolo.

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ABSTRACT – What is the quality of the mathematics knowledge of our pupils and our students – future teachers? Do they know how to apply such knowledge in real situations? Can they explain the concepts and procedures or do they just practice and automate the procedures without reasoning? These questions are often asked in the context of teaching mathematics and the education of future teachers. Considering the irreplaceable role of elementary (primary) schoolteachers in the process of teaching and learning mathematics, we set the goal of the research: to check the quality of our future primary school teachers' knowledge of the perimeter and area of a rectangle. Teachers' mathematical content knowledge must primarily be conceptual knowledge rather than procedural knowledge. The test results showed that the conceptual knowledge of future teachers is weaker than the procedural knowledge. Due to the obtained results and the role of textbooks as a basic learning resource, we have decided to also present the results of an analysis of the types of tasks related to the perimeter and area of a rectangle in elementary school textbooks.

1 Introduction

Although we live in a world of rapid technological development, which implies the availability of a large amount of information, the role of the teacher is irreplaceable in education, especially in the lower grades of elementary school. Effective mathematics teaching requires appropriate teacher competencies (Jukić Matić et al., 2020). This includes quality lesson planning and preparing the individual to successfully solve everyday problems even after completing formal education.

There has been a continuing interest in understanding and describing the mathematical content knowledge (MCK) and pedagogical content knowledge (PCK) of primary

mathematics teachers. In this paper, we analyze future primary teachers' content knowledge about the area and perimeter of a rectangle. In addition, we will review the content of the mathematics textbooks for elementary school, which is related to our topic, because the textbooks are an important component in the learning process, including the learning of mathematics. "Although textbooks describe the minimal effort that teachers and students should undertake in the learning process, they have a significant role" (Pratama and Retnawati, 2018, p. 6).

Theoretical framework

Shulman (1987) proposed a foundation for describing the knowledge base for teaching. He described subject content knowledge as a central feature and as the "amount and organization of knowledge in the mind of the teacher" (p. 9), and described PCK as the blending of content and pedagogy. Teacher's PCK is needed to teach different mathematical topics, making them comprehensible to learners; it is also necessary for understanding student misconceptions, knowing how topics are organized and taught, as well as influencing the ability to adjust lessons catering for all learners (Shulman, 1987). Relying on the work of Shulman (1987; 1998), as well as their own research, Ball et al. proposed a framework for distinguishing the different types of knowledge required for teaching mathematics: *Domains of Mathematical Knowledge for Teaching* (Ball et al., 2008). Their framework consisted of two broad categories: subject matter knowledge and pedagogical knowledge.

In this paper, we take mathematical content knowledge to be knowledge about the subject matter in mathematics, knowledge about its structure, concepts, facts, skills and definitions, as well as methods of justification and proof. Teachers' mathematical content knowledge must primarily be conceptual knowledge rather than procedural knowledge. Lipovec et al. (2015) confirm in their research that conceptual knowledge proves to be an effective source for procedural knowledge, so they suggest that teachers focus on the development of concepts. Teachers need a rich and deep understanding of their subject in order to respond to all aspects of their pupils' needs. Only when the teacher understands something well enough, are they able to teach others. They need to overcome the various obstacles that might otherwise deny their pupils access to knowledge (Yeo, 2008). Pedagogical content knowledge cannot be developed without strong subject content knowledge. Only with firm subject content knowledge (in our case, mathematical content knowledge) and pedagogical content knowledge is the teacher able to plan teaching and learning activities, choose appropriate examples and exercises, and organize their pupils' work in the classroom. Teachers' firm mathematical content knowledge and pedagogical content knowledge enable high-quality teaching of mathematics and the achievement of the outcomes prescribed by the curriculum. Some past studies (Vekić Kljaić and Lučić, 2021) emphasize the importance of the teacher's professional development and their (self-)reflection (Petlak, 2021) as an integral part of the design of the school curriculum.

Literature review

Studies have shown that novice teachers often struggle to represent concepts in an understandable manner to their students because they have little or no PCK at their disposal (Kagan, 1992; Reynolds, 1992). Yeo's study (2008) showed that a number of

primary teachers with weak mathematical content knowledge were focused on rules and procedures related to calculating the area while teachers with strong content knowledge provided a conceptual explanation for each example and exercise. Concerns within primary teacher education are even more prevalent. Several studies indicated that many primary teachers have a poor conceptual understanding of area, relying on rules and formulas, and have difficulties in explaining why these formulas work (Baturo and Nason, 1996; Berenson et al., 1997; Menon, 1998; Reinke, 1997). Baturo and Nason (1996) found that some primary teachers had poor knowledge of area, providing responses which were incorrect and rule-dominated. Berenson et al. (1997) in an international study of primary teachers' understanding of area required primary teachers to design a lesson plan introducing the concept of area to middle school students. The findings showed that many primary teachers had a primarily procedural knowledge of area, which was reflected in their procedural and formula-dominated lesson plans. In today's era of computerization and technology, "a conceptual approach to mathematics is more essential than a procedural approach" as the technical aspects of mathematical work are largely performed by computing technologies (Bergsten et al., 2016, p. 550). "Numerical and computational skills have become less important in mathematics education, and meta-mathematical, communicative skills are seen as more important, embodying what is thought of as "real" mathematical knowledge" (Österman and Bråting, 2019, p. 467).

Menon (1998) investigated 54 primary teachers' understanding of area and perimeter and found that they have a procedural understanding of area and perimeter rather than a conceptual and relational understanding. Tatto et al. (2012) found that, although primary teachers "were generally able to determine the areas and perimeters of simple figures" (p. 136), they "were likely to have more difficulty answering problems requiring more complex reasoning in applied or non-routine situations" (p. 137).

Sáenz (2009) tested 140 pre-service elementary teachers with various PISA tasks and observed difficulties in the conceptual knowledge of the respondents, which are related to a poor understanding of the concept of perimeter. He found that the "lack of conceptual profundity in understanding mathematical objects means that the only tools they used to tackle the mathematical tasks were elemental algorithms and stereotypical rules with a minimum of underlying conceptual knowledge" (p. 136).

On the other hand, research was done on the contents of textbooks with regard to tasks that require conceptual and/or procedural knowledge. The number of tasks that require a conceptual understanding is low compared to those that are solved by procedures (algorithms) (Ozer and Sezer, 2014). An analysis of Brazilian, Japanese and U.S. textbooks showed that Brazilian and U.S. textbooks contained a procedure-based approach, while Japanese textbooks highlighted conceptual understanding, which could be an essential differential to explain the Japanese results in PISA (Souza and Powell, 2021).

2 Methodology

As stated in the theoretical part of the paper, teachers' subject content knowledge and textbooks are essential factors in teaching and learning mathematics. The aim of the research is to determine future primary teachers' content knowledge of the concepts

of perimeter and area of a rectangle (or a square as a special type of rectangle) in terms of their conceptual and procedural knowledge. In addition, we analyzed elementary school mathematics textbooks to check if they include tasks related to the perimeter and area of rectangles that require conceptual knowledge, compared to those that require procedural knowledge.

The research included the following components: selecting the prescribed learning outcomes related to the concepts of perimeter and area of a rectangle; creating tasks for our respondents (future teachers) based on the outcomes; and determining the number of tasks about the perimeter and area of a rectangle in elementary school textbooks according to the type of knowledge required, i.e., conceptual and procedural knowledge. The research was conducted during the winter semester of the 2020/2021 academic year and involved 40 students in the Department of Teacher Education at the Faculty of Humanities and Social Sciences in Split, University of Split, i.e., future primary school teachers. The research instrument consisted of four short tests that included various arithmetic and geometry problems. Each test contained one problem related to the perimeter or area of a rectangle (or a square as a special type of rectangle). Those problems are presented in Table 1.

Table 1

Problems Involving the Perimeter and Area of a Rectangle

Problem	*Task*
Problem 1	Let one side of the rectangle be 10 cm long and the other 2 dm. Calculate the perimeter of that rectangle.
Problem 2	If we have a board 30 cm long, what are all the dimensional possibilities for making a wooden picture frame of a rectangular shape by cutting the board, if we know that the dimensions are given by natural numbers in cm? We do not want the frame to be narrower than 5 cm.
Problem 3	Calculate the area of a square with a side length of 5 cm. Calculate the area of a rectangle with side lengths of 2 dm and 15 mm.
Problem 4	On a bathroom floor of a rectangular shape, measuring 2 m × 1.5 m, we must place square tiles measuring 30 cm × 30 cm. How many tile pallets do we need if one pallet contains 20 tile pieces?

Problems 1 and 3 are given in the mathematical context and questions are explicitly stated. To solve those problems, students need to know the definition of the perimeter and area of a rectangle (or a square as a special type of rectangle). Moreover, it is possible to solve those tasks knowing only formulas for the perimeter and area of a rectangle and a square without a deeper understanding of the concepts of perimeter and area.

Therefore, Problems 1 and 3 are related to procedural knowledge about the perimeter and area of a rectangle (or a square). On the other hand, Problems 2 and 4 are given in a nonmathematical context. To solve those problems, students have to recognize the "hidden" concepts in their formulations. In Problem 2, students have to connect the frame of the picture with the perimeter of the rectangle. Problem 4 requires an identification of the given elements: the shape of the floor, its measure as a rectangle and

its area; the shape of the tile, its measure as a square and its area. This problem also requires knowledge of quotative division.

In parallel, mathematics textbooks for middle school were analyzed to check if there are problems involving the area and perimeter of rectangles that encourage conceptual and/or procedural knowledge.

3 Results and discussion

In the following passages, we will present the educational outcomes prescribed by the Croatian curriculum for the subject of mathematics, related to the perimeter and area of a rectangle (Ministarstvo znanosti i obrazovanja, 2019).

In the third grade, the student is expected to determine the perimeter of geometric figures (p. 41). The student draws a rectangle and a square of a certain side length, then estimates and calculates the perimeter. In the fourth grade, the concept of the area of squares and rectangles is introduced. The fourth-grade student gets to know the standard measures for the area and then measures the area of rectangular figures by covering the area with a unit square. At the end of the fourth grade, the student is expected *to compare the areas of figures and measure them with unit squares* (p. 50). In the fifth grade, the knowledge related to the calculation and application of the perimeter and area of squares and rectangles is expanded. In the recommendations for achieving the learning outcomes, teachers are suggested to use dynamic geometry programs and other appropriate and available interactive computer programs and tools, as well as educational games (p. 59), to evaluate and solve tasks with content related to the students' environment and to encourage them to create drawings composed of geometric figures and calculate their perimeters and areas (p. 62). In the seventh grade, the student is expected to justify the choice of strategy in a problem situation when calculating volume and area (p. 83). Therefore, the focus is not on evaluating calculation techniques, but on the student's logical thinking and ability to analyze problems. Assigning problem situations for perimeter and area calculation that concern problems from real life and encouraging students to find problem situations by themselves is recommended (p. 83). In the eighth grade, problems related to the square and rectangle are included when applying the Pythagorean theorem.

Given that the learning outcomes are directed towards the application and argumentation of the selection of strategies when calculating the area, the future teachers were assigned tasks that are described in the previous section. The percentages of solving each of the four tasks are presented in Table 2.

As shown in Table 2, Problems 1 and 3, which are given in a mathematical context and are related to procedural knowledge, have a 78.57 % and 73 % problem-solving success rate, respectively. On the other hand, Problems 2 and 4 are related to conceptual knowledge about the area and perimeter of a rectangle. Those problems are given in a nonmathematical context. They are related to real situations where the concepts of area and perimeter should be recognized and applied. Only 21.43 % and 35 % of respondents, respectively, solved these problems successfully.

Table 2

Percentage of Solving a Particular Problem

Problem	*Percentage of solving*
Problem 1	78.57%
Problem 2	21.43%
Problem 3	73%
Problem 4	35%

We will also comment on the most common errors. Among the surveyed future primary teachers, every second respondent did not even try to solve Problem 2; consequently, 64% of respondents did not successfully solve the problem. In Problem 1, as well as in Problem 4, the most common errors are related to the conversion of the units of measurement. In Problem 3, in addition to the error of calculating the perimeter instead of the area, 19% of students, future primary teachers, who did not successfully solve the task, were on the right track in terms of calculating the bathroom floor area and tile area, but they did not know how to connect those facts to get the number of required tiles and consequently calculate how many pallets to buy.

Furthermore, a t-test was used to determine the significance of the differences in successful solutions related to problem pairs: Problem 1 and Problem 2 that are related to the concept of the perimeter, and Problem 3 and Problem 4 that deal with the concept of the area. The results can be seen in Table 3.

Table 3

T-test Results

Problem pairs	*t-test results*
Problem 1 – Problem 2	$p = 0.00111$
Problem 3 – Problem 4	$p = 0.00024$

Statistically significant differences were detected in both pairs ($p < 0.05$): between Problems 1 and 2, and Problems 3 and 4. Thus, the success of students in solving procedural problems on the perimeter of a rectangle is statistically significantly better than their success in solving conceptual problems on the perimeter of a rectangle. The same is true for the concept of the area of a square and a rectangle: we see that the difference is statistically significant in favor of procedural tasks.

The above-mentioned results are certainly not satisfactory, as we could have predicted based on the reviewed literature and previous research (Isleyen and Isik, 2003; Khashan, 2014; Lauritzen, 2012). One of the possible causes of poor students' performance may be the low presence of conceptual tasks in elementary school mathematics textbooks; therefore, we reviewed mathematics textbooks for elementary school from two randomly selected Croatian publishers. The following table shows the results of an

overview of the number of conceptual and procedural tasks in which the perimeter and/or area of a rectangle (or a square as a special type of rectangle) are required.

Table 4

Number of Procedural and Conceptual Tasks about Perimeter and Area in Textbooks

Grade	*Conceptual tasks*		*Procedural tasks*	
	Publisher 1	*Publisher 2*	*Publisher 1*	*Publisher 2*
3	1	1	13	4
4	5	1	34	4
5	3	7	27	57
6	4	8	20	59
7	3	1	10	20
8	11	9	36	39

Our goal is not to compare textbooks from different publishers, but to observe the representation of the types of tasks according to the knowledge needed to solve them, i.e., conceptual and procedural knowledge. Below, we will also comment on the context of the reviewed tasks. In the third grade, conceptual knowledge of perimeter is required in tasks in which students are asked to calculate the length of yard fences, of a tablecloth lace border, a window rubber band, etc. Procedural tasks are those that ask students to draw, measure or calculate the perimeter. In the fourth grade, the area of a rectangle is studied. Students' conceptual knowledge is applied in tasks in which the calculation of the area of a rectangle is not explicitly required, but the tasks include calculating the area of a playground, vineyard, bathroom, tiles, etc. Assignments requiring procedural knowledge ask students to measure the lengths of the sides of a rectangle and then calculate its area, or calculate the area of the rectangle if its side lengths are given. In the math textbooks for middle school (from the fifth to the eighth grade) of Publisher 2, tasks that require the application of conceptual knowledge are included in the part of the lesson called "from the world of work" or "connect and apply". In the fifth-grade and sixth-grade textbooks, such tasks ask students to calculate the area and perimeter of a playground, field, room, garden, etc. For example: "How many pieces of an 0.4 m long corner lath should be bought to border a room of the given dimensions?". Some of those tasks are given only with text, and some contain an image of the object with the corresponding dimensions. The same types of assignments appear in the fifth-grade and sixth-grade textbooks of both publishers. Tasks that require procedural knowledge about the perimeter and area of a rectangle are prevailing in the fifth-grade and sixth-grade textbooks of both publishers. In those tasks, the lengths of the sides of a rectangle are given and students are asked to calculate its perimeter and area. In seventh-grade textbooks, tasks that require the application of conceptual knowledge about the perimeter and area of a rectangle include ratios, percentage and cost. In eighth-grade textbooks, students are asked to apply the Pythagorean theorem when calculating the area,

perimeter or diagonal of a TV screen, monitor, playground or pool, or the distance of a ship from a harbor, etc.

The following table shows the representation of conceptual tasks in relation to procedural ones in the analyzed textbooks.

Table 5

Representation of Conceptual Tasks

Grade	*Conceptual knowledge required to solve task*	
	Publisher 1	*Publisher 2*
3	7%	20%
4	13%	20%
5	10%	11%
6	17%	14%
7	23%	5%
8	23%	19%

The low presence of tasks that promote conceptual knowledge is noticeable in the analyzed textbooks. As teachers are often guided by textbooks when planning their lessons, they teach and demonstrate mathematical content using tasks from the textbook. Usually, students practice math using the textbook. "Giving excessive importance to the algorithm over the underlying concept is a particularly serious problem among those who are to be future teachers" (Sáenz, 2009, p. 135). Therefore, the question arises whether the low presence of this type of tasks is one of the causes of the poor performance of students in mathematics in general, but also of the poor performance of students in solving conceptual tasks compared with the success of students in solving procedural tasks.

4 Conclusion

Whilst learners of mathematics need only to be aware that they have gained sufficient knowledge to pass examinations (*learner-knowledge*), teachers need to have multiple and fluid conceptions of that knowledge to know how progress might be made through that knowledge (*teacher-knowledge*).

"Conceptual and procedural knowledge, as basic aspects of mathematical competencies, have been developing through informal, but primarily through formal experience and education. Therefore, it is important to encourage these two forms of knowledge in students through appropriate approaches, so that they could become adaptive experts, without fear of mathematics and with an image of themselves as competent mathematicians" (Putarek, 2018, p. 468).

Primary teachers with weak content knowledge, both conceptual and procedural, and those who are insecure about their mathematical content knowledge are predisposed to telling pupils rules and explaining procedures. "Teacher candidates need to possess a conceptual understanding of a problem regardless of whether it can be solved by applying a well-defined algorithm or the computational capabilities of a digital tool" (Abramovich, 2015, p. 48). On the other hand, teachers with strong mathematical content knowledge should have a solid background to develop pedagogical content knowledge, thus providing higher-quality knowledge to their pupils.

Our results show that future primary teachers need to work more on conceptual knowledge, which implies a greater verbalization of problem-solving procedures and its application in a real-world context. Contextual knowledge is "a very important tool for solving the most difficult tasks: those that demand something more than a simple reproduction of algorithms or formulas" (Sáenz, 2009, p. 135).

In addition, future teachers should master the methodical procedures of determining the perimeter as the sum of the length of all sides of a geometric figure, and the area as a measure of the figure in the plane. For perimeter, these procedures involve adding the lengths of rectangles, measuring their lengths, and finally summing the lengths of the sides. For area, these procedures include observing the unit of measurement for the area – the unit square, and ordering the unit squares on the surface of the rectangle, from which the formula for calculating the area is derived. If the knowledge of students (future teachers) remains at the procedural level, then there is a danger that their teaching will be focused on procedures and formulas, which will not lead to achieving the desired student learning outcomes.

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Znanje bodočih osnovnošolskih učiteljev o obsegu in ploščini pravokotnika

Učiteljski poklic je verjetno najstarejši poklic, katerega temelji so bili postavljeni že pred nastankom šole kot institucije. Družbene spremembe, zahteve sodobne družbe in globalizacija so spremenile naravo učiteljevega dela, ne pa tudi njegovega vpliva in odgovornosti za kakovost izobraževalnega sistema. Kakšna je kakovost matematičnega znanja naših učencev in naših študentov – bodočih učiteljev? Ali znajo to znanje uporabiti v dejanskih situacijah v realnem okolju? Ali lahko pojasnijo pojme in postopke ali samo vadijo in avtomatizirajo postopke brez razlage? To so vprašanja, ki se pogosto postavljajo v kontekstu pouka matematike in izobraževanja bodočih učiteljev. Aktualnost teh vprašanj nikoli ne zamre. V pričujočem članku pojem matematično znanje bodočih učiteljev razumemo kot znanje o matematičnih konceptih – poznavanje strukturiranja matematičnih konceptov in sposobnost argumentiranja vzročno-posledičnih zvez med njimi. Zato matematična znanja bodočih učiteljev vključujejo (fleksibilno) obvladovanje matematičnih dejstev: od definiranja matematičnih izrazov, navajanja in dokazovanja izrekov do uporabe matematičnih izrazov in pravil pri reševanju problemov (v okviru šolske matematike). Matematično znanje bodočih učiteljev bomo spremljali skozi priz-

mo klasifikacije na konceptualno in proceduralno znanje. Konceptualno znanje omogoča razumevanje pojmov, idej in zakonitosti, ki določajo neko območje teh odnosov med njimi. Proceduralno znanje, ki se nanaša na posedovanje veščin, poznavanje algoritmov in strategij reševanja nalog, omogoča hitro in učinkovito reševanje problemov. Znanje učitelja o matematični vsebini mora biti predvsem konceptualno in ne proceduralno. Lipovec idr. (2015) v svoji raziskavi potrdijo, da se konceptualno znanje izkaže kot učinkovit vir za proceduralno znanje, zato učiteljem predlagajo usmeritev pozornosti v razvoj konceptov. Na strani učiteljev je pomembno bogato in globoko razumevanje predmeta, da bi lahko odgovorili na vse vidike potreb svojih učencev. Šele takrat, ko učitelj snov dovolj dobro razume, lahko poučuje druge. Učiteljevo dobro poznavanje vsebin, ki jih poučuje, ter samozavest in samozavest pri prenašanju tega znanja na učence pozitivno vplivajo na uspeh učencev. Učiteljevo solidno matematično znanje je predpogoj za izgradnjo matematičnega znanja za poučevanje, kar vključuje sposobnost analiziranja učenčevega razmišljanja, ki je privedlo do napačnega odgovora, prepoznavanje, česa učenec ne razume, in odločitev, kako najbolje predstaviti matematično idejo, tako da učenci predstavljeno lahko razumejo.

Ob preučevanju številnih študij smo opazili problem slabega usvajanja geometrijskih pojmov in njihovih povezav. Ob upoštevanju omenjenega smo se odločili za pojma obseg in površina, ker smo na osnovi izkušenj in dela s študenti – bodočimi učitelji na fakulteti in učenci v šolah opazili napačna poimenovanja navedenih pojmov na splošno in neznanje ter negotovost pri geometriji precej bolj kot na področju aritmetike. Z dolgoletnim spremljanjem in analizo pouka matematike v osnovni šoli smo opazili, da je število ur geometrije manjše v primerjavi z drugimi področji matematike. Poleg tega smo ugotovili, da je z namenom realizacije vsebine o obsegu in površini poudarek največkrat na reševanju večjega števila nalog, v katerih je potrebno samo izračunati obseg in površino likov (ali dolžino stranice lika) za prenos danih podatkov v pripravljeno formulo.

Na podlagi pregledane literature smo ugotovili skladnost naših opažanj iz prakse z rezultati predhodnih raziskav o poznavanju pojmov obseg in ploščina pravokotnika (ter kvadrata kot vrste pravokotnika) pri bodočih učiteljev. Številne raziskave (Baturo in Nason, 1996; Berenson idr., 1997; Menon, 1998; Reinke, 1997; Yeo, 2008; Tatto idr., 2012; Sáenz, 2009) namreč kažejo, da pri učiteljih matematike prevladuje proceduralno znanje v primerjavi s konceptualnim. Vendar se v dobi informatizacije in sodobne tehnologije zdi, da je konceptualni pristop k matematični vsebini pomembnejši od proceduralnega. Glede na to, da se učitelji pri pripravi in izvajanju pouka opirajo na učbenike, potekajo tudi raziskave strukture nalog v učbenikih. Ena od nedavnih raziskav poudarja, da japonski učbeniki matematike spodbujajo konceptualni pristop, s katerim lahko razložimo uspeh japonskih učencev na testu PISA (Souza in Powell, 2021).

Cilj naše raziskave je ugotoviti znanje bodočih učiteljev o pojmih obseg in ploščina pravokotnika v luči pojmovnega in proceduralnega znanja. Prav tako smo želeli preveriti zastopanost nalog, povezanih z obsegom in ploščino pravokotnika, ki zahtevajo konceptualno znanje, v primerjavi s tistimi, ki zahtevajo proceduralno znanje, v učbenikih matematike za osnovno šolo.

Raziskava je obsegala naslednje sklope: izluščiti rezultate iz učnega načrta, povezane s pojmoma obseg in ploščina pravokotnika, in na njihovi podlagi izdelati naloge za študente, bodoče učitelje, ter določiti število nalog o obsegu in ploščini pravokotnika v učbenikih matematike, potrebnih za osnovno raven znanja: tako konceptualnega kot pro-

ceduralnega. Raziskava je bila izvedena v zimskem semestru študijskega leta 2020/2021 in je zajela 40 študentov, bodočih učiteljev. Raziskovalni instrument so sestavljali štirje kratki testi, sestavljeni iz različnih aritmetičnih in geometrijskih nalog. Vsak test je vseboval eno nalogo, ki se je nanašala na obseg ali ploščino pravokotnika (v posebnem primeru kvadrata). Proceduralno znanje je bilo potrebno za rešitev dveh nalog, ki sta bili podani v matematičnem kontekstu, medtem ko je bilo potrebno konceptualno znanje za rešitev preostalih dveh nalog, ki sta bili podani v nematematičnem kontekstu.

Analiza rezultatov je pokazala statistično značilne razlike med tipi nalog. Tako je uspeh učencev pri reševanju proceduralnih nalog, povezanih s pojmom obseg pravokotnika, statistično značilno boljši od uspeha pri reševanju konceptualnih nalog. Enako velja za naloge o površini kvadratov in pravokotnikov; razlika v uspešnosti je statistično pomembna v korist proceduralnih nalog. Glede na to, da so učni izidi o obsegu in površini v osnovni šoli (Ministarstvo znanosti i obrazovanja, 2019) usmerjeni v aplikacijo in argumentacijo teh konceptov, pridobljeni rezultati raziskave niso spodbudni. Rezultati upravičeno odpirajo vprašanje, kako lahko učitelji, ki nimajo zadostnega pojmovnega znanja o določenih matematičnih vsebinah, usmerjajo učence k doseganju načrtovanih učnih rezultatov.

Eden izmed možnih vzrokov slabega uspeha učencev in posledično tudi študentov – bodočih učiteljev bi lahko bila slaba zastopanost konceptualnih nalog v osnovnošolskih učbenikih matematike. Zato smo analizirali vsebine, vezane na obseg in površino, v učbenikih matematike za osnovno šolo dveh naključno izbranih hrvaških založnikov. Prešteli smo naloge glede na to, ali spodbujajo proceduralno ali konceptualno znanje, in navedli nekaj primerov iz učbenika. V analiziranih učbenikih je ugotovljena slaba zastopanost nalog, ki spodbujajo konceptualno znanje, v primerjavi s tistimi, ki iščejo proceduralno, in se giblje od 7 % do 23 %, odvisno od razreda in založnika. V učbenikih so pojmovna znanja potrebna pri nalogah o obsegu pravokotnika (kvadrata), kot so npr. računanje dolžine dvoriščne ograje, obrobljanje prta s čipko, okna z gumico in podobno, pojavijo se naloge za izračun površine parkirišča, vinograda, kopalnice, ploščic itd. Proceduralne so naloge tipa nariši, izmeri, izračunaj prostornino in/ali površino ali izračunaj površino pri že podanih dolžinah strani. Učitelji največkrat pri načrtovanju pouka pogosto sledijo učbenikom, saj matematične vsebine poučujejo in demonstrirajo s pomočjo nalog iz učbenika. Običajno učenci vadijo matematiko s pomočjo učbenika. Zato se zastavljata vprašanji, ali je slaba zastopanost te vrste nalog eden od vzrokov slabega uspeha učencev pri matematiki na splošno in ali je uspeh učencev pri reševanju konceptualnih nalog slab v primerjavi z uspehom učencev pri reševanju proceduralnih nalog. Navedeni tipi matematičnega znanja se razvijajo skozi neformalno ali predvsem formalno znanje in izobraževanje. Naši rezultati kažejo, da morajo učitelji primarnega poučevanja bolj delati na konceptualnem znanju, kar vključuje večjo verbalizacijo postopka reševanja problema ter njegovo uporabo v stvarnem kontekstu. Če znanje študentov (bodočih učiteljev) ostane na proceduralni ravni, obstaja nevarnost, da bo njihov pouk usmerjen na postopke in formule, kar ne bo privedlo do doseganja želenih rezultatov učenja učencev. Ko k naštetemu dodamo še učbenike, ki spodbujajo proceduralne naloge, postane doseganje želenih učnih rezultatov še težje in negotovo. Bogato znanje o matematičnih in pedagoških vsebinah učitelju omogoča kakovostno izvajanje pouka matematike in doseganje rezultatov predpisanega učnega načrta in programa. Dober učitelj uporablja učbenik kot pomožno in ne kot glavno

didaktično sredstvo. Učitelji z razvitim konceptualnim in proceduralnim znanjem lahko kritično pregledajo učbenik in poiščejo načine za odpravo morebitnih pomanjkljivosti z vidika spodbujanja določene vrste znanja.

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