

## Identifikacija dinamičnih lastnosti sestavljenih nosilnih strojnih delov

The Dynamic Properties of Composite Supporting Parts for Machines

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Cilj raziskave je bila definicija metode za primerjalno merjenje dinamičnih lastnosti lepljenih stenskih elementov (sestavljenih plošč), ki so namenjeni gradnji nosilnih delov strojev in naprav. Metoda omogoča razmeroma hitro in zanesljivo primerjanje dinamičnih lastnosti, točneje amplitud, frekvenc in dušenja modalnih načinov sestavljenih plošč. Vstopni signal pri določanju frekvenčnega odziva sestavljenih plošč je pomenil mehanski impuls, merjen z merilnikom pospeška, izstopni signal pa je bil rezultirajoči zvočni tlak na razdalji 1 m od preizkušane plošče. Uporabnost metode je ocena (i) izmer ter materiala sten, (ii) usmerjenosti jedra in (iii) vrste polnila pri izbiri optimalne strukture sestavljenih plošč za dano situacijo.

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(Ključne besede: plošče sestavljeni, lastnosti dinamične, ugotavljanje lastnosti, metode merilne)

Our aim was to define a method for comparing the measurements of the dynamic properties of glued wall elements (composite plates), intended for constructing the supporting parts of machine tools and devices. The method enables a relatively fast and reliable comparison of the dynamic properties: more precisely of the amplitudes, the frequencies and the damping of the corresponding modes of the composite plates. In the frequency response function (FRF) measurements the input signal was a mechanical impulse, measured by an accelerometer; and the output signal was the resulting sound pressure at a distance of 1 m from the specimen. The applicability of the method is an estimation of (i) the dimensions and the material of the walls, (ii) the core orientation, and (iii) the type of filling used in the optimum construction of composite plates for a given situation.

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(Keywords: composite plates, dynamic properties, identification of properties, measurement methods)

### 0 UVOD

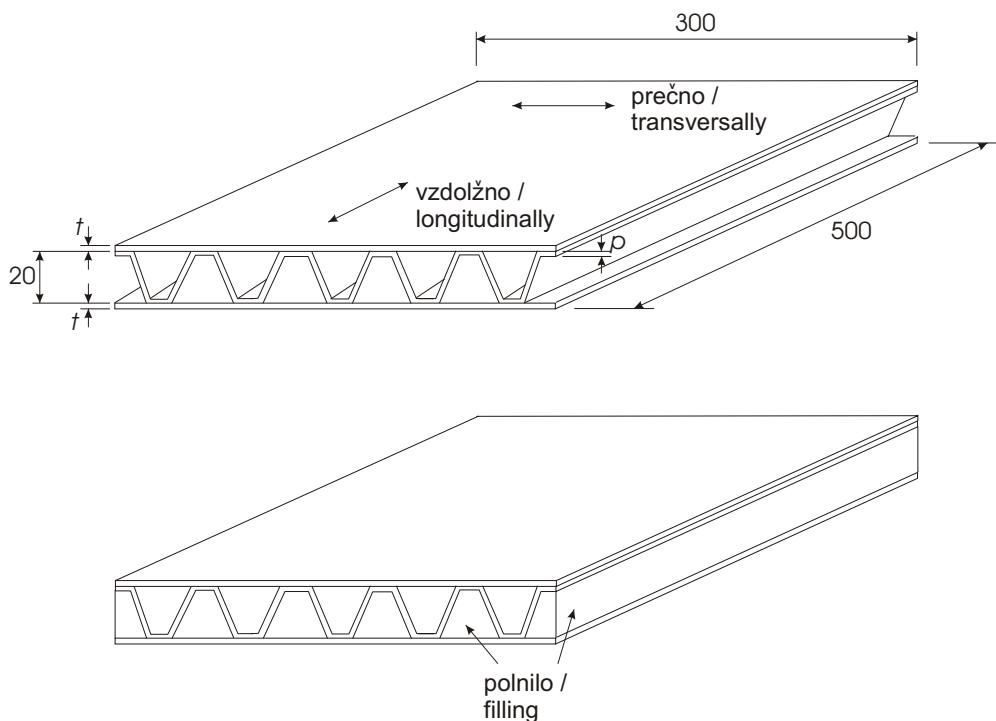
Povečanje natančnosti izdelave in krajšanje izdelovalnih krogov narekuje nenehen razvoj posameznih sklopov obdelovalnih strojev in naprav ([1] in [2]). Izboljšani rezalni materiali in spremenjeni pogonski sistemi zagotavljajo vse večje obdelovalne hitrosti. Tej usmeritvi morajo slediti tudi podajalne in strežne naprave ([3] in [4]). Vsi dinamični sklopi strojev in naprav so vgrajeni v nosilnih delih, ki prevzemajo statične in dinamične obremenitve zaradi obdelovalnega postopka [5]. Običajni materiali s svojimi primarnimi lastnostmi pomenijo predvsem na področju dinamičnega odziva določeno omejitve. Izhajajoč iz teh ugovovitev, se za najrazličnejše dele strojev in naprav vse več uporabljo različni drugačni materiali ([6] do [8]). Izboljšanje dinamičnih lastnosti pomeni zagotavljanje dinamične togosti in dušenja, ki v končni fazi pomenita pozitiven vpliv na natančnost obdelovanca.

### 0 INTRODUCTION

Both an increase in the accuracy and a reduction in the times needed for manufacturing processes dictate the development of the individual parts of machine tools and devices ([1] and [2]). Improvements on cutting tools and advances in machine-tool drives mean higher machining speeds. This is a trend that must be followed by feeding and manipulating devices ([3] and [4]). All the dynamic joints of machine tools and devices are built into their supporting parts, which statically and dynamically sustain the resulting loading that results from the machining process [5]. Due to their inherent properties many conventional materials are limited in terms of their dynamic response. Therefore, a variety of alternative materials for machine parts are taking their place ([6] to [8]). Improving the dynamic properties means ensuring the dynamic stiffness and damping, both of which can increase the accuracy of the workpiece.

Skladno s tem so bili tudi na Fakulteti za strojništvo v Ljubljani opravljeni razvojnорaziskovalni koraki na področju sestavljenih materialov in njihove uporabnosti za gradnjo nosilnih delov ([9] do [11]). Za identifikacijo dinamičnih lastnosti preskušancev smo uporabili sestavljene plošče [9] v različicah, katerih osnova je shematsko prikazana na sliki 1, njihove značilnosti pa v preglednici 1. Sestavljene plošče z debelino jedra 20 mm in ploskovno razsežnostjo 300 x 500 mm so bile izdelane iz jeklene in aluminijaste pločevine z debelino 1 in 2 mm. Jedro, ki predstavlja vmesni del (zlepil s stenama), je trapezno strukturirana pločevina, strmina trapeznih stranic jedra pa je  $60^\circ$ . Glede na največjo izmero elementa (500 mm) je bilo jedro vzdolžno ali prečno usmerjeno (sl. 1). Spodnji del slike 1 prikazuje primer, ko je med stenama oziroma jedrom sestavljene plošče polnilo (poliestrski beton, poliuretansko polnilo).

At the Faculty of Mechanical Engineering in Ljubljana we investigated some composite materials for use as supporting parts [9] to [11]. For the identification of the dynamic properties we used composite plates [9] of different shapes. These plates are shown schematically in Figure 1, and their characteristics are presented in Table 1. The composite plates, with a core thickness of 20 mm and surface dimensions of 300 x 500 mm, were made of steel and aluminium plates with thicknesses of 1 and 2 millimetres. The core, which is glued to both flatsheets, is a metal plate with the profile of a trapezoid and  $60^\circ$  side angles. The longest dimension of the element (500 mm) was oriented longitudinally or transversally (see Figure 1). The lower part of Figure 1 shows the composite with a filling (polyester concrete, polyurethane) between the flatsheets and the core.



Sl. 1. Analizirane sestavljene plošče  
Fig. 1. Analyzed composite plates

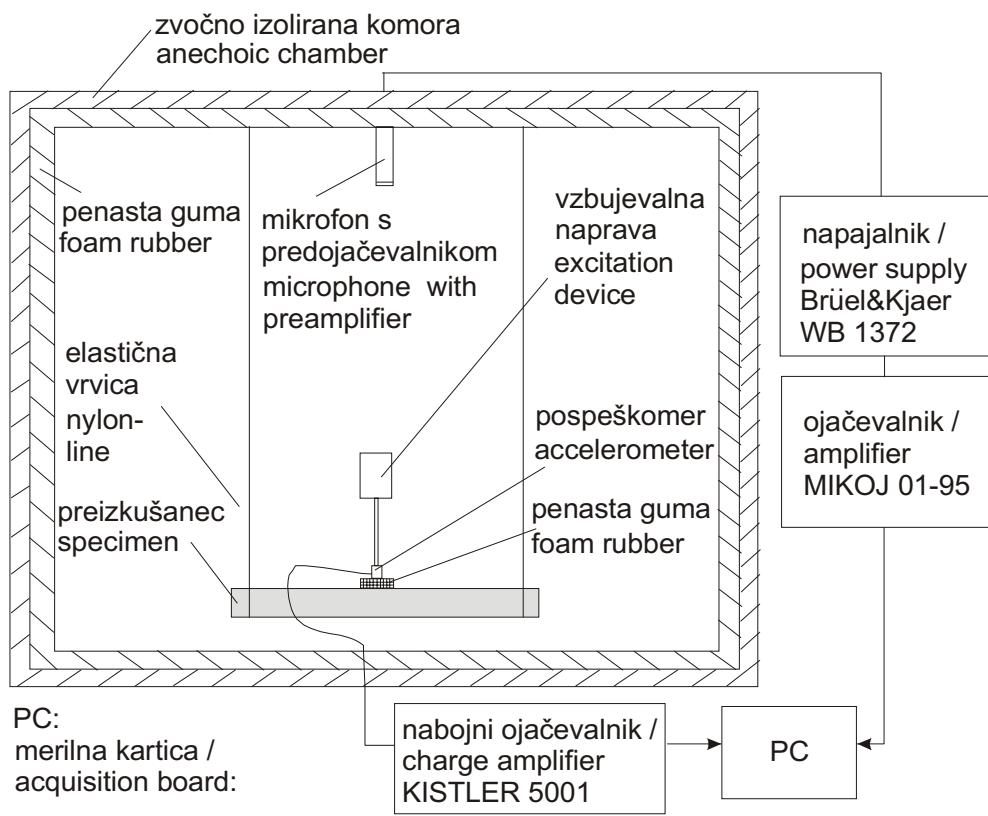
Preglednica 1. Značilnosti analiziranih sestavljenih plošč

Table 1. Characteristics of the analyzed composite plates

oznaka label	$t$ mm	$p$ mm	usmeritev jedra core orientation	polnilo filling
JE1V	1	1	vzdolžna / longitudinal	/
JE1P	1	1	prečna / transversal	/
JE2V	2	2	vzdolžna / longitudinal	/
JE2P	2	2	prečna / transversal	/
JE1VBE	1	1	vzdolžna / longitudinal	poliestrski beton polyester concrete
JE1VPO	1	1	vzdolžna / longitudinal	poliuretansko polnilo polyurethane

Eno od izhodišč gradnje sestavljenih plošč je bilo uporabiti čim manj materiala in s primerno strukturo dobiti optimalne nosilne lastnosti [12]. Pri tem smo uporabili osnovna pravila za t.i. lahko gradnjo nosilnih delov ([13] in [14]). Elementarne dele smo lepili v celoto, pri čemer so bili pri tehnologiji lepljenja upoštevani veljavni standardi [15]. Pred časom opravljeno dinamično testiranje lastnosti istih sestavljenih plošč [9] je bilo opravljeno z običajno metodo, opisano v literaturi [16]. Na kratko, frekvenčni odziv lepljenih plošč je bil dobljen kot razmerje med rezultirajočim pospeškom (hitrostjo) in sinusno vzbujevalno silo na njihovi geometrični sredini. K eksperimentalno dobljenim frekvenčnim odzivom so bile sintetizirane ustrezenne funkcije in iz njih ovrednotene dinamične značilnosti posameznih struktur [16]. Metoda je, tako glede potrebnih eksperimentalnih naprav kakor tudi po samem izvajanju testiranj, zahtevna. Z namenom, da bi zagotovili preprostješo, a še uporabno metodo, ki bi zagotovila vsaj primerjalno določanje dinamičnih lastnosti sestavljenih plošč, smo tokrat uporabili meritve frekvenčnega odziva z vstopnim mehanskim impulzom, izstopni signal pa je bil zvočni tlak na razdalji 1 m od plošče [17].

One of the construction targets for the composite was to use the minimum amount of material in order to achieve the optimum support characteristics [12]. This was achieved with the use of the so-called light construction of supporting parts ([13] and [14]). The elementary parts were glued into an assembly, where the basic rules of gluing technology were considered [15]. Recently performed dynamic tests on the same composite plates [9] were performed using a classical method described in the literature [16]. Briefly, a frequency response function (FRF) of the glued plates was obtained as a ratio of the resulting acceleration (velocity) to the sinusoidal input force at their geometrical centre. The corresponding functions were synthesised to the measured FRFs and from these functions the corresponding dynamic characteristics of the individual specimens were calculated [16]. In terms of experimental equipment and measurement procedure this method is complex. In order to achieve a simple and useful method for comparative measurements of the dynamic characteristics we used FRF measurements with mechanical excitation and the resulting sound pressure at 1 m from the specimen [17].



PC:  
merilna kartica /  
acquisition board:

National Instruments AT-A2150C (software LabWIEV)

mikrofon / microphone: Brüel&Kjaer (tip / type 4188)  
predojačevalnik / preamplifier: Brüel&Kjaer (tip / type 2671)  
pospeškomer / accelerometer: Brüel&Kjaer (tip / type 4374)

Sl. 2. Merilna oprema

Fig. 2. Measurement equipment

## 1 METODA IN REZULTATI

### 1.1 Merilna oprema in potek meritev

Merilna oprema, ki je bila uporabljena pri določanju relativne dinamične togosti preskušancev, je prikazana na sliki 2 [17]. Notranja stena komore je bila obložena s piramidno profilirano penasto gumo, ki je preprečevala odboje zvoka vzbujenega preskušanca. Preskušanci so bili med meritvami vpeti na dveh elastičnih vrvicah, ki sta v danih razmerah zagotavljali optimalno vpetje pri merjenju njihovega frekvenčnega odziva [16].

Mehansko vzbujanje preskušancev (sestavljenih plošč) je bilo izvedeno s tuljavo, podaljšanim jedrom in na njegovem koncu pritrjenim merilnikom pospeška, kakor je prikazano na sliki 3. Tu je prav tako prikazano mesto vzbujanja na ploščah ter tipična oblika impulznega signala pri vzbujanju plošč, kar je bilo doseženo z majhnim kosom penaste gume med merilnikom pospeška in preskušancem. Mesto vzbujanja na geometrični sredini sestavljenih plošč je bilo izbrano na podlagi predhodnih testov modalnih oblik [9]. Zaradi simetrične oblike preskušancev je namreč to mesto, vsaj za nekaj prvih modalnih načinov, kraj relativno velikih amplitud, torej relativno velike zastopanosti modalnih načinov. Po drugi strani pa kraj vzbujanja po načrtih ni imel bistvenega vpliva na potek meritev, saj je bil njihov končni rezultat razmerje med izstopnim in vstopnim signalom, ne pa absolutna vrednost enega ali drugega. V primeru vzbujanja na mestu z relativno majhno (veliko) zastopanostjo nekega modalnega načina je bil logično tudi odzivni zvočni tlak za ta modalni način majhen (velik). Med merjenjem frekvenčnega odziva (trajanje 1,024 s) preskušanec ni bil v stiku z 0,65-gramskim merilnikom pospeška. Ta je bil uporabljen namesto merilnika sile, kar pomeni, da so bile enote vstopnega signala  $\text{m/s}^2$ . Zaradi stalne mase merilnika pospeška so seveda te enote sorazmerne enotam newton.

Za vsak prekušanec je bilo opravljenih 20 posameznih meritev frekvenčnega odziva. Še pred analogno digitalno pretvorbo signala zvočnega tlaka so bile z nizkopasovnim filtrom na merilni kartici odstranjene vse frekvence, višje od 20 kHz. Nastavljena frekvenca vzorčenja signalov  $f_s$  je bila 16 kHz, kar pa v praksi za dano merilno kartico pomeni dejanskih 1,024 MHz [18]. Število digitalnih točk  $N$  je bilo 16 384, kar pri nastavljeni frekvenci vzorčenja ustreza dolžini snemanja signalov  $T = 1,024 \text{ s}$  [19]:

$$T = N / f_s \quad (1)$$

S tako izbranimi parametri snemanja signalov je bila zagotovljena kakovostna hitra Fourierjeva transformacija signalov. Pri tej mora namreč biti izpolnjen pogoj, da je število zajetih digitalnih točk  $N$  enako  $2^n$ , kjer je

## 1 METHODS AND RESULTS

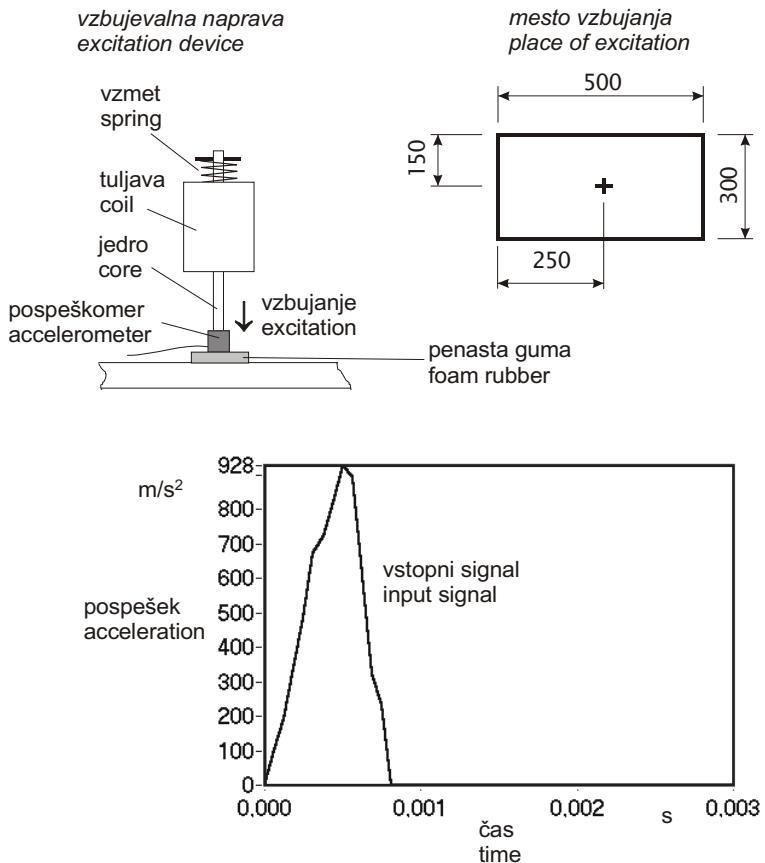
### 1.1 Measurement equipment and procedure

Figure 2 shows the measurement arrangement for determining the relative dynamic stiffness of the specimens [17]. In order to prevent sound reflection due to the excited specimen the inside of an anechoic chamber was lined with pyramid-shaped foam rubber. During the measurements of their FRF the specimens were suspended on two nylon lines, which provided optimum support under the experimental conditions [16].

As shown in Figure 3 the mechanical excitation of the composite plates was performed with a coil, a long core and an accelerometer fixed at the end of this core. Figure 3 also shows the position of the excitation and the shape of the impulse input signal, which was achieved by inserting a small piece of foam rubber between the accelerometer and the specimen. The position of excitation, at the geometrical centre of the composite plate, was selected on the basis of previous measurements of modal shapes [9]. Due to the symmetrical shape of the specimens this position is a place of relatively high amplitudes, at least for the modes with low frequencies. In other words, these modes are well represented at the selected position of excitation. On the other hand, the position of excitation presumably did not significantly influence the measurements since the final result was a ratio of the output to the input signal and did not involve their absolute magnitudes. In the case of excitation at the position with a relatively low (high) proportion of some mode, logically the response sound pressure for this mode is also low (high). During the FRF measurements with a duration of 1.024 s the specimen was not in contact with the 0.65-gram accelerometer. This was used instead of a force transducer, which means that the units of the input signal were  $\text{m/s}^2$ . Because of the constant mass of the accelerometer these units are proportional to Newtonian units.

For each specimen we performed 20 individual FRF measurements. Using a low-pass filter as part of the data-acquisition board meant that frequencies higher than 20 kHz were removed from the output signal (sound pressure) before the A/D conversion. The selected sampling frequency was 16 kHz, which actually means 1.024 MHz for the data-acquisition board that we used [18]. The number of discrete samples  $N$  was 16.384, which resulted in a total length of the signal in the time domain  $T = 1.024 \text{ s}$  [19]:

The selected conditions during the signal recording resulted in a qualitative fast Fourier transformation, where the number of discrete points  $N$  has to be  $2^n$ , and  $n$  is a posi-



Sl. 3. Vzbujevalna naprava, mesto vzbujanja in oblika vstopnega signala

Fig. 3. Excitation device, place of excitation and shape of input signal

$n$  celo pozitivno število (v našem primeru je  $n = 14$ ) [19]. Vsak časovni posnetek zvočnega tlaka je bil programsko obdelan še z 80-odstotno t.i. okensko funkcijo ("Force window"), s katero dosežemo končno vrednost amplitude signala enako nič [16]. Končni rezultat meritve posamezne sestavljenje plošče je bil enostranski amplitudni frekvenčni spekter [19], torej povprečje 20 posameznih spektrov v enakih razmerah merjenja. Amplitude izmerjenega povprečnega frekvenčnega odziva (t.j. frekvenčnega odziva) so za vse meritve izražene v enotah  $\text{Pa} / (\text{m/s}^2)$ , in sicer v vrednostih srednjega kvadratnega pogreška. Za vsak izraziti resonančni vrh v frekvenčnem odzivu je bila vrednost koherenčne funkcije med 0,95 in 1,0 za vse preskušance.

## 1.2 Analiza vpliva posameznih lastnosti sestavljenih plošč na njihove dinamične lastnosti

### 1.2.1 Model

Zvočno polje okoli telesa, ki vibrira, lahko predstavimo z vrsto krogelnih valov. Če je telo v primerjavi z oddaljenostjo od mesta opazovanja majhnih izmer, lahko zvočni tlak na mestu opazovanja zaradi teh valov opišemo z enačbo [20]:

$$p(t) = \frac{S \rho c z}{4\pi r_p} \sin z(ct - r_p) \quad (2),$$

tive integer (in our case  $n = 14$ ) [19]. Each signal of sound pressure was processed by an 80%-Force window function in order to suppress the amplitudes of the signal at its end [16]. The final result of each measurement was a one-sided amplitude frequency spectrum [19], an average of 20 individual spectra under the same measurement conditions. For the average frequency spectrum the amplitudes are expressed in units of  $\text{Pa} / (\text{m/s}^2)$  in rms values. The value of the coherence function was between 0.95 and 1.0 for each resonant peak in the FRF for all specimens.

## 1.2. Analysis of the influence of particular properties of composite plates on their dynamic characteristics

### 1.2.1. Model

The sound pressure around the vibrating body can be represented by a series of spherical waves. If the body is small in comparison to the distance between it and the place of observation, then the sound pressure at the place of observation due to these waves is given by [20]:

kjer so:  $S$  - največja emisija zraka v  $\text{m}^3/\text{s}$ ,  $\rho$  - gostota zraka,  $c$  - hitrost zvoka,  $z=2\pi/\lambda$  ( $\lambda$  je valovna dolžina),  $r_p$  - razdalja od izvora zvočnega tlaka,  $t$  - čas. Iz enačbe (2) vidimo, da se zvočni tlak spreminja obratno sorazmerno z oddaljenostjo od vira. Če v enačbi (2)  $p(t)$  nadomestimo s  $p'_n(t)$ , vstavimo  $z=2\pi/\lambda$  ter upoštevamo, da je  $\lambda=c/f$  ( $f$  pomeni frekvenco zvočnega valovanja), dobimo po okrajšavi za primer nedušenega nihanja:

$$p'_n(t) = \frac{S \rho f}{2 r_p} \sin \frac{2 \pi f}{c} (ct - r_p) \quad (3)$$

kjer  $p'_n(t)$  pomeni nedušen zvočni tlak  $p'(t)$ . Nedušen zvočni tlak  $p'(t)$  je posledica majhnega vira zvoka v obliki krogelnih valov, med drugim pa je odvisen tudi od oddaljenosti od vira zvoka in od frekvence zvočnega valovanja.

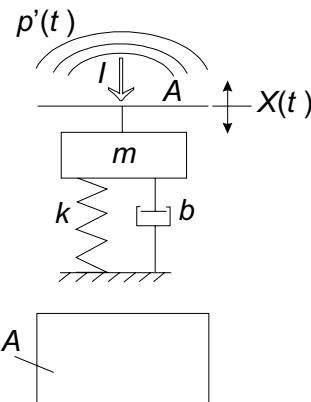
Rečemo lahko, da so izraziti frekvenčni vrhovi v frekvenčnem odzivu preskušancev posledica lastnih gibalnih oblik plošče. Vsaki lastni gibalni oblik ustreza modalna maso  $m$ , togost  $k$ , koeficient viskoznega dušenja  $b$  in površino sevanja  $A$ , kakor je prikazano na sliki 4. Tako je največja emisija zraka  $S$  v  $\text{m}^3/\text{s}$  glede na sliko 4 odvisna od ploskve s površino  $A$  in največje hitrosti nihanja te ploskve  $\dot{X}_{\max}$  [20]:

where  $S$  is the maximum rate of air emission of the small source,  $\rho$  is the density of the air,  $c$  is the velocity of sound,  $z=2\pi/\lambda$  ( $\lambda$  is the wavelength),  $r_p$  is the distance from the sound source, and  $t$  is time. We can see from equation (2) that the sound pressure is inversely proportional to the distance from the source. After replacing  $p(t)$  by  $p'_n(t)$  in equation (2) and taking into account that  $z=2\pi/\lambda$ , and  $\lambda=c/f$  ( $f$  is the frequency of sound waves), we obtain for the case of an undamped oscillation:

where  $p'_n(t)$  indicates the undamped sound pressure  $p'(t)$ . The undamped sound pressure  $p'_n(t)$  is a consequence of the small sound source in the shape of spherical waves and it is also dependent on the distance from the sound source and its frequency.

We can say that the significant frequency peaks in the FRF of the specimens correspond to the normal modes of the composite plates. Each normal mode has a modal mass  $m$ , a stiffness  $k$ , a coefficient of viscous damping  $b$ , and a sound-radiating surface  $A$ , as shown in Figure 4. According to Figure 4,  $S$  in  $\text{m}^3/\text{s}$  depends on a membrane with area  $A$  and the maximum velocity of the oscillation of this surface  $\dot{X}_{\max}$  [20]:

$$S = A \dot{X}_{\max} \quad (4)$$



Sl. 4. Enomasni mehansko-akustični sistem z membrano površine  $A$   
Fig. 4. One-mass mechanical-acoustical system with membrane of area  $A$

Ob upoštevanju enačbe (4) v enačbi (3), dobimo

$$p'_n(t, f = \text{konst}) = \frac{A \dot{X}_{\max} \rho f}{2 r_p} \sin \frac{2 \pi f}{c} (ct - r_p) \quad (5)$$

Ploskev  $A$  niha v skladu z značilnostmi impulzno vzbujenega sistema masa - dušilka - vzmet. Slika 4 kaže, da je brezmasna ploskev  $A$  togo pripeta na maso  $m$ . Torej je tlak  $p'_n(t)$  neposredno odvisen od  $m$ ,  $b$  in  $k$ , s katerimi lahko opišemo lastno dušeno nihanje sistemov drugega reda. Če je takšen sistem vzbujen z impulzom sile, je enačba gibanja ploskve  $A$  [21]:

Substituting Eq. (4) into Eq. (3), we obtain

Surface  $A$  vibrates according to the impulsively excited system consisting of a mass, damper and spring. Figure 4 shows that a massless membrane with area  $A$  is rigidly connected to the mass  $m$ . Thus,  $p'_n(t)$  is directly dependent on  $m$ ,  $b$  and  $k$ , which determine the natural damped oscillation of second-order systems. If such a system is excited by an impulse then the equation of motion for surface  $A$  is [21]:

$$m\ddot{X}(t) + b\dot{X}(t) + kX(t) = 0 \quad (6),$$

kjer sta  $\ddot{X}$  in  $\dot{X}$  pospešek in hitrost nihanja ploskve A. Rešitev enačbe (6) je [21]:

$$X(t) = e^{-\delta \omega_0 t} (X_z \cos \omega_{0d} t + \frac{\dot{X}_z + \delta \omega_0 X_z}{\omega_{0d}} \sin \omega_{0d} t) \quad (7),$$

kjer so:  $X_z$  in  $\dot{X}_z$  začetna amplituda in začetna hitrost nihanja ob prenehanju impulzne motnje,  $\omega_0 = \sqrt{k/m}$  lastna frekvenca nedušenega nihanja,  $\omega_{0d} = \sqrt{1-\delta^2} \cdot \omega_0$  frekvenca lastnega dušenega nihanja in  $\delta = b/2\sqrt{km}$  brezdimenzijski faktor viskoznega dušenja [21]. V enačbi (7) lahko začetno amplitudo štejemo za nič [21], začetno hitrost analiziranega sistema pa lahko izrazimo z upoštevanjem impulznega stavka:

$$I = G_2 - G_1 = m(\dot{X}_z - 0) \quad (8),$$

kjer so I - impulz sile,  $G_1$  in  $G_2$  pa gibalni količini na začetku in koncu delovanja impulza. Hitrost gibanja ploskve A je pred delovanjem impulzne motnje nič, po prenehanju impulza pa  $\dot{X}_z$  [21]. Torej je:

$$\dot{X}_z = I/m \quad (9).$$

Eqačba (7) se tako poenostavi v:

$$X(t) = e^{-\delta \omega_0 t} \left( \frac{I}{m \omega_{0d}} \sin \omega_{0d} t \right) \quad (10).$$

Iz enačbe (5) vidimo, da manjka člen za opis dušenja zvočnega tlaka  $p'(t)$  s slike 4, ki se je vsekakor pojavilo po impulznem vzbujanju plošče. Iz enačbe (10) vidimo, da je dušenje amplitudo nihanja hibridnega sistema, ki seva zvočni tlak  $p'(t)$ , lahko opisano s členom  $e^{-\delta \omega_0 t}$ . Če upoštevamo še, da frekvenca f pomeni pravzaprav frekvenco lastnega dušenega nihanja, tj.  $\omega_{0d}/2\pi$ , lahko zapišemo:

$$p'(t, \omega_{0d} = \text{konst}) = e^{-\delta \omega_0 t} \left( \frac{A \dot{X}_{\text{maks}} \rho \omega_{0d}}{4\pi r_p} \sin \frac{\omega_{0d}}{c} (ct - r_p) \right) \quad (11).$$

Iz enačbe (10) lahko izpeljemo prvi odvod amplitude nihanja in tako dobimo izraz za hitrost nihanja dušenega mehanskega sistema:

$$\dot{X}(t) = e^{-\delta \omega_0 t} \left( \frac{I}{m} \cos \omega_{0d} t - \frac{\delta I \omega_0}{m \omega_{0d}} \sin \omega_{0d} t \right) \quad (12).$$

Največja hitrost nihanja ploskve A se pojavi na začetku nihanja dušenega hibridnega sistema:

$$\dot{X}_{\text{maks}}(t=0) = \frac{I}{m} \quad (13).$$

Nazadnje lahko zvočni tlak  $p'(t)$  zapišemo kot:

where  $\ddot{X}$  and  $\dot{X}$  are the acceleration and velocity of surface A, respectively. The solution of equation (6) is [21]:

$$\dot{X}_z + \frac{\dot{X}_z + \delta \omega_0 X_z}{\omega_{0d}} \sin \omega_{0d} t \quad (7),$$

where  $X_z$  and  $\dot{X}_z$  are the initial amplitude and the initial velocity of the oscillation at the moment the sample disturbed:  $\omega_0 = \sqrt{k/m}$  is the natural frequency of the undamped oscillation;  $\omega_{0d} = \sqrt{1-\delta^2} \cdot \omega_0$  is the natural frequency of the damped oscillation; and  $\delta = b/2\sqrt{km}$  is the dimensionless viscous damping factor [21]. In equation (7) the initial amplitude may be considered as zero [21] and the initial velocity of the analysed system can be expressed from the condition of the conservation of momentum:

where I is a mechanical impulse, and  $G_1$  and  $G_2$  are the momenta at the beginning and at the end of the mechanical impulse action, respectively. The velocity of surface A before the action of the mechanical impulse is zero, and after the action it is  $\dot{X}_z$  [21]. Thus:

Equation (7) can be rewritten as:

We can see from expression (5) that a description of the damping of the sound pressure  $p'(t)$  from Figure 4 is not included. The damping definitely occurred after the impulse excitation of the composite plates. From equation (10) it is evident that the damping of the oscillation of a hybrid system, which radiates sound pressure  $p'(t)$ , can be expressed by  $e^{-\delta \omega_0 t}$ . Considering also that the frequency f is actually a natural frequency of the damped oscillation (i.e.,  $\omega_{0d}/2\pi$ ), we can write:

From equation (10) we can calculate the first derivative of the amplitude of oscillation, which means the velocity of the damped oscillation of the mechanical system:

The maximum velocity of oscillation of surface A occurs at the beginning of oscillatory motion of the damped hybrid system:

Finally, the sound pressure  $p'(t)$  can be written as:

$$p'(t, \omega_{0d} = \text{konst}) = p'(t) = e^{-\delta \omega_0 t} \left( \frac{AI \rho \omega_{0d}}{4\pi r_p m} \sin \frac{\omega_{0d}}{c} (ct - r_p) \right) \quad (14)$$

Analiza meritev je pokazala, da je dušenje  $\delta$  za vse sestavljeni plošče mnogo manjše od ena, torej lahko rečemo, da je  $\omega_0 \cong \omega_{0d}$ . Ob upoštevanju, da so  $I$ ,  $\rho$  in  $r_p$  enaki za vse meritve, iz enačbe (14) vidimo, da je amplituda ( $P$ ) posameznega resonančnega vrha iz frekvenčnega odziva impulzno vzbujenih plošč:

$$P = K \frac{A}{m} \sqrt{\frac{k}{m}} \quad (15),$$

kjer je  $K$  konstanta z vrednostjo  $(I\rho)/(4\pi r_p)$  in enotami  $\text{kg}^2 \cdot \text{s}/\text{m}^4$ . Absolutna vrednost konstante  $K$  je v nadaljnji analizi nepomembna, saj gre le za primerjavo posameznih frekvenčnih odzivov oziroma veličin, dobljenih iz teh odzivov.

Pozitivna lastnost nosilnih gradnikov strojev in naprav je poleg velike statične togosti tudi velika dinamična togost ob čim manjši masi. To pomeni veliko razmerje  $k/m$  iz enačbe (15). Seveda pa morajo biti amplitudi nihanja takšnih strojnih delov čim manjše, kar pomeni tudi čim manjše vrednosti za  $P$ , saj je moč zvočnega tlaka glede na jakost vzbujanja neposredno odvisna od moči vibracij preskušanih plošč. Iz enačbe (15) vidimo, da bosta oba pogoja izpolnjena pri razmeroma majhnem  $A/m$ , velikem  $k/m$  in njunem relativno majhnem zmnožku.

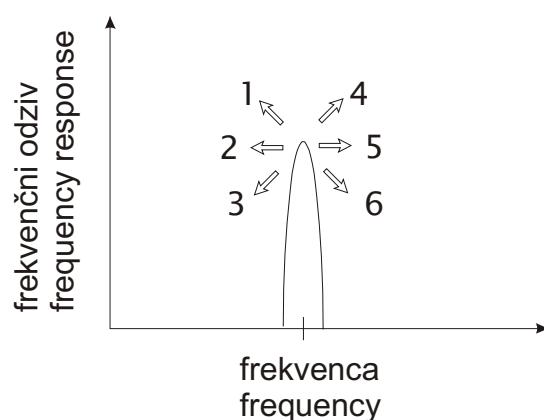
Sklenemo lahko, da je primerjavo dinamične togosti preskušanih plošč mogoče opraviti s primerjavo pripadajočih resonančnih vrhov v njihovem frekvenčnem odzivu, kar kaže slika 5. Glede na izhodiščni resonančni vrh pomeni možnost 1 najbolj neugodno kombinacijo modalne togosti in mase, možnost 6 pa najugodnejšo. Možnosti 2 in 3 lahko štejemo za neugodni, možnosti 4 in 5 pa za ugodni, kakor kaže slika 5. Dodati je treba, da je možnost 4 bolj ugodna, če povečanje amplitude resonančnega vrha ni večje od povečanja njegovega položaja (frekvence), kar logično izhaja iz enačbe (15).

The analysis of the measurements showed that the damping  $\delta$  is significantly smaller than one for all the composite plates, thus, we can say that  $\omega_0 \cong \omega_{0d}$ . Considering also that  $I$ ,  $\rho$  and  $r_p$  are the same for all measurements, we can conclude from equation (14) that the amplitude ( $P$ ) of the individual resonant peak in the FRF of impulsively excited plates is:

where  $K$  is a constant with magnitude  $(I\rho)/(4\pi r_p)$  in units of  $\text{kg}^2 \cdot \text{s}/\text{m}^4$ . Because the following analysis was focused only on a comparison between different FRFs and their corresponding parameters, the magnitude of the constant  $K$  is irrelevant.

In addition to a high static stiffness the dynamic stiffness of the supporting parts of machine tools and devices is also a positive feature. This indicates a high ratio  $k/m$  from equation (15). However, for such machine parts the amplitudes of oscillation should be relatively small, which indicates relatively small values for  $P$ . Namely, the intensity of the sound pressure with respect to the intensity of excitation depends directly on the intensity of the vibrations for the tested plates. We can see from equation (15) that both conditions will be fulfilled when  $A/m$  is relatively small,  $k/m$  is relatively high, and the product of both ratios is relatively small.

We can conclude that a comparison of FRFs and the corresponding resonant peaks allows us to compare the dynamic stiffness of the tested plates, as shown in Figure 5. With respect to the presented resonant peak alternative: 1 means the least favourable combination of modal stiffness and mass, whereas alternative 6 is the most favourable. As indicated in Figure 5, alternatives 2 and 3 can be seen as unfavourable, whereas alternatives 4 and 5 are favourable. It should be noted that alternative 4 is more favourable if the increase in the amplitude of the resonant peak is not higher than the increase in its position (frequency), which is logical according to equation (15). By anal-



Sl. 5. Relativni položaj in amplituda resonančnega vrha (1 do 3: neugodno, 4 do 6: ugodno, 6: najbolj ugodno)  
Fig. 5. Relative position and amplitude of resonant peak (1 to 3: unfavourable, 4 to 6: favourable, 6: most favourable).

Analogno je možnost 3 manj ugodna, če je zmanjšanje amplitude manjše od zmanjšanja frekvence resonančnega vrha.

### 1.2.2 Vpliv debeline sten in usmeritve jedra

Slika 6 prikazuje rezultate meritev za jeklene sestavljenne plošče v frekvenčnem obsegu 300 do 1300 Hz, kjer lahko analiziramo vpliv debeline sten in usmeritve jedra. Že na prvi pogled je očitno, da sta za vsako sestavljeni plošči značilna vsaj dva izrazita frekvenčna vrhova, ki jima lahko določimo frekvenco, amplitudo in dušenje. Vsi ti trije parametri ter razmerje  $A/m$  iz enačbe (15) so za pripadajoče preskušance za prva dva resonančna vrha zbrani v preglednici 2. Z veliko zanesljivostjo lahko trdimo, da je veliko razmerje  $A/m$  pokazatelj slabih dinamičnih lastnosti nosilnih gradnikov obdelovalnih strojev. Ti bi za vsak modalni način morali izkazovati veliko maso ( $m$ ) in logično majhne dinamično aktivne površine ( $A$ ), kar pomeni majhno razmerje  $A/m$ . Nadalje, v danih razmerah meritev je primerno predpostaviti vsaj približno enakost površin  $A$  ( $0,5 \text{ m} \times 0,3 \text{ m} = 0,15 \text{ m}^2$ ) za vse plošče. Meritve oblik modalnih načinov so namreč pokazale, da je vsaj pri modalnih načinih z razmeroma nizko frekvenco aktivna celotna površina za vse preskušance [9]. To pomeni, da lahko za vsak resonančni vrh izračunamo tudi relativne vrednosti  $m$  ter  $k$ , ki so prav tako prikazane v preglednici 2. Medtem ko sta bili amplituda  $P$  ter frekvenca  $f$  resonančnega vrha prebrani neposredno iz frekvenčnega odziva, je bil v analizi faktor viskoznega dušenja  $\delta$  izračunan kot [16]:

$$\delta \cong \frac{1}{2} \frac{f_2 - f_1}{f} \quad (16),$$

kjer  $f_1$  in  $f_2$  pomenita frekvenci nižje in višje od resonančne frekvence, kjer je amplituda resonančnega vrha enaka  $P/\sqrt{2}$  [16].

### 1.2.3 Vpliv polnila med jedrom in stenami sestavljeni plošče

Slika 7 in preglednica 3 prikazujeta rezultate meritev za jekleno sestavljeni ploščo z debelino sten 1 mm in vzdolžno usmeritvijo jedra za tri različne primere: (i) brez polnila, (ii) s poliesterskim betonom in (iii) s poliuretansko maso med obema stenama in jedrom (sl. 1).

## 2 ANALIZA REZULTATOV IN RAZPRAVA

Slike 6 in iz preglednice 2 vidimo, da vzdolžna usmeritev jedra ne glede na debelino stene pomeni bistveno večjo togost prvega modalnega načina v primerjavi s prečno usmeritvijo jedra. Točneje, v primeru plošče z debelino sten 1 mm je to razmerje

ogy, alternative 3 is less favourable if the decrease in the amplitude is smaller than the decrease in the frequency of the corresponding resonant peak.

### 1.2.2 The influence of the flatsheets' thickness and the core orientation

Figure 6 shows the results of measurements for steel composite plates in the frequency range from 300 to 1300 Hz, which allows us to analyse the influence of the flatsheets, thickness and the core orientation. At a first glance it is evident that at least two significant peaks with their corresponding frequencies, amplitudes and dampings are typical for each composite plate. For the first two resonant peaks these three parameters, and the  $A/m$  ratio from equation (15), are presented in Table 2 for the corresponding specimens. With considerable certainty we can say that the high  $A/m$  ratio indicates bad dynamic properties for the supporting elements of machine tools. These should have a large mass ( $m$ ) and, logically, small surfaces with dynamic activity ( $A$ ) for each mode; this means a small  $A/m$  ratio. Next, under our experimental conditions, it is reasonable to assume that the surfaces  $A$  ( $0,5 \text{ m} \times 0,3 \text{ m} = 0,15 \text{ m}^2$ ) are the same for all the plates. Namely, the measurements of modal shapes showed that at least for low-order modes the whole surface of the specimens was active [9]. This means that we can calculate relative values for  $m$  and  $k$  for each resonant peak. These values are also presented in Table 2. Whereas amplitude  $P$  and frequency  $f$  of the resonant peak were read directly from the FRF plot, the viscous damping factor  $\delta$  was calculated as [16]:

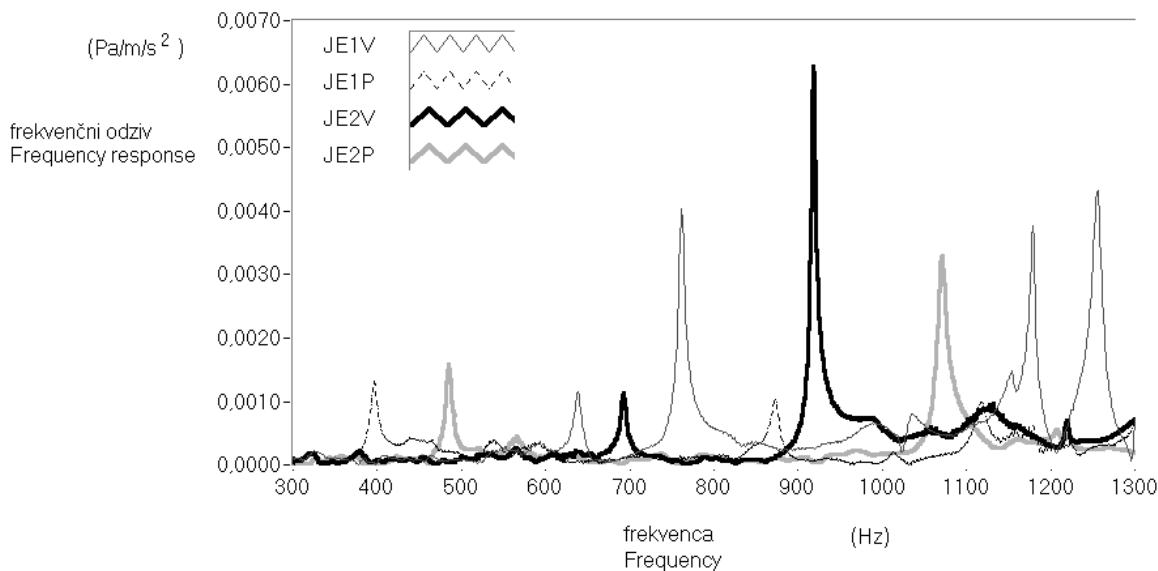
where  $f_1$  and  $f_2$  indicate frequencies (lower and higher than the resonant frequency) where the amplitude of the resonant peak is  $P/\sqrt{2}$  [16].

### 1.2.3 The influence of the filling between the core and the flatsheets of the composite plates

Figure 7 and Table 3 show the results on measurements for a steel composite plate with a flatsheet thickness of 1 mm and longitudinal orientation of core for three different cases: (i) without filling, (ii) with polyester concrete and (iii) with polyurethane filling between the core and the flatsheets (see Figure 1).

## 2 ANALYSIS OF RESULTS AND DISCUSSION

We can see from Figure 6 and Table 2 that apart from the flatsheet thickness the longitudinal core orientation results in a higher stiffness of the first mode in comparsion to the transversal orientation of the core. More precisely, in the case of a plate



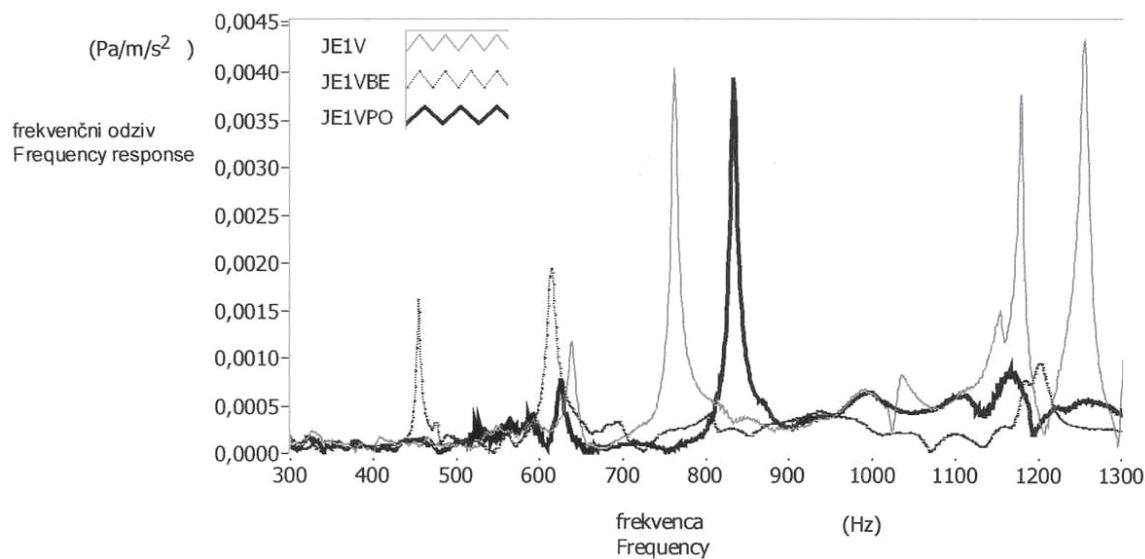
Sl. 6. Frekvenčni odziv plošč kot funkcija debeline sten in usmeritve jedra  
Fig. 6. Frequency response as a function of flatsheet thickness and core orientation

Preglednica 2. Prva dva resonančna vrha – analiza vpliva debeline sten in usmeritve jedra

Table 2. The first two resonant peaks – analysis of the influence of flatsheet thickness and core orientation

1. vrh / 1st peak	$P$ $\text{Pa}/\text{m}^2$	$\delta$	$f$ Hz	$\sqrt{k/m}$ rd/s	$A/\text{m}$ $\text{K}^{-1} \text{kg s}/\text{m}^2$	$m$ $\text{K m}^4/\text{kg s}$	$k^*$ $\text{K m}^4/\text{kg s}^3$
JE1V	0,00116	0,0062	638,0	4009,0	2,89E-7	518,4E3	8,33E12
JE1P	0,00130	0,0098	396,2	2489,0	5,22E-7	287,2E3	1,78E12
JE2V	0,00113	0,0045	692,4	4350,5	2,60E-7	577,5E3	10,9E12
JE2P	0,00158	0,0064	484,9	3046,7	5,19E-7	289,2E3	2,68E12
2. vrh / 2nd peak	$P$ $\text{Pa}/\text{m}^2$	$\delta$	$f$ Hz	$\sqrt{k/m}$ rd/s	$A/\text{m}$ $\text{K}^{-1} \text{kg s}/\text{m}^2$	$m$ $\text{K m}^4/\text{kg s}$	$k^*$ $\text{K m}^4/\text{kg s}^3$
JE1V	0,00400	0,0013	761,6	4785,0	8,36E-7	179,4E3	4,11E12
JE1P	0,00103	0,0055	872,5	5482,0	1,88E-7	798,3E3	24,0E12
JE2V	0,00630	0,0026	918,4	5770,5	10,92E-7	137,4E3	4,57E12
JE2P	0,00330	0,0045	1070,9	6728,7	4,90E-7	305,9E3	13,8E12

\* $A=0,15 \text{ m}^2$



Sl. 7. Frekvenčni odziv plošč kot funkcija polnila med stenama in jedrom  
Fig. 7. Frequency response as a function of the filling between the flatsheets and the core

Preglednica 3. Prva dva resonančna vrha – analiza vpliva polnila

Table 3. The first two resonant peaks – analysis of the influence of the filling

1. vrh / 1st peak	$P$ Pa/m/s <sup>2</sup>	$\delta$	$f$ Hz	$\sqrt{k / m}$ rd/s	$A/m$ $K^1 \text{ kg s/m}^2$	$m$ $K \text{ m}^4/\text{kg s}$	$k^*$ $K \text{ m}^4/\text{kg s}^3$
JE1V	0,00116	0,0062	638,0	4009,0	2,89E-7	518,4E3	8,33E12
JE1VBE	0,00162	0,0042	454,2	2853,8	5,67E-7	264,2E3	2,15E12
JE1VPO	0,00078	0,0064	625,5	3930,1	1,98E-7	755,8E3	11,7E12
2. vrh / 2nd peak	$P$ Pa/m/s <sup>2</sup>	$\delta$	$f$ Hz	$\sqrt{k / m}$ rd/s	$A/m$ $K^1 \text{ kg s/m}^2$	$m$ $K \text{ m}^4/\text{kg s}$	$k^*$ $K \text{ m}^4/\text{kg s}^3$
JE1V	0,00400	0,0013	761,6	4785,0	8,36E-7	179,4E3	4,11E12
JE1VBE	0,00195	0,0076	613,9	3857,2	5,05E-7	296,7E3	4,41E12
JE1VPO	0,00395	0,0044	833,0	5233,9	7,54E-7	198,7E3	5,44E12

 $*A=0,15 \text{ m}^2$ 

4,7:1, v primeru debeline sten 2 mm pa 4,1:1. To je v skladu s predhodnimi preskusi, pri katerih je bilo namesto impulznega vzbujanja plošč uporabljeno sinusno vzbujanje, frekvenčni odziv plošče pa je bil definiran kot razmerje med odzivnim pospeškom (hitrostjo) plošče in vzbujevalno silo na njeni geometrični sredini [9]. Primerjava med ploščama z debelino sten 2 in 1 mm pokaže, da je togost prvega modalnega načina v razmerju 1,5:1 za prečno usmeritev jedra in 1,3:1 za vzdolžno. Tudi to je v skladu s predhodnimi meritvami [9], sklenemo pa lahko, da za dano situacijo večja debelina sten pripomore k večanju togosti prvega modalnega načina opazno manj kot usmeritev jedra. Učinek usmerjenosti jedra je malenkostno večji pri bolj debeli steni sestavljene plošče. Faktor dušenja prvega modalnega načina je v splošnem večji za prečno usmerjenost jedra in za manjšo debelino sten. To je samo delno v skladu s predhodnimi meritvami [9], kjer se je izkazalo, da ima v primeru prečne usmerjenosti jedra plošča z 2-milimetrsko steno nekaj večje dušenje od tiste z 1-milimetrsko steno. Dodati pa je treba, da so v splošnem vrednosti faktorjev dušenja za vse plošče (prejšnje in zdajšnje meritve) veliko manjše od ena, torej je neskladje s predhodnimi meritvami praktično neznačilno.

Za drugi modalni način situacija ni analogna tisti za prvega, saj različno debeli plošči s prečno usmerjenostjo jedra izkazujeta neprimerno večjo togost od plošč z vzdolžno usmerjenostjo jedra. Takšni rezultati so v skladu s prejšnjimi ugotovitvami [9] in so pojasnjeni z obliko drugega modalnega načina, pri katerem prihaja do upogibanja okoli prečne osi plošče. Nadalje, togost tega modalnega načina je v primeru prečne postavitve jedra skoraj dvakrat večja za 1 mm debelo ploščo v primerjavi s ploščo z debelino sten 2 mm. Kljub temu, da so se frekvence gibalnih oblik ujemale s prejšnjimi raziskavami [9], pa je bila pri slednjih togost druge gibalne oblike (upogibanje okoli prečne osi) večja za večjo debelino stene pri enaki usmerjenosti jedra.

with a flatsheet thickness of 1 mm this ratio is 4.7:1, and in the case of a flatsheet thickness of 2 mm the ratio is 4.1:1. This is in agreement with previous experiments where sinusoidal excitation was used instead impulse the excitation, and FRF was defined as the ratio of the response acceleration (velocity) to excitation force at the geometrical centre of the plate [9]. For plates with a flatsheet thickness of 2 and 1 mm the stiffness of the first mode is 1.5:1 for a transversal core orientation, and 1.3:1 for a longitudinal orientation. This is also in agreement with previous measurements [9], thus we can conclude that for a given situation the flatsheet thickness is less significant for the first mode stiffness than core orientation. The effect of core orientation is slightly higher for the thicker flatsheet of the composite plate. The damping factor of the first mode is higher for the transversal core orientation and for the thinner flatsheets. This is only partly in agreement with previous measurements [9], where for a transversal core orientation the damping is slightly higher for the thicker (2 mm) than for the thinner (1 mm) composite plate. However, the values for the damping factors are significantly smaller than one for all plates, in general, as a result, this disagreement is insignificant.

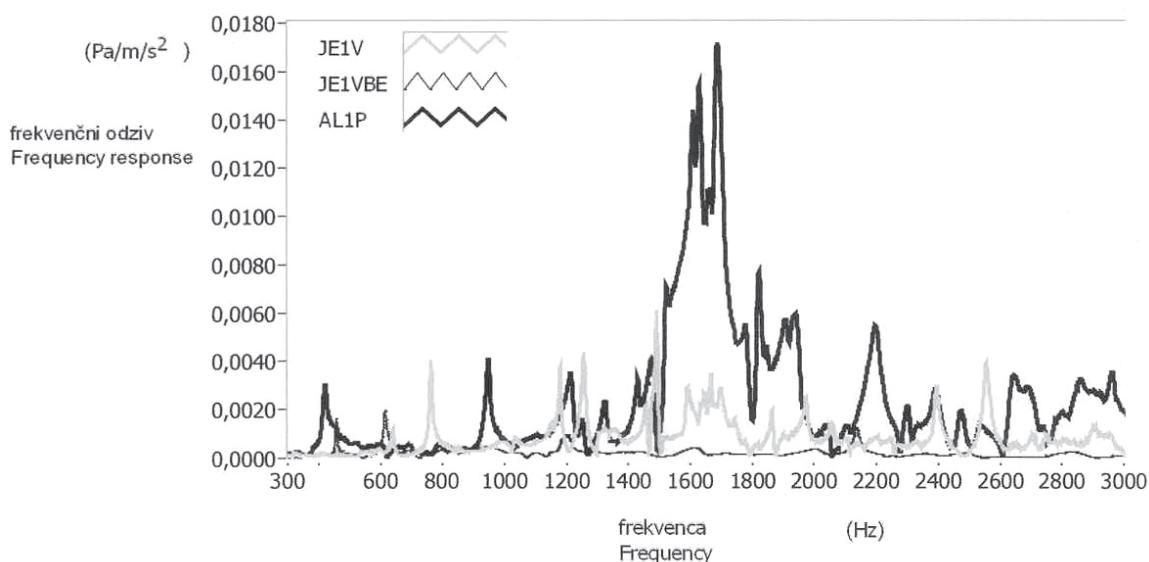
Because both the 1-mm- and 2-mm-thick plates with a transversal core orientation show a significantly higher stiffness in comparison to the plates with a longitudinal core orientation for the second mode, there is no analogy between the situation for the first and second mode. Such results are in agreement with previous findings [9] and are explained by the second mode shape, which is bending along the transversal direction of the plate. Next, for a transversal core orientation in the case of a flatsheet thickness of 1 mm the stiffness of this mode is almost 100% higher in comparison to the plate with a flatsheet thickness of 2 mm. In spite of the fact that the mode frequencies are in agreement with the previous tests [9] the latter indicate higher stiffness of the second mode (bending along the transversal axis) for a thicker plate in comparison to the thinner one for the same core orientation.

Velika masa poliestrskega betona je bistveno zmanjšala razmerje med togostjo in maso (nezaželeno) za oba prva modalna načina v primerjavi s ploščo brez polnila oziroma s poliuretanom. Za prvo gibalno obliko je togost plošče s poliestrskim betonom manjša od obeh drugih plošč, kar kaže relativno neučinkovitost betona v strukturi sestavljene plošče (predvidoma slaba oprijetost betona s stenami in jedrom ali podobno). V skladu s pričakovanji je, da je v višjem frekvenčnem področju (od 700 Hz navzgor) frekvenčni odziv plošče s poliestrskim betonom bistveno ugodnejši, torej manj intenziven, v primerjavi z obema primerjanima (sl. 7). Na dušenje prve gibalne oblike poliuretan praktično ni imel vpliva v primerjavi s ploščo brez polnila, pač pa je vplival na večje dušenje v primerjavi s ploščo s poliestrskim betonom. Enako kakor pri primerjavi plošč brez polnila, a z različno debelino sten in usmerjenostjo jedra, je treba na tem mestu poudariti, da so vse izmerjene vrednosti faktorja dušenja veliko manjše od ena, zato morebitne razlike praktično niso značilne. Izkazani vpliv betonskega polnila namreč ni v skladu s predhodnimi meritvami [9], ki so pokazale malenkostno večje dušenje plošče z betonom v primerjavi s ploščama s poliuretanskim polnilom in brez njega. Po drugi strani pa sta analizi drugega resonančnega vrha predhodnih [9] in zdajšnjih meritev v sozvočju, saj obe izkazujeta razmeroma veliko dušenje plošče z betonom v primerjavi s preostalima dvema ploščama. Nadalje, togost plošče z betonskim polnilom je največja, togost plošče brez polnila pa najmanjša [9].

Za boljšo fizikalno razlagajo dobljenih frekvenčnih odzivov je na sliki 8 prikazana njihova primerjava za sestavljene plošče JE1V, JE1VBE ter AL1P

In comparison to the plates without filling and with polyurethane filling a relatively large mass of the polyester concrete significantly reduced the ratio of stiffness to mass (which is undesired) for the first two modes. The relative inefficiency of the polyester concrete in the structure of the composite plate (presumably bad adhesion between the core and the flatsheets, or something similar) is evident in the lower stiffness of the first mode for this plate in comparison to another two plates. As expected, in a higher frequency range – from 700 Hz up – the FRF of the plate with polyester concrete is significantly more favourable, thus less intensive in comparison to the compared FRFs (see Figure 7). The damping of the first mode was practically independent of the polyurethane filling in comparison to the plate without it. However, in comparison to the polyester concrete the polyurethane filling resulted in a higher damping of this mode. In a comparison between the plates without filling and the plates with different thicknesses of flatsheets and core orientations, it should be noted here that the values of the damping factor are much less than one. Therefore, the measured differences are, in practise, not significant. Namely, the demonstrated effect of the polyester concrete is not in agreement with the previous measurements [9], which showed slightly higher damping of the composite plates with concrete in comparison to the plates without filling and with polyurethane filling. On the other hand, the analyses of the second resonant peak of the previous [9] and present measurements are in agreement, because they both indicate a relatively high damping of the plate with polyester concrete in comparison to the other two plates. The stiffness of the plate with the concrete filling is the highest, whereas the stiffness of the plate without the filling is the lowest [9].

For a better interpretation of the physical background of the measured FRFs Figure 8 shows a comparison of the composite plates: JE1V, JE1VBE and



Sl. 8. Primerjava s frekvenčnim odzivom aluminijaste sestavljeni plošče  
Fig. 8. A comparison with the frequency response of the aluminium composite plate

za širše frekvenčno področje. Slednja sestavljena plošča je geometrijsko enaka JE1P, le da je material sten in jedra aluminij. Poleg očitno neugodne lege prvega resonančnega vrha (razmeroma nizka frekvenca in visoka amplituda) je za aluminijasto sestavljeni plošči značilna razmeroma velika površina pod frekvenčnim odzivom. Za primerjavo, površine za plošče JE1V, JE1VBE ter AL1P so v razmerju 0,34 : 0,09 : 1. To pomeni relativno intenziven odziv (vibracije) aluminijaste plošče in majhen odziv sestavljeni plošče s poliestrskim betonom glede na JE1V. Glede na rezultate predstavljeni v 1.2.2 in 1.2.3, je logično, da je aluminijasta sestavljena plošča s prečno postavitvijo jedra od vseh preskušenih dinamično najbolj neugodna. Torej sta za opis dinamičnih lastnosti pomembna tako lega in amplituda prvega resonančnega vrha (zaželena majhna amplituda in visoka frekvenca) kakor tudi površina pod grafom celotnega frekvenčnega odziva (zaželena majhna površina). Kljub opazno večjemu dušenju (zaželeno) prvega resonančnega vrha za AL1P v primerjavi z drugima dvema sestavljenima ploščama lahko sklenemo, da je frekvenčni odziv aluminijastega preskušanca najslabši. Sestavljena plošča s poliestrskim betonom, kljub nekoliko neugodnejšemu prvemu (in drugemu) resonančnemu vrhu v primerjavi z JE1V, izkazuje skorajda ničen odziv v frekvenčnem področju, v katerem je odziv za JE1V in AL1P še relativno močan (nezaželeno).

Kljub dejству, da v poglavju 1.2.1 predstavljeni model predvideva relativno veliko oddaljenost ( $r_p$ ) od vira zvoka, v našem primeru to ni bilo najbolj idealno izpolnjeno. Oddaljenost mikrofona od preskušanca je bila namreč komaj dvakrat toliko, kolikor je njegova največja izmerna. To in pa različnost metod pri določanju dinamičnih lastnosti je najverjetnejši razlog za nekatera razhajanja pri rezultatih meritev.

### 3 SKLEP

Rezultati raziskave so v precejšnji meri v skladu s predhodnimi meritvami [9], pri katerih je bilo uporabljeno sinusno vzbujanje plošč, frekvenčni odziv pa je bil definiran kot razmerje med izstopnim pospeškom (hitrostjo) plošče in vstopno silo na njeni geometrični sredini. Kljub dejству, da v 1.2.1 predstavljeni model predvideva relativno veliko oddaljenost ( $r_p$ ) od vira zvoka, v našem primeru to ni bilo najbolj idealno izpolnjeno. Oddaljenost mikrofona od preskušanca je bila namreč komaj dvakrat toliko, kolikor je njegova največja izmerna, predvidoma pa to na meritve faktorja dušenja in resonančnih frekvenc ne vpliva [16]. Določene razlike pri rezultatih obeh postopkov (glej poglavje 2) lahko razložimo z različnim načinom vzbujanja, od katerega je odziv realnih (nelinearnih) sistemov načeloma močno odvisen [16].

AL1P, for a wide frequency range. The last of these composite plates is geometrically the same as JE1P; however, the material of the flatsheets and the core is aluminium. In addition to an unfavourable position of the first resonant peak (relatively low frequency and high amplitude) the aluminium plate indicates a relatively large area under its FRF. For comparison, the areas under the FRFs for plates JE1V, JE1VBE and AL1P are in the ratios: 0.34 : 0.09 : 1. This means a relatively intense response (vibrations) of the aluminium plate and a weak response of the plate with the polyester concrete in comparison to JE1V. According to results from sections 1.2.2 and 1.2.3 it is logical that the aluminium composite plate with a transversal core orientation is dynamically the worst. Therefore, not only the position, but also the amplitude (small amplitude and high frequency are desired) in addition to the area under the FRF (small area is desired), are important in defining the dynamic properties. Despite the higher damping (desired) of the first resonant peak for AL1P in comparison to the other two composed plates it is reasonable to conclude that the FRF of the aluminium specimen is the worst. Regardless of the slightly more unfavourable characteristics of the first (and also the second) resonant peak in comparison to JE1V, the plate with polyester concrete demonstrates an almost null response in the frequency range where this one is relatively strong for JE1V and AL1P (not desired).

Despite the fact that in section 1.2.1 the presented model presumes a relatively large distance ( $r_p$ ) from the sound source, this was not ideally fulfilled in our case. Namely, the distance between the microphone and the specimen was hardly twice its largest dimension. In addition to the differences between the methods for measuring the dynamic properties, this is the most probable reason for some dispersing conclusions.

### 3 CONCLUSION

The results of our research are mostly in agreement with previous measurements [9], where sinusoidal excitation of the plates was applied and the FRF was defined as a ratio of the output acceleration (velocity) to the input force plate at its geometrical centre. Despite the fact that in section 1.2.1 the presented model assumes a relatively large distance ( $r_p$ ) from the sound source, this was not ideally fulfilled in our case. Namely, the distance between the microphone and specimen was hardly twice its largest dimension. Presumably, this does not affect the measurements of damping factor and resonant frequencies [16]. Definite differences in the results of both methods (see section 3.) can be explained with the different types of excitation, which strongly affects the response of real (non-linear) systems [16].

Meritve frekvenčnega odziva sestavljenih plošč za gradnjo nosilnih delov obdelovalnih strojev so v splošnem pokazale značilen vpliv materiala, debeline sten, usmeritve jedra ter polnila med stenama in jedrom. Frekvenčni odziv je bil definiran kot razmerje med rezultirajočim zvočnim tlakom, merjenim 1 m od prostega vpetih plošč, in vstopnim mehanskim impulzom. Izraziti frekvenčni vrhovi v frekvenčnem odzivu so tako bili ponazorjeni z impulzno vzbujenim enomasnim mehansko-akustičnim sistemom, sestavljenim iz mase (diskretna masa  $m$ ), dušilke (koeficient viskoznega dušenja  $b$ ), vzmeti (togost  $k$ ) in brezmasne membrane s površino  $A$ .

Rezultati analize kažejo pomembno vlogo vseh analiziranih vplivov na gibalno obliko oziroma dinamične lastnosti sestavljenih plošč. Vzdolžna orientacija jedra na pravokotnih prekušancih se je izkazala za superiorno glede prve gibalne oblike, saj sta togost ter razmerje med togostjo in maso (frekvenca) te gibalne oblike večja v primerjavi s ploščami s prečno usmeritvijo jedra. Nasprotno se je izkazalo pri drugi gibalni obliki, tj. upogibanje okoli prečne osi plošče, in zanj prečna usmeritev jedra pomeni večjo togost in razmerje togosti proti masi v primerjavi z vzdolžno usmeritvijo jedra. Precej manjši vpliv od usmeritve jedra je na spremembo togosti prve gibalne oblike imela sprememba debeline sten sestavljenih plošč. V primerjavi z 1-milimetrsko steno ima plošča z 2-milimetrsko steno približno 30 do 50% večjo togost, medtem ko za drugo gibalno obliko ni bilo opaziti zanesljive odvisnosti med debelino stene in togostjo oblike. Sestavljena plošča s poliestrskim betonom kot polnilom je pokazala značilno majhno razmerje med togostjo in maso prvega (in drugega) modalnega načina v primerjavi s ploščo brez polnila. Toda ugodnost njenega frekvenčnega odziva v področju od 700 Hz navzgor, kjer izkazuje izrazito dinamično nedejavnost (nizko razmerje  $A/m$  za vse visoke modalne načine), je v primerjavi z vsemi drugimi preskušanci očitna. V splošnem je bil faktor dušenja analiziranih gibalnih oblik veliko manjši od ena, zato izmerjenim razlikam med njim pripadajočim dušenjem ni bila posvečena velika pozornost. Kljub temu je moč skleniti, da prečna usmeritev jedra ali manjša debelina sten pomenita večje dušenje za obe analizirani gibalni oblike. Dušenje prve gibalne oblike plošče s poliestrskim betonom je bilo nekaj manjše od dušenja plošče brez betona, medtem ko je za drugo gibalno obliko plošča z betonom izkazala precej večje dušenje. Plošča s poliuretanskim polnilom se je v vseh pogledih (dušenje, amplitudo) obnesla bolje kakor plošča brez polnila.

Sklenemo lahko, da predstavljena metoda kljub svoji poenostavitevi običajne oblikovne analize omogoča grobo identifikacijo dinamičnih lastnosti sestavljenih plošč. Kakor kaže, lahko z metodo bistveno poenostavimo izbiro optimalnih izmer sten,

FRF measurements of the composite plates intended for supporting parts of machine tools and devices demonstrated a significant influence of the material, the flatsheets, thickness, the core orientation and the filling between the flatsheets and the core. The FRF was defined as the ratio of the resulting sound pressure measured at 1 m from the free-supported plates to the input mechanical impulse. The significant resonant peaks in the FRF were approximated with an impulsively excited one-mass system (mechanical-acoustical) which consists of mass (discrete mass  $m$ ), damper (coefficient of viscous damping  $b$ ), spring (stiffness  $k$ ) and massless membrane with area  $A$ .

The results of the analysis point to an important role of all the analysed effects on the modal behaviour and on the dynamic properties of the composite plates. In comparison to the transversal core orientation the longitudinal one demonstrated the superiority of the rectangular-shaped specimens for the first mode. The reason for this is that the ratio of stiffness to mass (frequency) for this mode was higher for the plates with a longitudinal core orientation. In contrast, for the second mode, which is bending along the transversal axis of the plate, the transversal core orientation means both higher stiffness and ratio of stiffness to mass in comparison to the longitudinal core orientation. The effect of flatsheet thickness on the first mode stiffness was significantly lower than the effect of core orientation. In comparison to the flatsheet with a thickness of 1 mm the plate with a 2-millimetre flatsheet has approximately 30–50% higher stiffness, whereas for the second mode no significant correlation between flatsheet thickness and modal stiffness was measured. The composite plate with polyester concrete demonstrated a significantly lower ratio of stiffness to mass of the first (and also the second) mode in comparison to the composite plate without filling. However, the benefit of the plate with polyester concrete in the range from 700 Hz in comparison to the other plates is evident in the significant dynamic non-activity (low  $A/m$  ratio for all higher modes). In general, the damping factor of the analysed modes was significantly less than one, therefore, the measured differences in modal damping were not significant. Despite this, we can conclude that transversal rather than longitudinal or thinner rather than thicker flatsheet thicknesses mean higher damping for both the analysed modes. The damping of the first mode for the plate with polyester concrete was slightly lower in comparison to the plate without the filling, whereas for the second mode the concrete plate demonstrated significantly higher damping. The plate with polyurethane filling showed better performance in comparison to the plate without the filling in terms of damping and amplitudes.

It is reasonable to conclude that despite some simplifications of classical modal analysis, the presented method gives us a rough indication of the dynamic properties of composite plates. It seems that this method can avoid a complex modal analysis with

materiala, usmeritve jedra in polnila pri načrtovanju modulnih nosilnih delov strojev ter naprav, saj se izognemo zapleteni oblikovni analizi s problematičnim sinusnim vzbujanjem.

problematic sinusoidal excitation and significantly simplify the selection of flatsheet thickness, material, core orientation and filling during the construction of composite supporting parts of machine tools and devices.

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