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MATHEMATICAL MODEL FOR AN Al-COIL TEMPERATURE CALCULATION DURING HEAT TREATMENT

MATEMATIČNI MODEL ZA IZRAČUN TEMPERATURE V Al-KOLOBARJU MED TOPLOTNO OBDELAVO

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This paper presents the implementation of a mathematical model for an Al-coil's temperature evolution during heat treatment in a forced-circulation furnace. The coil's spatial-temperature evolution is calculated using the finite-difference method in the axial and radial directions, i.e., 2 dimensional. The model is verified by measuring the coil's temperature during reheating using 10 pre-installed thermocouples arranged in two lines of 5 thermocouples, each at a different coil radius. The sensitivities of the furnace-air temperature and the initial-coil temperature on the time to switch are determined, i.e., –52.5 s/°C and –55.5 s/°C, respectively. The mathematical model proved to be a multipurpose tool for different simulation-based reheating studies, while its industrial real-time application offers an even wider range of uses and capabilities: automatic coil-temperature control and observing reheating conditions on an industrial level.

Keywords: Al-coil, temperature, finite difference, mathematical model

Članek obravnava uporabo matematičnega modela razvoja temperature v Al-kolobarju med toplotno obdelavo v peči s prisiljeno cirkulacijo. Razvoj prostorske temperature v kolobarju je izračunan z uporabo metode končnih diferenc v osni in radialni smeri. Model je bil preizkušen z merjenjem temperature v kolobarju med ogrevanjem s predhodno vgrajenimi 10 termoelementi, razporejenimi v dveh vrstah po 5 na dveh različnih premerih kolobarja. Ugotovljena je bila občutljivost temperature atmosfere v peči in začetne temperature v kolobarju v odvisnosti od peči: –52,5 s/°C oziroma –55,5 s/°C. Matematični model se je izkazal kot večnamensko orodje za različne simulacije ogrevanja, medtem ko industrijska uporaba v realnem času ponuja še širše področje uporabe in zmogljivosti: avtomatsko kontrolo temperature kolobarja, opazovanje razmer pri ogrevanju na industrijskem nivoju.

Ključne besede: Al-kolobar, temperatura, končne diference, matematični model

1 INTRODUCTION

The description of physical processes using appropriate mathematical models is nowadays achieving practical usefulness in different fields.[1,2] One of these fields are certainly models for describing heat transfer, especially in metallic materials, where the metal's temperature drives, determines or triggers most material-transformation processes. Coil temperature is one of the most important reheating parameters for Al-coils. The precision of the reheating process is therefore limited by the precision of the coil's temperature data. The coil's temperature data can be obtained by measuring or from a calculation using mathematical models. Nowadays, furnace control systems optionally provide a control method known as the 'air-to-work ratio control system'.[3] This method uses two pairs of thermocouples to measure the 'work–coil' and 'air' temperatures and calculates the furnace-temperature set-point as a scaled difference of both. The method is efficient, but requires a proper measuring coil temperature. For this method, a hole is drilled into the coil and a thermocouple is mounted into it. If the location of the hole is correct and representative for the whole-coil temperature, the method is accurate. But the drilled hole represents a potential defect in the final strip. In any case, measuring the temperature of every coil in industry is not practical. A much more practical and non-destructive technique is to employ a mathematical model (MM) for the calculation and prediction of coil temperatures. For accurate MM coil-temperature predictions, (1) the measured air temperature in the furnace (which all furnaces already measure and use), (2) the coil-alloy and (3) the coil-dimension data are required. The rest of the required information is integrated into the MM during the development and calibration procedure. Thus the MM can be used instead of the measuring coil's temperature, but it should be capable of running in real-time. Another advantage of the MM for the coil's temperature is that the model can be used for optimizations and various other services, which are impossible without a MM. The fastest reheating of Al coils in reheating furnaces is achieved by an overshoot of the furnace-air temperature above the coil temperature.[3] The time from beginning of the reheating process until the time of decreasing the furnace temperature from T_{ot} is denoted as the time to switch, i.e., t_s (**Figure 1**).

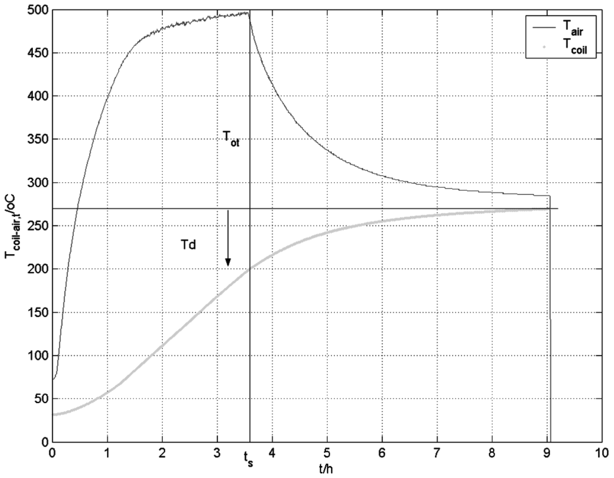

Figure 1: Furnace over-temperature; time to switch t_s
Slika 1: Višja temperatura zraka v peči; čas preklopa t_s

The aim of the work was to quantify the influence of the furnace over-temperature and the initial temperature of the coil on the time to switch (t_s) using a mathematical model.

2 MATHEMATICAL MODEL OF THE COIL TEMPERATURE

2.1 2D model of the Al-coil temperature using the finite-difference method

Heat transfer in solid state is mathematically expressed using the diffusion equation:

$$\frac{\partial T}{\partial t} = a\nabla^2 T;\ a = \frac{\lambda}{c_p \rho} \tag{1}$$

where T is the temperature, t is the time, λ is the thermal conductivity, c_p is the specific heat and ρ is the density. The temperature field is calculated in two dimensions, i.e., the radial r and the axial x directions.

Equation 1 for the selected r and x coordinate directions in a cylindrical coordinate system can be written as:

$$\begin{aligned}&\rho c_p(T)\frac{\partial T}{\partial t} = \\ &= \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial r}\left(\lambda(T)\frac{\partial T}{\partial r}\right) + \lambda(T)\frac{1}{r}\frac{\partial T}{\partial r}\end{aligned} \tag{2}$$

where the heating conditions in the angle ϕ direction are assumed to be symmetric and thus the second derivative $\partial^2 T/\partial\phi^2 = 0$ is zero. In equation 2, λ is a function of temperature, while the temperature field as well as the convective boundary conditions are not constant. In fact, the boundary conditions for equation (2) are a function of the measured values of the furnace-atmosphere-gas temperature and the equation is thus analytically unsolvable. A handy way for numerically solving eq. 2, bearing in mind that the boundary conditions (furnace temperatures) for real-time operation will be available (measured) one per calculation period, is the explicit method of finite difference.

For the used finite-difference method the calculation space is discretized in both time and space:

$$\{\Delta t(\text{sec}), \Delta x(\text{m}), \Delta r(\text{m})\} = \left\{1, \frac{21}{x}, \frac{21}{r_z - r_n}\right\},$$

so that the stability condition is satisfied[1] within the proposed coil dimensions and sample time. For each point on the grid, eq. 2 is rewritten as

$$\begin{aligned}&\rho c_p \frac{\Delta x \Delta r}{\Delta t}\left(T_{i,j}^{t+\Delta t} - T_{i,j}^{t}\right) = \frac{\lambda_x \Delta r}{\Delta x}\left(T_{i-1,j}^{t} - T_{i,j}^{t}\right) + \\ &+ \frac{\lambda_x \Delta r}{\Delta x}\left(T_{i+1,j}^{t} - T_{i,j}^{t}\right) + \frac{\lambda_r \Delta x}{\Delta r}\left(T_{i,j-1}^{t} - T_{i,j}^{t}\right) + \\ &+ \frac{\lambda_r \Delta x}{\Delta r}\left(T_{i,j+1}^{t} - T_{i,j}^{t}\right) + \frac{\lambda_r \Delta x}{\Delta r}\frac{1}{r}\left(T_{i,j+1}^{t} - T_{i,j}^{t}\right)\end{aligned} \tag{3}$$

According to the literature data,[4] the thermal conductivity of the Al-coils differs in the r and x directions due to the air-gap between the strip wraps and the thin oxide-layer of the strip surface. Thus, the thermal conductivities λ_r and λ_x are considered different, where λ_x equals the alloy data, while λ_r is lower then λ_x due to the air gap and the oil films between the coil wraps, which decreases the thermal conductivity in the radial direction. From eq. 3, the temperature at time $t + 1$ for each element on the grid (i, j) is expressed as:

$$\begin{aligned}&T_{i,j}^{t+\Delta t} = \frac{\lambda_x \Delta t}{c_p \rho}\left(\frac{T_{i-1,j}^{t} + T_{i+1,j}^{t} - 2T_{i,j}^{t}}{\Delta x^2}\right) + \\ &+ \frac{\lambda_r \Delta t}{c_p \rho}\left(\frac{T_{i,j-1}^{t} + T_{i,j+1}^{t} - 2T_{i,j}^{t}}{\Delta r^2}\right) + \\ &+ \frac{\lambda_r \Delta t}{c_p \rho}\frac{1}{r}\left(\frac{T_{i,j+1}^{t} - T_{i,j}^{t}}{\Delta r}\right) + T_{i,j}^{t}\end{aligned} \tag{4}$$

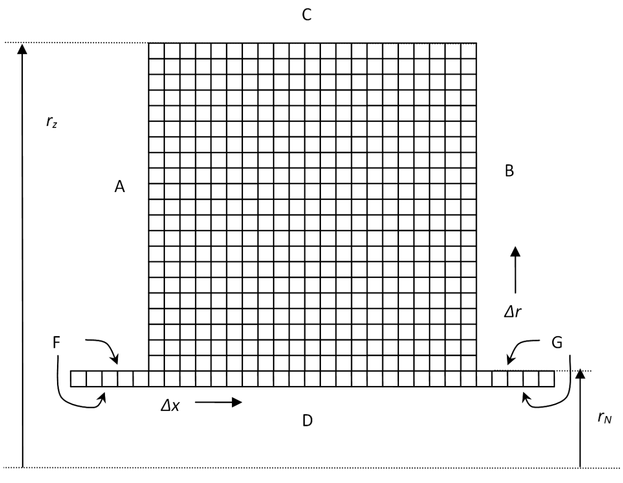

Figure 2: Coil dimensions, discretization grid, boundary lines and indexation of x–r cross-section
Slika 2: Dimenzije kolobarja, diskretizacijska mreža, robne ploskve in indeksiranje x–r prereza

Eq. 4 is recursive and is used for inner points of the calculation grid, while for the boundary points the equation is modified to consider the convective boundary conditions.[3]

2.2 Model calibration

The thermo-physical data of around 40 alloys were obtained using JMatPro software ver.4.0: ρ, c_p and λ in the desired temperature range. The MM was calibrated to the measured temperature values in 10 points, i.e., 5 points close to the outer radius r_z and the remaining 5 on the radius, which is in the middle of the coil's length. The air temperature in the furnace (boundary condition) was measured above the measured coil. The temperature of the heat treatment for Al-alloys rarely exceeds 550 °C, and thus convective heat transfer dominates the heat-transfer mode. The conductive heat-transfer mode is not present due to the stage design, while the radiative heat-transfer thermal mode is estimated to be sufficiently low compared to the conductive mode. The maximum ratio of the radiative-to-convective heat flux $\dot{q}_r/\dot{q}_c = A\varepsilon F\sigma(T^4_{\text{air}} - T^4_{\text{coil}}) / Ah_c(T_{\text{air}} - T_{\text{coil}})$ is estimated to be ≈ 0.031. For a furnace air temperature that is 5 K above the coil temperature of 550 K (close to the end of the reheating) this ratio is estimated to be ≈ 0.025. For the estimation, the following assumptions are considered: ε = 0.09, F = 1, h_c = 150, T_{air} = 823 K, T_{coil} = 293 K. The radiation heat transfer is thus neglected. Note that a forced-air-circulation furnace is studied. The calibration is made by changing the convective heat-transfer coefficient h_c on the boundary surfaces of the calculated cross-section area – see the lines of A, B, C, D, F and G in **Figure 2**, until a calculated and measured temperature match is achieved. When the calculated and measured temperature profiles match at this point, it can be concluded that the model is capable of describing the heat transfer for the considered conditions ($T_{\text{air}} = f(t)$, $h_c = f(T)$). To check the possible variability of the process, additional coil-temperature measurements were performed at the same position in the furnace and with the same coil, but with a modified $T_{\text{air}} = f(t)$ profile. Unfortunately, the reheating conditions for the measured coil were slightly modified (closed steel-coil on both sides) due to studies for improving the homogeneity conditions. The boundary conditions on the inner coil surface –D were therefore modified: h_c drops from 22 to 5. The model's validation is therefore performed with a modified h_c on boundary D and this is compared to the temperature measurement obtained with the closed steel-coil. A comparison is shown in **Figure 3** and it can be seen that the maximum absolute difference in the coil temperature is around ± 10 °C. However, for a strictly correct model validation, the process condition should remain intact, so that h_c would remain intact during the validation as well.

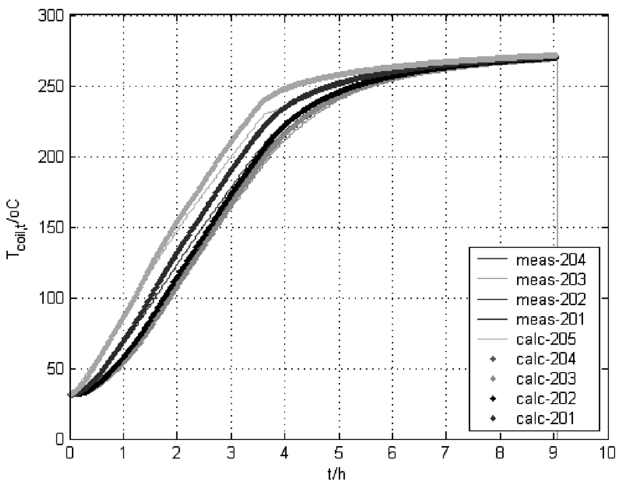

Figure 3: Comparison of measured and calculated coil temperatures. Note that the validation temperature measurements were obtained for slightly modified conditions on surface D.

Slika 3: Primerjava merjenih in izračunanih temperatur kolobarjev. Validacijske meritve temperature so bile merjene pri spremenjenih robnih pogojih na površini D.

3 USE OF THE MATHEMATICAL MODEL

3.1 Off-line calculations of the re-heating procedures

The MM was firstly applied for the prediction of the Al coil's temperature evolution for modified reheating conditions, e.g., modification of the desired end-temperature of the Al coil and the task was to modify the furnace time and temperature in such a way as to obtain a specific temperature homogeneity of the coil at the end of the treatment. To perform such a task, the boundary conditions – furnace air temperatures – need to be predicted (**Figure 4**).

This is achieved by employing another model, which predicted the furnace-air-temperature, closed-loop response, where the inputs are the furnace-temperature set-points. A first-order system[5] with a different time constant for increasing/decreasing the furnace temperatures is used to mathematically express the relation. The furnace temperature model is verified on 10 furnace-temperature profiles for various charging data (coil total mass from 22–78 t, 9 alloy grades, etc.). Due to the uncertainty of the model-obtained furnace air temperature (boundary conditions), the calculated coil tempera-

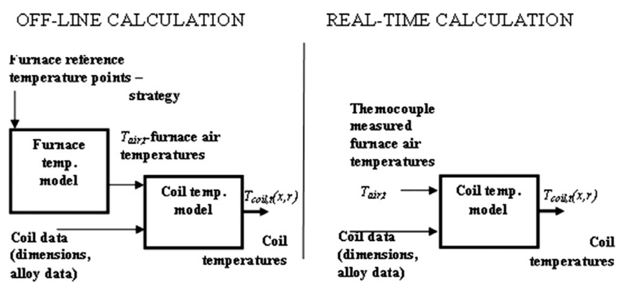

Figure 4: Furnace air temperature predictions for off-line calculations substitute measured temperatures, available in real-time calculations

Slika 4: Izračunane temperature zraka v peči v "off-line"-načinu nadomeščajo merjene temperature zraka, ki so v "real-time" načinu na voljo

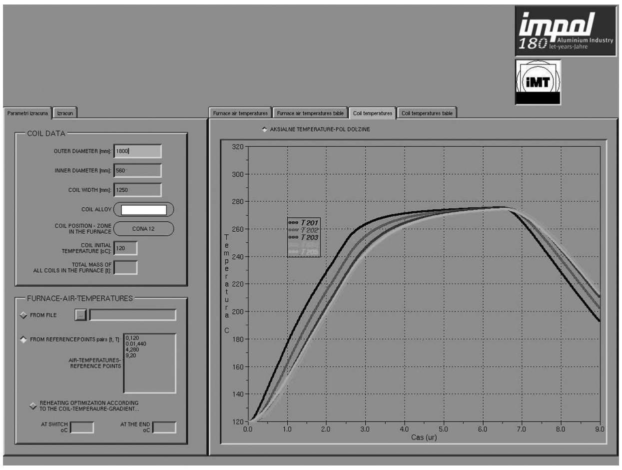

Figure 5: GUI for offline calculations of reheating procedures
Slika 5: Grafični uporabniški vmesnik za izračune ogrevanja kolobarjev v peči

ture for these boundary conditions decreases the accuracy of the coil temperatures compared to the measured boundary conditions. Note that the prediction-accuracy of the furnace-air temperature is ± 10 °C. The crucial benefit of the model-obtained furnace-temperatures (boundary condition) combined with the coil-temperature model is the capability for off-line simulations involving various conditions and parameter sets (coil dimensions, alloy data, coil initial temperature, furnace temperature profile during reheating, etc.), but at the cost of a slightly decreased coil-temperature accuracy. For the off-line calculations a Graphical User Interface (GUI) is provided (**Figure 5**).

3.2 Quantification of the furnace-air temperature and the initial coil temperature for the coil-temperature evolution

A frequent task in the Al-coil reheating process is the modification of the reheating parameters (furnace time and temperature) according to a specific variation of the parameters, e.g., variation of the initial coil temperature of the Al coils or a higher furnace over-temperature T_{ot}. The coil with an increased initial temperature reaches the desired temperature faster. A straightforward solution to such a problem is to employ the developed mathematical model, including a furnace-temperature model (**Figure 4**) and change the reheating parameters until the conditions are met (trial-and-error method). Another way is to quantify in advance the response of a certain parameter change on the reheating parameters. For industrial production, the quantification of such reheating shortening is beneficial. For each furnace over-temperature T_{ot} = (400, 420, 440, 460, 480) °C in set and independently for each initial coil temperature $T_{coil,0}$ = (0, 20, 40, 60, 80, 100, 120, 140, 160) °C, a simulation is performed and the resulting time to switch t_s is determined from the model results (**Figure 6**).

The t_s is actually determined by the model during the simulation, since the model also calculates the $T_{air,t}$ curve (**Figure 3**). The criterion to start decreasing the air temperature in the simulation model is when the Al-coil temperature in the node $T_{coil,t}$ (10, 10) is T_d = 30 °C under the desired final temperature of the coil, in this case (280–30) °C = 250 °C. Note that, different T_d values lead to different reheating profiles. Therefore, T_d is adjustable through the GUI. The node (10, 10) is in the middle of the coil (**Figure 2**). The other simulation parameters are constant: $T_{coil,0}$ = 20 °C, $T_{air,0}$ = 20 °C, coil dimensions r_z = 900 mm, r_N = 280 mm, x = 1250 mm, alloy EN 8079 AlFe1Si. The simulation is repeated for each T_{ot}. The obtained values are presented in **Figure 6**. The relation between T_{ot} and t_s is almost linear with a coefficient of –52.5 s/°C around the working point T_{ot} = 440 °C. The result shows that the increment of the furnace over-temperature for a single °C means t_s is shorter by 52.5 s. And conversely, the reduction of the furnace temperature by a single °C means a longer t_s for 52.5 s.

In the same way we estimated the influence of the initial coil temperature $T_{coil,0}$ on t_s. The relation between the initial coil temperature $T_{coil,0}$ and t_s is presented in **Figure 7**. The time to switch t_s is fairly linear, with a

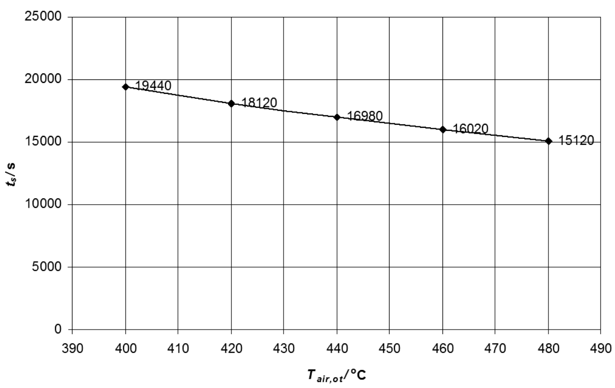

Figure 6: Influence of furnace over-temperature on the time to switch t_s
Slika 6: Vpliv višje temperature v peči na čas preklopa t_s

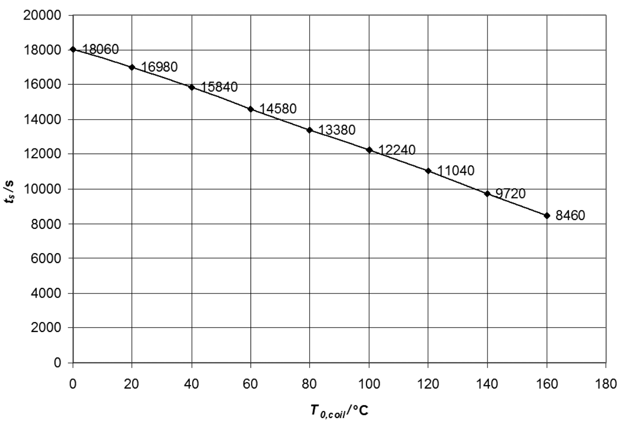

Figure 7: Influence of initial coil temperature $T_{coil,0}$ = (0 : 20 : 160) °C on the time to switch t_s
Slika 7: Vpliv začetne temperature kolobarja $T_{coil,0}$ = (0 : 20 : 160) °C na čas preklopa t_s

coefficient of –55.5 s/°C of the initial coil temperature. The simulation results show that an increment of +1 °C for the initial coil temperature means a shorter time 55.5 s to switch and, conversely, a reduction of –1 °C of the initial coil temperature means a longer time 55.5 s to switch.

3.3 Potential use of the model modified for a real-time calculation of the coil temperature

To run the developed model in real-time, accurate coil charging data, accurate air temperature in the furnace combined with the presented mathematical model are needed and thus provide the coil-temperature data without measuring the temperature in the coil. Note that the used explicit finite-difference method for the conduction calculation in the coil (eq. 4) is suitable for a real-time calculation, the only modification of the method is that the real-time simulation is interrupted after every iteration until the next measured furnace-air temperature is delivered to the model.

Unequal coil alloys, dimensions, initial temperature, malfunctions in the furnace and burners all lead to more or less unknown coil temperatures in the situation without either the coil temperature measurements or the real-time coil-temperature calculation. The MM obtained coil temperatures in such a situation are crucial for proper operator decisions and can be upgraded in the automatic coil-temperature control system. Furthermore, the stored real-time-calculated coil-temperatures can be used as documentation for coil-customer claims, research and development purposes. Real-time operation of the model is underway.

4 CONCLUSION

A mathematical model for an Al-coil temperature evolution during heat treatment proved to be an efficient and multi-purpose backbone tool for advanced planning, control and documentation of the Al-coil heat-treatment process. Employing the mathematical model, which provides coil-temperature knowledge, offers a simple, cost-effective and punctual tool for the determination of the proper furnace-temperature time settings during modified Al-coil reheating conditions.

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