IZVIRNI ZNANSTVENI ČLANEK – ORIGINAL SCIENTIFIC PAPER

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PRICE DETERMINATION IN A SIMPLE MARKET WITH AND WITHOUT SHORT SALES

Določanje cen na enostavnem trgu ob prisotnosti oziroma odsotnosti kratke prodaje

Abstract

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Miller (1977) argued that prohibiting or constraining short sales causes securities to be o verpriced because traders with low estimates of value do not participate in the market. Markets with short sales should, therefore, price securities correctly because all traders participate. This paper shows that Miller's (1977) result is not necessarily true. Even in the absence of short sales, securities may be overpriced, underpriced or correctly priced if bidders bid their estimate of the security's value. On the other hand, if short sales are allowed, the market will consistently exhibit underpricing. We also show how bidders may adjust their bids to account for the presence of the bias.

Key Words: Short sales, market microstructure, auctions.

Izvleček:

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Kot trdi Miller (1977), je rezultat prepovedi oziroma omejevanja kratke prodaje precenjena vrednost vrednostnih papirjev, saj trgovci, ki nižje vrednotijo le-te, ne sodelujejo na trgu. Trgi, na katerih je prisotna kratka prodaja, morajo torej določiti pravilno ceno vrednostnih papirjev zaradi sodelovanja vseh trgovcev. Pričujoči prispevek nakazuje, da rezultati raziskav, ki jih je opravil Miller (1977), nujno ne držijo. Tudi ob odsotnosti kratke prodaje so vrednostni papirji lahko precenjeni, podcenjeni ali ocenjeni na ustrezno vrednost, če ponudniki tržijo vrednostne papirje s svojo oceno njihove vrednosti. Kadar je kratka prodaja dovoljena, trg sistematično izkazuje podcenjenost. V članku tudi predstavljamo način, kako lahko ponudniki ustrezno prilagodijo svoje ponudbe v primeru omenjene pristranskosti.

Ključne besede: kratka prodaja, mikrostruktura trga, dražbe.

JEL Classification: G10, G12, G14

1 Introduction

In a frequently cited article Miller (1977) suggested that constraints on short sales can bias market prices of security upwards. According to Miller (1977) if short sales are costly or prohibited, the information held by potential short sellers is not reflected in the market price. Because only the opinions of the most optimistic traders are reflected in the price, it must be upward biased. The argument apparently presumes that all traders bid their best guess of the market price irrespective of the number of other participants. Miller (1977) implies that, in a market with unconstrained short sales, traders bidding their best guess would lead to an unbiased price.

This paper demonstrates a simple model in which:

1. Contrary to what Miller (1977) argues, the price in a simple market may be above, at or below intrinsic value when short sales are not permitted. The nature of the winner's curse depends on the number of units offered and the number of traders.
2. On the other hand, bias does not disappear with the introduction of short sales. Traders bidding their a-priori expected values lead to prices below the security's intrinsic value. Unless traders bid an amount *greater* than the *a-priori* expected value the market will *underprice* shares.
3. For a multi-unit auction both with and without short sales there are simple adjustments a bidder can make which offset the winner's curse.
4. The riskiness of the price, in a certain sense, depends on whether short sales are permitted and prices may be either more or less risky when short sales are allowed.

I use some basic results from auction pricing theory. There appears to be no prior examination of the properties of auctions, or of the winners curse in particular, when short sales are allowed. Nor is this surprising. For most applications of auction theory: oil leases, spectrum, etc. no short sales are possible. In security markets, however, short sales are common, although the use of short sales is frequently criticized, most especially by those whose securities are being shorted. On the other hand Miller (1977) and others argue that prohibiting short sales prevents valuable information from reaching the market. It is worthwhile, therefore, to examine what theory says about the impact of short sales on prices.

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Section 2, below summarizes the history of the winner's curse and its relation to pricing and short sales. Section 3 introduces the model and applies it to markets with and without short sales when traders naively bid their best estimate of the price. Section 4 derives the optimal strategic bid for traders who wish to avoid the winners curse. Section 5 examines the role of short sales in reducing (or not) uncertainty about value and section 6 is a conclusion.

2 The Winner's Curse and Short Sales

The literature discussing phenomenon of the winner's curse goes back to Capen, Clapp and Campbell (1971) and bidding for oil contracts. The authors, oil economists, described the problem of bidding when the value of the auctioned object (in this case offshore oil tracts) is unknown. Naive bidding of the estimated value leads to the contract going to the most optimistic bidder—who, on average, will be bidding too great a value. Even if each bidder's estimate is unbiased, the value of the maximum bid is biased upward. Hence the winner's curse—the winner will almost always overpay. The winner's curse is discussed at length in the auction literature (see, for example, Milgrom (1989) for an overview). Kagel and Levin (2002) provide evidence of its existence and persistence in experimental contexts.

Miller (1977, 2000, 200a) in a series of papers has argued that the winner's curse applies in securities markets. If short sales are limited and costly, only the most optimistic investors will purchase securities and only their valuations will be reflected in security prices. Duffie, Garleanu, and Pederson (2002) present a somewhat more technical model in which searching for securities to short and paying a lending fee leads to higher prices. Chen, Hong, and Stein (2002) also model a market with limited short sales. They predict that when breadth of ownership is low, prices should be high relative to fundamentals. They find, empirically, lack of breadth is associated with undeperformance. Further empirical work by Jones and Lamont (2002) shows that stocks which were most expensive to short during the period from 1926 through 1933 had high valuations and low returns.

Since options can be used to substitute for short sales there has been some interest in examining the impact of the introduction of option trading on security sales. Figlewski and Webb (1993), Ofek Richardson and Whitelaw (2004), and Danielsen and Sorescu (2001) find evidence that options trading seems to be a substitute for short sales in mitigating mis-pricing of securities.

The argument for mispricing depends upon the existence of a variety of opinions about price. Diether, Malloy and Scherbina (2002) show that dispersion of analysts' forecasts about future earnings has predictive power with respect to future returns.

In a somewhat related article Cohen, Diether, and Malloy (2007) show that shifts in shorting demand predict future stock returns.

3 Market Prices With and Without Short Sales

We now look at a small model which will demonstrate that the simple conclusions in the previous literature don't hold, even in rather straightforward circumstances, when multiple units are being sold. We will see that price may be biased up or down even without short sales and that the existence of short sales will bias the price downward. We will first examine the no short sale case, then introduce short sales, and finally costly short sales.

3.1 No Short Sales

As Miller (1977) noted, the existence of the winner's curse problem is well established. If all traders in an auction receive noisy but unbiased signals of the value of a single unit of an auctioned security, and if all traders bid the value of their signals, the winning trader will probably find that the price paid exceeds the value of the security. This result does not necessarily extend to markets in which multiple units are being sold. We will examine this with a very simple multi-unit auction market dating back to Vickrey (1961).

Suppose that there are N traders who are competing to purchase m ($m < N$) units of a security. Each trader may buy only one unit of the security[1] . Each security will have a payoff of V euros. Think, perhaps, of a set of m envelopes each of which contains V euros in cash. V is unobservable, but each trader gets a noisy signal of the value. For trader i the signal is $s_i = V + \varepsilon_i$ where ε_i is a noise term unique to trader i and drawn from a uniform distribution with a range of $-\varepsilon$ to $+\varepsilon$. Thus, the signal, though noisy, is unbiased. The traders are risk-indifferent and the interest rate is zero. They would, if the actual payoff of the asset were known, be willing to buy it for exactly V. Instead, the traders must submit limit buy orders in a market, each giving the value which is the maximum which the trader would pay for a unit of the security given the signal received. The top m bidders will each receive one unit and will pay the mth highest bid price. In auction-theory parlance we have a multi-unit, common-value, sealed-bid auction in which traders are constrained to purchase only one unit. Common value in this context means that all participants will get the same (but, as yet, unknown) benefit, V, from winning the auction—in contrast, for example, to an art auction in which the desirability of a painting would be different to different bidders. Vickrey's (1961) model, though very similar in structure to this one was not a common-value model.

3.1.1 A Single Share Sold

Consider first the situation, already well analyzed in the winner's curse literature, in which $m = 1$. If traders naively bid their signals, the price for the unit will be the value of the highest signal. If N is large, this will almost certainly be greater than V—hence the winner's curse.

[1] This is significant constraint on the model. Multiple unit auctions where buyers can purchase more than one unit can yield interesting results. The price, however, is substantial additional complexity to the model (see, e.g. Tenorio (1999)).

The expected value of the price will be the expected value of the Nth order statistic of the bids[2]. The kth order statistic from a sample of size N drawn from a standard uniform distribution (i.e. a uniform distribution between 0 and 1) takes a beta distribution with parameters k and $N+1-k$. The mean of the kth order statistic, $x_{(k)}$, from N draws is, therefore:

$$E\left[x_{(k)}\right] = \frac{k}{N+1} \tag{1}$$

And its variance is:

$$\sigma^2 x_{(k)} = \frac{k(N+1 - k)}{(N+1)^2(N+2)} \tag{2}$$

The signals are drawn from a uniform distribution ranging from $V-\varepsilon$ to $V+\varepsilon$, a range of 2ε. If all traders bid their signals, the expected value of their winning bid for the single share on offer is, therefore:

$$E[p] = V + \frac{N-1}{N+1}\varepsilon \tag{3}$$

with variance:

$$\sigma_p^2 = \frac{N}{(N+1)^2(N+2)}4\varepsilon^2 \tag{4}$$

This was first shown, apparently, by Vickrey (1961). The amount of the winner's curse is given by the term in equation (1). The greater the number of traders and the greater the variability of the signal, the greater is the winner's curse.

3.1.2 Multiple Shares Sold

Suppose now that m shares are available. Again, each trader will bid for one share and the bid will equal the trader's signal. The market clearing bid will be the $N-m+1$ order statistic. This will have expected value:

$$E[p_{N-m+1}] = V + \frac{N-2m+1}{N+1}\varepsilon \tag{5}$$

with variance:

$$\sigma_{p\,N-m+1}^2 = \frac{(N-m+1)m}{(N+1)^2(N+2)}\,4\varepsilon^2 \tag{6}$$

The winner's curse term in (6), $\frac{N-2m+1}{N+1}\varepsilon$, is an increasing function of N and a decreasing function of m. Note that the sign of the winner's curse term is ambiguous. If $m > \frac{N+1}{2}$ the bias is negative and the winner's curse turns into a winner's blessing. The expected price is less than V, the value of the security. The marginal investor's bid is in the lower half of the distribution of bids and, therefore, there will be a downward bias to the price. Thus, even in a world in which short sales are prohibited and traders naively bid their signal, it is not clear that there is an upward bias to price. If a sufficiently large amount of the security is on offer the price will be biased downward.

3.2 Short Sales

Suppose now that short sales are possible. In a short sale the short seller's profit or loss is the sale price, p, less the realized value of the security, V. In this model we assume that each trader will buy *or sell* one unit of the security and, furthermore, they will buy at any price below their bid and sell short at any price above their bid. Therefore, every trader will be either long or short after the auction. We don't worry about the ability to borrow the security. This is something like "naked shorting" see Culp and Heaton (2007) or a bucket shop.

Consider, as a starting point, a forward market. Every long position must be offset by a short position. If all traders bid the value of the signal they receive the market clearing price will be close to the median signal value which should be very nearly an unbiased estimate of the true value of the traded good. In this case, the winner's curse disappears. If the number of traders is odd, there will be $\frac{N-1}{2}$ traders long and the same number short. The price should be the bid of the trader who is neither long nor short–the median of the sample of signals.[3]

Now suppose some quantity of the security is also thrown into the bidding. In other words, some buyers will actually get the security while others will get a payment equal to the difference between the security value and the market-clearing price. The long traders will have a payoff of $V-p$ where p is the market clearing price. The short traders will receive or pay $p-V$. For N traders and m units offered, if N and m are both even or both odd and if every trader can only buy or sell one unit there will be $m + \frac{N-m}{2} = \frac{N+m}{2}$ traders long and $\frac{N-m}{2}$ traders short. m long traders will have a claim on the securities and $\frac{N-m}{2}$ traders will have long claims against the $\frac{N-m}{2}$ short traders. The market-clearing price is the $\frac{N-m}{2}+1$ order statistic of the distribution of prices quoted—the lowest price bid by a long trader. If the traders bid their signals, this will also be the order $\frac{N-m}{2}+1$ statistic of the signals.

The expected value of the price is:

$$E[p] = V + \frac{1-m}{N+1}\varepsilon \tag{7}$$

2 The definition of order statistic in the auction literature is sometimes different than that in the statistics literature. In statistics the first order statistic is the smallest number, the second order statistic is the second-smallest number and the nth order statistic is the nth from smallest number in the data. In the auction literature the notation is sometimes reversed so that the first order statistic is the largest number in the data set. In this paper we use the statistics norm: i.e. the higher the order statistic the larger the number.

3 If there is an even number of traders, $\frac{N}{2}$ traders will be long and the same number will be short. The price will be approximately the $\frac{N}{2}$ th order statistic of the set of signals. The median of the prices (mean of $\frac{N}{2}$ and $\frac{N}{2}+1$ order statistics) will clear the market as will any price between the two statistics.

and the variance is:

$$\sigma_p^2 = \frac{(N-m+2)(N+m)}{(N+1)^2(N+2)}\varepsilon^2 \tag{8}$$

The winner's curse term is now negative for $m > 1$ and the winner's curse once again becomes a winner's blessing. The long traders would profit at the expense of the shorts. Contrary to Miller (1977), prices in a market with unconstrained short sales would be biased if traders bid their signals. The greater is m, the greater the bias. The greater is the uncertainty about the signal the the greater is the bias and the larger the number of traders, the smaller is the bias.

Thus, for markets in which short sales are prohibited prices may be biased either up or, if the number of shares to be auctioned is large relative to the number of traders, down. In markets with short sales prices are always biased down with the size of the bias depending on the size of the offering relative to the number of traders. In all cases, the determining factor for pricing is the marginal investor's bid. If the marginal investor is the median investor, there is no bias. If the marginal investor is below the median the price will be biased downward and if the marginal investor is above the median the price will be biased upwards.

3.3 Limited Short Sales

The notion of short sales used in the previous section was extremely strong—anyone who is not long must be short. Suppose that there are limits to short sales. It is well known that some stocks are difficult to short because of a limited supply of shortable stock. Consider, therefore, a modification of the previous model in which the number of share which can be shorted is limited to $S < \frac{N-m}{2}$ shares.

In this case, the number of traders holding long positions would be $m+S$ and the expected market clearing price would be given by the expected value of the $N-(m+S)+1$ order statistic:

$$E[p] = V + \frac{N-2(m+S)+1}{N+1}\varepsilon \tag{9}$$

As in the multiple share case, the sign of the bias term is may be positive or negative. In this case the bias is positive if:

$$m + S > \frac{N+1}{2} \tag{10}$$

and negative otherwise. The larger the number of shares which are available for sale, the lower the price.

3.4 Costly Short Sales

Alternatively, suppose short sales are costly. Suppose, in other words, that there are costs to short sales which are not incurred by stock purchasers. If there is a cost of c euros to sell stock short, a trader with a signal of will be willing to buy at , but will only go short at . In other words, there is a bid-ask spread. There will, therefore, be some traders who are neither long nor short. If the market-clearing price is p, all traders who receive a signal between p and $p-c$ will be out of the market. Given the uniform distribution of signals the expected number of traders who are neither long nor short is $N\frac{c}{2\varepsilon}$; the expected order statistic for the price is:

$$N - m - \frac{N - (m + \frac{c}{2\varepsilon}N)}{2} + 1$$

and the expected value of the price is

$$E[p] = V + N\frac{c}{2(N+1)} + \frac{1-m}{N+1}\varepsilon \tag{11}$$

Note that the difference between this price and the price with zero-cost short sales is an amount that depends on the cost of shorting and the number of traders, but not the uncertainty about the signal.The price is driven up because fewer shares are available for short sale. The traders whose signals lie in the interval between $p-c$ and p are not shorting. The bias term is indeterminate, but increasing in c.

If there are a fixed number of shortable shares as well as a cost to shorting the price will be the same as in the case where there was a fixed number of shortable shares and no trading costs. The price is determined by the number of shares traders can buy, and in this model it will be unchanged by trading costs. There may be different traders shorting, but the price will be the same.

4 Rational Bidding Adjustments

As Figlewski (1981) has pointed out, if a trader knows that winning an auction implies an expected loss, a rational trader will adjust the bid price to reflect the potential for loss. In other words, the greater the number of competitors or the greater the uncertainty about the true value of the security the lower will be the amount bid relative to the unbiased signal. Rational bidders' bids in markets without short sales should be consistently below the signal value by an amount which reflects uncertainty about value and the number of traders. Bidders can thus compete without expecting to lose money if they win. Rational bidding can offset the winner's curse. Therefore, the market clearing price in a long-only market should, on average, equal the true value of the offered security. In a competitive market with rational traders the winning bid should be an unbiased estimate of the cash value of the security. There should be no upward bias because of the lack of short sales.

4.1 Optimal Bidding for a Single Share—No Short Sales

Consider first the situation, already well analyzed in the auction-theory literature, in which m=1. To avoid the winners curse, the traders must shade their bids. In other words a trader must ask "if I receive the highest signal, by how much will it be biased upward? " The bias only matters if the trader's bid is the winning bid.

Therefore, if only one unit of the security is on offer, the competitive, zero expected profit bid for trader i, b_i, is the signal, s_i, less the expected value of ε_i conditional on the signal being the maximum:

$$b_i = s_i - (\frac{N}{N+1}2\varepsilon - \varepsilon) \quad (12)$$

Thus the well known result (see, e.g. Kagel and Levin (2002)):

$$b_i = s_i - \frac{N-1}{N+1}\varepsilon \quad (13)$$

The term in parentheses in equation (3) adjusts the expected value of the Nth order statistic from N draws (the maximum) from a standard uniform distribution to a uniform distribution with range 2ε and median zero. The bid is less than the signal by the expected difference between the maximum signal and V in a market with N traders. This is the bias term from equation 3 above. Thus, the greater the number of traders and the greater the uncertainty about the value the greater will be the amount by which a trader must shave the bid in order to avoid the winner's curse, while still having a chance to win.

4.2 Optimal Bidding with Multiple Shares and no Short Sales

Suppose now that m is greater than one. The winning bid will be the $N–m+1$ order statistic of the bids (i.e. the $N–m+1$th number starting from the minimum). The trader will bid the signal less the bias term from equation (5), above:

$$b_i = s_i - \frac{N-2m+1}{N+1}\varepsilon \quad (14)$$

The adjustment term is decreasing in m.

Note that if $m > \frac{N+1}{2}$ the amount bid will be less than the signal, if $m = \frac{N+1}{2}$ it is optimal to bid the exact value of the signal and if $m > \frac{N+1}{2}$, the equilibrium strategy is to bid *more* than the signal rather than less. In other words, the price may be shaded either up or down, relative to the signal, depending on the quantity on offer.

If $m < \frac{N+1}{2}$, traders might prefer to be in a world in which everyone bid their signal. In a world where traders did not bias their bids upward the winner would, on average, earn a pure profit. But all traders would be motivated to increase their bids to increase the probability of winning until they are bidding the amount in equation (14) and a zero expected profit equilibrium is reached.

Although the analytics of this model with ε assumed to be drawn from a uniform distribution are easy, a similar result holds for any symmetric, zero mean distribution for which the expected value is defined–e.g. the normal. Symmetry implies that the median equals the mean. Depending on whether $m - \frac{N+1}{2}$ is positive or negative, the expected value of the bias will be negative or positive. In a zero-profit equilibrium, the traders must make bids which are larger or smaller, respectively, than the value of their signals. For most distributions there is no simple, closed-form analog to equation (14), but similar results can be determined numerically.

4.3 Optimal Bidding with Short Sales

If short sales are possible, once again, traders have an incentive to fade their bids *upwards.* The resulting market equilibrium bids will be the signals adjusted upwards by a value determined by the expected value of the $\frac{N-m}{2}+1$ order statistic.

Using equation (1) we see that the expected value of the $\frac{N-m}{2}+1$ order statistic of the sample of signals, drawing again from a noise distribution running from −ε to +ε is:

$$E\left[s_{(\frac{N-m}{2}+1)}\right] = V + \frac{N-m+2}{N+1}\varepsilon - \varepsilon \quad (15)$$

The analog of equation (13) gives the optimal bid, contingent on signal :

$$b_i = s_i + \frac{m-1}{N+1}\varepsilon \quad (16)$$

It is always necessary to bid *more* than the signal in a zero expected profit equilibrium by an amount given by the bias term in equation (7).[4]

Clearly, the greater the number of traders, the less the bias. The greater the number of units offered, the greater the bias and the noisier the signal, the greater the bias (because the dispersion of bids will be greater).

The equilibrium price will be the same zero-profit price that prevails in the absence of short trading, but traders, instead of quoting a price less than the value of the signal they receive will quote a higher price.

Once again, the fundamental conclusion, that the existence of short sales implies systematic biasing upwards of bids, holds *mutatis mutandis* for any zero expected value, symmetric distribution of signaling errors. Since the winning bid must always be an order statistic less than the median, it must always be necessary to adjust the bid upward by the absolute value of the expected bias. While the analytics of dealing with other distributions would be more complicated, often involving values which may be only determined numerically, the qualitative result would be the same.

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[4] If N is even and m is odd or vice-versa, m traders will end up with orders filled from the stock of securities and $\frac{N-m-1}{2}$ traders will end up with long positions against the same number of short positions. The market price should equal the bid price of the single trader whose order is not filled, i.e. the $\frac{N-m-1}{2}+1$ order statistic of the distribution of bids. The expected value of the signal received by this trader is:

$$\left[s_{(\frac{N-m-1}{2}+1)}\right] = V + \frac{N-m+1}{N+1}\varepsilon - \varepsilon$$

and the market equilibrium bid for trader i is, therefore:

$$b_i = s_i + \frac{m}{N+1}\varepsilon$$

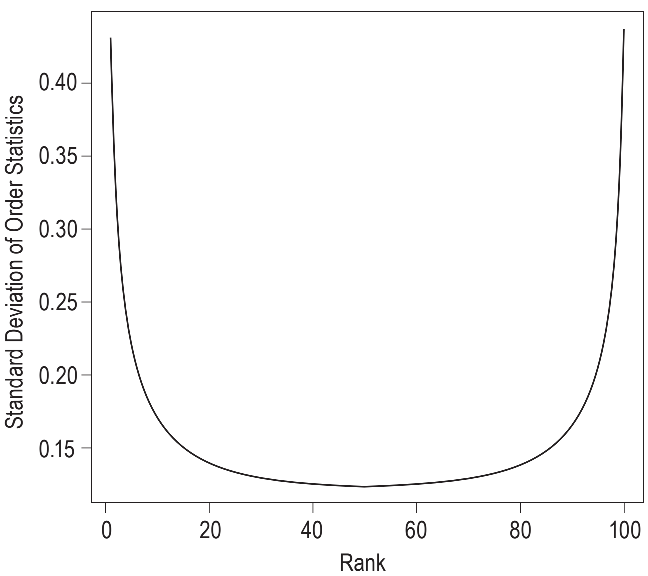
We can contrast the no short sale and short sale conditions. If short sales are prohibited, the bias the traders use to construct their bids depends on the number of shares offered. If $m < \frac{N+1}{2}$ traders will need to bid an amount less than the value of an unbiased signal to avoid falling prey to the winner's curse. For larger values of m the bid should be equal to or greater than the value of the signal.If short sales are permitted, traders should *always* bid an amount greater than an unbiased signal. With and without short sales, The greater the noise in the signal the larger must be the absolute size of the bias imposed on the bid. Without short sales the greater the number of traders the larger is the bias (algebraically, if $m < \frac{N+1}{2}$ it falls in absolute value). If short sales are permitted, the bias is smaller, the greater the number of traders. With no short sales the greater the number of shares of the security offered, the smaller, algebraically, is the bias. With short sales, the greater the number of shares offered, the greater is the bias.

Both with and without short sales, rational behavior on the part of traders will cause the market-clearing price to, on average, equal the realized value of the security. Thus, the possibility of short sales does not resolve in any simple way the mispricing problem. It will still be necessary for traders to adjust their bids.

5 Risk and the Short Sale

Might it be possible that introducing short sales improves markets in another way? If, as Miller (1977) suggests, allowing short sales brings more information to the market, might it not be true that there is less uncertainty about the price? Might the resulting price be, on average, closer to the intrinsic value of the security? The market price, after all, in both the short sale and non-short sale cases is an order statistic from a sample distribution which must be adjusted for bias if the traders make such an adjustment. Sampling error will cause the market price to differ from the intrinsic value–the median of the distribution from which the traders' signals were drawn.

Figure 1: *Standard Deviation of Order Statistics Generated from Normal Distribution. 10,000 sets of 100 random variates were generated from the normal distribution. The 100 order statistics were calculated for each set. The standard deviation of each order statistic was calculated.*



We are interested in the difference between the market-clearing price and the final value V of the security. If traders appropriately shade their bids, the expected value of this difference should be zero, but the realized value may be positive or negative. Consider, therefore, the variance of the difference. Since the bias term and V are non-stochastic, it suffices to consider the variance of the order statistic.

If we continue to use the model in the previous section with a uniform distribution of noise in the signal, the variance is lowest when the expected value of the winner's signal is near the extremes of the distribution. Recall the variance given in equation (6):

$$\sigma^2_{p\,N-m+1} = \frac{(N-m+1)m}{(N+1)^2(N+2)}4\varepsilon^2$$

This function of m has its minima over the range from 1 to N at 1 and N. The maximum is near the middle of the range at $\frac{N+1}{2}$. Note that this pattern does not depend upon whether or not traders shade their bids. Thus, in a market for a single item, prohibiting short sales would have the effect of *reducing* the uncertainty about the winning bid. This occurs because of the upper boundary on the order statistic imposed by the uniform distribution. If we have a large number of traders it will be very likely that the maximum signal will be close to $V+\varepsilon$ and subtracting ε from the signal should give a good estimate of V. On the other hand, if the number of shares being sold is large enough, allowing short sales can reduce the variance of the resulting price by bringing the order statistic closer to its lower limit.

The behavior of the variance in the case of the uniform distribution is not reflected by cases of other distributions. The existence of upside and downside limits causes it to be anomalous. In the more realistic case where the noise in the signal is modeled as a normal distribution without an upper bound the situation is quite different. In this case, unfortunately, there is, apparently, no simple closed-form expression for the value of the standard deviation of the value of a rank-order statistic. It is, however, easy enough to simulate the values. 10,000 sets of 100 random draws were generated from a standard normal distribution. The 100 order statistics were calculated for each set and the standard deviation of each order statistic was calculated.

Figure 1 shows the standard deviations of various order statistics. Note the U shape of the curve with a minimum at the median of the distribution. A market design which sets the price based on extreme signals (e.g. auctioning one unit in a market where short sales are prohibited) will result in substantially greater variability of the price about V than will an auction in which the price is set based on a signal close to the median (e.g. auctioning one unit in a market in which short sales are permitted).

On the other hand, this does not mean that permitting short sales always reduces the uncertainty about the resulting price-value difference. Suppose, for example that $m = \frac{N}{2}$. In this case, in the absence of short sales, the price-determining bid will be close to the median and the standard deviation will be at its minimum. If short sales are permitted, the price determining bid will be close to the 25th percentile and the standard deviation of the value-price difference will be correspondingly greater.

Thus, markets in which short sales are permitted may or may not have prices which are closer to the intrinsic value. The result depends on the structure of the market—the number of traders and the number of securities on offer. The "additional information" provided by the short sellers may actually cause greater uncertainty about the difference between the market equilibrium price and the security's intrinsic value.

6 Conclusion

In a competitive market for securities, it is not the most optimistic traders who set the price—they will be long at any reasonable price. It is not the most pessimistic traders who set the price—they would be short (if short sales are allowed) or out of the market at any reasonable price. It is the trader on the margin who sets the price. Changing rules about short sales can, indeed, change the identity of the marginal trader and, absent strategic bidding, will reduce the price. It will not, however, guarantee that the market will, on average, correctly price securities. The sign of the bias may change, but bias will remain unless traders are sophisticated enough to bid strategically.

In this paper I have presented a simple model of a market in which, even in the absence of short sales, the price may be greater or less than the intrinsic value of the security if traders naively bid their unbiased signals. With the addition of short sales, the bias in the price becomes consistently negative. With the further addition of limitations on short sales, the negative bias is reduced and, again may be positive or negative.

Clearly, in all cases, traders who trade strategically—who attempt to avoid the winner's curse could cause the expected price to equal the value of the security.

The impact of permitting short sales on the uncertainty about the price is ambiguous.

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