

4 Emergent SUSY Theories: QED, SM & GUT

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Abstract. It might be expected that only global symmetries are fundamental symmetries of Nature, whereas local symmetries and associated massless gauge fields could solely emerge due to spontaneous breaking of underlying spacetime symmetries involved, such as relativistic invariance and supersymmetry. This breaking, taken in the form of the nonlinear σ -model type pattern for vector fields or superfields, puts essential restrictions on geometrical degrees of freedom of a physical field system that makes it to adjust itself in such a way that its global internal symmetry G turns into the local symmetry G_{loc} . Remarkably, this emergence process may naturally be triggered by spontaneously broken supersymmetry, as is illustrated in detail by an example of a general supersymmetric QED model which is then extended to electroweak models and grand unified theories. Among others, the U(1) × SU(2) symmetrical Standard Model and flipped SU(5) GUT appear preferable to emerge at high energies.

Povzetek. Avtor dokazuje, da so samo globalne simetrije temeljne simetrije narave, lokalne simetrije in njim pridružena umeritvena polja pa se pojavljajo samo zaradi spontane zlomitve simetrije prostora-časa, kot sta relativistična invarianca in supersimetrija. Avtor uporabi nelinearni model σ za obravnavo sistema z relativistično simetrijo in supersimetrijo. Ugotavlja, da zlomitev relativistične invariance ter supersimetrije bistveno zoži geometrijske prostostne stopnje vektorskih in superpolja, zato se ta odzove s spremembo globalne notranje simetrije G v lokalno simetrijo G_{loc}. Spontano zlomljena supersimetrija lahko naravno sproži ta proces porajanja. Avtor ta spontani prehod ilustrira na primeru splošnega modela supersimetrične kvantne elektordinamike. Ta model razširi tudi na elektrošibki *standardni model* ter na teorije, imenovane "velike teorije poenotenja" (GUT). Pri visokih energijah se pri takih zlomitvah simetrij pojavita prav *standardni model* in "prekucnjen" (flipped) model SU(5).

4.1 Introduction

We all believe that internal gauge symmetries form the basis of modern particle physics being most successfully realized within the celebrated Standard Model (SM) of quarks and leptons and their fundamental strong, weak and electromagnetic interactions. At the same time, local gauge invariance, contrary to a global symmetry case, may look like a cumbersome geometrical input rather than a "true" physical principle, especially in the framework of an effective quantum field theory (QFT) becoming, presumably, irrelevant at very high energies. In this connection,

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one could wonder whether there is any basic dynamical reason that necessitates gauge invariance and the associated masslessness of gauge fields as some emergent phenomenon arising from a more profound level of dynamics. By analogy with a dynamical origin of massless scalar particle excitations, which is very well understood in terms of spontaneously broken global internal symmetries [1], one could think that the origin of massless gauge fields as vector Nambu-Goldstone (NG) bosons are related to the spontaneous Lorentz invariance violation (SLIV) that is the minimal spacetime global symmetry underlying particle physics. This well-known approach providing a viable alternative to quantum electrodynamics [2], gravity [3] and Yang-Mills theories [4] has a long history started over fifty years ago, though has been significantly revised in the recent years [5–8].

4.1.1 An emergence conjecture

Directly or indirectly, the approach mentioned includes several key points which in a conventional QFT framework may be formulated nowadays in the following way (see [9] and comprehensive references therein):

- Only global symmetries are fundamental symmetries of Nature. Local symmetries and associated massless gauge vector (tensor) fields could only emerge due to some phase transition producing them as appropriate Nambu-Goldstone modes,
- The underlying Lorentz invariance is proposed to be spontaneously broken since only spacetime symmetry breaking could basically provide an existence of vector (tensor) emerging modes which mediate all interactions involved,
- The theory itself is proposed to be "physically" viable in the sense that any appropriate initial value condition (IVC), which determines the subsequent dynamical evolution of a physical field system, is uniquely satisfied. This means in turn that an interacting field system can not be superfluously restricted in the number of physical degrees of freedom in order to remain physical,
- Together, they naturally lead to the gauge symmetry emergence (GSE) conjecture which I will follow throughout the paper: Let there be given an interacting field system containing some vector field (or vector field multiplet) A_μ together with fermion (ψ), scalar (φ) and other matter fields in an arbitrary relativistically invariant Lagrangian L(A_μ, ψ, φ, ...) which possesses only global Abelian or non-Abelian internal symmetry G. Suppose that an underlying relativistic invariance of this field system is spontaneously broken in terms of the "length-fixing" covariant constraint put on vector fields,

$$A_{\mu}A^{\mu} = n^2 M^2 \tag{4.1}$$

(where M stands for the proposed SLIV scale, while n_{μ} is a properly-oriented unit Lorentz vector, $n^2 = n_{\mu}n^{\mu} = \pm 1$). If this constraint is preserved under the time development given by the field equations of motion, then in order to be protected from further reduction in degrees of freedom this system will modify its global symmetry G into a local symmetry G_{loc} , that will in turn convert the vector field constraint itself into a gauge condition thus virtually resulting in gauge invariant and Lorentz invariant theory.

To see how technically a global internal symmetry may be converted into a local one, let us consider in some detail the question of consistency of a possible constraint for a general 4-vector field A_{μ} with its equation of motion in an Abelian symmetry case, G = U(1). In the presence of the SLIV constraint $C(A) = A_{\mu}A^{\mu} - n^2M^2 = 0$ (4.1), it follows that the equations of motion can no longer be independent. The important point is that, in general, the time development would not preserve the constraint. So the parameters in the starting Lagrangian have to be adjusted in such a way that effectively we have one less equation of motion for the vector field A_{μ} not to be superfluously restricted. This means that there should be some relationship given by a functional equation $F(C = 0; E_A, E_{\psi}, ...) = 0$ between all the vector and matter field Eulerians involved¹ which are individually satisfied on the mass shell. According to Noether's second theorem [10] such a relationship gives rise to an emergence of local symmetry for the field system considered provided that the functional F satisfies the same symmetry requirements of Lorentz and translational invariance, as well as all the global internal symmetry requirements, as the general starting Lagrangian does.

In this way, the nonlinear SLIV condition (4.1), due to which true vacuum in the theory is chosen and massless gauge fields are generated, may provide a dynamical setting for all underlying internal symmetries involved through the GSE conjecture [9]. One might think that the length-fixing vector field constraint (4.1) itself first introduced by Nambu in a conventional QED framework [11] (for some extensions and generalizations, see also [12–17]) does not especially stand out in the present context. Actually, it seems that the GSE conjecture might be equally formulated for any type of covariant constraint, say for the spin-1 vector field condition, $\partial_{\mu}A^{\mu} = 0$ [18]. However, as is generally argued in [9], the SLIV constraint (4.1) appears to be the only one whose application leads to a full conversion of an internal global symmetry G into a local symmetry G_{1oc} that forces a given field system to remain always physical. Other constraints could only lead to partial gauge invariance being broken by some terms in an emerging theory.

Based upon the SLIV constraint (4.1), the starting vector field A_{μ} may be expanded around the true vacuum configuration in the theory,

$$A_{\mu} = a_{\mu} + n_{\mu}\sqrt{M^2 - n^2a^2} , \ n_{\mu}a_{\mu} = 0 \ (a^2 \equiv a_{\mu}a^{\mu}) , \eqno(4.2)$$

which means that it develops the vacuum expectation value (VEV) $\langle A_{\mu} \rangle = n_{\mu}M$. Meanwhile, its a_{μ} components which are orthogonal to the Lorentz violating direction n_{μ} describe a massless vector NG boson being an eventual gauge field (photon) candidate.

4.1.2 Gauge invariance versus spontaneous Lorentz violation

One can see that the gauge theory framework, be it taken from the outset or emerged, makes in turn spontaneous Lorentz violation to be physically unobserv-

¹ The field Eulerians (E_A , E_{ψ} , ...) are determined, as usual, $(E_A)^{\mu} \equiv \partial L/\partial A_{\mu} - \partial_{\nu}[\partial L/\partial (\partial_{\nu}A_{\mu})]$, and so forth.

able both in Abelian and non-Abelian symmetry case. In substance, the essential part of the SLIV pattern (4.2), due to which the vector field $A_{\mu}(x)$ develops the VEV M, may itself be treated as a pure gauge transformation with a gauge function linear in coordinates, $\omega(x) = n_{\mu}x^{\mu}M$. This is what one could refer to as the generic non-observability of SLIV in gauge invariant theories. I shall call it the "inactive" SLIV in contrast to the "active" SLIV case where physical Lorentz invariance could effectively occur. From the present standpoint, the only way for an active SLIV to occur would be if emergent gauge symmetries presented above were slightly broken at small distances. This could inevitably happen, for example, in a partially gauge invariant theory which might appear if the considered field system could become "a little unphysical" at distances being presumably controlled by quantum gravity [19]. One may think that quantum gravity could in principle hinder the setting of the required IVC in the appropriate Cauchy problem (thus admitting a superfluous restriction of vector fields) due to the occurrence of some gauge-noninvariant high-order operators near the Planck scale. As a consequence, through special dispersion relations appearing for matter and gauge fields, one is led a new class of phenomena which could be of distinctive observational interest in particle physics and astrophysics. They include a significant change in the Greizen-Zatsepin-Kouzmin cutoff for ultra-high energy cosmic-ray nucleons, stability of high-energy pions and W bosons, modification of nucleon beta decays, and some others just in the presently accessible energy area in cosmic ray physics [19] (for many phenomenological aspects, see pioneering works [20,21]).

4.1.3 SUSY profile of emergent theories

The role of Lorentz invariance may change, and its spontaneous violation may not be the only reason why massless photons and other gauge fields could dynamically appear, if spacetime symmetry is further enlarged. In this connection, special interest is related to supersymmetry which has made a serious impact on particle physics in the last decades (though has not been yet discovered). Actually, as we will see, the situation is changed dramatically in the SUSY inspired emergent gauge theories. In sharp contrast to non-SUSY analogs, it appears that the spontaneous Lorentz violation caused by an arbitrary potential of vector superfield $V(x, \theta, \theta)$ never goes any further than some nonlinear gauge condition put on its vector field component $A_{\mu}(x)$ associated with a photon or any other gauge field. Remarkably, this condition coincides, as we shall see below, with the SLIV constraint (4.1) given above in the GSE conjecture. This allows to think that physical Lorentz invariance is somewhat protected by SUSY, thus only requiring the "condensation" of the gauge degree of freedom in the vector field A_{μ} . The point is, however, that even in the case when SLIV is not physical it inevitably leads to the generation of massless photons as vector NG bosons provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance.

While there are many papers in the literature on Lorentz noninvariant extensions of supersymmetric models (for some interesting ideas, see [22,23] and references therein), an emergent gauge theory in a SUSY context has only recently been introduced [9,24]. Actually, the situation was shown to be seriously changed in a SUSY context which certainly disfavors some emergent models considered above. It appears that, while the constraint-based models of an inactive SLIV successfully matches supersymmetry, the composite and potential-based models of an active SLIV leading to physical Lorentz violation cannot be conceptually realized in the SUSY context. The reason is that, in contrast to an ordinary vector field theory where all kinds of polynomial terms $(A_{\mu}A^{\mu})^n$ (n = 1, 2, ...) can be included into the Lagrangian in a Lorentz invariant way, SUSY theories only admit the bilinear mass term $A_{\mu}A^{\mu}$ in the vector field potential energy. As a result, without a stabilizing high-linear (at least quartic) vector field terms, the potential-based SLIV never occurs in SUSY theories. The same could be said about composite models [2-4] as well: a fundamental Lagrangian with multi-fermi current-current interactions can not be constructed from any matter chiral superfields. So, all the models mentioned above, but the constraint-based models determined by the GSE conjecture (4.1), are ruled out in the SUSY framework and, therefore, between the two basic SLIV versions, active and inactive, SUSY unambiguously chooses the inactive SLIV case.

4.1.4 Outline of the paper

The paper is organized in the following way. In the next section 2 I consider supersymmetric QED model extended by an arbitrary polynomial potential of massive vector superfield that breaks gauge invariance in the SUSY invariant phase. However, the requirement of vacuum stability in such class of models makes both supersymmetry and Lorentz invariance to become spontaneously broken. As a consequence, the massless photino and photon appear as the corresponding Nambu-Goldstone zero modes in an emergent SUSY QED, and also a special gauge invariance is simultaneously generated. Due to this invariance all observable relativistically noninvariant effects appear to be completely cancelled out and physical Lorentz invariance is recovered. Further in section 3, all basic arguments developed in SUSY QED are generalized successively to the Standard Model and Grand Unified Theories (GUTs). For definiteness, I focus on the $U(1) \times SU(N)$ symmetrical theories. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case one only has the SUSY invariant phase in the theory that makes it inappropriate for an outgrowth of an emergence process. As possible realistic realizations, the Standard Model case with the electroweak $U(1) \times SU(2)$ symmetry and flipped SU(5) GUT including some immediate applications are briefly discussed. And finally in section 4, I summarize the main results and conclude.

The present talk is complimentary to my last year talk in Bled [25]. Some more detail can also be found in the recent extended paper [9].

4.2 Emergent SUSY theories: a QED primer

In contrast to attempts simply probing physical Lorentz noninvariance through some SM extensions [8,20] with hypothetical external vector (tensor) field backgrounds originated around the Planck scale, we will principally focus here on a spontaneous Lorentz violation in an ordinary Standard Model framework itself. Particularly, we will try to extend an emergent SM with electroweak bosons appearing as massless vector NG modes to their supersymmetric analogs [9,24]. Such theories seem to open a new avenue for exploring the origin of gauge symmetries. Indeed, as I discussed at the previous workshop [25], the emergent SUSY theories, in contrast to the non-SUSY ones, could naturally have some clear observational signature. Actually, we have seen above that ordinary emergent gauge theories are physically indistinguishable from the conventional ones unless gauge invariance becomes broken being caused by some high-dimension couplings. Meanwhile, their SUSY counterparts - supersymmetric QED, SM and GUT - can be experimentally verified in another way. The point is that they generically emerge only if supersymmetry is spontaneously broken in a visible sector in order to ensure stability of the underlying theory. Therefore, the verification of emergent theories is now related to an inevitable emergence of a goldstino-like photino state in the SUSY particle spectrum at low energies, while physical Lorentz invariance may be still left intact.

4.2.1 Spontaneous SUSY violation

Since gauge invariance is not generically assumed in an emergent approach, all possible gauge-noninvariant couplings could in principle occur in the theory in a pre-emergent phase. The most essential couplings, as I discussed earlier [25], appear to be the vector field self-interaction terms triggering an emergence process in non-SUSY theories. Starting from this standpoint, I consider a conventional supersymmetric QED being similarly extended by an arbitrary polynomial potential of a general vector superfield $V(x, \theta, \overline{\theta})$ which in the standard parametrization [26] has a form

$$\begin{split} V(x,\theta,\overline{\theta}) &= C + i\theta\chi - i\overline{\theta}\overline{\chi} + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta\theta}S^* \\ &-\theta\sigma^{\mu}\overline{\theta}A_{\mu} + i\theta\theta\overline{\theta\lambda'} - i\overline{\theta\theta}\theta\lambda' + \frac{1}{2}\theta\theta\overline{\theta\theta}D', \end{split} \tag{4.3}$$

where its vector field component A_{μ} is usually associated with a photon. Note that, apart from an ordinary photino field λ and an auxiliary D field, the superfield (4.3) contains in general some additional degrees of freedom in terms of the dynamical C and χ fields and nondynamical complex scalar field S (I have used the brief notations, $\lambda' = \lambda + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\overline{\chi}$ and $D' = D + \frac{1}{2}\partial^{2}C$ with $\sigma^{\mu} = (1, \overrightarrow{\sigma})$ and $\overline{\sigma}^{\mu} = (1, -\overrightarrow{\sigma})$). The corresponding Lagrangian can be written as

$$L = L_{SQED} + \frac{1}{2}D^2 + \sum_{k=1} b_k V^k |_D$$
(4.4)

where, besides a standard SUSY QED part, new potential terms are presented in the sum by corresponding D-term expansions $V^k|_D$ of the vector superfield (4.3) into the component fields (b_k are some constants). It can readily be checked that the first term in this expansion is the known Fayet-Iliopoulos D-term, while other

terms only contain bilinear, trilinear and quartic combination of the superfield components A_{μ} , S, λ and χ , respectively.

Actually, the higher-degree terms only appear for the scalar field component C(x). Expressing them all in terms of the C field polynomial

$$P(C) = \sum_{k=1}^{k} \frac{k}{2} \mathfrak{b}_k C^{k-1}(x)$$
(4.5)

and its first three derivatives

$$P'_{C} \equiv \frac{\partial P}{\partial C}, \ P''_{C} \equiv \frac{\partial^{2} P}{\partial C^{2}}, \ P''_{C} \equiv \frac{\partial^{3} P}{\partial C^{3}}$$
 (4.6)

one has for the whole Lagrangian L

$$L = L_{SQED} + \frac{1}{2}D^{2} + P\left(D + \frac{1}{2}\partial^{2}C\right)$$

+ $P_{C}^{\prime}\left(\frac{1}{2}SS^{*} - \chi\lambda^{\prime} - \overline{\chi}\overline{\lambda^{\prime}} - \frac{1}{2}A_{\mu}A^{\mu}\right)$
+ $\frac{1}{2}P_{C}^{\prime\prime}\left(\frac{i}{2}\overline{\chi\chi}S - \frac{i}{2}\chi\chi S^{*} - \chi\sigma^{\mu}\overline{\chi}A_{\mu}\right) + \frac{1}{8}P_{C}^{\prime\prime\prime}(\chi\chi\overline{\chi\chi}).$ (4.7)

As one can see, extra degrees of freedom related to the C and χ component fields in a general vector superfield V(x, θ , $\overline{\theta}$) appear through the potential terms in (4.7) rather than from the properly constructed supersymmetric field strengths, as appear for the vector field A_µ and its gaugino companion λ .

Note that all terms in the sum in (4.4) except Fayet-Iliopoulos D-term explicitly break gauge invariance. However, as we will see later in this section, the special gauge invariance constrained by some gauge condition will be recovered in the Lagrangian in the broken SUSY phase. Furthermore, as is seen from (4.7), the vector field A_{μ} may only appear with bilinear mass term in the polynomially extended superfield Lagrangian (4.4) in sharp contrast to the non-SUSY theory case where, apart from the vector field mass term, some high-linear stabilizing terms necessarily appear in a similar polynomially extended Lagrangian. This means in turn that physical Lorentz invariance is still preserved. Actually, only supersymmetry appears to be spontaneously broken in the theory.

Indeed, varying the Lagrangian L with respect to the D field we come to

$$\mathsf{D} = -\mathsf{P}(\mathsf{C}) \tag{4.8}$$

that finally gives the following potential energy for the field system considered

$$U(C) = \frac{1}{2} [P(C)]^2 .$$
(4.9)

The potential (4.9) may lead to spontaneous SUSY breaking in the visible sector provided that the polynomial P (4.5) has no real roots, while its first derivative has,

$$P \neq 0$$
, $P'_C = 0$. (4.10)

This requires P(C) to be an even degree polynomial with properly chosen coefficients b_k in (4.5) that will force its derivative P'_C to have at least one root, $C = C_0$, in which the potential (4.9) is minimized. Therefore, supersymmetry is spontaneously broken and the C field acquires the VEV

$$\langle C \rangle = C_0 , P'_C(C_0) = 0.$$
 (4.11)

As an immediate consequence, that one can readily see from the Lagrangian L (4.7), a massless photino λ being Goldstone fermion in the broken SUSY phase make all the other component fields in the superfield $V(x, \theta, \overline{\theta})$ including the photon to also become massless. However, the question then arises whether this masslessness of the photon will be stable against radiative corrections since gauge invariance is explicitly broken in the Lagrangian (4.7). I show below that it could be the case if the vector superfield $V(x, \theta, \overline{\theta})$ would appear properly constrained.

4.2.2 Instability of superfield polynomial potential

Let us first analyze possible vacuum configurations for the superfield components in the polynomially extended QED case taken above. In general, besides the "standard" potential energy expression (4.9) determined solely by the scalar field component C(x) of the vector superfield (4.3), one also has to consider other field component contributions into the potential energy. A possible extension of the potential energy (4.9) seems to appear only due to the pure bosonic field contributions, namely due to couplings of the vector and auxiliary scalar fields, A_{μ} and S, in (4.7)

$$U_{tot} = \frac{1}{2}P^2 + \frac{1}{2}P'_C \left(A_{\mu}A^{\mu} - SS^*\right)$$
(4.12)

rather than due to the potential terms containing the superfield fermionic components. It can be immediately seen that these new couplings in (4.12) can make the potential unstable since the vector and scalar fields mentioned may in general develop any arbitrary VEVs. This happens, as emphasized above, due the fact that their bilinear term contributions are not properly compensated by appropriate four-linear field terms which are generically absent in a SUSY theory context.

4.2.3 Stabilization of vacuum by constraining vector superfield

The only possible way to stabilize the theory seems to seek the proper constraints on the superfield component fields (C, A_{μ} , S) themselves rather than on their expectation values. This will be done again through some invariant Lagrange multiplier coupling simply adding its D term to the above Lagrangian (4.4, 4.7)

$$L_{tot} = L + \frac{1}{2}\Lambda (V - C_0)^2|_D , \qquad (4.13)$$

where $\Lambda(x, \theta, \overline{\theta})$ is some auxiliary vector superfield, while C_0 is the constant background value of the C field which minimizes the potential U (4.9). Accordingly, the potential vanishes for the supersymmetric minimum or acquires some positive

value corresponding to the SUSY breaking minimum (4.10) in the visible sector. I shall consider both cases simultaneously using the same notation C_0 for either of the background values of the C field.

Writing down the Lagrange multiplier D term in (4.13) through the component fields

$$C_{\Lambda}, \chi_{\Lambda}, S_{\Lambda}, A^{\mu}_{\Lambda}, \lambda'_{\Lambda} = \lambda_{\Lambda} + \frac{i}{2} \sigma^{\mu} \partial_{\mu} \overline{\chi}_{\Lambda}, D'_{\Lambda} = D_{\Lambda} + \frac{1}{2} \partial^{2} C_{\Lambda}$$
 (4.14)

and varying the whole Lagrangian (4.13) with respect to these fields one finds the constraints which appear to put on the V superfield components [25]

$$C = C_0, \ \chi = 0, \ A_{\mu}A^{\mu} = SS^*.$$
 (4.15)

They also determine the corresponding D-term (4.8), $D = -P(C_0)$, for the spontaneously broken supersymmetry. As usual, I only take a solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (4.14). This will provide a ghost-free theory with a positive Hamiltonian.

Finally, implementing the constraints (4.15) into the total Lagrangian L_{tot} (4.13, 4.7) through the Lagrange multiplier terms for component fields, we come to the emergent SUSY QED appearing in the broken SUSY phase

$$L^{em} = L_{SQED} + P(C)D + \frac{D_{\Lambda}}{4}(C - C_0)^2 - \frac{C_{\Lambda}}{4}(A_{\mu}A^{\mu} - SS^*) .$$
 (4.16)

The last two term with the component multiplier functions C_{Λ} and D_{Λ} of the auxiliary superfield Λ (4.14) provide the vacuum stability condition of the theory. In essence, one does not need now to postulate from the outset gauge invariance for the physical SUSY QED Lagrangian L_{SQED} . Rather, one can derive it following the GSE conjecture (section 1.1) specified for Abelian theory. Indeed, due to the constraints (4.15), the Lagrangian L_{SQED} is only allowed to have a conventional gauge invariant form

$$L_{SQED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^{\mu}\partial_{\mu}\overline{\lambda} + \frac{1}{2}D^{2} \tag{4.17}$$

Thus, for the constrained vector superfield involved

$$\widehat{V}(x,\theta,\overline{\theta}) = C_0 + \frac{i}{2}\theta\theta S - \frac{i}{2}\overline{\theta\theta}S^* - \theta\sigma^{\mu}\overline{\theta}A_{\mu} + i\theta\theta\overline{\theta\lambda} - i\overline{\theta\theta}\theta\lambda + \frac{1}{2}\theta\theta\overline{\theta\theta}D, \quad (4.18)$$

we have the almost standard SUSY QED Lagrangian with the same states - a photon, a photino and an auxiliary scalar D field - in its gauge supermultiplet, while another auxiliary complex scalar field S gets only involved in the vector field constraint in (4.15). The linear (Fayet-Iliopoulos) D-term with the effective coupling constant $P(C_0)$ in (4.16) shows that supersymmetry in the theory is spontaneously broken due to which the D field acquires the VEV, $D = -P(C_0)$. Taking the nondynamical S field in the constraint (4.15) to be some constant background field we come to the SLIV constraint (4.1) underlying the GSE conjecture. As is seen from this constraint in (4.16), one may only have the time-like SLIV in a SUSY

framework but never the space-like one. There also may be a light-like SLIV, if the S field vanishes². So, any possible choice for the S field corresponds to the particular gauge choice for the vector field A_{μ} in an otherwise gauge invariant theory. So, the massless photon appearing first as a companion of a massless photino (being a Goldstone fermion in the visible broken SUSY phase) remains massless due to this recovering gauge invariance in the emergent SUSY QED. At the same time, the "built-in" nonlinear gauge condition in (4.16) allows to treat the photon as a vector Goldstone boson induced by an inactive SLIV.

4.3 On emergent SUSY Standard Models and GUTs

4.3.1 Potential of Abelian and non-Abelian vector superfields

Now, we extend our discussion to the non-Abelian internal symmetry case given by some group G with generators t^p

$$[t^{p}, t^{q}] = if^{pqr}t^{r}, \ Tr(t^{p}t^{q}) = \delta^{pq} \ (p, q, r = 0, 1, ..., \Upsilon - 1)$$
(4.19)

where f^{pqr} stand structure constants, while Υ is a dimension of the G group. This case may correspond in general to some Grand Unified Theory which includes the Standard Model and its possible extensions. For definiteness, I will be further focused on the $U(1) \times SU(N)$ symmetrical theories, though any other non-Abelian group in place of SU(N) is also admissible. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case supersymmetry does not get spontaneously broken in a visible sector that makes it inappropriate for an outgrowth of an emergence process³. So, the theory now contains the Abelian vector superfield V, as is given in (4.3), and non-Abelian superfield multiplet \mathbf{V}^p

$$\mathbf{V}^{p}(\mathbf{x},\theta,\overline{\theta}) = \mathbf{C}^{p} + \mathrm{i}\theta\chi^{p} - \mathrm{i}\overline{\theta}\overline{\chi}^{p} + \frac{\mathrm{i}}{2}\theta\theta\mathbf{S}^{p} - \frac{\mathrm{i}}{2}\overline{\theta\theta}\mathbf{S}^{*p} - \theta\sigma^{\mu}\overline{\theta}\mathbf{A}^{p}_{\mu} + \mathrm{i}\theta\theta\overline{\theta}\overline{\lambda'}^{p} - \mathrm{i}\overline{\theta\theta}\theta\lambda'^{p} + \frac{1}{2}\theta\theta\overline{\theta\theta}\mathbf{D}'^{p}, \qquad (4.20)$$

where its vector field components \mathbf{A}^p_{μ} are usually associated with an adjoint gauge field multiplet, $(\mathbf{A}_{\mu})^i_j \equiv (\mathbf{A}^p_{\mu} t^p)^i_j$ (i, j, k = 1, 2, ..., N; p, q, r = 1, 2, ..., N²-1). Note that, apart from the conventional gaugino multiplet λ^p and the auxiliary fields \mathbf{D}^p , the superfield \mathbf{V}^p contains in general the additional degrees of freedom in terms of the dynamical scalar and fermion field multiplets \mathbf{C}^p and χ^p and nondynamical complex scalar field \mathbf{S}^p . Note that for the non-Abelian superfield components I use hereafter the bold symbols and take again the brief notations, $\lambda'^p = \lambda^p + \frac{i}{2}\sigma^{\mu}\partial_{\mu}\overline{\chi}^p$ and $\mathbf{D}'^p = \mathbf{D}^p + \frac{1}{2}\partial^2 \mathbf{C}^p$.

² Indeed, this case, first mentioned in [11], may also mean spontaneous Lorentz violation with a nonzero VEV $\langle A_{\mu} \rangle = (\widetilde{M}, 0, 0, \widetilde{M})$ and Goldstone modes $A_{1,2}$ and $(A_0 + A_3)/2$ $-\widetilde{M}$. The "effective" Higgs mode $(A_0 - A_3)/2$ can be then expressed through Goldstone modes so as the light-like condition $A_{\mu}A^{\mu} = 0$ to be satisfied.

³ In principle, SUSY may be spontaneously broken in the visible sector even in the pure non-Abelian symmetry case provided that the vector superfield potential includes some essential high-dimension couplings.

Augmenting the SUSY and $U(1) \times SU(N)$ invariant GUT by some polynomial potential of vector superfields V and V^p one comes to

$$\mathcal{L} = \mathcal{L}_{SGUT} + \frac{1}{2}D^2 + \frac{1}{2}D^pD^p + [\xi V + b_1V^3/3 + b_2V(VV) + b_3(VVV)/3]_D \quad (4.21)$$

where ξ and $b_{1,2,3}$ stand for coupling constants, and the last term in (4.21) contains products of the Abelian superfield V and the adjoint SU(N) superfield multiplet $V_j^i \equiv (V^p t^p)_j^i$. The round brackets denote hereafter traces for the superfield V_j^i

$$(\mathbf{VV...}) \equiv \mathrm{Tr}(\mathbf{VV...}) \tag{4.22}$$

and its field components (see below). For simplicity, we restricted ourselves to the third degree superfield terms in the Lagrangian \mathcal{L} to eventually have a theory at a renormalizible level. Furthermore, I have only taken the odd power superfield terms that provides, as we see below, an additional discrete symmetry of the potential with respect to the scalar field components in the V and **V**^p superfields

$$C \rightarrow -C, \quad C^{p} \rightarrow -C^{p}.$$
 (4.23)

Finally, eliminating the auxiliary D and D^p fields in the Lagrangian \mathcal{L} we come to the total potential for all superfield bosonic field components written in terms of traces mentioned above (4.22)

$$\mathcal{U}_{tot} = \mathcal{U}(C, C) + \frac{1}{2} b_1 C(A_{\mu}A^{\mu} - S_{\alpha}S_{\alpha}) + \frac{1}{2} b_2 C[(A_{\mu}A^{\mu}) - (S_{\alpha}S_{\alpha})] \\ + \frac{1}{2} b_2 [A_{\mu}(A^{\mu}C) - S_{\alpha}(S_{\alpha}C)] + \frac{1}{2} b_3 [(A_{\mu}A^{\mu}C) - (S_{\alpha}S_{\alpha}C)]. \quad (4.24)$$

Note that the potential terms depending only on scalar fields C and $C_j^i \equiv (C^a t^a)_j^i$ are collected in

$$\mathcal{U}(\mathbf{C},\mathbf{C}) = \frac{1}{8} [\xi + b_1 \mathbf{C}^2 + b_2 (\mathbf{C}\mathbf{C})]^2 + \frac{1}{2} [b_2^2 \mathbf{C}^2 (\mathbf{C}\mathbf{C}) + b_2 b_3 \mathbf{C} (\mathbf{C}\mathbf{C}\mathbf{C}) + \frac{1}{4} b_3^2 (\mathbf{C}\mathbf{C}\mathbf{C}\mathbf{C})]$$
(4.25)

and complex scalar fields S_α and S^p_α $(\alpha=1,2)$ are now taken in the real field basis like as

$$S_1 = (S + S^*)/2, \quad S_2 = (S - S^*)/2i$$
, (4.26)

an so on. One can see that all these terms are invariant under the discrete symmetry (4.23), whereas the vector field couplings in the total potential U_{tot} (4.24) break it. However, they vanish when the V and V^p superfields are properly constrained that we actually confirm in the next section.

Let us consider first the pure scalar field potential \mathcal{U} (4.25). The corresponding extremum conditions for C and C^{α} fields are,

$$\mathcal{U}_{C}^{\prime} = b_{1}(\xi + b_{1}C^{2})C + b_{2}(b_{1} - 2b_{2})C(\mathbf{CC}) = 0,$$

Tr($\mathcal{U}_{C_{i}^{i}}^{\prime}$) = $3b_{2}C(\mathbf{CC}) + b_{3}(\mathbf{CCC}) = 0,$ (4.27)

respectively. As shows the second partial derivative test, the simplest solution to the above equations

$$\mathbf{C}_0 = \mathbf{0} , \ \mathbf{C}_j^i = \mathbf{0} \tag{4.28}$$

provides, under conditions put on the potential parameters,

$$\xi, \ b_1 > 0, \ b_2 \ge 0 \ \text{ or } \ \xi, \ b_1 < 0, \ b_2 \le 0$$
(4.29)

its global minimum

$$\mathcal{U}(\mathbf{C},\mathbf{C})_{\min}^{\alpha s} = \frac{1}{8}\xi^2 . \qquad (4.30)$$

This minimum corresponds to the broken SUSY phase with the unbroken internal symmetry $U(1) \times SU(N)$ that is just what one would want to trigger an emergence process. This minimum appears in fact due to the Fayet-Iliopoulos linear term in the superfield polynomial in (4.21). As can easily be confirmed, in absence of this term, namely, for $\xi = 0$ and any arbitrary values of all other parameters, there is only the SUSY symmetrical solution with unbroken internal symmetry

$$\mathcal{U}(\mathbf{C}, \mathbf{C})_{\min}^{sym} = \mathbf{0}.$$
(4.31)

Interestingly, the symmetrical solution corresponding to the global minimum (4.31) may appear for the nonzero parameter ξ as well

$$C_0^{(\pm)} = \pm \sqrt{-\xi/b_1}, \ C_j^i = 0$$
 (4.32)

provided that

$$\xi b_1 < 0$$
. (4.33)

However, as we saw in the QED case, in the unbroken SUSY case one comes to the trivial constant superfield when all factual constraints are included into consideration [25] and, therefore, this case is in general of little interest.

4.3.2 Constrained vector supermultiplets

Let us now take the vector fields A_{μ} and A^{p}_{μ} into consideration that immediately reveals that, in contrast to the pure scalar field part (4.25), U(C, C), the vector field couplings in the total potential (4.24) make it unstable. This happens, as was emphasized before, due the fact that bilinear term VEV contributions of the vector fields A_{μ} and A^{p}_{μ} , as well as the auxiliary scalar fields S_{α} and S^{p}_{α} , are not properly compensated by appropriate four-linear field terms which are generically absent in a supersymmetric theory framework.

Again, as in the supersymmetric QED case considered above, the only possible way to stabilize the ground state (4.28, 4.29, 4.30) seems to seek the proper constraints on the superfields component fields (C, \mathbf{C}^{p} ; A_{μ} , \mathbf{A}^{p} ; S_{α} , $\mathbf{S}^{\mathrm{p}}_{\alpha}$) themselves rather than on their expectation values. Provided that such constraints are physically realizable, the required vacuum will be automatically stabilized. This will be done again through some invariant Lagrange multiplier couplings simply adding their D terms to the above Lagrangian (4.21)

$$\mathcal{L}_{tot} = \mathcal{L} + \frac{1}{2}\Lambda(V - C_0)^2|_{D} + \frac{1}{2}\Pi(VV)|_{D}$$
, (4.34)

where $\Lambda(x, \theta, \overline{\theta})$ and $\Pi(x, \theta, \overline{\theta})$ are auxiliary vector superfields. Note that C₀ presented in the first multiplier coupling is just the constant background value of the

C field for which the potential part $\mathcal{U}(C, \mathbb{C})$ in (4.24) vanishes as appears for the supersymmetric minimum (4.31) or has some nonzero value corresponding to the SUSY breaking minimum (4.30) in the visible sector.

I will consider both cases simultaneously using the same notation C_0 for either of the potential minimizing values of the C field. The second multiplier coupling in (4.34) provides, as we will soon see, the vanishing background value for the non-Abelian scalar field, $\mathbf{C}^{\alpha} = 0$, due to which the underlying internal symmetry $U(1) \times SU(N)$ is left intact in both unbroken and broken SUSY phase. The Lagrange multiplier terms presented in (4.34) have in fact the simplest possible form that leads to some nontrivial constrained superfields $V(x, \theta, \overline{\theta})$ and $\mathbf{V}^{p}(x, \theta, \overline{\theta})$. Writing down their invariant D terms through the component fields one finds the precisely the same expression as in the SUSY QED [25] case for the Abelian superfield V and the slightly modified one for the non-Abelian superfield \mathbf{V}^{α}

$$\begin{split} \Pi(\mathbf{V}\mathbf{V})|_{\mathrm{D}} &= C_{\Pi} \left[\mathbf{C}\mathbf{D}' + \left(\frac{1}{2}\mathbf{S}\mathbf{S}^{*} - \chi\lambda' - \overline{\chi}\overline{\lambda'} - \frac{1}{2}\mathbf{A}_{\mu}\mathbf{A}^{\mu} \right) \right] \\ &+ \chi_{\Pi} \left[2\mathbf{C}\lambda' + \mathrm{i}(\chi\mathbf{S}^{*} + \mathrm{i}\sigma^{\mu}\overline{\chi}\mathbf{A}_{\mu}) \right] + \overline{\chi}_{\Pi} \left[2\mathbf{C}\overline{\lambda'} - \mathrm{i}(\overline{\chi}\mathbf{S} - \mathrm{i}\chi\sigma^{\mu}\mathbf{A}_{\mu}) \right] \\ &+ \frac{1}{2}S_{\Pi} \left(\mathbf{C}\mathbf{S}^{*} + \frac{\mathrm{i}}{2}\overline{\chi}\overline{\chi} \right) + \frac{1}{2}S_{\Pi}^{*} \left(\mathbf{C}\mathbf{S} - \frac{\mathrm{i}}{2}\chi\chi \right) \\ &+ 2A_{\Pi}^{\mu}(\mathbf{C}\mathbf{A}_{\mu} - \chi\sigma_{\mu}\overline{\chi}) + 2\lambda_{\Pi}'(\mathbf{C}\chi) + 2\overline{\lambda}_{\Pi}'(\mathbf{C}\overline{\chi}) + \frac{1}{2}D_{\Pi}'(\mathbf{C}\mathbf{C}) \quad (4.35) \end{split}$$

where the pairly grouped field bold symbols mean hereafter the SU(N) scalar products of the component field multiplets (for instance, $\mathbf{CD}' = \mathbf{C}^{p}\mathbf{D}'^{p}$, and so forth) and

$$C_{\Pi}, \chi_{\Pi}, S_{\Pi}, A_{\Pi}^{\mu}, \lambda_{\Pi}' = \lambda_{\Pi} + \frac{i}{2} \sigma^{\mu} \partial_{\mu} \overline{\chi}_{\Pi}, D_{\Pi}' = D_{\Pi} + \frac{1}{2} \partial^{2} C_{\Pi}$$
 (4.36)

are the component fields of the Lagrange multiplier superfield $\Pi(x, \theta, \overline{\theta})$ in the standard parametrization (4.20).

Varying the total Lagrangian (4.34) with respect to the component fields of both multipliers, (4.14) and (4.36), and properly combining their equations of motion we find the constraints which appear to put on the V and V^{α} superfields components [9]

$$C = C_0, \ \chi = 0, \ A_{\mu}A^{\mu} = S_{\alpha}S_{\alpha},$$

$$C^{p} = 0, \ \chi^{p} = 0, \ (A_{\mu}A^{\mu}) = (S_{\alpha}S_{\alpha}), \ \alpha = 1, 2.$$
(4.37)

As before in the SUSY QED case, one may only have the time-like SLIV in a supersymmetric $U(1) \times SU(N)$ framework but never the space-like one (there also may be a light-like SLIV, if the S and S fields vanish). Also note that we only take the solution with initial values for all fields (and their momenta) chosen so as to restrict the phase space to vanishing values of the multiplier component fields (4.14) and (4.36) that will provide a ghost-free theory with a positive Hamiltonian. Again, apart from the constraints (4.37), one has the equations of motion for all fields involved in the basic superfields $V(x, \theta, \overline{\theta})$ and $V^p(x, \theta, \overline{\theta})$. With vanishing

multiplier component fields (4.14) and (4.36), as was proposed above, these equations appear in fact as extra constraints on components of the V and V^p superfields. Indeed, equations of motion for the S_{α} , χ and C fields, on the one hand hand, and for the S_{α}^{p} , χ^{p} and C^p fields, on the other, are obtained by the corresponding variations of the total Lagrangian \mathcal{L}^{tot} (4.34) including the potential (4.24).

They are turned out to be, respectively,

$$\begin{split} S_{\alpha}C_{0} &= 0, \ \lambda C_{0} = 0, \ (\xi + b_{1}C_{0}^{2})C_{0} = 0, \\ S_{\alpha}^{p}C_{0} &= 0, \ \lambda^{p}C_{0} = 0, \ b_{2}[A_{\mu}A_{j}^{\mu} - S_{\alpha}S_{\alpha}_{j}^{i}] + b_{3}[(A_{\mu}A^{\mu})_{j}^{i} - (S_{\alpha}S_{\alpha})_{j}^{i}] = (4.38) \end{split}$$

where the basic constraints (4.37) emerging at the potential U(C, C) extremum point (C_0 , $C_0^p = 0$) have been also used for both broken and unbroken SUSY case. Note also that the equations for gauginos λ and λ^p in (4.38) are received by variation of the potential terms in (4.21) containing fermion field couplings

$$\begin{aligned} \mathfrak{U} &= \mathfrak{b}_{1} \mathcal{C}(\chi \lambda' + \overline{\chi} \overline{\lambda'}) + \mathfrak{b}_{2} \mathcal{C}[(\chi \lambda') + (\overline{\chi} \overline{\lambda'})] \\ &+ \frac{1}{2} \mathfrak{b}_{2}[\chi(\lambda' \mathbf{C}) + \overline{\chi}(\overline{\lambda'} \mathbf{C}) + \lambda'(\chi \mathbf{C}) + \overline{\lambda'}(\overline{\chi} \mathbf{C})] \\ &+ \mathfrak{b}_{3}(\chi \lambda' \mathbf{C}) + (\overline{\chi} \overline{\lambda'} \mathbf{C})] . \end{aligned}$$

$$(4.39)$$

One can immediately see now that all equations in (4.38) but the last equation system turn to trivial identities in the broken SUSY case (4.28) in which the corresponding C field value appears to be identically vanished, $C_0 = 0$. In the unbroken SUSY case (4.32), this field value is definitely nonzero, $C_0 = \pm \sqrt{-\xi/b_1}$, and the situation is radically changed. Indeed, as follows from the equations (4.38), the auxiliary fields S(x) and S^p , as well as the gaugino fields $\lambda(x)$ and $\lambda^p(x)$ have to be identically vanished. This causes in turn that the gauge vector fields field A_{μ} and A^p_{μ} should also be vanished according to the basic constraints (4.37). So, we have to conclude, as in the SUSY QED case, that the unbroken SUSY fails to provide stability of the potential (4.12) even by constraining the superfields V and V^p and, therefore, only the spontaneously broken SUSY case could in principle lead to a physically meaningful emergent theory.

4.3.3 Broken SUSY phase: an emergent $U(1) \times SU(N)$ theory

With the constraints (4.37) providing vacuum stability for the total Lagrangian \mathcal{L}_{tot} (4.34) we eventually come to the emergent theory with a local $U(1) \times SU(N)$ symmetry that appears in the broken SUSY phase (4.28). Actually, implementing these constraints into the Lagrangian through the Lagrange multiplier terms for component fields one has

$$\mathcal{L}^{em} = \mathcal{L}_{SGUT} + \frac{1}{2} \xi D + \frac{D_{\Lambda}}{4} (C - C_0)^2 - \frac{C_{\Lambda}}{4} (A_{\mu} A^{\mu} - SS^*) + \frac{D_{\Pi}}{4} (CC) - \frac{C_{\Pi}}{4} (A_{\mu} A^{\mu} - SS^*)$$
(4.40)

with the multiplier component functions C_{Λ} and D_{Λ} of the auxiliary superfield Λ (4.14) and component functions C_{Π} and D_{Π} of the auxiliary superfield Π (4.36)

presented in the Lagrangian (4.34). Again, with these constraints and the GSE conjecture (section 1.1) specified for non-Abelian theories, one does not need to postulate gauge invariance for the physical SUSY GUT Lagrangian \mathcal{L}_{SGUT} from the outset. Instead, one can derive it starting from an arbitrary relativistically invariant theory. Indeed, even if the Lagrangian \mathcal{L}_{SGUT} is initially taken to only possess the global U(1) × SU(N) symmetry it will tend to uniquely acquire a standard gauge invariant form

$$\mathcal{L}_{SGUT} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\lambda \sigma^{\mu} \partial_{\mu} \overline{\lambda} + \frac{1}{2} D^{2} -\frac{1}{4} F^{p\mu\nu} F^{p}_{\mu\nu} + i\lambda^{p} \sigma^{\mu} \mathcal{D}_{\mu} \overline{\lambda}^{p} + \frac{1}{2} D^{p} D^{p}$$
(4.41)

where the conventional gauge field strengths for both U(1) and SU(N) part and terms with proper covariant derivatives for gaugino fields λ^p necessarily appear [9]. Again as in the pure Abelian case, for the respectively constrained vector superfields V and V^p we come in fact to a conventional SUSY GUT Lagrangian with a standard gauge supermultiplet containing gauge bosons A_{μ} and A^p , gauginos λ and λ^p , and auxiliary scalar D and D^p fields, whereas other auxiliary scalar fields S_{α} and S^p_{α} get solely involved in the Lagrange multiplier terms (4.41). Actually, the only remnant of the polynomial potential of vector superfields V and V^p (4.21) survived in the emergent theory (4.40) appears to be the Fayet-Iliopoulos D-term which shows that supersymmetry in the theory is indeed spontaneously broken and the D field acquires the VEV, D = $-\frac{1}{2}\xi$.

Let us show now that this theory is in essence gauge invariant and the constraints (4.37) on the field space appearing due to the Lagrange multiplier terms in (4.34) are consistent with supersymmetry. Namely, as was argued in [25] (see also [9]), though constrained vector superfield (4.18) in QED is not strictly compatible with the linear superspace version of SUSY transformations, its supermultiplet structure can be restored by appropriate supergauge transformations. Following the same argumentation, one can see that similar transformations keep invariant the constraints (4.37) put on the vector fields A_{μ} and A^{p} . Leaving aside the U(1) sector considered in [25] in significant details, I will now focus on the SU(N) symmetry case with the constrained superfield V^{p} transformed as

$$\mathbf{V}^{\mathrm{p}} \rightarrow \mathbf{V}^{\mathrm{p}} + \frac{\mathrm{i}}{2} (\mathbf{\Omega} - \mathbf{\Omega}^{*})^{\mathrm{p}}$$
 (4.42)

The essential part of this transformation which directly acts on the vector field constraint

$$\mathbf{A}^{\mathbf{p}}_{\mathbf{\mu}}\mathbf{A}^{\mathbf{p}\,\mathbf{\mu}} = \mathbf{S}^{\mathbf{p}}\mathbf{S}^{*\mathbf{p}} \tag{4.43}$$

has the form

$$\mathbf{V}^{\mathrm{p}} \rightarrow \mathbf{V}^{\mathrm{p}} + \frac{\mathrm{i}}{2} \theta \theta F^{\mathrm{p}} - \frac{\mathrm{i}}{2} \overline{\theta} \overline{\theta} F^{*\mathrm{p}} - \theta \sigma^{\mu} \overline{\theta} \partial_{\mu} \boldsymbol{\phi}^{\mathrm{p}}$$
(4.44)

where the real and complex scalar field components, φ^p and F^p , in a chiral superfield parameter Ω^p are properly activated. As a result, the corresponding vector and scalar component fields, A^p_{μ} and S^p_{α} , in the constrained supermultiplet V^p transform as

$$\mathbf{A}^{\mathrm{p}}_{\mu}
ightarrow \mathbf{a}^{\mathrm{p}}_{\mu} = \mathbf{A}^{\mathrm{p}}_{\mu} - \partial_{\mu} \phi^{\mathrm{p}}, \ \ \mathbf{S}^{\mathrm{p}}
ightarrow \mathbf{s}^{\mathrm{p}} = \mathbf{S}^{\mathrm{p}} + \mathbf{F}^{\mathrm{p}} \ .$$
 (4.45)

One can readily see that our basic Lagrangian \mathcal{L}^{em} (4.40) being gauge invariant and containing no the auxiliary scalar fields \mathbf{S}^{p} is automatically invariant under either of these two transformations individually. In contrast, the supplementary vector field constraint (4.43), though it is also turned out to be invariant under supergauge transformations (4.45), but only if they act jointly. Indeed, for any choice of the scalar $\boldsymbol{\varphi}^{p}$ in (4.45) there can always be found such a scalar F^{a} (and vice versa) that the constraint remains invariant. In other words, the vector field constraint is invariant under supergauge transformations (4.45) but not invariant under an ordinary gauge transformation. As a result, in contrast to the Wess-Zumino case, the supergauge fixing in our case will also lead to the ordinary gauge fixing. We will use this supergauge freedom to reduce the scalar field bilinear $\mathbf{S}^{p} \mathbf{S}^{*p}$ to some constant background value and find a final equation for the gauge function $\boldsymbol{\varphi}^{p}(\mathbf{x})$. It is convenient to come to real field basis (4.26) for scalar fields \mathbf{S}^{p}_{α} and \mathbf{F}^{p}_{α} ($\alpha = 1, 2$), and choose the parameter fields \mathbf{F}^{a}_{α} as

$$\mathbf{F}^{\mathbf{p}}_{\alpha} = \mathbf{r}_{\alpha} \boldsymbol{\epsilon}^{\mathbf{p}} (\mathbf{M} + \mathbf{f}), \ \mathbf{r}_{\alpha} \mathbf{s}^{\mathbf{p}}_{\alpha} = 0, \ \ \mathbf{r}^{2}_{\alpha} = 1, \ \boldsymbol{\epsilon}^{\mathbf{p}} \boldsymbol{\epsilon}^{\mathbf{p}} = 1$$
(4.46)

so that the old S^p_{α} fields in (4.45) are related to the new ones s^p_{α} in the following way

$$\mathbf{S}^{\mathrm{p}}_{\alpha} = \mathbf{s}^{\mathrm{p}}_{\alpha} - \mathbf{r}_{\alpha} \boldsymbol{\epsilon}^{\mathrm{p}} (\mathbf{M} + \mathbf{f}), \ \mathbf{r}_{\alpha} \mathbf{s}^{\mathrm{p}}_{\alpha} = \mathbf{0}, \ \mathbf{S}^{\mathrm{p}}_{\alpha} \mathbf{S}^{\mathrm{p}}_{\alpha} = \mathbf{s}^{\mathrm{p}}_{\alpha} \mathbf{s}^{\mathrm{p}}_{\alpha} + (\mathbf{M} + \mathbf{f})^{2}.$$
(4.47)

where **M** is a new mass parameter, $\mathbf{f}(\mathbf{x})$ is some Higgs field like function, \mathbf{r}_{α} is again the two-component unit "vector" chosen to be orthogonal to the scalar \mathbf{s}_{α}^{p} , while $\boldsymbol{\epsilon}^{p}$ is the unit SU(N) adjoint vector. This parametrization for the old fields \mathbf{S}_{α}^{p} formally looks as if they develop the VEV, $\langle \mathbf{S}_{\alpha}^{p} \rangle = -\mathbf{r}_{\alpha} \boldsymbol{\epsilon}^{p} \mathbf{M}$, due to which the related SO(2) × SU(N) symmetry would be spontaneously violated and corresponding zero modes in terms of the new fields \mathbf{s}_{α}^{p} could be consequently produced (indeed, they they never appear in the theory). Eventually, for an appropriate choice of the Higgs field like function $\mathbf{f}(\mathbf{x})$ in (4.47)

$$\mathbf{f} = -\mathbf{M} + \sqrt{\mathbf{M}^2 - \mathbf{s}^p_\alpha \mathbf{s}^p_\alpha} \tag{4.48}$$

we come in (4.43) to the condition

$$\mathbf{A}^{\mathrm{p}}_{\mathrm{\mu}}\mathbf{A}^{\mathrm{p}\,\mathrm{\mu}} = \mathbf{M}^2 \,. \tag{4.49}$$

leading, as in the QED U(1) symmetry case [25], exclusively to the time-like SLIV.

Remarkably, thanks to a generic high symmetry of the constraint (4.49) one can apply the emergence conjecture with dynamically produced massless gauge modes to any non-Abelian internal symmetry case as well, though SLIV itself could produce only one zero vector mode. The point is that although we only propose Lorentz invariance SO(1,3) and internal symmetry U(1) × SU(N) of the Lagrangian \mathcal{L}^{em} (4.40), the emerged constraint (4.49) possesses in fact a much higher accidental symmetry SO(Υ , 3 Υ) determined by the dimension $\Upsilon = N^2 - 1$ of the SU(N) adjoint representation to which the vector fields \mathbf{A}^{p}_{μ} belong⁴. This

⁴ Actually, a total symmetry even higher if one keeps in mind both constraints (4.1) and (4.49) put on the vector fields A_{μ} and A^{a}_{μ} , respectively. As long as they are independent the related total symmetry is in fact SO(1, 3) × SO(Υ , 3 Υ) until it starts breaking.

symmetry is indeed spontaneously broken at a scale **M** leading exclusively to the time-like SLIV case, as is determined by the positive sign in the SUSY SLIV constraint (4.49). The emerging pseudo-Goldstone vector bosons may be in fact considered as candidates for non-Abelian gauge fields which together with the true vector Goldstone boson entirely complete the adjoint multiplet of the internal symmetry group SU(N). Remarkably, they remain strictly massless being protected by the simultaneously generated non-Abelian gauge invariance. When expressed in these zero modes, the theory look essentially nonlinear and contains many Lorentz and CPT violating couplings. However, as in the SUSY QED case, they do not lead to physical SLIV effects which due to simultaneously generated gauge invariance appear to be strictly cancelled out.

As in the pure QED case, one can calculate the gauge functions $\varphi^{p}(x)$ comparing the relation between the old and new vector fields in (4.45) with a conventional SLIV parametrization for non-Abelian vector fields [9]

$$\mathbf{A}^{p}_{\mu} = \mathbf{a}^{p}_{\mu} + \mathbf{n}^{p}_{\mu}\sqrt{\mathbf{M}^{2} - \mathbf{n}^{2}\mathbf{a}^{2}} , \ \mathbf{n}^{p}_{\mu}\mathbf{a}^{p\mu} = 0 \quad (\mathbf{a}^{2} \equiv \mathbf{a}^{p}_{\mu}\mathbf{a}^{p\mu}).$$
(4.50)

They are expressed through the non-Abelian Goldstone and pseudo-Goldstone modes a^p_μ

$$\boldsymbol{\phi}^{\mathrm{p}} = \boldsymbol{\varepsilon}^{\mathrm{p}} \int^{x} d(\boldsymbol{n}_{\mu} \boldsymbol{x}^{\mu}) \sqrt{\mathbf{M}^{2} - \boldsymbol{n}^{2} \boldsymbol{a}^{2}} \,. \tag{4.51}$$

Here n_{μ} is the unit Lorentz vector being analogous to the vector introduced in the Abelian case (4.2), which is now oriented in Minkowskian spacetime so as to be "parallel" to the vacuum unit n_{μ}^{p} matrix. This matrix can be taken in the "two-vector" form

$$\mathbf{n}^{\mathrm{p}}_{\mathrm{\mu}} = \mathbf{n}_{\mathrm{\mu}} \boldsymbol{\varepsilon}^{\mathrm{p}} , \ \boldsymbol{\varepsilon}^{\mathrm{p}} \boldsymbol{\varepsilon}^{\mathrm{p}} = 1 \tag{4.52}$$

where ε^{p} is the unit SU(N) group vector belonging to its adjoint representation.

4.3.4 Some immediate outcomes

Quite remarkably, an obligatory split symmetry form $U(1) \times SU(N)$ (or $U(1) \times G$, in general) of plausible emergent theories which could exist beyond the prototype QED case, leads us to the standard electroweak theory with an $U(1) \times SU(2)$ symmetry as the simplest possibility. The potential of type (4.21) written for the corresponding superfields requires spontaneous SUSY breaking in the visible sector to avoid the vacuum instability in the theory. Eventually, this requires the SLIV type constraints to be put on the hypercharge and weak isospin vector fields, respectively,

$$B_{\mu}B^{\mu} = M^2$$
, $W^p_{\mu}W^{p\mu} = M^2$ (p = 1, 2, 3). (4.53)

These constraints are independent from each other and possess, as was generally argued above, the total symmetry SO(1,3) × SO(3,9) which is much higher than the actual Lorentz invariance and electroweak U(1) × SU(2) symmetry in the theory. Thanks to this fact, one Goldstone and three pseudo-Goldstone zero vector modes b_{μ} and w_{μ}^{p} are generated to eventually complete the gauge multiplet of

the Standard Model

$$B_{\mu} = b_{\mu} + n_{\mu} \sqrt{M^2 - b_{\mu} b^{\mu}}, \ n_{\mu} b_{\mu} = 0,$$

$$W^{p}_{\mu} = w^{p}_{\mu} + n_{\mu} \epsilon^{p} \sqrt{M^2 - w^{q}_{\mu} w^{q\mu}}, \ n_{\mu} w^{p\mu} = 0$$
(4.54)

where the unit vectors n_{μ} and ε^{p} are defined in accordance with a rectangular unit matrix n_{μ}^{p} taken in the two-vector form (4.52). The true vector Goldstone boson appear to be some superposition of the zero modes b_{μ} and w_{μ}^{3} . This superposition is in fact determined by the conventional Higgs doublet in the model since just through the Higgs field couplings these modes are only mixed [19]. Thus, when the electroweak symmetry gets spontaneously broken an accidental degeneracy related to the total symmetry of constraints mentioned above is lifted. As a consequence, the vector pseudo-Goldstones acquire masses and only photon, being the true vector Goldstone boson in the model, is left massless. In this sense, there is not much difference for a photon in emergent QED and SM: it emerges as a true vector Goldstone boson in both frameworks.

Going beyond the Standard Model we unavoidably come to the flipped SU(5)GUT [27] as a minimal and in fact distinguished possibility. Indeed, the U(1) symmetry part being mandatory for emergent theories now naturally appears as a linear combination of a conventional electroweak hypercharge and another hypercharge belonging to the standard SU(5). The flipped SU(5) GUT has several advantages over the standard SU(5) one: the doublet-triplet splitting problem is resolved with use of only minimal Higgs representations and protons are naturally long lived, neutrinos are necessarily massive, and supersymmetric hybrid inflation can easily be implemented successfully. Also in string theory, the flipped SU(5)model is of significant interest for a variety of reasons. In essence, the abovementioned natural solution to the doublet-triplet splitting problem without using large GUT representations is in the remarkable conformity with string theories where such representations are typically unavailable. Also, in weakly coupled heterotic models, the flipped SU(5) allows to achieve gauge coupling unification at the string scale 10¹⁷ GeV if some extra vector-like particles are added. They are normally taken to transform in the 10 and $\overline{10}$ representations, that is easy to engineer in string theory.

So, supersymmetric emergent theories look attractive both theoretically and phenomenologically whether they are considered at low energies in terms of the Standard Model or at high energies as the flipped SU(5) GUTs being inspired by superstrings.

4.4 Summary

As we have seen above, spontaneous Lorentz violation in a vector field theory framework may be active as in the composite and potential-based models leading to physical Lorentz violation, or inactive as in the constraint-based models resulting in the nonlinear gauge choice in an otherwise Lorentz invariant theory. Remarkably, between these two basic SLIV versions SUSY unambiguously chooses the inactive SLIV case. Indeed, SUSY theories only admit the bilinear mass term in the vector field potential energy. As a result, without a stabilizing quartic vector field terms, the physical spontaneous Lorentz violation never occurs in SUSY theories. Hence it follows that the composite and potential-based SLIV models can in no way be realized in the SUSY context. This may have far-going consequences in that supergravity and superstring theories could also disfavor such models in general.

Though, even in the case when SLIV is not physical it inevitably leads to the generation of massless photons as vector NG bosons provided that SUSY itself is spontaneously broken. In this sense, a generic trigger for massless photons to dynamically emerge happens to be spontaneously broken supersymmetry rather than physically manifested Lorentz noninvariance. To see how this idea might work we have considered supersymmetric QED model extended by an arbitrary polynomial potential of a general vector superfield that induces spontaneous SUSY violation in the visible sector, and gauge invariance gets broken as well. Nevertheless, the special gauge invariance is in fact recovered in the broken SUSY phase that universally protects the photon masslessness.

All basic arguments developed in SUSY QED were then generalized to Standard Model and Grand Unified Theories. Remarkably, thanks to a generic high symmetry of the length-fixing SLIV constraint (4.49) put on the vector fields the emergence conjecture with dynamically produced massless gauge modes can be applied to any non-Abelian global internal symmetry case due to which it gets converted into to the local one. For definiteness, we have focused above on the $U(1) \times SU(N)$ symmetrical theories. Such a split group form is dictated by the fact that in the pure non-Abelian symmetry case one only has the SUSY invariant phase in the theory that would make it inappropriate for an outgrowth of an emergence process. As we briefly discussed, supersymmetric emergent theories look attractive both theoretically and phenomenologically whether they are considered at low energies in terms of the Standard Model or at high energies as the flipped SU(5) GUTs inspired by superstrings.

However, their most generic manifestations, as I discussed here in Bled about a year ago [25] (for more details, see also [9]), is related to a spontaneous SUSY violation in the visible sector that seems to open a new avenue for exploring the origin of gauge symmetries. Indeed, the photino emerging due to this violation will be then mixed with another goldstino which stems from a spontaneous SUSY violation in the hidden sector. Eventually, it largely turns into light pseudogoldstino whose physics seems to be of special interest. Such pseudo-Goldstone photinos might appear typically as the eV scale stable LSP or the electroweak scale long-lived NLSP, being accompanied by a very light gravitinos in both cases. Their observation could shed some light on an emergence nature of gauge symmetries.

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