Reducing the Number of Solutions in the Unit Commitment Problem Using Variations with Repetition

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Abstract. *In this paper, a new approach has been presented to reduce the number of solutions in the unit commitment (UC) problem, using variations with repetition, which can not be used in power system exploitation, due to the impossibility of meeting the loads and limitations associated with the minimal operation time after the start-up and minimal pause time after shutting down thermal units (minimum up/down). Each variation that repeats is representative of the corresponding drive status and the number of thermal units from the economic dispatching (ED) process will be executed for a given time interval. The problem is formulated as a complex mathematical optimization task that includes both binary (power status of thermal units, on/off) and real variables (production of active power of engaged thermal units). The suggested approach is tested on a system with ten thermal units showing very good performances.*

1 Introduction

In everyday life, we often find a series of problems whose solution is necessary for normal function of power system. By developing the power of today's computers, part of these problems has become trivial. However, a large set of problems still poses an extremely difficult challenge. These are the problems that we can not solve with today's computer development by using complete enumeration (counting) of possible solutions, due to extensive requirements at the time of the budget. One of the typical representatives of this type of problem is NP-hard problems, which is also a UC problem. Due to the large number of thermal units of different characteristics, as well as the dynamics of the system, this is a very complex task of optimization, so the researchers' efforts are focused on seeking a suboptimal solution, with moderate demands on the time of budget and computer resources.

The most commonly used methods for solving UC problems are list priority methods [1], dynamic programming method [2], Lagrange relaxation problem [3], Benders decomposition [4]. Recent results also include some heuristic methods such as taboo search, simulated annealing, neural networks [5], genetic algorithms [6], and fuzzy logic [7].

2 UC problem formulation

The UC problem aims at minimizing the total production cost over the scheduling horizon satisfying all constraints. In addition, the total production cost comprises fuel cost, which are related to operation of thermal units, start-up costs and shut down costs. In this sense, the general form of the UC problem can be described as:

$$\min F_{TC} = \sum_{t=i}^{T}\sum_{i=1}^{N}\left[F_i(t)\cdot u_i(t) + SC_i(t)\cdot v_i(t)\right] \quad (1)$$

where:
i : Index of thermal unit
t : Index of hour
F_{TC} : Total production cost
$F_i(t)$: Fuel cost for thermal unit i at hour t
$u_i(t)$: i-th thermal unit status at hour t
(ON: $u_i(t)=1$; OFF: $u_i(t)=0$)
$SC_i(t)$: Start-up cost for thermal unit i
$v_i(t)$: i-th thermal unit start-up/shut down status at hour t (if the thermal unit started $v_i(t)=1$, otherwise $v_i(t)=0$)
N : Number of thermal units
T : Scheduling horizon

The main component of the operating costs is the i-th unit cost function $F_i(t)$ which can be viewed as a quadratic function of the power output at hour t:

$$F_i(t) = a_i + b_i P_i(t) + c_i P_i^2(t) \quad (2)$$

where:
$P_i(t)$: Output power of i-th thermal unit at hour t
a_i, b_i and c_i: Cost coefficients of the i-th thermal unit

The start-up costs in the thermal units change from some of the highest values when the thermal units start from a completely cold state, to considerably lower values if the start-up is carried out after short cooling of the thermal units. In other words, the start-up costs are the functions of the cooling time of the boiler and mathematically can be presented in the form of

$$SC_i(t) = \begin{cases} hc_i : T_i^{off} \le X_i^{off}(t) \le T_i^{off} + cs_i \\ cc_i : X_i^{off} > T_i^{off} + cs_i \end{cases} \quad (3)$$

where:

T_i^{off} : Minimum down time of i-th thermal unit

$X_i^{off}(t)$: Duration that the i-th thermal unit is continuously OFF

hc_i : Hot start-up cost of i-th thermal unit

cc_i : Cold start-up cost of i-th thermal unit

cs_i : Cold start-up hours of i-th thermal unit

Note that if the number of hours less than cs_i we use hot start-up costs otherwise we use cold start-up costs.

Different constraints of the system are to be satisfied while solving the complex UC problem. Moreover, the constraints that must be fullfiled at specific time are as follows:

- System power balance

$$\sum_{i=1}^{N} P_i(t) = P_D(t) \quad (4)$$

where:

$P_D(t)$: Power demand at hour t

- Minimum up/down time

$$\begin{cases} T_i^{on} \le X_i^{on}(t) \\ T_i^{off} \le X_i^{off}(t) \end{cases} \quad (5)$$

where:

T_i^{on}: Minimum up time of i-th thermal unit

$X_i^{off}(t)$: Duration that the i-th thermal unit is continuously ON

- Output limit of thermal units

$$P_i^{\min} \le P_i(t) \le P_i^{\max} \quad (6)$$

where:

$P_i^{\max}$: Maximum power output for i-th thermal unit

$P_i^{\min}$: Minimum power output for i-th thermal unit

After the defining the UC problem together with the function of total production costs (1), its components (2) and (3), as well as different constraints (4) – (6), there is need to choose an appropriate method for solving the UC problem.

3 Variations with repetition

When the ordering of objects matters, and an object can be choosen more than once, we are talking about variations with repetition, and the number of variations is:

$$V_n^k = n^k \quad (7)$$

where:

n: Number of objects from which we can choose

k: Number of objects we can choose

For instance, the first element can be chosen in n ways, the second element can be chosen in n ways, the third element in n ways, etc., k-th element in n ways, because it is allowed to repeat the elements from the set.

Since the operating status of the i-th thermal unit at hour t (ON: 1; OFF: 0) is used in solving the UC problem, it is n = 2, while the whole number of repeating variations for a one-hour period is defined by the expression:

$$\underbrace{(0\,0\,0\ldots0\,0)}_{1},\underbrace{(0\,0\,0\ldots0\,1)}_{2},\ldots,\underbrace{(1\,1\,1\ldots1\,1)}_{2^k} \quad (8)$$

4 Solution methodology

The daily load curve is characterized by a big disparity between minimal and maximum loads despite efforts to make the load as balanced as possible. At all times, the load value must be fullfiled by using a production of thermal units. This means that just a few number of variations with repetition will be used, due to the impossibility of loads to meet its value and constraints associated with the minimal operation time after the start-up and minimal pause time after shutting down thermal units.

In order to obtain an optimum or suboptimal solution, the initial solution of the problem is represented by a binary matrix of dimension T x N where all elements are assigned the value 1 (all thermal units are ON) as shown in Table 1.

Table 1. Initial solution

	1	**2**	**3**	**⋯**	**N**
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
⋮	1	1	1	1	1
T	1	1	1	1	1

Each row vector in the solution matrix is the representative of the corresponding drive state of the thermal unit for the time interval T and indicates the number of thermal units over which the economic dispatching process will be executed [8]. For the first hour, after the ED procedure, a low-cost solution is adopted and replaced in the initial solution, and verifies whether the solution is applicable. If the solution is applicable, the ED process is executed for the second hour and replaced in the lowest cost solution solution matrix, and it is again verified whether the solution is applicable or not. This process is repeated for all hours in the time interval T. If the solution is not applicable for a given hour, then it is replaced by the first higher occurance for that hour and so until an applicable solution is obtained.

5 Results

To illustrate the efficiency of the proposed method as well as solving technique, testing was performed on a test system whose data is given in Table 2 [6] and the 24-hour interval.

The optimum number of thermal units involved is continually changing with the time-changing load. In practice, it is common that an one hour is the minimum time required to switch the selected thermal unit from off to on, so it can be stated that at any time step (one hour), the assumption of constant load could be supposed. This represents the conversion of a continual problem into a discrete problem as shown in Figure 1.

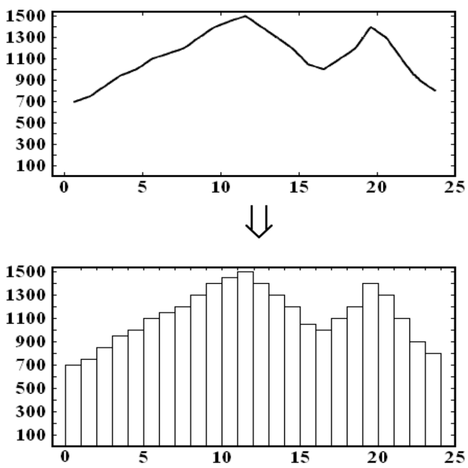

Figure 1. Conversion of continuous into discrete problem

The exact solution of the problem can only be obtained by fully enumerating (counting) the possible working conditions of the thermal units, which is impossible to achieve on systems of realistic size due to the large number of requests at the time of the calculation.

In order to reduce the number of variations with repetition in the UC problem with the operating status {0,1} and the total number (k = 10) of the thermal units in the system for a time period of one hour, it is possible to obtain:

$$\overline{V}_2^{10} = 2^{10} = 1024 \text{ variations with repetition}$$
$$\underbrace{(0000000000)}_{1}, \underbrace{(0000000001)}_{2}, ..., \underbrace{(1111111111)}_{1024}$$

Because most of the above variations are not applicable to a power system exploitation due to impossibility of loading, Table 3 shows the number of variations with repetition that are applicable in the power system exploitation for different loads occurring in the system.

Table 3. Number of variations with repetition that are applicable

Hour	Load (MW)	Number
1	700	671
2	750	623
3	850	498
4	950	359
5	1000	314
6	1100	242
7	1150	222
8	1200	187
9	1300	121
10	1400	53
11	1450	34
12	1500	18
13	1400	53
14	1300	121
15	1200	187
16	1050	276
17	1000	314
18	1100	242
19	1200	187
20	1400	53
21	1300	121
22	1100	242
23	900	422
24	800	564

Table 2. Parameters of thermal units

Thermal Units No.	a_i	b_i	c_i	P_i^{min}	P_i^{max}	T_i^{on}	T_i^{off}	hc_i	cc_i	cs_i	Initial state	
1	1000	16.19	0.00048	150	455	8	8	4500	9000	5	8	+ (ON)
2	970	17.26	0.00031	150	455	8	8	5000	1000	5	8	+ (ON)
3	700	16.60	0.00200	20	130	5	5	550	1100	4	-5	- (OFF)
4	680	16.50	0.00211	20	130	5	5	560	1120	4	-5	- (OFF)
5	450	19.70	0.00398	25	162	6	6	900	1800	4	-6	- (OFF)
6	370	22.26	0.00712	20	80	3	3	170	340	2	-3	- (OFF)
7	480	27.74	0.00079	25	85	3	3	260	520	2	-3	- (OFF)
8	660	25.92	0.00413	10	55	1	1	30	60	0	-1	- (OFF)
9	665	27.27	0.00222	10	55	1	1	30	60	0	-1	- (OFF)
10	670	27.79	0.00173	10	55	1	1	30	60	0	-1	- (OFF)

An example of the calculation costs using the described method for 24-hour interval is given in Table 4.

6 Discussion

In addition to the fact that the problem UC is a highly complex mathematical optimization task that includes both binary and real variables, the additional complexity of the problem is the presence of dynamic constraints on thermal units. In order to solve these problems, the number of solutions in the UC problem has been reduced and from the obtained results it is possible to see that by applying the initial solution in which all the thermal units are in operation, using the variations with repetition it is possible to find the applicable solutions much faster than the initial solution is given by default. By limiting itself to choose the lowest cost solution in a given hour, a path that matches an optimal or suboptimal solution is chosen.

7 References

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Table 4. Operating and startup costs, load and schedule for 24 hours time frame

Hour	Operation cost ($)	Start up cost ($)	Load (MW)	Generation schedule									
				1	2	3	4	5	6	7	8	9	10
1	13683	0	700	455	245	0	0	0	0	0	0	0	0
2	14554	0	750	455	295	0	0	0	0	0	0	0	0
3	16302	0	850	455	395	0	0	0	0	0	0	0	0
4	18638	1120	950	455	365	0	130	0	0	0	0	0	0
5	19513	0	1000	455	455	0	130	0	0	0	0	0	0
6	21860	1800	1100	455	455	0	130	60	0	0	0	0	0
7	22879	0	1150	455	435	0	130	110	0	0	0	0	0
8	23918	0	1200	455	455	0	130	160	0	0	0	0	0
9	26184	1100	1300	455	455	130	130	130	0	0	0	0	0
10	28768	340	1400	455	455	130	130	162	68	0	0	0	0
11	30583	520	1450	455	455	130	130	162	80	38	0	0	0
12	32542	60	1500	455	455	130	130	162	80	33	55	0	0
13	29222	0	1400	455	455	130	130	162	0	68	0	0	0
14	26184	0	1300	455	455	130	130	130	0	0	0	0	0
15	23918	0	1200	455	455	0	130	160	0	0	0	0	0
16	20896	0	1050	455	440	0	130	25	0	0	0	0	0
17	20020	0	1000	455	390	0	130	25	0	0	0	0	0
18	21860	0	1100	455	455	0	130	60	0	0	0	0	0
19	23918	0	1200	455	455	0	130	160	0	0	0	0	0
20	30485	920	1400	455	455	0	130	162	80	63	55	0	0
21	27167	0	1300	455	455	0	130	162	73	25	0	0	0
22	22546	0	1100	455	455	0	130	0	35	25	0	0	0
23	17795	0	900	455	445	0	0	0	0	0	0	0	0
24	15427	0	800	455	345	0	0	0	0	0	0	0	0
Total	**548862**	**5860**	**27100**										