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14 Vector and Scalar Gauge Fields with Respect to $d = (3+1)$ in Kaluza-Klein Theories and in the *Spin-charge-family theory*

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Abstract. This contribution is to prove that in the Kaluza-Klein like theories the vielbeins and the spin connection fields — as used in the *spin-charge-family* theory — lead in $d = (3+1)$ space to equivalent vector (and scalar) gauge fields. The authors demonstrate this equivalence in spaces with the symmetry: $g_{\alpha\beta} = \eta_{\alpha\beta} e$, for any scalar function e of the coordinates x^α.

Povzetek. Prispevek dokazuje na posebnem primeru izometričnih prostorov, da vodijo v teorijah Kaluza-Kleinovega tipa vektorski svežnji in spinske povezave (uporabljene v teoriji *spinov-nabojev-družin*) v prostoru z $d = (3+1)$ do ekvivalentnih vektorskih (in skalarnih) umeritvenih polj. Avtorja demonstrirata enakovrednost obeh pristopov za prostore s simetrijo: $g_{\alpha\beta} = \eta_{\alpha\beta}\, e$, kjer je e poljubna skalarna funkcija koordinat x^α.

14.1 Introduction

This contribution is to demonstrate that in spaces with the symmetry of metric tensor $g_{\alpha\beta} = \eta_{\alpha\beta} e$, where $\eta_{\alpha\beta}$ is the diagonal matrix and e any scalar function of the coordinates, both procedures - the ordinary Kaluza-Klein procedure with vielbeins and the procedure with the spin connections used in the *spin-charge-family* theory - lead in $d = (3+1)$ to the same gauge vector and scalar fields.

In the starting action of the *spin-charge-family* theory[1–3] fermions interact with the vielbeins $f^\alpha{}_a$ and the two kinds of the spin-connection fields - $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ - the gauge fields of $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$ and $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$, respectively.

$$\begin{aligned}\mathcal{A} = & \int d^d x\, E\, \frac{1}{2}\, (\bar{\psi}\, \gamma^a p_{0a} \psi) + \text{h.c.} + \\ & \int d^d x\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \end{aligned} \tag{14.1}$$

here $p_{0a} = f^\alpha{}_a p_{0\alpha} + \frac{1}{2E}\{p_\alpha, E f^\alpha{}_a\}_-$, $p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}$, $R = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\, (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^c{}_{b\beta})\} + \text{h.c.}$, $\tilde{R} = \frac{1}{2}\{f^{\alpha[a} f^{\beta b]}\, (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\, \tilde{\omega}^c{}_{b\beta})\} +$ h.c.. The action introduces two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$,

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+\,. \tag{14.2}$$

$f^{\alpha}{}_{a}$ are vielbeins inverted to $e^{a}{}_{\alpha}$, Latin letters $(a, b, ..)$ denote flat indices, Greek letters $(\alpha, \beta, ..)$ are Einstein indices, $(m, n, ..)$ and $(\mu, \nu, ..)$ denote the corresponding indices in $(0, 1, 2, 3)$, $(s, t, ..)$ and $(\sigma, \tau, ..)$ denote corresponding indices in $d \geq 5$:

$$e^{a}{}_{\alpha} f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}\,, \quad e^{a}{}_{\alpha} f^{\alpha}{}_{b} = \delta^{a}_{b}\,, \tag{14.3}$$

$E = \det(e^{a}{}_{\alpha})$. The action $\mathcal{A}$ offers the explanation for all the properties of the observed fermions and their families and of the observed vector gauge fields, the scalar higgs and the Yukawa couplings.

The spin connection fields and the vielbeins are related fields, and if there are no spinor (fermion) sources both kinds of the spin connection fields are expressible with the vielbeins. In Ref. [2] (Eq. (C9)) the expressions related the spin connection fields of both kinds with the vielbeins and the spinor sources are presented.

We prove in this contribution that in the spaces with the maximal number of the Killing vectors [4] (p. 333–340) and no spinor sources either the vielbeins or the spin connections can be used in Kaluza-Klein theories [5] to derive all the vector and scalar gauge fields. We present below the relation among the $\omega_{ab\alpha}$ fields and the vielbeins with no sources present, which is relevant for our discussions ([2], Eq. (C9)).

$$\begin{aligned}
\omega_{ab}{}^{e} = {} & \frac{1}{2E}\{e^{e}{}_{\alpha}\,\partial_{\beta}(Ef^{\alpha}{}_{[a}f^{\beta}{}_{b]}) - e_{a\alpha}\,\partial_{\beta}(Ef^{\alpha}{}_{[b}f^{\beta e]}) \\
& - e_{b\alpha}\partial_{\beta}(Ef^{\alpha[e}f^{\beta}{}_{a]})\} \\
& - \frac{1}{d-2}\{\delta^{e}_{a}\frac{1}{E}\,e^{d}{}_{\alpha}\partial_{\beta}(Ef^{\alpha}{}_{[d}f^{\beta}{}_{b]}) - \delta^{e}_{b}\frac{1}{E}e^{d}{}_{\alpha}\partial_{\beta}(Ef^{\alpha}{}_{[d}f^{\beta}{}_{a]})\}\,,
\end{aligned} \tag{14.4}$$

(The expression for the spin connection fields carrying family quantum numbers is in the case that there are no spinor sources identical with the right hand side of Eq. 14.4.) One notices that if there are no spinor sources, carrying the spinor quantum numbers S^{ab}, then ω_{abc} is completely determined by the vielbeins (and so is $\tilde{\omega}_{abc}$).

14.2 Proof that spin connections and vielbeins lead to the same vector gauge fields in $d = (3+1)$

We discuss relations between spin connections and vielbeins when there are no spinor sources present in order to prove that both ways, either using the vielbeins or using the spin connection, lead to equivalent vector gauge fields.

Let the space manifest the rotational symmetry, determined by the infinitesimal coordinate transformations of the kind

$$x'^{\mu} = x^{\mu}\,, \quad x'^{\sigma} = x^{\sigma} + \varepsilon^{st}(x^{\mu})\,E^{\sigma}_{st}(x^{\tau}) = x^{\sigma} - i\varepsilon^{st}(x^{\mu})\,M_{st}\,x^{\sigma}\,, \tag{14.5}$$

where $M^{st} = S^{st} + L^{st}$, $L^{st} = x^{s}p^{t} - x^{t}p^{s}$, S^{st} concern internal degrees of freedom of boson and fermion fields, $\{M^{st}, M^{s't'}\}_{-} = i(\eta^{st'}M^{ts'} + \eta^{ts'}M^{st'} - \eta^{ss'}M^{tt'} - \eta^{tt'}M^{ss'})$. From Eq. (14.5) then follows that

$$-i\,M_{st}\,x^{\sigma} = E^{\sigma}_{st} = x_{s}\,f^{\sigma}{}_{t} - x_{t}\,f^{\sigma}{}_{s}\,, \tag{14.6}$$

and correspondingly $M_{st} = E^{\sigma}_{st} p_{\sigma}$. One derives, when taking into account the last relation and the commutation relations among generators of the infinitesimal rotations, the relation

$$E^{\sigma}_{st} p_{\sigma} E^{\tau}_{s't'} p_{\sigma} - E^{\sigma}_{s't'} p_{\sigma} E^{\tau}_{st} p_{\sigma} = -i(\eta_{st'} E^{\tau}_{ts'} + \eta_{ts'} E^{\tau}_{st'} - \eta_{ss'} E^{\tau}_{tt'} - \eta_{tt'} E^{\tau}_{ss'}) p_{\sigma}\,. \tag{14.7}$$

Let the corresponding background field ($g_{\alpha\beta} = e^{a}{}_{\alpha}\, e^{a}{}_{\beta}$) be

$$e^{a}{}_{\alpha} = \begin{pmatrix} \delta^{m}{}_{\mu} & e^{m}{}_{\sigma} = 0 \\ e^{s}{}_{\mu} & e^{s}{}_{\sigma} \end{pmatrix}, \quad f^{\alpha}{}_{a} = \begin{pmatrix} \delta^{\mu}{}_{m} & f^{\sigma}{}_{m} \\ 0 = f^{\mu}{}_{s} & f^{\sigma}{}_{s} \end{pmatrix}, \tag{14.8}$$

so that the background field in $d = (3+1)$ is flat. From $e^{a}{}_{\mu} f^{\sigma}{}_{a} = \delta^{\sigma}_{\mu} = 0$ it follows

$$e^{s}{}_{\mu} = -\delta^{m}_{\mu} e^{s}{}_{\sigma} f^{\sigma}{}_{m}\,. \tag{14.9}$$

This leads to

$$g_{\alpha\beta} = \begin{pmatrix} \eta_{mn} + f^{\sigma}{}_{m} f^{\tau}{}_{n} e^{s}{}_{\sigma} e_{s\tau} & -f^{\tau}{}_{m} e^{s}{}_{\tau} e_{s\sigma} \\ -f^{\tau}{}_{n} e^{s}{}_{\tau} e_{s\sigma} & e^{s}{}_{\sigma} e_{s\tau} \end{pmatrix}. \tag{14.10}$$

One can check properties of $f^{\sigma}{}_{m}\delta^{m}_{\mu}$ under general coordinate transformations $x'^{\mu} = x'^{\mu}(x^{\nu})$, $x'^{\sigma} = x'^{\sigma}(x^{\tau})$ $\left(g'_{\alpha\beta} = \frac{\partial x^{\rho}}{\partial x'^{\alpha}} \frac{\partial x^{\delta}}{\partial x'^{\beta}} g_{\rho\delta}\right)$

$$f'^{\sigma}{}_{m}\delta^{m}_{\mu} = \frac{\partial x^{\nu}}{\partial x'^{\mu}} \frac{\partial x'^{\sigma}}{\partial x^{\tau}} f^{\tau}{}_{\nu}\,. \tag{14.11}$$

Let us introduce the field $\Omega^{st}{}_{m}(x^{\nu})$ as follows

$$f^{\sigma}{}_{m} := -\frac{1}{2}\, E^{\sigma}_{st}(x^{\tau})\, \Omega^{st}{}_{m}(x^{\nu})\,. \tag{14.12}$$

From Eqs. (14.11,14.12) follow the transformation properties of $\Omega^{st}{}_{m}$ under the coordinate transformations of Eq. (14.5)

$$-E^{\sigma st}\,\delta_0\,\Omega_{stm} = -E^{\sigma st}\{-\varepsilon_{st,m} + i2(\varepsilon_{s}{}^{s'}\,\Omega_{s'tm} - \varepsilon_{t}{}^{s'}\,\Omega_{s'sm})\}\,. \tag{14.13}$$

If we look for the transformation properties of the superpositions of the fields Ω_{stm}, which are the gauge fields of let say τ^{Ai} with the commutation relations $\{\tau^{Ai}, \tau^{Bj}\}_{-} = i\delta^{A}_{B}\, f^{Aijk}\tau^{Ak}$, where $\tau^{Ai} = C^{Ai}{}_{st} M^{st}$, under the coordinate transformations of Eq. (14.5), one finds for the corresponding superposition of the fields Ω_{stm} the transformation properties

$$\delta_0\, A^{Ai}{}_{m} = \varepsilon^{Ai}{}_{,m} + i f^{Aijk} A^{Aj}_{m}\, \varepsilon^{Ak}\,. \tag{14.14}$$

Let us use the expression for $f^{\sigma}{}_{m}$ from Eq. (14.12) in Eq. (14.4) to see the relation among ω_{stm} and $f^{\sigma}{}_{m}$. One finds

$$\begin{aligned} \omega_{stm} = \frac{1}{2E}\{ & f^{\sigma}{}_{m}\,[e_{t\sigma}\partial_{\tau}(E f^{\tau}{}_{s}) - e_{s\sigma}\,\partial_{\tau}(E f^{\tau}{}_{t})] \\ & + e_{s\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m} f^{\tau}{}_{t} - f^{\tau}{}_{m} f^{\sigma}{}_{t})] - e_{t\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m} f^{\tau}{}_{s} - f^{\tau}{}_{m} f^{\sigma}{}_{s})]\}\,. \end{aligned} \tag{14.15}$$

For $\Omega_{stm} = \Omega_{stm}(x^n)$ (as assumed above) and for $f^{\sigma}{}_s = f\delta^{\sigma}_s$ (which requires $e^s{}_{\sigma} = f^{-1}\delta^s_{\sigma}$) it follows for any f

$$\omega_{stm} = \Omega_{stm}. \tag{14.16}$$

Statement: Let the space with $s \geq 5$ *have the symmetry allowing the infinitesimal transformations of the kind*

$$x'^{\mu} = x^{\mu}\,, \quad x'^{\sigma} = x^{\sigma} - i \sum_{A,i,s,t} \varepsilon^{Ai}(x^{\mu})\, c_{Ai}{}^{st} M_{st}\, x^{\sigma}\,, \tag{14.17}$$

then the vielbein $f^{\sigma}{}_m$ in Eq. (14.8) manifest in $d = (3 + 1)$ the vector gauge fields $\mathcal{A}^{Ai}_m$

$$f^{\sigma}{}_m = i \sum_A \vec{\tau}^{A\sigma}{}_{\tau}\, \vec{\mathcal{A}}^A_m x^{\tau}, \tag{14.18}$$

where

$$\begin{aligned}
\tau^{Ai} &= \sum_{Ai} c^{Ai}{}_{st}\, M^{st}\,, \\
\{\tau^{Ai}, \tau^{Bj}\}_- &= i f^{Aijk} \tau^{Ak}\, \delta^{AB}\,, \\
\vec{\tau}^A &= \vec{\tau}^{A\sigma}\, p_{\sigma} = x^{\tau} \vec{\tau}^{A\sigma}{}_{\tau}\, p_{\sigma} \\
A^{Ai}_m &= \sum_{st} c^{Ai}{}_{st} \omega^{st}{}_m\, x^{\tau}\,,
\end{aligned} \tag{14.19}$$

while $\omega^{st}{}_m$ is determined in Eq. (14.15).

We shall prove this statement in the case, when the space $SO(7, 1)$ breaks into $SO(3, 1) \times SU(2) \times SU(2)$. One finds for the two $SU(2)$ generators

$$\begin{aligned}
\vec{\tau}^1 &= \frac{1}{2}\,(M^{58} - M^{67}, M^{57} + M^{68}, M^{56} - M^{78}) \\
\vec{\tau}^2 &= \frac{1}{2}\,(M^{58} + M^{67}, M^{57} - M^{68}, M^{56} + M^{78})\,,
\end{aligned} \tag{14.20}$$

and for the corresponding gauge fields

$$\begin{aligned}
\vec{A}^1_a &= \frac{1}{2}\,(\omega_{58a} - \omega_{67a}, \omega_{57a} + \omega_{68a}, \omega_{56a} - \omega_{78a}) \\
\vec{A}^2_a &= \frac{1}{2}\,(\omega_{58a} + \omega_{67a}, \omega_{57a} - \omega_{68a}, \omega_{56a} + \omega_{78a})\,.
\end{aligned} \tag{14.21}$$

One derives (Ref. [2], Eq. (11))

$$\begin{aligned}
\vec{\tau}^1 &= \vec{\tau}^{1\sigma}\, p_{\sigma} = \vec{\tau}^{1\sigma}{}_{\tau} x^{\tau}\, p_{\sigma}\,, \\
\vec{\tau}^2 &= \vec{\tau}^{2\sigma}\, p_{\sigma} = \vec{\tau}^{2\sigma}{}_{\tau} x^{\tau}\, p_{\sigma}\,, \\
\vec{\tau}^{1\sigma}{}_{\tau} &= \frac{i}{2}\,(e^5{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 5} - e^6{}_{\tau} f^{\sigma 7} + e^7{}_{\tau} f^{\sigma 6}, \\
&\quad e^5{}_{\tau} f^{\sigma 7} - e^7{}_{\tau} f^{\sigma 5} + e^6{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 6}, \\
&\quad e^5{}_{\tau} f^{\sigma 6} - e^6{}_{\tau} f^{\sigma 5} - e^7{}_{\tau} f^{\sigma 8} + e^8{}_{\tau} f^{\sigma 7}), \\
\vec{\tau}^{2\sigma}{}_{\tau} &= \frac{i}{2}\,(e^5{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 5} + e^6{}_{\tau} f^{\sigma 7} - e^7{}_{\tau} f^{\sigma 6}, \\
&\quad e^5{}_{\tau} f^{\sigma 7} - e^7{}_{\tau} f^{\sigma 5} - e^6{}_{\tau} f^{\sigma 8} + e^8{}_{\tau} f^{\sigma 6}, \\
&\quad e^5{}_{\tau} f^{\sigma 6} - e^6{}_{\tau} f^{\sigma 5} + e^7{}_{\tau} f^{\sigma 8} - e^8{}_{\tau} f^{\sigma 7})\,.
\end{aligned} \tag{14.22}$$

The expressions for ${f^{\sigma}}_m$ are correspondingly as follows

$$ {f^{\sigma}}_m = i\,(\vec{\tau}^{1\sigma}{}_{\tau}\,\vec{\mathcal{A}}^1_m + \vec{\tau}^{2\sigma}{}_{\tau}\,\vec{\mathcal{A}}^2_m)\,x^{\tau}\,. \tag{14.23}$$

Expressing the two $SU(2)$ gauge fields, $\vec{A}^1_m$ and $\vec{A}^2_m$, with ω_{stm} as required in Eqs. (14.21), and then using for each ω_{stm} the expression presented in Eq. (14.15), in which ${f^{\sigma}}_m$ is replaced by the relation in Eq. (14.23), while one takes for ${f^{\sigma}}_s$ $= f\delta^{\sigma}_s$, for any f, while then ${e^s}_{\mu} = -\delta^m_{\mu}{e^s}_{\sigma}{f^{\sigma}}_m$, Eq. (14.9), it follows after a longer but straightforward calculation that

$$\begin{aligned} \vec{A}^1_m &= \vec{\mathcal{A}}^1_m\,,\\ \vec{A}^2_m &= \vec{\mathcal{A}}^2_m\,. \end{aligned} \tag{14.24}$$

One obtains this result of any component of $A^{1i}_m and A^{2i}_m$, $i = 1, 2, 3$ separately.

It is not difficult to generalize this poof to any isometry of the space with $s \geq 5$ of any dimensional space, where then

$$ {f^{\sigma}}_m = -i\sum_A \vec{A}^A_m \vec{\tau}^{A\sigma}{}_{\tau} x^{\tau}\,, \tag{14.25}$$

where $\vec{A}^A_m$ are the superposition of ${\omega^{st}}_m$, $A^{Ai}_m = {c^{Ai}}_{st}{\omega^{st}}_m$, which demonstrate the symmetry of the space with $s \geq 5$.

This completes the proof of the above statement.

14.3 Conclusions

We presented the proof, that in spaces without fermion sources either the vielbeins or the spin connections lead in $d = (3 + 1)$ to the equivalent vector gauge fields. The proof offers indeed no surprise due to the fact that the spin connection fields ω_{abc} are expressible with the vielbeins as presented in (Eq. (14.4). This is true also for the scalar gauge fields, although not discussed in this contribution.

The proof is true for any f which is a scalar function of the coordinates x^{σ}, $\sigma \geq 5$. We have shown in Ref. [7,6] that for $f = (1 + \frac{\rho^2}{(2\rho_0)^2})$ the symmetry of the space with the coordinate x^{σ}, $\sigma = (5), (6)$, is a surface S^2, with one point missing.

14.4 Appendix: Derivation of the equality $\vec{A}^1_m = \vec{\mathcal{A}}^1_m$

We demonstrate for the particular case A^{11}_m, equal to $\omega_{58a} - \omega_{67a}$, Eq. (14.21), that this A^{11}_m is equal to $\mathcal{A}^{11}_m$, appearing in Eq. (14.23)

$$ {f^{\sigma}}_m = i\sum_A \mathcal{A}^{Ai}_m {\tau^{Ai\sigma}}_{\tau} x^{\tau}\,. \tag{14.26}$$

When using Eq. (14.15) for $A^{11}_{m} = \omega_{58a} - \omega_{67a}$ we end up with the expression

$$
\begin{aligned}
A^{11}_{m} = \frac{i^2}{2}\frac{1}{2E}\Big\{ & f^{\sigma}{}_{m}[e^{8}{}_{\sigma}\partial_{\tau}(Ef^{\tau 5}) - e^{5}{}_{\sigma}\partial_{\tau}(EF^{\tau 8})] \\
& -f^{\sigma}{}_{m}[e^{7}{}_{\sigma}\partial_{\tau}(Ef^{\tau 6}) - e^{6}{}_{\sigma}\partial_{\tau}(EF^{\tau 7})] \\
& +e^{5}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 8}) - f^{\tau}{}_{m}f^{\sigma 8}] - e^{6}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 7}) - f^{\tau}{}_{m}f^{\sigma 7}] \\
& -e^{8}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 5}) - f^{\tau}{}_{m}f^{\sigma 5}] + e^{7}{}_{\sigma}\partial_{\tau}[E(f^{\sigma}{}_{m}f^{\tau 6}) - f^{\tau}{}_{m}f^{\sigma 6}]\Big\}.
\end{aligned}
\tag{14.27}
$$

We must insert for $f^{\sigma}{}_{m}$ the expression from Eq. (14.23). We obtain

$$
\begin{aligned}
A^{11}_{m} = -\frac{1}{2}\frac{1}{2E}\sum_{i}\mathcal{A}^{1i}_{m}\Big\{ & \tau^{1i\sigma}{}_{\tau'}x^{\tau'}[e^{8}{}_{\sigma}\partial_{\tau}(Ef^{\tau 5}) - e^{5}{}_{\sigma}\partial_{\tau}(EF^{\tau 8}) \\
& -e^{7}{}_{\sigma}\partial_{\tau}(Ef^{\tau 6}) + e^{6}{}_{\sigma}\partial_{\tau}(EF^{\tau 7})] \\
& +e^{5}{}_{\sigma}\delta^{\tau'}_{\tau}E(f^{\tau 8}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 8}\tau^{1i\sigma}{}_{\tau'}) + e^{5}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 8}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 8}\tau^{1i\sigma}{}_{\tau'})] \\
& -e^{6}{}_{\sigma}E(f^{\tau 7}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 7}\tau^{1i\sigma}{}_{\tau'}) - e^{6}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 7}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 7}\tau^{1i\sigma}{}_{\tau'})] \\
& -e^{8}{}_{\sigma}E(f^{\tau 5}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 5}\tau^{1i\sigma}{}_{\tau'}) - e^{8}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 5}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 5}\tau^{1i\sigma}{}_{\tau'})] \\
& +e^{7}{}_{\sigma}E(f^{\tau 6}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 6}\tau^{1i\sigma}{}_{\tau'}) + e^{7}{}_{\sigma}x^{\tau'}\partial_{\tau}[E(f^{\tau 6}\tau^{1i\sigma}{}_{\tau'} - f^{\sigma 6}\tau^{1i\sigma}{}_{\tau'})]\Big\}.
\end{aligned}
\tag{14.28}
$$

We can write Eq. (14.28) in a compact way as follows

$$
A^{11}{}_{m} = -\frac{1}{2}\frac{1}{2E}\sum_{i}\mathcal{A}^{1i}{}_{m}\mathcal{C}^{1i}\,, \tag{14.29}
$$

where $\mathcal{C}^{1i}$ can be read off Eq. (14.28). Taking into account in Eqs. (14.22, 14.28) that $f^{\sigma}{}_{s} = f\delta^{\sigma}_{s}$ and $e^{s}{}_{\sigma} = f\delta^{s}_{\sigma}$ we find that most of terms in $\mathcal{C}^{11}$ cancel each other. The only term, which remains, originates in terms from coordinate derivatives, leading to

$$
\mathcal{C}^{11} = 0 + Ef(\tau^{11\,58} - \tau^{11\,67} - \tau^{11\,85} + \tau^{11\,76})\,, \tag{14.30}
$$

while we found that $\mathcal{C}^{12} = 0 = \mathcal{C}^{13}$.

Recognizing that $\mathcal{C}^{2i}$ contribute to A^{11}_{m} nothing, we can conclude that $A^{11}{}_{m} = \mathcal{A}^{11}{}_{m}$.

One easily see that to the expressions for $A^{Ai}{}_{m}$ only $\mathcal{C}^{Ai}$ contribute, while all $\mathcal{C}^{Bj}$, $B \neq A$ and $j \neq i$ contribute nothing. This completes the proof that $\vec{A}^{A}{}_{m} = \vec{\mathcal{A}}^{A}{}_{m}$, for all the gauge fields $\vec{A}^{A}{}_{m}$ of the charges $\vec{\tau}^{A}$, Eq. (14.22).

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