Testing the Performance of Cubic Splines and Nelson-Siegel Model for Estimating the Zero-coupon Yield Curve

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Abstract

Understanding the relationship between interest rates and term to maturity of securities is a prerequisite for developing financial theory and evaluating whether it holds up in the real world; therefore, such an understanding lies at the heart of monetary and financial economics. Accurately fitting the term structure of interest rates is the backbone of a smoothly functioning financial market, which is why the testing of various models for estimating and predicting the term structure of interest rates is an important topic in finance that has received considerable attention for many decades. In this paper, we empirically contrast the performance of cubic splines and the Nelson-Siegel model by estimating the zero-coupon yields of Austrian government bonds. The main conclusion that can be drawn from the results of the calculations is that the Nelson-Siegel model outperforms cubic splines at the short end of the yield curve (up to 2 years), whereas for medium-term maturities (2 to 10 years) the fitting performance of both models is comparable.

Keywords: Cubic splines; Nelson-Siegel; yield curve; zero-coupon bonds; term structure of interest rates

1 Introduction

The purpose of this paper is to construct a zero-coupon yield curve based on the data for coupon yields, price, and maturity of Austrian government bonds. The models used are the Nelson-Siegel and cubic splines. Forecasting the term structure of interest rates is a prerequisite for managing investment portfolios, pricing financial assets and their derivatives, calculating risk measures, valuing capital goods, managing pension funds, formulating economic policy, making household finance decisions, and managing fixed-income wealth. Prices of fixed-income securities such as swaps, bonds, and mortgage-backed securities depend on the yield curve. The yields on default-free government bonds that have different maturities, when examined together, reveal information about forward rates, which can predict real economic activity and are, hence, of interest for policymakers, market participants, and economists. For instance, forward rates are often inputs to pricing models and may indicate market expectations of the movement of inflation rates and currency appreciation/depreciation rates in the future. Understanding the relationship between interest rates and the term to maturity of securities is a prerequisite for developing and testing financial theory and, therefore, lies at the heart

of monetary and financial economics. Accurately fitting the term structure of interest rates is the backbone of a smoothly functioning financial market, which is why refining yield curve modeling and forecasting methods is an important topic in finance that has received considerable attention for many decades (De Rezende & Ferreira, 2013; Diebold, Li, & Yue, 2008; Exterkate, van Dijk, Heij, & Groenen, 2013; Ioannides, 2003; Jordan & Mansi, 2003; Linton, Mammen, Nielsen, & Tanggaard, 2001; Rugengamanzi, 2013).

The yield curve is a graphical representation of the term structure of interest rates (i.e., a one-to-one relationship between yields and corresponding maturities of default-free zero-coupon securities issued by sovereign lenders). The term structure of interest rates contains information about the yields of zero-coupon bonds[1] of various maturities at a certain date. Constructing the term structure of interest rates is not a straightforward task due to the scarcity of zero-coupon bonds on the market, which represent the essential part of the term structure of interest rates. The majority of bonds traded in the market bear coupons. The yields to maturity on coupon-bearing bonds, whose maturities or coupons differ, are not immediately comparable. As a result, a uniform way of measuring the term structure of interest rates is needed: The spot interest rates[2] (i.e., the yields earned on bonds that pay no coupon) must be estimated from coupon bond prices of bonds with different maturities by using interpolation methods, such as polynomial splines (e.g., cubic splines) and parsimonious functions (e.g., Nelson-Siegel). This is how the yield curve of zero-coupon bonds is constructed (Christensen, Diebold, & Rudebusch, 2011; Christofi, 1998; Gauthier & Simonato, 2012; Luo, Han, & Zhang, 2012; Teichmann & Wüthrich, 2013; Yu & Zivot, 2011).

The most widely used models for estimating the zero-coupon yield curve are Nelson-Siegel and cubic splines. For instance, the central banks of Belgium, Finland, France, Germany, Italy, Norway, Spain, and Switzerland use the Nelson-Sigel model or some type of its enhanced extension to fit and forecast yield curves (BIS, 2005). The European Central Bank uses the Sonderlind-Svensson model, an extension of the Nelson-Siegel model, to estimate yield curves in the Eurozone (Coroneo, Nyholm & Vidova-Koleva, 2011).

The remainder of the paper is organized as follows. Section 2 gives an overview of the existing literature and relevant research studies. Section 3 presents the data; Section 4 lays out the methodology and the results. Section 5 concludes the paper.

2 Literature Review

The yield curve estimation methods originated in McCulloch's (1975) cubic splines and in Nelson and Siegel's (1987) parsimonious function. Most of the research studies on the term structure of interest rates build on these two methods and propose improvements and extensions. McCulloch (1975) modeled the discount curve with a spline. The fitted discount curve gives a poor fit of the yield curves, most notably at longer maturities. Nelson-Siegel's parsimonious function allows for various shapes of the yield curve. The forward rates are a solution to a second-order differential equation. The forward rate curve under McCulloch's method is not smooth, whereas the forward rate curve under Nelson-Siegel's method is smooth, but still unable to accurately price instruments at the longest end of the yield curve (Rugengamanzi, 2013).

The Nelson-Siegel type of models are relatively efficient in capturing the general shapes of the yield curve, which is why they are extensively used by central banks and market practitioners. Nevertheless, the Nelson-Siegel type of models are still inferior to the dynamic term structure models, like the quadratic or affine term structure models or the forward rate-based arbitrage-free model introduced by Heath, Jarrow, and Morton (1992). Jordan and Mansi (2003) used five distinct yield curve-smoothing methods to derive spot rates from on-the-run treasuries. All methods used the bootstrapping[3] technique either in discrete time or in continuous time. Yield curve-smoothing methods based on continuous-time bootstrapping deliver a superior approximation of the term structure of interest rates to those methods that employ the discrete-time bootstrapping technique. Of the five yield curve-smoothing methods, the linear spline interpolation produces the worst results; the second-worst method is cubic splines; and the two best methods are Mansi-Phillips and Nelson-Siegel.

Diebold and Li (2006) developed a dynamic version of the Nelson-Siegel model. They showed that the three factors present in the Nelson-Siegel model can be interpreted as the level, slope, and curvature of the yield curve. They also corroborated that the dynamic model improves the forecasting accuracy. Diebold, Rudebusch, and Aruoba (2006) combined the Diebold-Li model with macroeconomic variables to analyze the relationship between the yield curve and the economy. Diebold et al. (2008) extended the dynamic Diebold-Li model to a global context. A large set of country yield curves was modeled in a setting that allows for both country-specific and global factors. The researchers found that the global yield level and slope factors exist and can

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[1] Another name for "zero-coupon bond" is discount bond.

[2] The terms "yield to maturity on a zero-coupon bond," "zero-coupon interest rate," "spot interest rate" and "zero-coupon yield" are synonyms; they all describe the same aspect of reality.

[3] "Bootstrapping" refers to calculating spot rates from bond yields in an iterative manner (Jordan et al., 2003).

explain a substantial proportion of variation in country bond yields. Christensen, Diebold, and Rudebusch (2009) proposed a generalized arbitrage-free Nelson-Siegel model using five factors. Christensen et al. (2011) added the arbitrage-free restriction to the Diebold-Li model, thereby creating an affine arbitrage-free Nelson-Siegel model (AFNS). The results show an improvement in the model's predictive performance. Yu and Zivot (2011) empirically tested Diebold and Li's dynamic Nelson-Siegel three factor model and found that the dynamic Diebold-Li factor AR(1) model is superior to other models on the out-of-sample forecast accuracy. Luo et al. (2012) compared the forecasting ability of the Diebold-Li, dynamic Svennson, and dynamic Björk and Christensen models for the term structure of Chinese treasury yields. The results showed that all three models fit the data very well and that more flexible models produced superior in-sample fitting performance.

Gauthier and Simonato (2012) developed linearized algorithms for estimating spot interest rate term structures. These algorithms converge much faster while retaining the important characteristics of the original approaches. These algorithms are superior in that they enable the inclusion of prior information about some of the parameters, thereby enhancing the precision of the estimated spot rate curves. Using the Brazilian yield curve data, De Rezende and Ferreira (2013) compared the in-sample adjustment and the out-of-sample forecasting performance of four different Nelson-Siegel type models: Nelson-Siegel, Bliss, Svensson, and a new five-factor model that is an extension of the Svensson model and could improve the fitting flexibility. The introduction of the fifth factor into the model produced the best in-sample fitting, but poor out-of-sample forecasting. Exterkate et al. (2013) investigated whether the inclusion of additional macroeconomic information into the Nelson-Siegel model results in improved yield curve forecasts. In general, the forecasts could not be improved in stable times (e.g., 1994–1998): When the yields are not volatile, the dynamic Nelson-Siegel model produces good yield curve forecasts. The inclusion of additional macroeconomic variables can substantially improve the yield curve forecast accuracy when the yields are volatile (e.g., 2008–2009).

3 Data

We empirically tested the fitting performance of the Nelson-Siegel model and cubic splines on the data for Austrian government bonds. The data were retrieved from Bloomberg[4] on October 8, 2013(see Table 1). All prices are in euros.

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[4] We do not have access to more recent data, as Bloomberg terminal subscription costs around $20,000 per user per annum, which is why the license is mostly bought by and available in major institutions only.

Table 1. Data on Austrian Government Bonds (retrieved from Bloomberg on October 8, 2013).

AT - Benchmark	Maturity	Coupon (%)	Bid (EUR)	Ask (EUR)
1Y BUND	20.10.2014	3,40	103,32	103,35
2Y BUND	15.07.2015	3,50	105,72	105,77
3Y BUND	15.09.2016	4,00	110,14	110,18
4Y BUND	15.09.2017	4,30	113,39	113,46
5Y BUND	15.01.2018	4,65	115,53	115,58
6Y BUND	15.03.2019	4,35	116,25	116,29
7Y BUND	15.07.2020	3,90	114,85	114,91
8Y BUND	15.09.2021	3,50	112,44	112,50
9Y BUND	20.04.2022	3,65	113,35	113,42
10Y BUND	20.10.2023	1,75	95,98	96,02
13Y BUND	15.03.2026	4,85	125,63	125,81

Source: Bloomberg.

The raw data given in Table 1 was used to calculate the accrued interest and the dirty price (i.e., the market value, the present value). The dirty price is a sum of the clean price (i.e., the average of the "bid quote" and the "ask quote") and the accrued interest. The accrued interest is the interest that a bond holder would have obtained in theory between the last coupon date of each bond and the current date (which we assumed to be October 8, 2013—the day the data were extracted).

The Nelson-Siegel model and cubic splines were applied to the data in Table 1 to estimate the zero-coupon bond yield curve.

4 Methodology and Results

The two main categories of methods for estimating a yield curve are the spline methods and the parsimonious methods. The spline methods use a piecewise polynomial function (usually a cubic one) to approximate the yield curve. Cubic splines were first introduced by McCulloch (1975) and subsequently improved by Fisher, Nychka, and Zervos (1995), Waggoner (1997), and Anderson and Sleath (1999). The parsimonious methods (such as Nelson-Siegel model [1987]; Nelson-Siegel-Svensson model [Sevensson, 1995], and models described by Wiseman [1994] and Bjork and Christensen [1997]) approximate the yield curve by estimating the parameters in a single parametric function. With spline methods, it is possible to capture almost any shape of the yield curve, whereas parsimonious methods can capture only yield curves obeying certain financial constraints. Both groups of methods have specific advantages and disadvantages, but none of them seems to be able to consistently

outperform the other (Manousopoulos & Michalopoulos, 2009). In this paper, we employed the most representative type of each method (cubic splines and Nelson-Siegel) with the goal of estimating the zero-coupon yield curve.

4.1 Cubic splines model

The cubic splines method, developed by McCulloch (1971) and McCulloch (1975), divides the zero-coupon yield curve into distinct intervals. In each of these intervals, a cubic spline acts as vertebra in the vertebrate spinal column. If the yield curve is divided into $k–1$ knots, then we need k parameters to describe the entire zero-coupon yield curve. The optimal parameters are obtained by constructing the matrix A, defined in the continuation of the paper. The optimal parameters and the optimal zero-coupon yield curve minimize the discrepancy (the error) between the model price and the market price of government bonds (McCulloch, 1975; Jankowitsch & Pichler, 2003; Rugengamanzi, 2013).

To fit the observed market data for government bond yield curve, McCulloch (1971) used the discount function presented in (1):

$$D(t) = 1 + \sum_{j=1}^{k} f_j(t) \cdot a_j,\tag{1}$$

where a_j are the parameters that need to be estimated. For $j < k$, $f_j(t)$ is a cubic polynomial defined as follows:

I.   When $t < d_{j-1}$:

$$f_j(t) = 0\tag{2}$$

II. When $d_{j-1} \leq t < d_j$:

$$f_j(t) = \frac{(t - d_{j-1})^3}{6 \cdot (d_j - d_{j-1})}.\tag{3}$$

III. When $d_j \leq t < d_{j+1}$:

$$f_j(t) = \frac{c^2}{6} + \frac{c \cdot e}{2} + \frac{e^2}{2} - \frac{e^3}{6 \cdot (d_{j+1} - d_j)},\tag{4}$$

where c = $d_j - d_{j-1}$ and e = $t - d_j$.

IV. When $d_{j+1} \leq t$:

$$f_j(t) = (d_{j+1} - d_{j-1}) \cdot \left[\frac{2 \cdot d_{j+1} - d_j - d_{j-1}}{6} + \frac{t - d_{j+1}}{2} \right].\tag{5}$$

Two additional conditions have to be met:

a. When j = 1:
$$d_j = d_{j-1} = 0\tag{6}$$

b. When j = k:
$$f_j(t) = t\tag{7}$$

In $f_j(t)$, t stands for time and j is a knot number ($j = 1, …, k–1$). Because the number of knots is equal to $k–1$, we need k parameters. The knots are denoted as d_j. We used four parameters ($k = 4$) and three knots ($k–1 = 3$). We set the first knot equal to zero ($d_1 = 0$), the second knot equal to five ($d_2 = 5$), and the third knot equal to 13 ($d_3 = 13$), which corresponds to the maturity of the last bond (McCulloch, 1975; Jankowitsch & Pichler, 2003; Rugengamanzi, 2013).

The zero-coupon yields are calculated from the discount function defined by (1) as follows:

$$r(t) = \left(\frac{1}{D(t)}\right)^{\frac{1}{t}} - 1.\tag{8}$$

In order to calculate the parameters a_j (which are needed in (1) and (8)), we first need to construct matrix A:

$$A = (X \cdot X^T)^{-1} \cdot X^T \cdot Y,\tag{9}$$

where X is a matrix and Y is a matrix. X^T and Y^T are the transposed matrices of matrices X and Y. The matrix A has j rows and one column. The element a_{j1} of matrix A (where $j = 1, …, k$) is an optimal parameter a_j, needed to calculate the discount function, as defined by (1).

The matrix X is given as follows:

$$x_{ij} = \sum_{h=1}^{R_i} Z_i(h) \cdot f_j(w_i(h))\tag{10}$$

for every i=1,…,n and for every j=1,…,k,

where R_i = the number of future cash flows of bond i,
   $Z_i(h)$ = a future cash flow h of bond i,
   $w_i(h)$ = the time to maturity of a next cash flow h of bond i in years, and
   $f_j(t)$ = a function of time t, defined by equations (2) to (5).

The matrix Y is given as follows:

$$y_i = P_i - \sum_{h=1}^{R_i} Z_i(h) \text{ for every } i = 1,…,n,\tag{11}$$

where P_i = the dirty price (the present value, the market value) of bond i.

The dirty price (the present value, the market value) P_i of bond is defined as follows:

P_i=clean price from Bloomberg+accrued interest.   (12)

The accrued interest is defined as follows:

$$
\begin{array}{ccc}
\text{accrued} \\
\text{interest}
\end{array} = \begin{array}{c}\text{the next cash flow} \\ \text{(coupon amount)}\end{array} \times \begin{array}{c}\text{time that has passed} \\ \text{between the last coupon} \\ \text{payment and today} \\ \hline \text{time between two} \\ \text{consecutive coupon} \\ \text{payments}\end{array}
$$

(13)

Table 2. Estimated Discount Factors (D(t)) and Zero-coupon Yields for t = 1, 2, ..., 10.

t	D(t)	r(t)	r(t) in %
1	1,00056	-0,00056	-0,05606
2	0,99443	0,00279	0,27943
3	0,98238	0,00594	0,59424
4	0,96517	0,00890	0,89031
5	0,94355	0,01169	1,16896
6	0,91830	0,01431	1,43070
7	0,89024	0,01675	1,67481
8	0,86020	0,01900	1,90021
9	0,82901	0,02105	2,10539
10	0,79751	0,02288	2,28837

The final results obtained by empirically testing the cubic splines model (equations (1) to (13)) on data given in Table 1 (Austrian government bonds) are summarized in Table 2.

Figure 1. The zero-coupon yield curve, estimated with the cubic splines model.
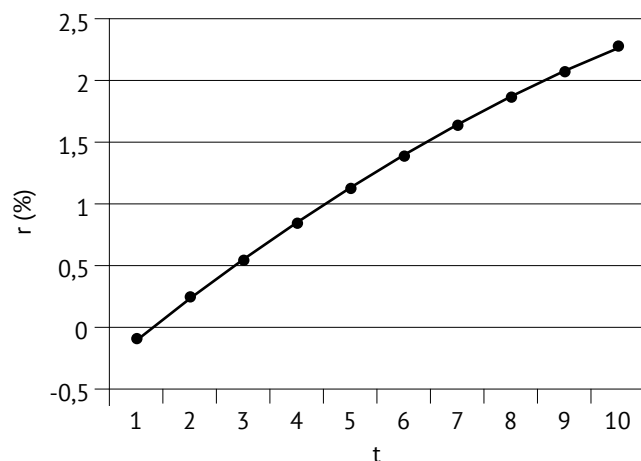
The estimated zero-coupon yield curve is displayed in Figure 1.

4.2 Nelson-Siegel model

Nelson and Siegel (1987) developed a parsimonious function to model forward rates. The zero-coupon yield (the spot rate) can be derived as follows:

$$
r(t) = \beta_1 + \beta_2 \cdot \frac{1 - e^{-\frac{t}{\tau_1}}}{\frac{t}{\tau_1}} + \beta_3 \cdot \left(\frac{1 - e^{-\frac{t}{\tau_1}}}{\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_1}} \right) =
$$

$$
= \beta_1 + (\beta_2 + \beta_3) \cdot \frac{1 - e^{-\frac{t}{\tau_1}}}{\frac{t}{\tau_1}} - \beta_3 \cdot e^{-\frac{t}{\tau_1}}, \qquad (14)
$$

where t = time to maturity of a bond (in years),
    β_1 = parameter beta 1 (the level factor),
    β_2 = parameter beta 2 (the slope factor),
    β_3 = parameter beta 3 (the curvature factor),
    τ_1 = parameter tau 1 (the rate of exponential decay), and
    e = exponential function.

The parameters β_1, β_2, β_3, and τ_1 can be calculated with the Excel add-in "Solver" by minimizing the sum of squared residuals between the dirty price (market value, present value) of the bonds and the model price of the bonds. (The dirty price is a sum of the clean price, retrieved from Bloomberg, and accrued interest.) The price of zero-coupon securities for time t is calculated as follows:

$$
P(t) = e^{\frac{-r(t) \cdot t}{100}}. \qquad (15)
$$

The market value (MV) of a bond i according to the Nelson-Siegel model is calculated as follows:

$$
MV\ Nelson\ Siegel\ (bond_i) = \sum_{h=1}^{R_i} Z_i(h) \cdot P(w_i(h)), \quad (16)
$$

where R_i = the number of future cash flows of bond i,
    $Z_i(h)$ = a future cash flow h of bond i,
    $w_i(h)$ = the time to maturity of a future cash flow h of bond i in years, and
    $P(t)$ = the present value of a zero-coupon security with nominal value 1 and maturity in time t.

The error of the Nelson-Siegel model for bond i is defined as follows:

The error of the model for bond i =
= (dirty price of bond$_i$ – MV Nelson Siegel of bond$_i$)^2 (17)

Table 3. Estimated Zero-coupon Yields for t = 1, 2, ..., 10.

t	r(t) in %
1	0.11
2	0.30
3	0.57
4	0.86
5	1.15
6	1.42
7	1.67
8	1.89
9	2.09
10	2.26

Figure 2. The zero-coupon yield curve, estimated with the Nelson-Siegel model.
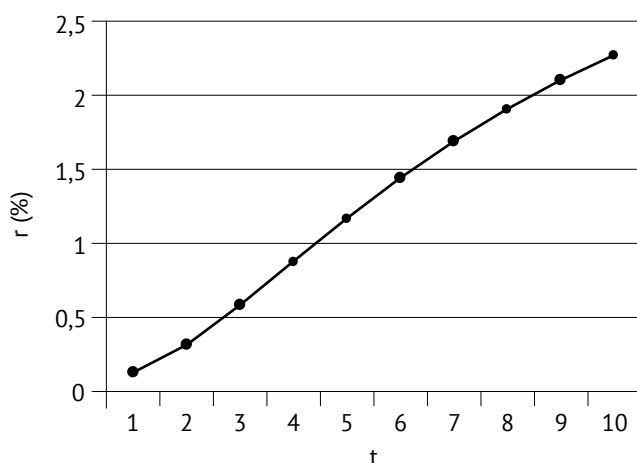
The final results obtained by empirically testing the cubic splines model (equations (14) to (17)) on data given in Table 1 (Austrian government bonds) are summarized in Table 3.

The estimated zero-coupon yield curve is displayed in Figure 2.

4.3 Comparison of the two models

Our coupon bond price estimation results (Tables 4 and 5) are comparable with the in-sample coupon bond price estimation results of Jordan and Mansi (2003; see Table 2). Jordan and Mansi's (2003) results show that, at the short end of the yield curve (0 to 5 years), the Nelson-Siegel model outperforms cubic splines; the same is true for the intermediate range (5 to 10 years). Our findings are similar to Jordan and Mansi's (2003) findings in that the Nelson-Siegel model performed better than cubic splines at the short end of the yield curve (up to 5 years) and in the intermediate range (5–10 years); however, the supremacy of the Nelson-Siegel model over cubic splines was more pronounced for short-term maturities than for medium-term maturities.

The cubic splines and Nelson-Siegel estimates of zero-coupon yields are summarized and compared in Table 6. The Nelson-Siegel model outperformed cubic splines at the short end of the yield curve (up to 2 years), whereas for medium-term maturities (2 to 10 years), the fitting performance of both models was comparable.

Table 4. Comparison of Cubic Splines and Nelson-Siegel Estimates of Bond Prices.

Maturity of the bond	Actual bond price	Cubic splines bond price estimation	Nelson-Siegel bond price estimation	Cubic splines squared price error*	Nelson-Siegel squared price error**
1 year	106.62	106.85	106.68	0.05061	0.00308
2 years	106.56	106.63	106.55	0.00532	0.00006
3 years	110.41	110.24	110.31	0.02882	0.01081
4 years	113.70	113.60	113.69	0.00949	0.00001
5 years	118.94	118.89	118.97	0.00301	0.00082
6 years	118.74	118.76	118.77	0.00058	0.00134
7 years	115.79	115.90	115.84	0.01271	0.00242
8 years	112.69	112.67	112.60	0.00038	0.00797
9 years	115.10	115.26	115.21	0.02767	0.01350
10 years	97.69	97.47	97.52	0.04878	0.02825
13 years	128.47	128.52	128.54	0.00213	0.00422
				Sum: 0.18951	**Sum:** 0.07246

Notes. *Cubic splines squared price error is equal to the squared difference between the actual bond price and the cubic splines bond price estimation.

**Nelson-Siegel squared price error is equal to the squared difference between the actual bond price and the Nelson-Siegel bond price estimation.

Table 5. Comparison of Cubic Splines and Nelson-Siegel Squared Price Errors for Short-term and Medium-term Maturities.

Maturity	Sum of squared price errors, cubic splines	Sum of squared price errors, Nelson-Siegel
Up to 5 years	0.0972*	0.01477*
> 5–10 years	0.09012**	0.05348**

Notes. *Sum of squared price errors for maturities from 1 to 5 years.
**Sum of squared price errors for maturities from 6 to 10 years.

**Table 6.** *Comparison of Cubic Splines and Nelson-Siegel Estimates of Zero-coupon Yields.*

t	Cubic splines estimates of zero-coupon yields in %	Nelson-Siegel estimates of zero-coupon yield in %	Absolute difference between the two model estimates in basis points
1	-0.06	0.11	17
2	0.28	0.30	2
3	0.59	0.57	2
4	0.89	0.86	3
5	1.17	1.15	2
6	1.43	1.42	1
7	1.67	1.67	0
8	1.90	1.89	1
9	2.11	2.09	2
10	2.29	2.26	3
Sum of absolute differences in basis points:			33

The term structure of interest rates, as estimated by the two models, is displayed in Figure 3.

5 Conclusion

In this paper, we empirically contrasted the performance of cubic splines and the Nelson-Siegel model by estimating the zero-coupon yields of Austrian government bonds. The main

Figure 3. The term structure of interest rates, as estimated with cubic splines (grey line) and Nelson-Siegel (black line).
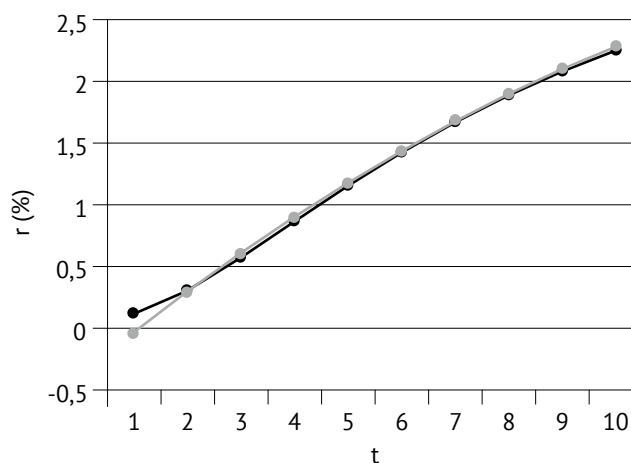
conclusion drawn from the results of the calculations was that the Nelson-Siegel model outperformed cubic splines at the short end of the yield curve (up to 2 years), whereas for medium-term maturities (2 to 10 years) the fitting performance of both models was comparable. In estimating the term structure of interest rates, we employed the simplest versions of the two models without any further extensions. In reality, Nelson-Siegel is one of the most widely used models for deriving the zero-coupon yield curve; however, the central banks and more sophisticated commercial banks use its enhanced versions.

Our study is limited in that we test the fitting performance of the models on the data of government bonds from only one country (Austria). In order to further substantiate our findings, the fitting performance of the models could be tested on a wider set of data, such as government bond data from various countries.

As shown in the theoretical part of this paper, the researchers recently derived the dynamic and no-arbitrage improved versions of the Nelson-Siegel model, which are characterized by a greater estimate precision and forecast accuracy. Further research studies are warranted to evaluate and analyze the performance of the more recent versions of the model.

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Testiranje učinkovitosti modela kubičnih zlepkov in Nelson-Sieglovega modela pri ocenjevanju krivulje donosa brezkuponskih vrednostnih papirjev

Izvleček

Razumevanje razmerja med obrestnimi merami in časom do dospelosti vrednostnih papirjev je osnovni pogoj za razvoj in ovrednotenje pravilnosti finančne teorije. Ta tematika je zato v osrčju monetarne in finančne ekonomije. Natančno prilagajanje terminske strukture obrestnih mer je hrbtenica tekoče delujočega finančnega trga. To je razlog, da je testiranje različnih modelov, ki ocenjujejo in napovedujejo terminsko strukturo obrestnih mer, na področju financ pomembna vsebina, ki je že nekaj desetletij deležna precejšnje pozornosti. V tem članku empirično primerjamo učinkovitost modela kubičnih zlepkov in Nelson-Sieglovega modela, tako da ocenimo donosnost brezkuponskih avstrijskih državnih vrednostnih papirjev. Ključni sklep, ki ga lahko izpostavimo na podlagi dobljenih rezultatov, je, da Nelson-Sieglov model bolje aproksimira brezkuponsko krivuljo donosa na kratkem koncu (do dveh let), na srednjem delu krivulje donosa (od dveh do desetih let) pa med rezultati obeh modelov ni bistvenih razlik.

Ključne besede: kubični zlepki, Nelson-Sieglov model, krivulja donosa, brezkuponske obveznice, terminska struktura obrestnih mer