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An Analysis of Business Cycle Fluctuations in Slovenia and the Euro Area

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Abstract

This paper aims to analyse business cycle dynamics in Slovenia and the euro area. In order to thoroughly tackle the research topic, several modelling methodologies are proposed. First, the characteristics of contractions and expansions and the question of business cycle synchronization between the two focal economies are investigated through the lens of the most common non-parametric approach. Against this background, the most relevant parametric modelling alternatives are closely considered in order to challenge the findings coming from the non-parametric modelling concepts. Second, an analysis of the duration dependence of the particular phases of the business cycle is presented. By utilizing the most relevant (non-parametric) weak-form and strong-form testing procedures, we find some evidence of positive duration dependence of contractions in the case of Slovenia, and largely inconclusive results for positive duration dependence of expansions in the case of euro area. Lastly, given the potential usefulness of the Markov-switching (MS) modelling concept for the real-time business cycle analysis, we utilize Markov-switching Bayesian dynamic factor model (MS-BDFM) in order to infer the probability of the low growth regime and the probability of current quarter negative growth in Slovenia and the euro area.

JEL Classification Numbers: C11, C14, C24, C25, C32, C34, C38, C41, E32.

Keywords: business cycle fluctuations, modified Bry-Boschan algorithm, univariate Markov-switching models, logit models, weak-form and strong-form duration dependence tests, Markov-switching Bayesian dynamic factor models.

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Povzetek

V članku je analizirana dinamika poslovnega cikla v Sloveniji in evrskem območju. Za namen obravnave raziskovalne teme je predlaganih več metodologij modeliranja. Najprej so značilnosti krčenj in ekspanzij gospodarske aktivnosti ter vprašanje sinhronizacije poslovnega cikla med obravnavanima gospodarstvoma raziskani z uporabo najpogostejšega neparametričnega pristopa. Dodatno so podrobno obravnavane tudi najpomembnejše alternativne parametrične metode modeliranja, z namenom izzvanja rezultatov in ugotovitev, izhajajoč iz neparametričnih konceptov modeliranja. V drugem delu analize je preučena časovna odvisnost trajanja posamezne faze poslovnega cikla. Uporaba najpomembnejših (neparametričnih) šibkih in močnih postopkov testiranja prikaže obstoj pozitivne časovne odvisnosti trajanja krčenj gospodarske aktivnosti v primeru Slovenije in razmeroma šibko pozitivno časovno odvisnost trajanja ekspanzij gospodarske aktivnosti v primeru evrskega območja. Glede na potencialno uporabnost koncepta modeliranja Markovih prehodov (MS) za analizo stanja poslovnega cikla v realnem času, zadnji del članka predstavi Bayesov dinamični faktorski model Markova (MS-BDFM). Rezultati uporabe modela so prikazani kot verjetnosti pojava režima nizke rasti in verjetnosti negativne rasti tekom tekočega četrtletja v primeru Slovenije in evrskega območja.

1 Introduction

A widely accepted theoretical view on the dynamics of the modern economies is that they fluctuate around some trend growth rate. In that context the negative oscillations are known as phases of contraction, while positive ones are considered to be expansions. Since the contractionary episodes are usually accompanied by higher unemployment rate, lower real income, decreasing availability of the profitable economic opportunities, and a significant decline in production and sales of firms, the understanding of the fluctuations in the business cycles has an important implications for the welfare of all economic agents. Investigating business cycle characteristics and their synchronicity in related economies, examining the duration dependencies of the business cycle phases, and the analysis of the mechanics that define the current state of the business cycle are therefore the basis for informed macroeconomic policy decisions.

The primary goal of this paper is to apply existing modelling methodologies to thoroughly analyse the broader topic of business cycle fluctuations in Slovenia and the euro area. At the outset, the process of measuring the business cycles requires defining the criteria for the detection of turning points, the yardstick that will be used to uniquely characterize and examine the exact length of business cycle phases and of complete cycles, as well as to answer the question of business cycle synchronization between the two observed economies. The idea followed in the first part of the paper closely resembles the dating methodologies of the National Bureau of Economic Research (NBER) for the United States and the Centre for Economic Policy and Research (CEPR) for the euro area, and is discussed in detail by Bry and Boschan (1971) and Harding and Pagan (2002, 2003). Against this background, the parametric modelling alternatives, initially proposed by Hamilton (1989) and Estrella and Mishkin (1998), are considered in order to challenge the findings coming from the non-parametric modelling concepts. Next, the discussion of the duration dependence of particular phases of the business cycle is based on the most relevant (non-parametric) weak-form and strong-form testing procedures, suggested first by Diebold and Rudebusch (1990, 1991), Mudambi and Taylor (1991, 1995), Pagan (1998), and Ohn et al. (2004). Lastly, the analysis of the current state of the business cycle requires a combination of the two previously considered distinct fields of research. By joining together the most up-to-date mixed-frequency dynamic factor modelling concepts (e.g. Giannone et al., 2008; Camacho et al., 2013; Bańbura & Modugno, 2014 and Poncela et al., 2021) and Hamilton's (1989) idea of Markov-switching (MS), we arrive at the mixed-frequency Markov-switching dynamic factor modelling (MS-DFM) framework, which was, in the Bayesian setting, originally developed by Chauvet and Piger (2008). To the best of our knowledge, no similar analysis utilizing a data set for a small open economy in the monetary union has been done on such a wide range of research topics.

The rest of the paper is organized as follows. Section 2 presents a short literature review of the most important findings of previous relevant works. Section 3 discusses the most common non-parametric (i.e. modified Bry-Boschan (MBBQ) algorithm) and parametric (i.e. univariate MS and logit models) modelling frameworks for business cycle dating. Section 4 introduces the most relevant weak-form and strong-form tests for the analysis of the presence of duration dependence in various phases of the business cycles. Due to the rather short duration of the sample based on quarterly frequency, the conclusions presented for Slovenia and the euro area are based on a monthly aggregate economic activity series. Drawing on the findings in Section 3, Section 5 examines the potential framework for predicting the current state of the business cycle. The usefulness of the MS-DFM concept in a Bayesian setting is discussed by providing some empirical evidence on the probability of low (current quarter negative) quarter-on-quarter real GDP growth.[1] Finally, Section 6 ends this work with the conclusion.

2 Literature review

The tradition of distinguishing between periods of expansion and contraction in an economy has a fairly long history. Based on Burns and Mitchell's (1946) definition of business cycles, and the need to understand the transition between the business cycle phases, NBER (Moore & Zarnowitz, 1986) and CEPR (`Hyperlink`) established their own chronologies of turning point dates at which the shifts between the expansion and contraction phases occur. However, due to several drawbacks of the developed methods, namely the lack of transparency and reproducibility of both dating approaches (as both are based on the consensus of the respective committee members), the absence of the re-examination of business cycle phases in the case of revisions in the macroeconomic time series, and the substantial delay in the determination of the exact peak and trough dates, some alternative, more formal rules to date such turning points emerged. The most representative among the so called non-parametric approaches to business cycle dating is the algorithmic method proposed by Bry and Boschan (1971), which was later revived in the works of Harding and Pagan (2002, 2003). A similar path has been taken by a number of non-parametric studies to establish historical, in-sample chronologies of business cycle turning points that could to a great extent mimic the approach by NBER or CEPR (e.g. Vishwakarma, 1994; Kose et al., 2003, 2008, 2012; Artis et al., 2005; Harding & Pagan, 2006; Fushing et al., 2010; Stock & Watson, 2010, 2014; Berge & Jordá, 2011; Ng, 2014 and Giusto & Piger, 2017). Although the underlying logic of all these non-parametric dating algorithms is practical and clear, they share an important disadvantage. This is related to

[1] Real GDP as well as all other variables considered in the current research are seasonally adjusted.

the sensitivity of turning point dates with respect to the censoring parameters, namely the minimum duration of a phase, the minimum duration of a complete cycle, and the specific threshold parameter. All of the these methods thus require a lot of fine tuning, and often also an expert's judgement regarding the timing of the particular turning point.

On the other hand, there has been a rapid growth in the development of parametric dating modelling concepts in the past three decades, that was especially prominent with regard to non-linear time series models. When compared to the non-parametric methods, the main advantages of the parametric concepts are related to the probabilistic description of the occurrence of the particular phase of the business cycle, on the one hand, and to the possibility of predicting the current (and future) states of the world on the other. Reflecting these advancements, the development of the non-linear framework has mainly followed three branches of research. Conceptually closest to the underlying logic of non-parametric concepts are the works by Goldfeld and Quandt (1973), Cosslett and Lee (1985), and Hamilton (1989). These studies suggest that the behaviour of macroeconomic time series may exhibit different patterns (i.e. regimes) over time, where the dates of a change in regime are endogenously determined by the estimated model parameters. A particular design of a switching mechanism is such that combines two or more dynamic models via a hidden first order Markov process, which governs the random behaviour of the state variable.[2] This makes the model more suitable to capture complex dynamic patterns. The ability of the modelling tool to address the characteristics of the business cycle asymmetries has made it quite useful to investigate the role of non-linearity in identifying, monitoring, and dating the turning points of the business cycle, formally shown in the number of works (e.g. Hamilton, 1989; Hansen, 1992; Diebold et al., 1994; Krolzig, 1997; Filardo & Gordon, 1998; Moolman, 2004; Doornik, 2013; Dufrénot & Keddad, 2014; Aastveit et al., 2016 and Di Giorgio, 2016).

Another strand of literature on business cycle assessment is focused on dynamic factor models (DFM) that were initially proposed by Stock and Watson (1989, 1991, 1993) and later improved in a number of other works by considering richer and more timely information sets, as well as more sophisticated and efficient estimation techniques, such as an application of a state-space system and the Kalman filtering procedures.[3] Regarding the simultaneous treatment of co-movement in macroeconomic time series and the changes in growth regimes, Diebold and Rudebusch (1996) is considered to be a pioneering work. A combination of the linear coincident indicator approach by Stock

[2]Given such Markovian property of the state variable, a particular structure of the model may prevail for a random period of time, and is then replaced by another set of equations when a switch takes place.

[3]For an extensive list of references see Bańbura et al. (2011, 2013), Camacho et al. (2013), and Poncela et al. (2021).

and Watson (1991) and the MS concept proposed by Hamilton (1989) was then also considered in research by Chauvet (1998), Kim and Nelson (1998), and Kaufmann (2000). Nevertheless, given their mathematical form, the early MS-DFMs posses two important disadvantages. Firstly, their underlying design is constructed to handle balanced sets of macroeconomic indicators and as such is not suitable to monitor economic activity in real-time, which is characterized by the lack of synchronicity in the arrival of new macroeconomic information. And secondly, the proposed setting is not suited for handling macroeconomic indicators of different frequencies. More recently, a mixed-frequency data extension of the basic MS-DFM has been considered by Chauvet and Hamilton (2006), Chauvet and Piger (2008), Hamilton (2011), Camacho et al. (2014, 2018), Carstensen et al. (2020), Leiva-León et al. (2020), and Baumeister et al. (2022). The works based on this extension demonstrate the ability of such models to provide reliable real-time signals regarding the dynamics in business cycles. In addition, due to recently accepted stylized facts, univariate or multivariate (i.e. DFM) MS modelling frameworks have been augmented by allowing for time variation in long-term real GDP growth (Eo & Kim, 2016; Antolin-Diaz et al., 2017; Leiva-León et al., 2020 and Baumeister et al., 2022), MS volatility of shocks (Bai & Wang, 2011) or both (Giordani et al., 2007 and Doz et al., 2020).

A somewhat stand-alone body of literature on business cycles focuses on binary outcome models (i.e. probit and logit models) for predicting the occurrence of contractionary episodes. The main reason for such a distinction from the MS literature lies in the fundamentally different assumptions about the nature of the state variable. The (non-linear) binary outcome models assume that, with an absolute certainty, the underlying state of the world is known for each observation in the sample, therefore considering the state variable as given. As such, the most common approaches with regard to assigning the relevant state value (i.e. 0 for expansions and 1 for contractions) for each observation in the sample rely either on the official chronologies of turning point dates reported by NBER and CEPR, or use the non-parametric approach by Bry and Boschan (1971). The obtained state variable is thereafter modelled directly as a dependent variable in a corresponding binary outcome model, embodying either standard normal or logistic distribution functional form. Early research that strongly advocated the aforementioned modelling framework originates in the works of Estrella and Mishkin (1998) and Dueker (1997), who successfully model the probability of contractions. Their findings were later confirmed by Wright (2006) and Rudebusch and Williams (2009), who focus on simple binary outcome models, while most of the other works (e.g. Chauvet & Potter, 2002, 2005, 2010; Kauppi & Saikkonen, 2008; Katayama, 2010; Hamilton, 2011; Berge, 2015; Fossati, 2015; Owyang et al., 2015 and Meller & Metiu, 2017) arrive at similar conclusions by using various modelling extensions.

The main focus of the literature that examines the duration dependence of business cycle phases aims at answering whether expansions or contractions in economic activity are more likely to end as they become older. Standard statistical tools to make inferences about the probability of moving from one phase of the business cycle to another typically use the official chronologies of turning point dates reported by NBER and CEPR, or utilize the non-parametric approach by Bry and Boschan (1971) to identify the aforementioned chronologies. In studying business cycle duration dependence for expansions and contractions, the first strand of research mainly focuses on non-parametric duration models and utilizes various testing procedures in order to arrive at the conclusions (Diebold & Rudebusch, 1990, 1991; Diebold et al., 1990; Mudambi & Taylor, 1991, 1995; Pagan, 1998; Mills, 2001; du Plessis, 2004, 2006; Ohn et al., 2004 and Astolfi et al., 2015). Another strand of research applies parametric duration models, most often some sort of Weibull model (Diebold et al., 1990; Sichel, 1991; Diebold et al., 1993; Zuehlke, 2003; Davig, 2007 and Castro, 2010, 2013, 2015). Alternatively, some authors have tried to model the business cycle as an outcome of a Markov process that switches between the states of expansion and contraction (e.g. Durland & McCurdy, 1994; Kim & Nelson, 1998; Lam, 2004 and Iiboshi, 2007), by applying multinomial regime switching logit models (e.g. Di Venuto & Layton, 2005; Layton & Smith, 2007 and de Bondt and Vermeulen, 2020), or by utilizing an approach relying on a Poisson process, called the modulate power law process (e.g. Zhou & Rigdon, 2008).

3 Modelling methodologies for business cycle dating

The following section investigates the existing modelling tools for examining the dating of the business cycle dynamics. We start by laying out the most common non-parametric approach to business cycle dating and obtain the chronology of turning point dates. We then discuss in detail the most important business cycle characteristics. These include average duration and average amplitude of the two phases, cumulative movements within each phase, asymmetries between the two phases (i.e. contractions and expansions), coefficients of the variation (CVs) for all the aforementioned measures, and the degree of steepness of each phase. The analysis is done for Slovenia and the euro area by using a combination of turning point dates and the logarithm of considered economic activity series. In light of the main considerations about the non-parametric methods, described in Section 2, we also separately discuss also the results based on the relevant business cycle dating parametric modelling concepts among the univariate approaches.

3.1 Modified Bry-Boschan algorithm

The method discussed in this subsection relies on a classical (non-parametric) approach to business cycle dating, which attempts to separate expansions from contractions and relies on the methodology developed by Bry and Boschan (1971) and Harding and Pagan (2002, 2003).[4] The pattern recognition algorithm initially investigates and detects potential peaks and troughs of the respective real activity series,[5] where the actual set of turning points at the end also takes into account also some other relevant business cycle criteria (i.e. censoring rules). The formal exposition of the algorithm is further presented in the following way:

- Identifying the turning points ⇒ a solution to apply in this step is to treat peaks (troughs) as local maxima (minima) in the series y_t. Hence, by defining a binary variable $\wedge_t$ ($\vee_t$), which takes the value of unity where there is a peak (trough) at t and zero otherwise, the definition of peaks can be written as (Harding & Pagan, 2003, 2006):

$$\wedge_t = \mathbf{1}\left\{(y_{t-l}, \ldots, y_{t-1}) < y_t > (y_{t+1}, \ldots, y_{t+l})\right\} \tag{1}$$

  while troughs can be defined in the following way:

$$\vee_t = \mathbf{1}\left\{(y_{t-l}, \ldots, y_{t-1}) > y_t < (y_{t+1}, \ldots, y_{t+l})\right\} \tag{2}$$

  In order to describe the interval over which the local extrema are expected to occur, we choose a value for the parameter l. In order to stick to the main findings of Burns and Mitchell's (1946) business cycle dating procedure for quarterly data, we set this value to 2.

- Ensuring the succession of turning points ⇒ the second step of the MBBQ algorithm ensures that peaks and troughs, as identified above, must alternate and thereby uniquely mark the beginning and end of a particular phase of the business cycle. Since the resulting turning points in our case are used to construct a business cycle chronology, the requirement for alternating turning points is essential to identify the period between the peak and the trough with a contraction and the period between the trough and the peak as an expansion. In the case of multiple consecutive maxima (minima), the highest (lowest) maxima (minima) in

[4] The MATLAB implementation program was developed by Engel (2005).

[5] We follow previous similar research and use logarithms of real GDP for Slovenia and the euro area in this specification. This macroeconomic variable is widely acknowledged as the most important indicator of aggregate economic activity.

the set of potential turning points is chosen. Formally, the alternation of the phases can be constructed by introducing the notion of a business cycle state, S_t^*, that takes the value 1 in expansions and 0 in contractions. The business cycle state is thus related to the local peaks and troughs ($\wedge_t$ and $\vee_t$) by the following formula (Aastveit et al., 2016):

$$\begin{gathered} S_t^* = S_{t-1}^* \left(1 - \wedge_{t-1}\right) + \left(1 - S_{t-1}^*\right) \vee_{t-1} \\ S_t = 1 - S_t^* \end{gathered} \tag{3}$$

The recursion given by Equation 3 is feasible if the initial states S_1^* (S_1) and S_2^* (S_2) can be clearly determined.[6] The most convenient way of doing this is to use Equations 1 and 2 to determine the first turning point in the data. If it is a peak then the economy starts in an expansion (i.e. $S_1^* = S_2^* = 1$ or $S_1 = S_2 = 0$), while the detection of a trough as the first turning point implies that economy starts in a contraction (i.e. $S_1^* = S_2^* = 0$ or $S_1 = S_2 = 1$).

- Imposing an additional set of rules (i.e. censoring rules) $\Rightarrow$ King and Plosser (1994), Simkins (1994), and Harding and Pagan (2002, 2003, 2006) define some additional criteria that help distinguish turning points:
  - The minimum duration of a phase $\Rightarrow$ for quarterly data an expansion or a contraction should last at least 2 periods;
  - The minimum duration of a complete cycle $\Rightarrow$ for quarterly data trough-peak-trough or peak-trough-peak cycles should last at least 4 periods;
  - A specific threshold parameter $\Rightarrow$ when the quarterly growth rate in the series exceeds 10.4% in absolute terms the algorithm automatically assumes the beginning of a new phase, regardless of the length of the previous phase.

The method described above provides a good approximation of the chronologies of turning point dates reported by NBER and CEPR, where both committees use more comprehensive methods that include not only quarterly GDP data but also key GDP components, employment and industrial activity, among the other variables. Nevertheless, due to the caveats of non-parametric methods, already discussed in the previous section, the current research also considers some parametric modelling alternatives to business cycle turning points dating.

[6] It has to be pointed out that the MMBQ algorithm cannot identify turning points at the very beginning and end of the observed sample, as there is no previous or subsequent information for these observations.

3.2 Univariate Markov-switching models

Following the introductory discussion on the non-parametric methods' shortcomings in Section 2, the current subsection focuses on the most common business cycle dating parametric modelling concepts among the univariate approaches (i.e. MS and logit models). The MS models are characterized by a non-linear specification that captures the asymmetry in the business cycle dynamics which is explicitly driven by the heterogeneous states of the world. Based on the pioneering work of Lindgren (1978), Hamilton (1989), Clements and Krolzig (1998), Kim and Nelson (1999), and Kim and Piger (2002), our analysis considers a process given by the following exposition:[7]

$$y_t = \alpha_{s_t} + \varepsilon_t \tag{4}$$

In Equation 4, $t = 1, \ldots, T$, y_t is a quarter-on-quarter real GDP growth rate, α_{s_t} is the MS (i.e. regime switching) intercept, $s_t = \{0, 1, \ldots, N\}$ represents an N states ergodic and aperiodic Markov chain process, and ε_t is an error term that follows a generalized error distribution ($\mathcal{GED}$) with the mean 0, variance $\sigma^2_{s_t}$ and the shape or tail-thickness parameter κ (Giller, 2005). Since such a model setting allows α_{s_t} and $\sigma^2_{s_t}$ to switch states, there are N different values for them, each corresponding to a unique regime. In our specific case, we allow for the model given by Equation 4 to have two heterogeneous states:

$$\begin{aligned} y_t &= \alpha_0 + \varepsilon_t \quad \text{for state 0} \\ y_t &= \alpha_1 + \varepsilon_t \quad \text{for state 1} \end{aligned} \tag{5}$$

where:

$$\begin{aligned} \varepsilon_t &\sim \mathcal{GED}\left(0, \sigma^2_0, \kappa\right) \quad \text{for state 0} \\ \varepsilon_t &\sim \mathcal{GED}\left(0, \sigma^2_1, \kappa\right) \quad \text{for state 1} \end{aligned} \tag{6}$$

The representation in Equations 5 and 6 suggests two distinct processes for the dependent variable y_t. When the current value of unobserved (latent) process s_t for time t is equal to 0 (1), the expected value of the dependent variable is α_0 (α_1), while the variance of the error term is σ^2_0 (σ^2_1). The different characteristics of the error term in the two distinct states of the world represent the different levels of uncertainty related to them. To date, no information regarding the exact switching from one state to another has yet been provided. As it happens in a stochastic manner, described by a first order, two-state Markov process, the switch from one state to another can be described in a probabilistic fashion:

[7] The MATLAB implementation program was developed by Perlin (2015).

$$p_{ij} = \mathbb{P}\left(s_t = j | s_{t-1} = i, \Omega_{t-1}\right) = \mathbb{P}\left(s_t = j | s_{t-1} = i\right) \tag{7}$$

where in our case $i, j = \{0, 1\}$ and Ω_{t-1} denote the observed information until time $t-1$. The dynamics behind the switching process is therefore known and driven by the following transition matrix, which controls the probabilities of making a switch from one state to the other:

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \tag{8}$$

In Equation 8, the elements in row i and column j control the probability of a switch from state i to state j. For example, if for a certain time $t-1$ the state of the world is 0, the probability of a switch from state 0 to state 1 between time $t-1$ and t is given by p_{01}. On the other hand, a probability of remaining in state 0 is determined by p_{00}. In order to ensure that the parameters in the transition matrix are not free to vary, we have to impose reasonable restrictions on them. Since the elements of the transition matrix are probabilities, their values have to be between 0 and 1 and need to sum to 1 in each row (i.e. $p_{i0} + p_{i1} = 1$).

Regarding the estimation procedure, in general there exist two different methods, maximum likelihood and Bayesian inference, and the latter utilizes the Gibbs sampling method. The analysis in this subsection relies on the maximum likelihood approach, while Section 5 uses the benefits of Bayesian techniques when predicting the state of the business cycle.[8] The log-likelihood function of the model described by Equations 5 and 6 is constructed in the following way (Giller, 2005):

$$\ln L = \sum_{t=1}^{T} \ln \left[\frac{2^{-(\kappa+1)}}{\sigma_{s_t} \Gamma\left(\kappa+1\right)} \exp\left(-\frac{1}{2} \left| \frac{y_t - \alpha_{s_t}}{\sigma_{s_t}} \right|^{\frac{1}{\kappa}} \right) \right] \tag{9}$$

[8] According to Doz et al. (2020) there are two important advantages of choosing the Bayesian approach in Section 5. First, despite the multivariate real-time framework for predicting the state of the business cycle, the Bayesian estimation techniques ensure modularity, which allows for a relatively straightforward extension of the standard MS-DFM (e.g. with different magnitudes of contractions). Second, the inference on the Markov-switching variable S is significantly simplified, as its repetitive drawing is conditioned on the generated underlying factor, which is treated as an observed monthly variable. The utilization of both quarterly and monthly variables in the model therefore does not cause any complications in the process of drawing the Markov-switching variable S. In the case of the maximum likelihood estimation (Camacho et al., 2014, 2018), the Markov-switching variable S is directly related to the observable variables. When operating with mixed-frequencies, the distribution of observable variables depends on (potentially large number of) lags of the Markov-switching variable, resulting in the "curse of dimensionality" problem.

However, as the two considered states of the world s_t are unknown, the model cannot be estimated by directly maximizing Equation 9 as a function of the parameter vector $\Theta = [\alpha_0, \alpha_1, \sigma_0^2, \sigma_1^2, \kappa]'$. Instead, its log-likelihood function has to be modified in the following way (Frühwirth-Schnatter, 2006):

$$\ln L = \sum_{t=1}^{T} \ln \sum_{j=0}^{1} \left[f\left(y_t | s_t = j, \Omega_{t-1}; \Theta^*\right) \mathbb{P}\left(s_t = j | \Omega_{t-1}\right)\right] \tag{10}$$

where $f\left(y_t | s_t = j, \Omega_{t-1}; \Theta^*\right)$ is the probability density function of y_t for state j, conditional on observed information in time $t-1$ and $\Theta^* = [\alpha_0, \alpha_1, \sigma_0^2\ ,$ $, \sigma_1^2, \kappa, p_{00}, p_{11}]'$ is an extended parameter vector. Equation 10 therefore suggests that the full log-likelihood function of the model is a weighted average of the probability density functions in each state of the world with the weights corresponding to the state probabilities. Nevertheless, Equation 10 can be only applied once the estimates of unobserved state probabilities are obtained. Based on the available set of information the application of the Hamilton filter allows us to calculate the filtered probabilities of each state, conditioned on the arrival of new information. More details on the aforementioned iterative algorithm are available in the Appendix A.1.[9]

Regarding the inclusion of additional explanatory variables, a simple extension of the model presented in Equation 4 is expressed in the following way:

$$y_t = \alpha_{s_t} + \beta_1 x_{1,t} + \varepsilon_t \tag{11}$$

where $x_{1,t}$ in our case presents the non-switching explanatory variable related to production in the aggregate industrial sector (excl. construction).[10] The motivation for its inclusion dates back to works of Kuznets (1949) and Chenery (1960), with some recent research done by Hassani et al. (2009, 2019). Moreover, as the same monthly aggregate activity series is also applied in Section 4's duration dependence exercise, we consider such an addition to the original model as a relevant robustness check. The extended model also allows for a parameter κ, that determines the shape or tail-thickness in the $\mathcal{GED}$ distribution, to switch between the states. The rest of the notation used

[9]As it is evident from the procedure's description, $\mathbb{P}\left(s_t = j | \Omega_{t-1}\right)$ is a function of $\mathbb{P}\left(s_{t-1} = j | \Omega_{t-1}\right)$, which enables us to recursively derive the state probabilities, needed in the estimation of the remaining parameters of the model that maximize Equation 10.

[10]To be more specific, in order to preserve as much month-on-month dynamics as possible in our extended version of the univariate MS model, we use average (i.e. compounded) month-on-month growth rates of industrial production index (excl. construction). Such a transformation technique is applied in certain analytical exercises, for example by Busaba et al. (2015), Jurado et al. (2015), and Garín et al. (2019). Nevertheless, we acknowledge that the quarter-on-quarter growth rates are more widely used.

in the extended model remains the same as already discussed above, while its theoretical background is in line with the derived set of equations (Equations 5 to 10).

3.3 Logit models

The second type among the popular parametric methods are logit models, whose primary purpose is to provide an estimated response probability of a specific state of the world that is expected to occur. The choice of the modelling framework goes in line with the ideas provided by Estrella and Mishkin (1998), Dueker (1997), Layton and Katsuura (2001), Wright (2006), and Rudebusch and Williams (2009), with the authors opting for either probit or logit model specifications. In our case, we adopt the application of the standard logistic distribution that enables us to model heavier tails (Verbeek, 2017).[11]

The logit model considered here emerges from the underlying latent variable representation of the model. Letting y^* be an unobserved (i.e. latent) variable, we can write the underlying process as follows:

$$y_t^* = \beta_0 + \sum_{l=0}^{4} \beta_{1,l} x_{1,t-l} + \varepsilon_t = \beta_0 + \boldsymbol{x_{t-l}\beta} + \varepsilon_t \tag{12}$$

where $y_t = \mathbf{1}\{y_t^* > 0\}$ is introduced to define the binary outcome.[12] The $\mathbf{1}[\cdot]$ denotes the index function which takes on the value 1 if the event in brackets occurs and 0 otherwise (Wooldridge, 2019). Two separate logit model specifications are further considered, one where $x_{1,t-l}$ denotes regressors related to the current value and the lags of quarter-on-quarter real GDP growth, and second with the $x_{1,t-l}$ denoting regressors related to the current value and lags of the average month-on-month growth rates of industrial production index (excl. construction). Both logit model specifications are expected to well describe the developments in the S_t state series obtained from the MBBQ algorithm analysis, and at the same time represent the same information set used in the case of univariate MS models. The decision to also incorporate the lagged values of the explanatory variable goes in line with the stream of literature on early warning systems (e.g. Lo Duca & Peltonen, 2013 and Caggiano et al., 2016). In order to be consistent with the duration of a complete cycle, discussed in the Subsection 3.1, the maximum number of lags l is set to 4. To further shorten the notation we write $\boldsymbol{x_{t-l}\beta} = \sum_{l=0}^{4} \beta_{1,l} x_{1,t-l}$. From Equation 12, we can derive the response probability for y_t in the following way:

[11] A similar approach was taken in the previous section by considering the $\mathcal{GED}$ distribution of error terms in a univariate MS framework.

[12] As already discussed in Section 2, the binary outcome that is applied in the logit framework is assumed to be known for each observation in the sample. In our case, it corresponds to the S_t state series obtained from the MBBQ algorithm analysis.

$$
\begin{aligned}
p\left(\boldsymbol{x_{t-l}}\right) = \mathbb{P}\left(y_t = 1|\boldsymbol{x_{t-l}}\right) = \mathbb{P}\left(y_t^* > 0|\boldsymbol{x_{t-l}}\right) = \mathbb{P}\left(\beta_0 + \boldsymbol{x_{t-l}\beta} + \right. \\
\left. + \varepsilon_t > 0|\boldsymbol{x_{t-l}}\right) = \mathbb{P}\left(-\varepsilon_t < \beta_0 + \boldsymbol{x_{t-l}\beta}|\boldsymbol{x_{t-l}}\right) = F\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right)
\end{aligned} \tag{13}
$$

where $F(\cdot)$ denotes the distribution function of $-\varepsilon_t$ or, in the most common case of a symmetric distribution, the distribution function of ε_t (Verbeek, 2017). In the course of a linear probability model, $F(\beta_0 + \boldsymbol{x_{t-l}\beta})$ is expressed as $\beta_0 + \boldsymbol{x_{t-l}\beta}$ which has a number of shortcomings. The most serious one is based on the need to ensure that predictions from this model will eventually look like probabilities, as we cannot restrict $\beta_0 + \boldsymbol{x_{t-l}\beta}$ to the open unit interval without bounding values of $\boldsymbol{x_{t-l}}$ and making certain restrictions on $\boldsymbol{\beta}$. To overcome the aforementioned issue, we introduce logit models, which ensure that $0 < p(\boldsymbol{x_{t-l}}) < 1$ by specifying $F(\cdot)$ as a cumulative distribution function of a standard logistic distribution (Verbeek, 2017; Green, 2018 and Wooldridge, 2019):

$$
F\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right) = \Lambda\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right) = \frac{\exp\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right)}{1 + \exp\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right)} \tag{14}
$$

To estimate the function above, we use the maximum likelihood approach which addresses the non-linear nature of the logit models. Given the assumption that each observation is treated as a single draw from a Bernoulli distribution, the model with response probability $F(\beta_0 + \boldsymbol{x_{t-l}\beta})$ and independent observations leads to the following log-likelihood function (Cameron & Trivedi, 2005):

$$
\begin{aligned}
\ln L = \sum_{t=1}^{T} \left\{ y_t \ln\left(\frac{\exp\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right)}{1 + \exp\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right)}\right) + \right. \\
\left. + \left(1 - y_t\right) \ln\left[1 - \frac{\exp\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right)}{1 + \exp\left(\beta_0 + \boldsymbol{x_{t-l}\beta}\right)}\right] \right\}
\end{aligned} \tag{15}
$$

3.4 Empirical analysis

Based on our analysis we identify peaks and troughs in the logarithm of real GDP series for Slovenia and the euro area, covering the 1995Q1-2020Q1 period. Such a decision about the length of the sample preserves the comparability of the results across all the applied modelling frameworks in the following sections, and also takes into account the increased possibility of revisions for the rest of the quarters in 2020 and 2021 in the upcoming months.

Table 1: Peak and trough dates for Slovenia and the euro area based on the MBBQ algorithm

Slovenia		Euro area	
Peaks	Troughs	Peaks	Troughs
	start of sample		start of sample
2008Q2	2010Q1	2008Q1	2009Q2
2011Q1	2012Q4	2011Q3	2013Q1
end of sample		end of sample	

Source: *Own calculations.*

Table 1 reveals that in the period between 19995Q1 and the 2020Q1, the MBBQ algorithm identifies two periods of contraction in economic activity (i.e. a double-dip recession) in both Slovenia as well as in the euro area. The first period in Slovenia begins after 2008Q2 and lasts until 2010Q1. The results for the euro area are similar, except that the first contraction period begins one quarter earlier and ends two quarters before Slovenia's first contraction. As far as the second crisis period is concerned, the contractions in economic activity are broadly of the same length. The second contraction period in Slovenia begins after 2011Q1 and lasts until 2012Q4, while in the case of the euro area it begins after 2011Q3 and ends in 2013Q1. The analysis of expansions in the two observed economies identifies three such episodes. The first period lasts at least from the first available observation in the sample (1995Q1 in both cases) until the first peak, which is identified in 2008Q2 in the case of Slovenia and in 2008Q1 in the case of the euro area. The second period starts from the first trough, which is identified in 2010Q1 in the case of Slovenia and in 2009Q2 in the case of the euro area, and ends with the second identified peak (2011Q1 and 2011Q3 in the case of Slovenia and the euro area, respectively). The last period lasts from the second identified trough (2012Q4 and 2013Q1 in the case of Slovenia and the euro area) until at least the last available observation in the sample, which is 2020Q1 for both economies.

We now look at the important business cycle characteristic of Slovenia and the euro area. Using the combined information from the turning point dates and the logarithm of economic activity series, we examine various features that are well defined in the literature, which include the average duration and the average amplitude of the two phases, the cumulative movements within each phase, the asymmetries between the two phases, the CVs for all the aforementioned measures, and the average degree of steepness of each phase (Harding & Pagan, 2002, 2006; Camacho et al., 2008; Claessens et al., 2012 and Ingram, 2015). The mathematical formulation and the graphical visualization of the most important business cycle characteristics discussed in continuation are provided in Appendix A.2 and Appendix A.3 (Figure A.1).

Table 2: Business cycle characteristics of Slovenia and the euro area based on the MBBQ algorithm

	Slovenia		Euro area	
	Contractions	Expansions	Contractions	Expansions
Avg. duration ($\overline{D_p}$)	7.00	28.67	5.50	29.67
Avg. amplitude ($\overline{A_p}$)	-7.41	24.82	-3.47	14.92
Avg. cumulative movements ($\overline{C_p}$)	-33.49	545.51	-10.14	349.18
Avg. excess movements ($\overline{E_p}$)	2.54	12.01	-7.07	20.59
CV of duration (CV_{D_p})	0.00	0.85	0.13	0.73
CV of amplitude (CV_{A_p})	-0.49	1.09	-0.66	0.98
CV of excess movements (CV_{E_p})	19.94	1.64	-0.54	1.40
Avg. degree of steepness ($\overline{S_p}$)	-1.06	0.87	-0.63	0.50

Source: *Own calculations.*

In terms of average duration of each phase in Slovenia and the euro area, the results in Table 2 reveal that the differences between the two economies are relatively small (7.00 quarters vs. 5.50 quarters in the case of contractions, and 28.67 quarters vs. 29.67 quarters in the case of expansions). Greater heterogeneity is evident when studying the average amplitude and the average cumulative movements in each phase. Both measures indicate that the Slovenian economy contracts more significantly during the time of declining economic activity (-7.41% vs. -3.47% in the case of the average amplitude, and -33.49% vs. -10.14% in the case of the average cumulative movements). The reason for such discrepancies may be related to the different sizes of the two economies and levels of economic development. In addition Slovenia, as a small open economy, is highly affected by global macroeconomic developments and as such is more subject to the adverse impacts of a general decline in economic activity (López, 2015 and Corsetti et al., 2016). On the other hand, and relating to the "plucking" theory (Dupraz et al., 2019 and Tasci & Zevanove, 2019) and the catching-up nature of the Slovene economy, the expansions of economic activity in Slovenia are stronger compared to those in the euro area (24.82% vs. 14.92% in the case of the average amplitude, and 545.51% vs. 349.18% in the case of the average cumulative movements). When considering the average excess movements in each phase, the measure shows significant asymmetries in the shapes of the trajectories of the logarithm of real GDP in each phase. The sign of the average excess movements in contractionary episodes is positive in the case of Slovenia (2.54%), implying the convex shape of the phase, while the opposite holds for the euro area (-7.07%). Regarding the expansions, the sign of the average excess movements is positive for Slovenia (12.01%) as well as for the euro area (20.59%), indicating that the shape of the phase is convex in both considered economies. Looking at the CVs, the expansion values indicate more diversity in terms of duration (0.00 vs. 0.85 in the case of Slovenia and 0.13 vs. 0.73 in the case of the euro area) and the amplitude (-0.49 vs. 1.09 in the case of Slovenia and -0.66 vs. 0.98 in the case of the euro area) when compared to the respective calculations for contractions. This holds for both considered economies. Lastly, the contractions (-1.06 vs. -0.63) as well as the expansions (0.87 vs. 0.50) in Slovenia seem to be on average somewhat steeper than in the case of the euro area.

The results obtained from the MBBQ algorithm analysis also enable us to analyse the degree of synchronization in the business cycle fluctuations between Slovenia and the euro area. By following Harding and Pagan (2002, 2006), Artis et al. (2005), and Claessens et al. (2012) the concordance index (CI) measure of the business cycle synchronization between the pair of the considered economies is defined in the following way:

$$CI = \frac{1}{T}\left[\sum_{t=1}^{T} S_{x,t}S_{y,t} + \sum_{t=1}^{T}\left(1 - S_{x,t}\right)\left(1 - S_{y,t}\right)\right] \tag{16}$$

where $t = 1, \ldots, T$, while $S_{x,t}$ and $S_{y,t}$ refer to the MBBQ algorithm binary indicators of the state of the economy, available for Slovenia and the euro area. The examined measure of the business cycle synchronization specifies the average amount of time in which the two economies are found to be in the same phase of the business cycle. The indicator takes values between 0 and 1, where 1 denotes perfect overlap of the phases and 0 would indicate that the economies are always in opposite phases. The results based on the concordance index are normally compared to some conventional measure such as the Pearson correlation coefficient (ρ).

Table 3: Correlation coefficients and concordance index for business cycle fluctuations in Slovenia and the euro area

	Euro area		
	ρ (logarithms)	ρ (y-o-y growth rates)	CI (using MBBQ algorithm results)
Slovenia	0.99	0.83	0.93

Source: *Own calculations.*

The results in Table 3 confirm the high synchronization of the business cycle fluctuations between Slovenia and the euro area, as both Pearson correlation coefficients for the real GDP series in logarithms and y-o-y growth rates, and concordance index based on the MMBQ algorithm, demonstrate values that are significantly greater than 0.80. This can be related to the strong integration of the small and open Slovenian economy in the wider economy of the euro area.

Table 4: Estimated parameters of the basic univariate MS models for Slovenia and the euro area

Parameter	Estimated value (SE)	
	Slovenia	Euro area
α_0	0.94*** (0.08)	0.51*** (0.05)
α_1	-0.74 (0.66)	-0.22 (0.20)
σ_0^2	0.83*** (0.30)	0.03 (0.03)
σ_1^2	12.13* (6.14)	0.27 (0.39)
κ	0.26** (0.13)	0.88*** (0.25)
p_{00}	0.94*** (0.25)	0.94*** (0.26)
p_{11}	0.73 (NA)	0.77 (NA)
Exp. duration of contractions	3.65	4.39
Exp. duration of expansions	17.95	16.45
Observations	100	100

Note: *Standard errors in parenthesis. *** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$.*
Source: *Own calculations.*

Table 5: Estimated parameters of the extended univariate MS models for Slovenia and the euro area

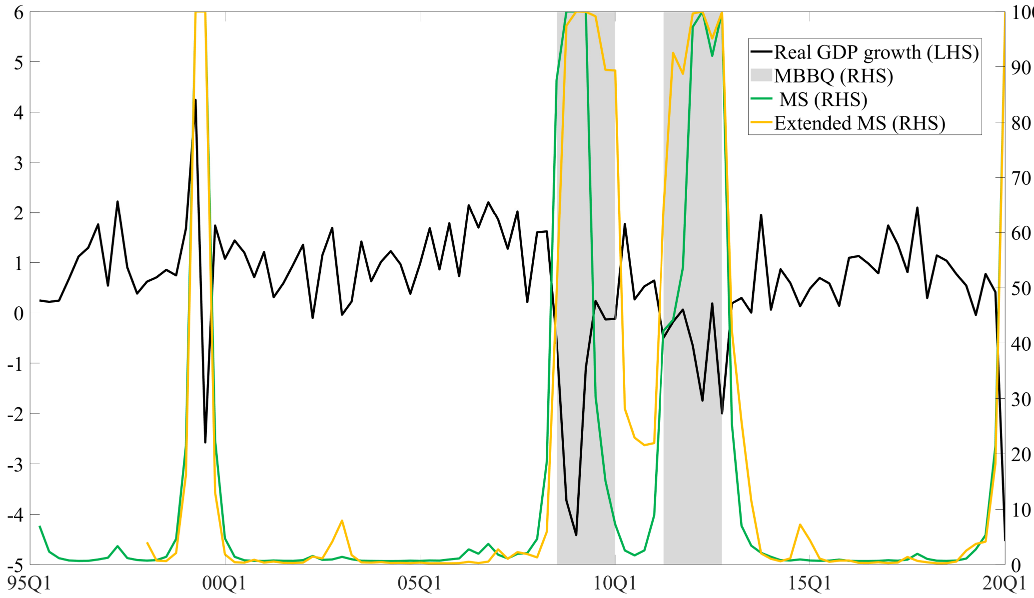
Parameter	Estimated value (SE)	
	Slovenia	Euro area
α_0	0.80*** (0.07)	0.45*** (0.03)
α_1	-0.62*** (0.00)	-0.16*** (0.00)
σ_0^2	0.32* (0.17)	0.02 (0.02)
σ_1^2	0.06 (0.07)	0.01 (0.03)
κ_0	0.49*** (0.15)	0.71*** (0.23)
κ_1	1.35*** (0.26)	1.34*** (0.46)
β_1	0.43*** (0.00)	0.55*** (0.00)
p_{00}	0.94*** (0.27)	0.89*** (0.25)
p_{11}	0.80 (NA)	0.65 (NA)
Exp. duration of contractions	5.04	2.88
Exp. duration of expansions	16.63	9.10
Observations	89	100

Note: *Standard errors in parenthesis. *** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$.*
Source: *Own calculations.*

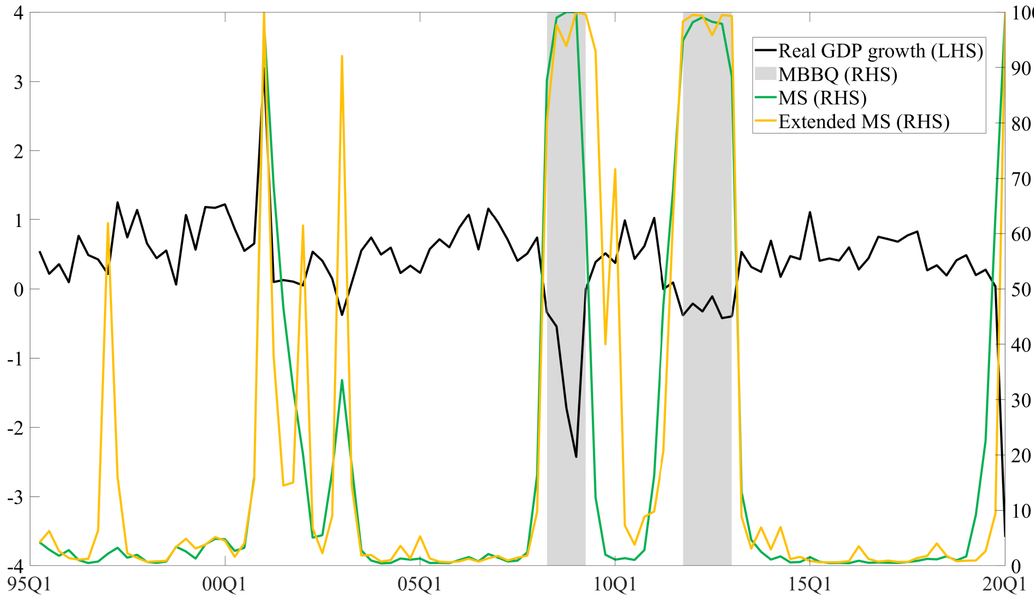
Turning to the results based on the parametric modelling alternatives to the MBBQ algorithm, Tables 4 and 5 show the estimated parameter values of the basic and the extended versions of a univariate MS models. Both considered versions use quarter-on-quarter real GDP growth rates from 1995Q2 to 2020Q1 for both examined economies as a dependent variable in the regression, while the extended version also includes average month-on-month growth rates of the industrial production index (excl. construction) from 1998Q1 to 2020Q1 for Slovenia and from 1995Q2 to 2020Q1 for the euro area as an additional explanatory variable. As evident from the results, the average quarter-on-quarter real GDP growth rates (α_0 and α_1) differ across the two states of the world in the case of both considered economies. In the basic (the extended) univariate MS models, growth rates in the expansionary regime amount to 0.94 (0.80) and 0.51 (0.45) in the case of Slovenia and the euro area, respectively, while in the contractionary phase, the values amount to -0.74 (-0.62) and -0.22 (-0.16) for the two observed economies. Regarding the assessment of the variance of the error term (σ_0^2 and σ_1^2), the results in both versions of the model for Slovenia to some extent confirm the adjustment for different levels of uncertainty in each state of the world. This is, however, not supported by the euro area models. Additionally, there exist some evidence to consider the $\mathcal{GED}$ distribution of the error term, as the shape or tail-thickness parameter (κ) turns out statistically significantly different from 0 in all model specifications. The transition probabilities for both considered economies demonstrate that expansionary phases are strongly persistent, as the probability of remaining in the expansion (p_{00}) is in all considered cases greater than 0.85. In the case of contractions, these values are somewhat lower, with the probability of remaining in the contractionary phase (p_{11}) varying between 0.65 and 0.80. Given the estimated transition probabilities, the expected durations of contractions and expansions are further obtained. While the expected lengths of contractions for the two observed economies seem to be somewhat shorter but relatively close to the results obtained from the MBBQ algorithm, the expected durations of expansions deviate rather more, especially in the case of the euro area. These discrepancies are thoroughly discussed below as they are closely related to the detection of additional short-lived episodes of low or negative growth rates in both considered economies. Lastly, the extended version of the basic MS model also supports the inclusion of production in the aggregate industrial sector (excl. construction) as a relevant monthly aggregate activity series. The importance of the aforementioned measure is further tested in a regression where the shape or tail-thickness parameter (κ) is not allowed to change between the two states of the world. Table A.1 in Appendix A.3 provides evidence for the two observed economies. In the case of Slovenia, the results suggest that the variation in the shape or tail-thickness parameter (κ) contributes more notably to the statistical significance of the parameters related to the average quarter-on-quarter real GDP growth in the contractionary phase (α_1), and the variance of the error term in the expansionary regime (σ_0^2). On the other hand, the euro area figures show that the industrial production (excl. construction) alone already provides a statistically significant parameter of the average quarter-on-quarter real GDP growth in the contractionary phase (α_1). The results in Table 5 further improve once the shape or tail-thickness parameter (κ) is allowed to change between the two states of the world.

Figure 1: MBBQ algorithm vs. (basic and extended) univariate MS estimation results for Slovenia and the euro area – full-sample estimates

(a) Slovenia (q-o-q real GDP growth in %, probability of contraction in %)



(b) Euro area (q-o-q real GDP growth in %, probability of contraction in %)



Source: *Own calculations.*

Next, Figure 1 shows the quarter-on-quarter real GDP growth rate together with the MBBQ algorithm results and the smoothed probability of the contraction derived from the (basic and extended) univariate MS models. The results confirm the findings of the initial analysis using the MBBQ algorithm, as both the basic and the extended univariate MS models identify the maximum probability of a contraction for the period of the global financial crisis (first shaded area), and the second contractionary wave, as induced by the sovereign debt crisis (second shaded area). However, due to different characteristics of the MBBQ algorithm and the univariate MS modelling concept,[13] the latter modelling infrastructure detects one additional period of low growth rate in the case of Slovenia,[14] and two such episodes in the case of euro area.[15] In addition, as a result of the unavailability of real GDP data for 2020Q2 and the application of specific censoring rules, the MBBQ algorithm cannot identify a contraction at the end of the observed sample. In contrast, all univariate MS specifications clearly indicate a spike in probability of the manifestation of a contraction related to the COVID-19 pandemic.

Table 6: Estimated parameters of time series logit model specifications for Slovenia and the euro area

Variables	Estimated coefficients (SE)			
	Logit (real GDP)		Logit (IP (BCD))	
	Slovenia	Euro area	Slovenia	Euro area
$x_{1,t}$	-1.15** (0.48)	-0.79 (0.58)	-1.16** (0.54)	-0.42 (0.57)
$x_{1,t-1}$	-1.06** (0.50)	-5.01*** (1.60)	-1.26** (0.61)	-3.36** (1.46)
$x_{1,t-2}$	-0.82* (0.43)	NA (NA)	-0.41 (0.32)	-1.55** (0.77)
$x_{1,t-3}$	NA (NA)	NA (NA)	-0.94* (0.55)	NA (NA)
$x_{1,t-4}$	NA (NA)	NA (NA)	-0.93* (0.51)	NA (NA)
Constant	-0.91** (0.46)	-1.40*** (0.47)	-1.20*** (0.40)	-2.48*** (0.47)
Observations	98	99	85	99
LR χ^2 test	42.76	37.45	29.24	25.08
$p > \chi^2$	0.00	0.00	0.00	0.00
Pseudo R^2	0.53	0.54	0.39	0.36

Note: *Standard errors in parenthesis. *** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$.*
Source: *Own calculations.*

Comparing the performance of both parametric modelling concepts and

[13] The MBBQ algorithm assumes censoring rules, which affect the construction of the state variable.

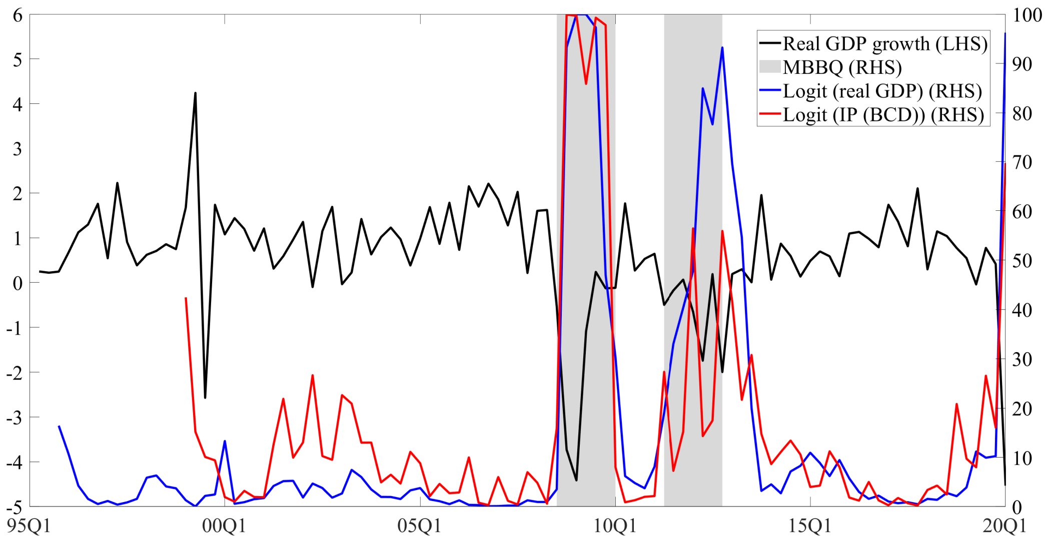
[14] This period is associated with the structural break in productivity growth in 1999Q3 mentioned by Sila et al. (2015).

[15] These are related to the prolonged period of sluggish growth driven by the oil price surge, crisis in the information and communications technology sector and the slowdown of economic activity in the United States in 2001, and a global economic uncertainty tied to the Iraq conflict at the beginning of 2003 (European Commission, 2001, 2003). Nevertheless, none of the additional episodes of low or even negative growth rates in the case of the euro area are identified by the CEPR dating committee. This mainly reflects the more comprehensive definition of the recessions by the committee, which takes into account not only quarterly GDP developments but also employment and other measures of aggregate economic activity for the euro area as a whole. In light of this, the univariate MS models offer a useful modelling alternative for dating business cycle turning points, but cannot serve as a replacement for the committee's official procedure.

the MBBQ algorithm, Table 6 displays the estimation results of the time series logit model specifications, determined on the basis of the lag length selection criteria from Table A.2 in Appendix A.3. Binary dependent variable applied in all considered models is the S_t state series obtained from the MBBQ algorithm analysis. The explanatory variables in the two logit model specifications (for both considered economies) correspond to the current value and the lags of quarter-on-quarter real GDP growth rates and average month-on-month growth rates of industrial production index (excl. construction). The length of all used series is the same as in the previous exercise. The interpretation of the estimated coefficients follows a somewhat different rationale than in the case of univariate MS models, as we can only comment the direction of the effect of the change in the regressor on the realization of contraction in the two observed economies.[16] The results of all model specifications suggest that the increase in the quarter-on-quarter real GDP growth rates or average month-on-month growth rates in the industrial production index (excl. construction) decreases the probability of contraction. The same reasoning holds for all considered lags as well. Regarding the overall statistical significance of the models, the LR χ^2 test results imply the rejection of the null hypothesis, meaning that the regressors used contribute significantly to the better fit of the model.

Figure 2: MBBQ algorithm vs. logit estimation results for Slovenia and the euro area – full-sample estimates

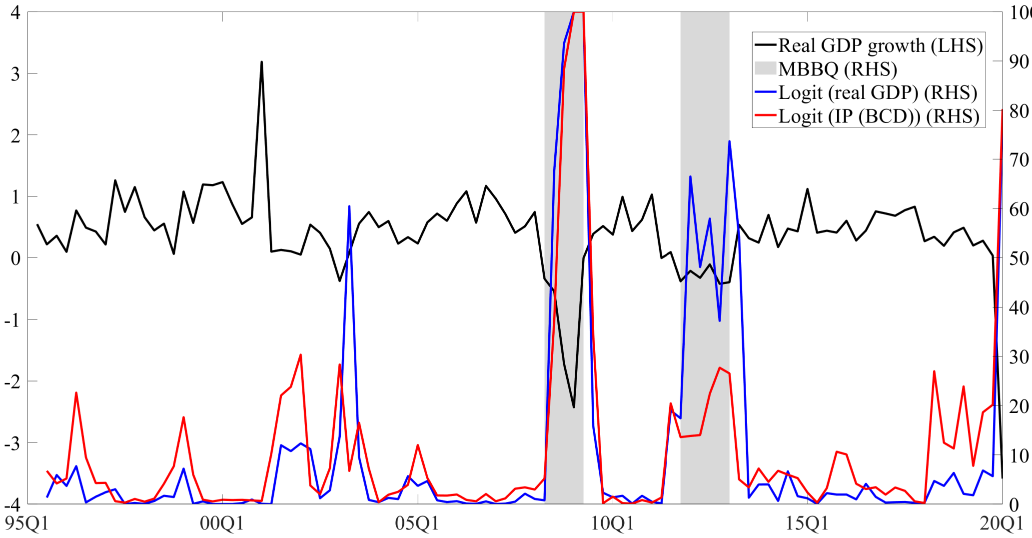
(a) Slovenia (q-o-q real GDP growth in %, probability of contraction in %)



[16]Since the primary focus of the current analysis is obtaining the predicted response probabilities of the occurrence of contraction, the discussion of the marginal effects in all model specifications is excluded from the analysis.

Figure 2: MBBQ algorithm vs. logit estimation results for Slovenia and the euro area – full-sample estimates (contd.)

(b) Euro area (q-o-q real GDP growth in %, probability of contraction in %)



Source: *Own calculations.*

Lastly, Figure 2 demonstrates the predicted response probabilities of the time series logit model specifications for Slovenia and the euro area. The results show that all logit model specifications are successful at detecting the slowdown in economic activity related to the impact of the global financial crisis in both observed economies (first shaded area), with the length of the corresponding period captured quite adequately. However, compared to the results shown in Figure 1, the aforementioned modelling concepts struggle to fully identify the sovereign debt crisis (second shaded area) as another episode of subdued economic growth. Similarly as in the case of univariate MS models, all the logit model specifications reveal the surge in predicted response probability at the end of the observed sample, which is related to the COVID-19 pandemic.

4 Testing for duration dependence[17]

In this section we test for the presence of duration dependence in business cycle phases for Slovenia and the euro area. Specifically, we aim to examine

[17]The analysis in this section was conducted in collaboration with Peonare Caka (Banka Slovenije) within the framework of WGF Expert Team on Business Cycle Drivers. WGF stands for the Working Group on Forecasting, which is one of the three working groups reporting to the Monetary Policy Committee (MPC) and is composed of European Central Bank's (ECB's) and euro area National Central Bank's (NCB's) experts. Its main responsibilities have been thoroughly discussed by the European Central Bank (2016).

the question, already raised by Sichel (1991), as to whether the probability of exiting a cycle phase depends upon the time spent in the particular phase. In doing so, the state series S_t from the previous section is applied, aggregated in a way that represents duration data (i.e. time spent in each phase of the business cycle). Such an approach is advocated by Diebold and Rudebusch (1990, 1991) and Ohn et al. (2004), who divide the T observations into N phases and label D_n as the duration of time spent in the n^{th} phase.[18] Following their setting, the duration dependence can be examined by comparing the probability density function of D_n to the one assumed under duration independence. By defining a random sample of N observations $(D_1, D_2, \ldots, D_N)$ from a continuous distribution $F(\cdot)$, such that $F(a) = 0$ for $a < 0$, and the probability density function of a variable from the random sample defined above as $f(d)$, the hazard function is formulated in the following way (Ohn et al., 2004):

$$h(d) = \frac{f(d)}{G(d)} \tag{17}$$

where $h(d)$ is the hazard (i.e. failure) rate and $G(d) = \mathbb{P}(D \geq d)$ is the survival function. For a small Δ, $h(d)\,\Delta$ is the probability that the contraction (expansion) ends during the interval $(d, d+\Delta)$, assuming it lasts until time d. In the case of no duration dependence, the hazard rate is equal to some constant value θ, and thus does not depend on d:

$$H_0 : h(d) = \theta \tag{18}$$

which holds for all $d > 0$ (Ohn et al., 2004 and Cardinale & Taylor, 2009). Given that we are operating with a continuous random variable, the application of exponential probability density function for $f(d)$[19] results in a constant hazard rate $h(d) = \lambda$, which is of fundamental importance for most weak-form tests, applied in the following subsections.[20] On the other hand, all strong-form tests and certain weak-form tests,[21] take into account the fact that duration data on business cycles is collected at discrete time intervals. In this case the geometric probability density function of D_n[22] gives us the constant hazard rate $h(d) = p$, and is therefore more appropriate when there is a duration independence (Ohn et al., 2004). The following subsections briefly overview the general characteristics of the applied weak- and strong-form tests.

[18] In our specific case, we assume that D_n represents the time spent in one of the two phases of the business cycle.

[19] The exponential probability density function is defined as $f(d) = \lambda \exp(-\lambda d)$ for scale parameter $\lambda > 0$.

[20] These are W, $W^*(D_0)$, Z, $Z(D_0)$, and Z^* tests.

[21] These are the MT, GMD, SB, and χ^2 tests.

[22] The geometric probability density function is defined as $\mathbb{P}(D = d) = (1-p)^d\, p$ for the probability $0 < p \leq 1$.

Given that the quarterly frequency sample for Slovenia and the euro area is too short for such analysis, we use a measure related to production in the aggregate industrial sector (excl. construction) as a relevant monthly aggregate economic activity series. This is also in line with the previous works on this topic.

4.1 Weak-form and strong-form tests

One set of tests for duration dependence stems from the research by Diebold and Rudebusch (1990) and Astolfi et al. (2015), which underlines the exponential distribution under the null hypothesis of a constant hazard rate assumption in continuous time. Another set of procedures, developed by Mudambi and Taylor (19991, 1995) and Pagan (1998), consider the geometric distribution under the null hypothesis, compatible with a constant hazard rate assumption in discrete time. All the aforementioned non-parametric weak-form tests largely involve the comparison of sample moments with those implied by the probability density function that satisfy the null hypothesis of a constant hazard rate (Ohn et al., 2004). The current exercise considers the following tests, with their detailed descriptions available in Appendix B.1:

- The Shapiro-Wilk test (W) $\Rightarrow$ the W test statistic is defined as (Shapiro & Wilk, 1972):

$$W = \frac{\left[\left(\sum_{n=1}^{N} \frac{D_n}{N}\right) - D_1\right]^2}{\frac{N-1}{N}\sum_{n=1}^{N}\left[D_n - \left(\sum_{n=1}^{N}\frac{D_n}{N}\right)\right]^2} \tag{19}$$

  where $D_1 \leq D_2 \leq \cdots \leq D_N$ are durations in each phase, rearranged in an ascending order and N denotes the number of contractions (expansions).

- The Stephens test ($W^*(D_0)$) $\Rightarrow$ the $W^*(D_0)$ test statistic is given by the following formula (Stephens, 1978):

$$W^*(D_0) = \frac{\left[\sum_{n=1}^{N}(D_n - D_0)\right]^2}{N\left\{(N+1)\sum_{n=1}^{N}(D_n - D_0)^2 - \left[\sum_{n=1}^{N}(D_n - D_0)\right]^2\right\}} \tag{20}$$

  where D_0 is the assumed known minimum duration of a particular phase and the rest of the notation corresponds to that utilized in Equation 19.

- The Brain-Shapiro test (Z) $\Rightarrow$ the Z test statistic is formulated as (Stephens, 1978 and Brain & Shapiro, 1983):

$$Z = \frac{\sum_{n=1}^{N-1} \tilde{n} \hat{Y}_{n+1}}{\sum_{n=1}^{N-1} Y_{n+1} \sqrt{\sum_{n=1}^{N-1} \frac{\tilde{n}^2}{N(N-1)}}} \tag{21}$$

  where Y_n are the normalized spacings between the ordered durations, defined for $n = 2, \ldots, N$ as:

$$Y_n = (N - n + 1)(D_n - D_{n-1}) \tag{22}$$

  $\tilde{n} = n - N/2$, $\hat{Y}_n = Y_n - \overline{Y}$ denotes the de-meaned variables, and the rest of the notation corresponds to that utilized in Equations 19 and 20.

- The modified Brain-Shapiro test with minimum duration ($Z(D_0)$) $\Rightarrow$ an extension of the Brain-Shapiro test that employs the assumed known minimum duration D_0 in a similar way as the Stephens test.

- The modified Brain-Shapiro test for duration distributions associated with the non-linear hazard functions (Z^*) $\Rightarrow$ another extension of the Brain-Shapiro test, given by the following formula:

$$Z^* = Z^2 + Z_q^2 \tag{23}$$

  where Z_q is computed in the same way as Z from Equation 21 with $\tilde{n}$ being defined as $\tilde{n} = (n - N/2)^2 - [N(N-2)/12]$ and the rest of the notation corresponding to that utilized in Equations 19, 20, and 21.

- The Mudambi-Taylor test (MT) $\Rightarrow$ the MT test statistic is defined as (Mudambi & Taylor, 1991):

$$MT = \sqrt{N}\left[\frac{\bar{Q}}{S_Q} - 1\right] \tag{24}$$

  where $\bar{Q}$ and S_Q are the mean and the standard deviation of the transformed durations (i.e. deviations of durations from the assumed known minimum duration $Q_n = D_n - D_0$) and the rest of the notation corresponds to that utilized in Equations 19, 20, 21, and 23.

- The GMD test (GMD) $\Rightarrow$ the GMD test statistic can be expressed in the following way (Mudambi & Taylor, 1995):

$$GMD = \frac{1}{N}\sum_{n=1}^{N}\left(D_n - \bar{Q}\right)^2 - \bar{Q}^2 - \bar{Q} \tag{25}$$

  where the notation corresponds to that utilized in Equations 19, 20, 21, 23, and 24.

- The state-based test (SB) $\Rightarrow$ based on a simple linear regression of the state variable S_t^*, as defined in the previous Section, on the lag of the duration variable of contractions (expansions) D_{t-1} (Pagan, 1998, Ohn et al., 2004 and Cardinal & Taylor, 2009):

$$S_t^* = \beta_0 + \beta_1 D_{t-1} + \varepsilon_t \tag{26}$$

  where $t = 1, \ldots, T$ and $\mathbb{E}_{t-1}(\varepsilon_t) = 0$. Under the null hypothesis, the test requires the estimated coefficient associated with the lagged duration variable of phases to be zero (i.e. $\beta_1 = 0$).

As a representative of the strong-form tests, we use the approach employed in Diebold and Rudebusch (1991) and Ohn et al. (2004). This approach is similar to the ones suggested by the works of McCulloch (1975), Savin (1977), and de Leeuw (1987). It involves a comparison of a non-parametric (i.e. empirical) estimate of a probability density function from the data with the hypothesized parametric form, which in this case corresponds to the geometric probability density function under the null hypothesis. The employed χ^2 goodness-of-fit test has the following form:

$$\chi^2 = \sum_{b=1}^{K^*}\frac{\left(O_b - E_b\right)^2}{E_b} \tag{27}$$

where O_b denotes the observed number of elements in the bin b and E_b is the expected number of elements in the bin b, given the geometric distribution under the null hypothesis.[23] Further details about the testing procedure are more thoroughly discussed in Appendix B.2.

[23]The expected number of elements in the bin b is obtained by averaging across 1,000,000 Monte Carlo simulated samples from the geometric distribution.

4.2 Empirical analysis

The length of contractions and expansions is obtained on the basis of the monthly version of the MBBQ algorithm described in detail in Section 3. Due to the unavailability of a sufficiently long real GDP series at the quarterly frequency for both studied economies, we use a variable related to the production in the aggregate industrial sector (excl. construction) to proxy real activity developments. The monthly variable is available from 1998M01 (1995M01) for Slovenia (the euro area) up until 2020M06. Given the monthly setting, we mainly follow the previous research by Berge and Jordà (2011) and Martínez-García et al. (2015) in order to construct the following censoring rules, which broadly correspond to the ones presented in the case of the quarterly setting in Section 3:

- Turn-phase window width ⇒ the identification of local extrema (i.e. contractions and expansions) is studied in the 4-month intervals.
- Minimum duration of a phase ⇒ the contractions and expansions are set to last at least 4 months.
- Minimum length of a complete cycle ⇒ a trough-peak-trough or a peak-trough-peak cycles should last at least 12 months.
- Specific threshold parameter ⇒ if the monthly growth rate in the series exceeds 10.4% in absolute terms, the algorithm automatically assumes a new phase has started, regardless of the length of the previous phase.

Table 7: Duration dependence data for Slovenia and the euro area

Peaks	Troughs	Durations of contractions	Durations of expansions
		Slovenia	
start of sample	1999M04	15	
2000M12	2001M11	11	20
2002M04	2003M06	14	5
2008M06	2009M04	10	60
2010M12	2011M08	8	20
2012M08	2013M05	9	12
2020M02	end of sample	4	81
Observations		7	6
Avg. duration		10.14	33.00
SD of durations		3.72	30.32
Min. duration		4.00	5.00
Max. duration		15.00	81.00
		Euro area	
	start of sample		
1995M05	1996M04	11	4
1998M07	1998M12	5	27
2000M12	2003M06	30	24
2008M04	2009M04	12	58
2011M05	2013M01	20	25
2014M04	2014M08	4	15
2017M12	end of sample	30	40
Observations		7	7
Avg. duration		16.00	27.57
SD of durations		10.91	17.39
Min. duration		4.00	4.00
Max. duration		30.00	58.00

Source: *Own calculations.*

The results of the analysis are presented as duration dependence data for Slovenia and the euro area (Table 7), and are further used in all of the testing procedures mentioned above. In addition, Table B.1 in Appendix B.3 provides more information about the business cycle characteristics of Slovenia and the euro area based on the analysis of the production in the aggregate industrial sector (excl. construction). Compared to the figures in Table 2, those in Table B.1 suggest somewhat stronger cyclicality, which is reflected in the shorter durations of both phases of the business cycle, the more pronounced average amplitude and average cumulative movements, and the higher duration and amplitude CVs, particularly in the case of contractions in the two observed economies. In addition, in contrast to the results based on the logarithm of real GDP in Table 2, the euro area's average excess movements in the expansionary episode of the business cycle imply a concave shape of the phase. The two major contractionary episodes in Slovenia as well as in the euro area are nevertheless also reasonably captured when analysing the production in the aggregate industrial sector (excl. construction).

Turning back to the duration dependence data, the summary statistics in

Table 7 suggest the existence of seven contractions in both observed economies, and the manifestation of six (seven) expansions in Slovenia (the euro area). In terms of the average duration of contractions and expansions, the difference between the two economies is relatively small (10.14 months vs. 16.00 months in the case of contractions, and 33.00 months vs. 27.57 months in the case of expansions). Greater heterogeneity is illustrated by the standard deviation of the durations. Based on simple calculations, it is evident that the durations of contractions in Slovenia are considerably less diverse than in the case of the euro area, while the opposite holds for the durations of expansions. The preliminary findings, based on the simple measures discussed above, therefore indicate a potential duration dependence of contractions in the case of Slovenia and of expansions in the case of the euro area. In addition, the information on the minimum duration serves as an upper bound, taken into account when specifying the assumed known minimum duration D_0 in Tables 8 and 9.[24]

Table 8: Weak-form and strong-form test results for contractions in Slovenia and the euro area

D_0	W	$W^*(D_0)$	Z	$Z(D_0)$	Z^*	MT	GMD	SB	χ^2
					Slovenia				
/	0.5313**	NA	-2.0356**	NA	6.1435**	NA	NA	3.1630***	NA
1	NA	0.4689**	NA	-1.8194**	NA	3.8637***	-80.898	NA	8.9766**
2	NA	0.4118**	NA	-1.6051**	NA	3.1517**	-62.6122	NA	7.4565**
3	NA	0.3501**	NA	-1.3308*	NA	2.4397**	-46.3265	NA	5.9429**
4	NA	0.2849	NA	-0.9671	NA	1.7278*	-32.0408	NA	4.4534
					Euro area				
/	0.2353	NA	-0.3689	NA	1.3707	NA	NA	1.4193	NA
1	NA	0.2161	NA	-0.4640	NA	0.9923	-138.0000	NA	0.1610
2	NA	0.1937	NA	-0.2425	NA	0.7497	-108.0000	NA	0.2278
3	NA	0.1716	NA	0.0131	NA	0.5072	-80.0000	NA	0.3193
4	NA	0.1500	NA	0.3112	NA	0.2647	-54.0000	NA	0.4397

Note: **** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$.*
Source: *Own calculations.*

[24] The study of various sub-samples in order to control for possible heterogeneity across the business cycles (as initially done by Blanchard & Watson, 1986) is due to the limited sample size for both studied economies not feasible.

Table 9: Weak-form and strong-form test results for expansions in Slovenia and the euro area

D_0	W	$W^*(D_0)$	Z	$Z(D_0)$	Z^*	MT	GMD	SB	χ^2
Slovenia									
/	0.2047	NA	0.2062	NA	0.0425	NA	NA	0.4874	NA
1	NA	0.1604	NA	0.2880	NA	0.1359	-290.0000	NA	2.0452
2	NA	0.1520	NA	0.3986	NA	0.0551	-226.0000	NA	2.1360
3	NA	0.1437	NA	0.5166	NA	-0.0257	-164.0000	NA	2.2287
4	NA	0.1356	NA	0.6427	NA	-0.1065	-104.0000	NA	2.3274
5	NA	0.1276	NA	0.7779	NA	-0.1873	-46.0000	NA	2.4486
Euro area									
/	0.3574*	NA	-1.3849*	NA	3.6715	NA	NA	1.6538	NA
1	NA	0.2541	NA	-0.8177	NA	1.3977	-473.5102	NA	2.5038
2	NA	0.2398	NA	-0.7103	NA	1.2456	-420.3673	NA	2.3579
3	NA	0.2256	NA	-0.5941	NA	1.0934	-369.2245	NA	2.2132
4	NA	0.2114	NA	-0.4681	NA	0.9412	-320.0816	NA	2.0790

Note: **** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$.*
Source: *Own calculations.*

The results of the applied weak- and strong-form tests are found in Tables 8 and 9, separately covering contractions and expansions. As in the case of Diebold and Rudebusch (1990), Mudambi and Taylor (1991,1995), and Ohn et al. (2004), we consider various assumed values of the known minimum durations D_0, which are set to be at most the observed minimum given in the Table 7. The argument for doing this lies in the uncertainty associated with the precise timing of the turning points. Regarding the conclusions based on statistical inference, the exact finite sample critical values are available in the cases of the W, $W(D_0)$, Z, $Z(D_0)$, Z^*, and SB test statistics,[25] while in the case of MT, GMD, and χ^2 test statistics only the asymptotic distributions are known. To construct the finite sample critical values for the latter cases, we generate 1,000,000 samples of size N from a geometric distribution with the parametric parameter p fixed at the maximum likelihood estimator.[26] In each case an assumed value of known minimum duration D_0 is also taken into account. The critical values for a given test statistic are then obtained from the 1,000,000 ordered realized values.

The results based on the analysis of contractions show some evidence of positive duration dependence in the case of Slovenia. More specifically, the

[25]These are the critical values of the exponential distribution (tabulated by Shapiro and Wilk), the standard normal distribution, the χ^2 distribution, and the standard t distribution, respectively.

[26]To be more explicit, the maximum likelihood estimator for p is equal to:

$$p_{ML} = \frac{1}{(\overline{D} - D_0) + 1}$$

in the case of MT, GMD, and χ^2 tests.

results of the W test statistic indicate a significant departure from the exponentiality under the null hypothesis. This is also confirmed by the significant deviations of Z, Z^*, and SB test statistics from the standard normal and standard t distributions under the respective null hypotheses. Furthermore, when conditioning on the assumed value of known minimum duration D_0, $W(D_0)$, $Z(D_0)$, MT, and χ^2 test statistics broadly confirm the aforementioned findings. On the other hand, the results of the GMD test statistic are statistically insignificant. Considering the analysis of expansions, the results of the exercise remain largely inconclusive for positive duration dependence in the case of the euro area. More specifically, only the results based on the W and Z tests indicate slight departures from the exponentiality and normality under the respective null hypotheses. The rest of the test statistics, including the procedures that contain information on the assumed value of known minimum duration D_0, largely produce statistically insignificant results. A caveat to the results for the duration dependence of contractions and expansions in the case of both studied economies is that the analyses are based on a relatively small sample sizes, which may lead to biased results.

5 Predicting the current state of the business cycle

In this section we propose a potential framework for analysing and predicting the current state of the business cycle. In this context two main defining characteristics of business cycles, as characterized by Burns and Mitchell (1946), have to be addressed, namely the co-movement across real activity indicators and the non-linear dynamics in the contractionary and expansionary phases. Our choice of the modelling methodology predominantly draws from the empirical results in Section 3, which demonstrated the potential usefulness of the MS concept in modelling the business cycle dynamics of the observed economy. However, the correct assessment of the economic conditions in real-time and the earlier detection of negative developments in the macroeconomic environment also require a richer and more timely information set that would address the specific characteristics of multivariate data generating process (e.g. variables sampled at different frequencies), publication lags, and ragged edges. Based on the available literature,[27] the real-time macroeconomic analysis is most sufficiently addressed by DFM as the state-space system and the Kalman filtering techniques make it possible to relate a potentially large number of observable macroeconomic variables with a few common unobserved components (i.e. extracted factors) in a single system. Moreover, recursive updating of the state variables when new information become available is relatively straightforward.

[27]For an extensive list of references see Bańbura et al. (2011, 2013), Camacho et al. (2013), and Poncela et al. (2021).

5.1 Markov-switching Bayesian dynamic factor models

This class of modelling framework expands on the ideas first put forward by Kim and Yoo (1995), Diebold and Rudebusch (1996), Chauvet (1998), and Kim and Nelson (1998). A mixed-frequency data extension of these models is considered in Chauvet and Hamilton (2006), Chauvet and Piger (2008), Camacho et al. (2014, 2018), Carstensen et al. (2020), Leiva-León et al. (2020), and Baumeister et al. (2022). Given that the current exercise employs Bayesian estimation methods, special attention is paid to the research of Kim and Nelson (1998), Chauvet and Piger (2008), Doz et al. (2020), Leiva-León et al. (2020), and Baumeister et al. (2022), which demonstrates some promising results. In order to account for the changed data generation process in the two observed economies after the global financial crisis, and consequently the rise in heterogeneity in contractionary business cycle phases, the common factor follows a MS dynamics with time-varying means. This modelling solution follows the work of Giordani et al. (2007) and Eo and Kim (2016), which builds on a univariate setting, and of Doz et al. (2020), Leiva-León et al. (2020), and Baumeister et al. (2022), where the authors use a multivariate setting. Furthermore, to be able to summarize the information contained in the set of macroeconomic indicators, the proposed DFM infrastructure is used to construct a common factor that serves as an index that proxies business cycle fluctuations in each considered economy. Due to the importance of the factor estimation for the entire analysis, we first discuss the dynamics of the latent common factor and then describe how it is extracted from the observed data.

Following the setting proposed by Leiva-León et al. (2020), the common factor, f_t, adheres to non-linear dynamics and is expressed in the following way:

$$f_t = \alpha_0 \left(1 - s_t\right) + \alpha_1 s_t + s_t z_t + \varepsilon_t \tag{28}$$

where $s_t = \{0, 1\}$ is an unobservable variable which follows a two-state Markov chain process. Similarly as in Section 3, the evolution of s_t can be summarized by the transition probabilities (Equation 7) grouped in a transition matrix (Equation 8), which controls the probability of a switch from state i to state j.[28] ε_t is an error term that follows a normal distribution ($\mathcal{N}$) with a mean of 0 and variance of σ_f^2. Lastly, the variable z_t in Equation 28 follows another latent process that evolves, following the propositions in Eo and Kim (2016) and Leiva-León et al. (2020), as:

$$z_t = s_t z_{t-1} + \left(1 - s_t\right) e_t \tag{29}$$

[28]In our case $i, j = \{0, 1\}$ adhere to a normal (i.e. expansionary) state and to an abnormal (i.e. contractionary) state, respectively.

where in the abnormal state (i.e. $s_t = 1$) the value of z_t remains fixed at z_{t-1} and, in line with Equation 28, impacts the dynamics of the common factor. On the other hand, when the economy is in the normal state (i.e. $s_t = 0$), z_t follows an $\mathcal{N}$ distributed stochastic process e_t with mean 0 and variance σ_e^2 and has, according to Equation 28, no impact on the common factor. The mechanism behind Equation 29 is therefore constructed in such a way as to capture the realization of contractions of different magnitudes. In this setting the common factor has the constant mean α_0 in normal state, but during each contractionary episode, the mean of the common factor α_1 is adjusted by the unique value of z_t, corresponding to the mean of the contractionary episode estimated from the observed data.

To extract a common factor from a set of macroeconomic indicators we use a state-space form that relates the signal and state equations in a single system (following Doz et al., 2020 and Leiva-León et al. 2020). In a signal equation, each real activity indicator is decomposed into a common component (i.e. common factor) and an idiosyncratic component. However, due to the mixed-frequency environment, indicators cast at different frequencies are treated separately. In particular, in the case of monthly frequency the indicators are expressed as:

$$y_{k,t}^m = \lambda_k f_t + u_{k,t} \tag{30}$$

where $k = 1, \ldots, 4$ is the number of monthly real activity indicators used in the current research,[29] λ_k denotes the corresponding factor loadings, and $u_{k,t}$ stands for the idiosyncratic components. For indicators at the quarterly frequency, we use the Mariano and Murasawa (2003) approach instead. The quarter-on-quarter growth rates, defined in terms of (unobserved) month-on-month growth rates, are correspondingly expressed in the following way:

$$y_{l,t}^q = \frac{1}{3}y_{l,t}^m + \frac{2}{3}y_{l,t-1}^m + y_{l,t-2}^m + \frac{2}{3}y_{l,t-3}^m + \frac{1}{3}y_{l,t-4}^m \tag{31}$$

where $l = 1$ is the number of quarterly real activity indicators in our model, namely the quarter-on-quarter real GDP growth rate. Given that the month-on-month growth rates in Equation 31 have the same decomposition as in Equation 30, the quarterly growth rate can be further expressed as a combination of the idiosyncratic component and the common factor in the following way:

[29] Monthly real activity indicators are transformed to month-on-month growth rates.

$$y^q_{l,t} = \lambda_l\left(\frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4}\right) + \frac{1}{3}u_{l,t} + \frac{2}{3}u_{l,t-1} + \\ + u_{l,t-2} + \frac{2}{3}u_{l,t-3} + \frac{1}{3}u_{l,t-4} \tag{32}$$

Lastly, each individual component contains specific characteristics that are exclusively related to a particular macroeconomic indicator, and (generally) follow an autoregressive process of the order P:

$$u_{h,t} = \psi_{h,1}u_{h,t-1} + \cdots + \psi_{h,P}u_{h,t-P} + \epsilon_{h,t} \tag{33}$$

where $h = k + l$ and the error term, $\epsilon_{h,t}$, follows an $\mathcal{N}$ distribution with the mean of 0 and variance of σ^2_h. The number of lags P is set to 2 in the case of monthly variables,[30] while the individual component of the quarterly variable is further restricted to follow a white noise process (based on the work of Leiva-León et al., 2020).

In the estimation procedure for obtaining the parameters and latent variables, we employ the Bayesian methods which are more comprehensively discussed in Appendix D.1. Their use is justified by the underlying non-linearities in the model and the risk of the "curse of dimensionality" (Pelagatti, 2015). The set of standard prior distributions applied in the Gibbs sampling procedure is the following.

[30]The choice of lag order is based on studies of Stock and Watson (1991), Chauvet (1998), and Kim and Nelson (1998).

Table 10: Prior parameter values

Parameter	Prior density type	1^{st} moment	2^{nd} moment
	Slovenia		
α_0	Normal ($\mathcal{N}$)	0.90	0.00
α_1	Normal ($\mathcal{N}$)	-0.85	0.00
σ_f^2	Inverse gamma ($\mathcal{IG}$)	$T \cdot 6/10$	$T \cdot 6/10 - 1$
σ_e^2	Inverse gamma ($\mathcal{IG}$)	$T \cdot 6/10$	$(T \cdot 6/10 - 1)/10$
σ_h^2	Inverse gamma ($\mathcal{IG}$)	$T/100$	$(T/100 - 1)/10$
λ_h	Normal ($\mathcal{N}$)	0.00	1.00
$\psi_{h,P}$	Normal ($\mathcal{N}$)	0.00	1.00
p_{00}	Beta ($\mathcal{BE}$)	90.00	10.00
p_{11}	Beta ($\mathcal{BE}$)	90.00	10.00
	Euro area		
α_0	Normal ($\mathcal{N}$)	0.50	0.00
α_1	Normal ($\mathcal{N}$)	-0.45	0.00
σ_f^2	Inverse gamma ($\mathcal{IG}$)	$T \cdot 6/10$	$T \cdot 6/10 - 1$
σ_e^2	Inverse gamma ($\mathcal{IG}$)	$T \cdot 6/10$	$(T \cdot 6/10 - 1)/10$
σ_h^2	Inverse gamma ($\mathcal{IG}$)	$T/100$	$(T/100 - 1)/10$
λ_h	Normal ($\mathcal{N}$)	0.00	1.00
$\psi_{h,P}$	Normal ($\mathcal{N}$)	0.00	1.00
p_{00}	Beta ($\mathcal{BE}$)	90.00	10.00
p_{11}	Beta ($\mathcal{BE}$)	90.00	10.00

Source: *Own calculations.*

The specified parameter values in Table 10 indicate the tight priors put on the average quarter-on-quarter real GDP growth rates during the normal and abnormal episodes (α_0 and α_1), which reflect the historical (i.e. pre-COVID-19) trimmed mean growth rate developments in the two states for both observed economies.[31] The main drivers behind the economic fluctuations are considered to be the shocks affecting the common component (σ_f^2 and σ_e^2), which are assumed to be relatively stronger and more volatile than the perceived shocks in the idiosyncratic components (σ_h^2). The rest of the prior parametrization closely follows the work of Kim and Nelson (1998), Chauvet and Piger (2008), Leiva-León et al. (2020), and Baumeister et al. (2022).

5.2 Empirical analysis

The choice of the variables for both studied economies in general follows the previous research of Stock and Watson (1989, 1991), Kim and Nelson (1998), Chauvet and Piger (2008), Camacho et al. (2018), and Leiva-León et al. (2020), which largely coincide with the national accounts configuration. More precisely, the only work that derives the MS-BDFM framework for the euro

[31]The trimmed mean growth rates are calculated by excluding the highest and lowest quarter-on-quarter real GDP growth rates when computing mean growth rates in both the normal and abnormal episodes.

area is the paper by Leiva-León et al. (2020). This makes their analysis a reliable starting point for choosing the set of variables.

In addition to the information on the quarter-on-quarter real GDP growth rate, we aim to utilize two monthly variables capturing supply side developments and two monthly variables related to the demand side dynamics of both studied economies. Since the main goal is to obtain a common factor which is strongly correlated with the quarter-on-quarter real GDP growth rate, the selection criterion for monthly indicators is based on their pairwise correlations with the quarter-on-quarter real GDP growth rate, taking into account their respective pre-COVID-19 samples.[32] The results are reported in Table C.1 in Appendix C.2. Given the desired configuration of the date set, the month-on-month growth rates of the industrial production index (excl. construction) are used as a first supply side variable. In the case of both considered economies, the developments in the industrial production index (excl. construction) reflect stronger pairwise correlations with the quarter-on-quarter real GDP growth rates rather than the dynamics of the two alternative monthly supply side aggregates, namely the volume index of production in construction and the volume index of production in (wholesale and retail) trade and services activities (Table C.1). The month-on-month growth rates for total imports of goods are considered as the first demand side variable that captures the developments in the domestic demand. Some studies listed above utilize sales in retail trade as a relevant domestic demand indicator instead, but the results in Table C.1 show that the developments in total imports of goods better correspond to the quarter-on-quarter real GDP growth rate than the dynamics in the deflated turnover in retail trade. This holds for both studied economies. In order to capture the dynamics in external demand as closely as possible, the month-on-month growth rates of total exports of goods seem to be a natural choice for our second demand side variable in the case of both observed economies.[33] A potential alternative would be to consider a monthly indicator that captures the macroeconomic developments in the main trading partner. In the case of both studied economies, the month-on-month growth rates of the industrial production index (excl. construction) in Germany and the United States are considered,[34] although compared to the dynamics of the total exports of goods they exhibit lower pairwise correlations with the quarter-on-quarter real GDP growth rate. A candidate for the last monthly variable of choice would be ideally connected to developments in industrial production

[32]In order to calculate the required pairwise correlations, the utilized monthly variables are aggregated to their quarterly counterparts. Furthermore, soft (i.e. survey) indicators are transformed to quarter-on-quarter differences, while other variables are cast in quarter-on-quarter growth rates.

[33]In the case of euro area, total exports and imports of goods are constructed from the respective extra- and intra-euro area aggregates.

[34]Germany and the United States are the main trading partners of Slovenia and the euro area, respectively.

index (excl. construction), which is found to be a relatively important supply side variable in the case of both observed economies. Table C.1 therefore reports the pairwise correlations with the quarter-on-quarter real GDP growth rate for the set of potential indicators, including the soft (i.e. survey) indicators on the economic sentiment, industrial confidence, and the subcomponents of industrial confidence, related to the assessment of (export) order-book levels, the production expectations for the months ahead, the production trend observed in recent months, and the assessment of stock of finished products. Among the indicators mentioned above, the strongest support is found for the assessment of order-book levels which enters in the modelling framework in month-on-month differences. Some of studies mentioned above alternatively include certain monthly labour market indicators. In our case potential candidates for Slovenia as well as the euro area are the subcomponent of the industrial confidence indicator, describing employment expectations for the months ahead, and the harmonized unemployment rate, based on the Labour Force Survey (LFS). In the case of both observed economies, the two proposed alternatives exhibit weaker pairwise correlations with the quarter-on-quarter real GDP growth rates than the order-book levels, therefore indicating less support for the inclusion of labour market information in the final set of variables. Non surprisingly, we end up with the same list of employed variables for Slovenia as well as for the euro area, which to some extent reflects the high synchronization of the business cycle fluctuations between the studied economies. For the purpose of the analysis, the following data vintage is taken into account (Table 11).

Table 11: List of variables utilized in the MS-BDFM for Slovenia and the euro area

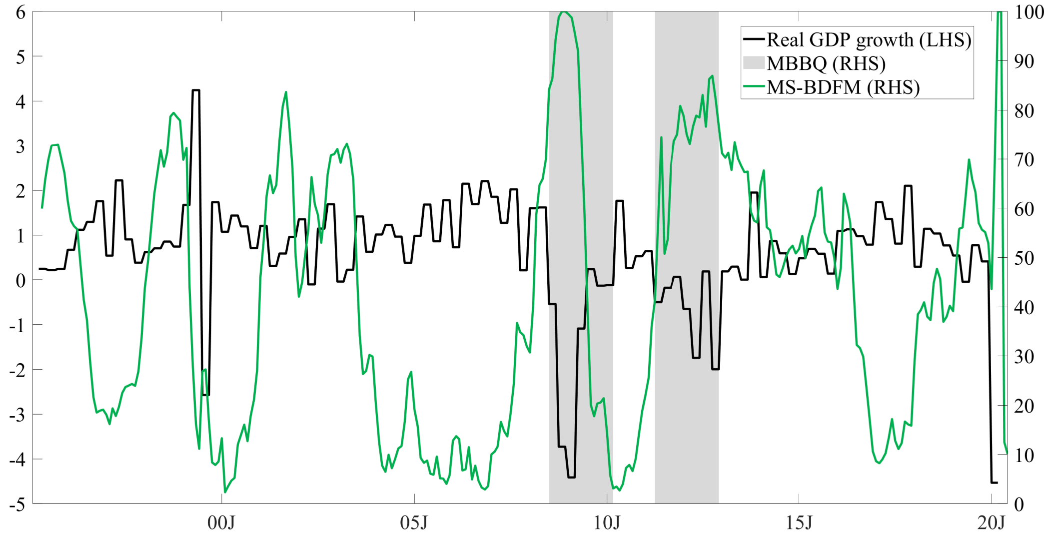
Variable	Frequency	Sample size
Slovenia		
Real GDP	Quarterly	1995Q2-2020Q1
Industrial production index (excl. construction)	Monthly	1998M02-2020M06
Total imports of goods	Monthly	1999M02-2020M05
Total exports of goods	Monthly	1999M02-2020M05
Industrial orders	Monthly	1995M05-2020M06
Euro area		
Real GDP	Quarterly	1995Q2-2020Q1
Industrial production index (excl. construction)	Monthly	1995M02-2020M06
Intra+extra euro area imports of goods	Monthly	1999M02-2020M05
Intra+extra euro area exports of goods	Monthly	1999M02-2020M05
Industrial orders	Monthly	1995M02-2020M06

Source: *Eurostat.*

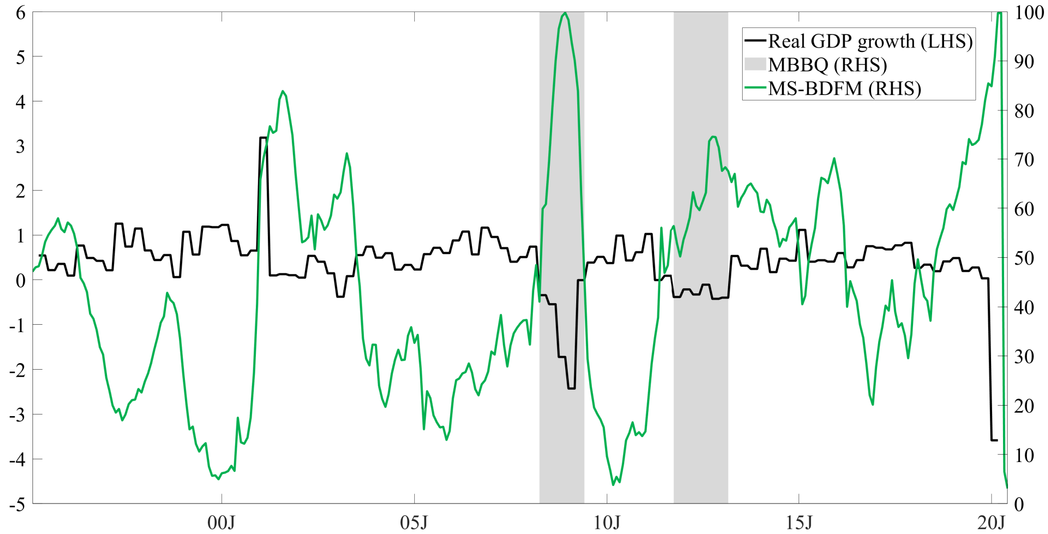
The most current research as well as results discussed above demonstrate that the set of variables in Table 11 is fairly representative to capture the current conditions in the economy and to identify the probability of contraction in real-time. Moreover, national accounts type of variables are also convenient in apprehending the different types of contractions, as they capture the general fluctuations in the observed economy and are therefore not biased towards the fluctuations in a specific sector or commodity.

Figure 3: MBBQ algorithm vs. MS-BDFM estimation results for Slovenia and the euro area (low growth regime) – full-sample estimates

(a) Slovenia (q-o-q real GDP growth rate in %, probability of contraction in %, probability of low growth regime in %)



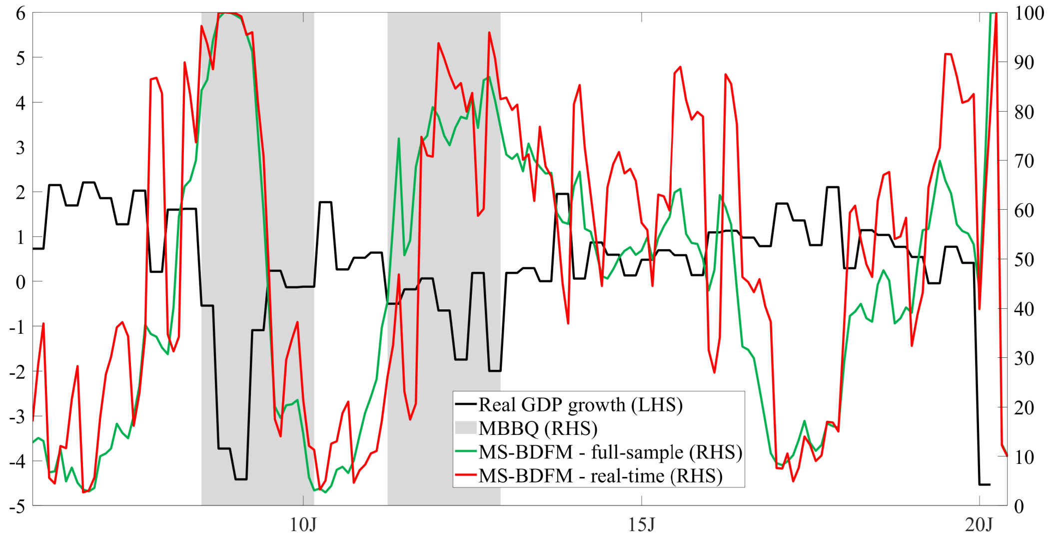
(b) Euro area (q-o-q real GDP growth rate in %, probability of contraction in %, probability of low growth regime in %)



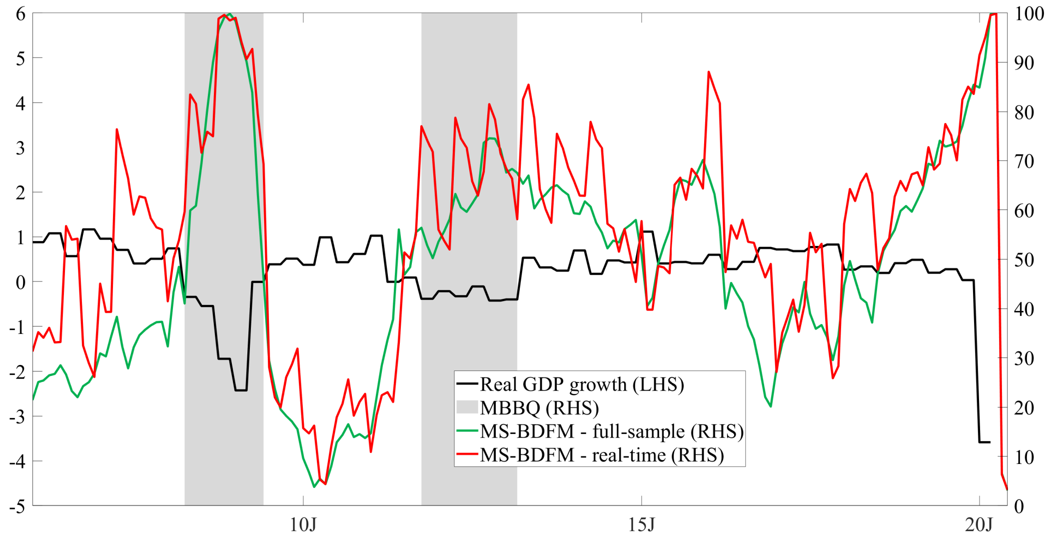
Source: *Own calculations.*

Figure 4: MS-BDFM estimation results for Slovenia and the euro area (low growth regime) – full-sample estimates full-sample vs. real-time estimates

(a) Slovenia (q-o-q real GDP growth rate in %, probability of contraction in %, probability of low growth regime in %)



(b) Euro area (q-o-q real GDP growth rate in %, probability of contraction in %, probability of low growth regime in %)



Source: *Own calculations.*

Following the method described in the previous subsection, Figure 3 plots the probability of the low growth rate regime in Slovenia and the euro area. The low and normal growth rate regimes are distinguished by the specific threshold, reached by the smoothed probabilities shown in the Figure 3, which corresponds to 50%.[35] In the case of both studied economies, the applied modelling concept accurately identifies the maximum probability of the low growth rate regime. The classified periods mainly refer to the evident slowdown in economic activity, related to the global financial crisis (first shaded area), and the second contractionary wave, induced by the sovereign debt crisis (second shaded area). There are also some additional episodes where the quarter-on-quarter real GDP growth rate is perceived to be low or negative, which closely correspond to the periods obtained by the univariate MS modelling framework in Section 3. From the perspective of the current state prediction, the episode at the end of the observed sample is of the most interest. The significantly elevated probabilities of the low growth regime are clearly visible in 2020M03 and 2020M04, demonstrating the manifestation of the contraction related to the COVID-19 pandemic. Nevertheless, the last two months (i.e. 2020M05 and 2020M06) already show a substantial improvement in month-on-month developments in the majority of monthly real activity variables, thereby indicating the decline in the depicted probabilities. As the tool's main goal is to provide reliable real-time signals, the stability of the full-sample estimates is further tested by performing a pseudo real-time out-of-sample recursive exercise.[36] First, the MS-BDFM is estimated from 1995M05 (1995M02) to 2005M12 in the case of Slovenia (the euro area). Second, the recursive estimations that take into account the pseudo real-time ragged edge pattern prevailing at the time of obtaining the full-sample estimates (Table 11) are performed from 2006M01 to 2020M06, adding one quarter of information at every iteration. The estimated real-time probabilities of the low growth regime in Slovenia and the euro area are depicted in Figure 4. The results obtained from the exercise show the success of the demonstrated approach in capturing different types of contractions[37] in the real-time environment which is of key importance for the policymaking decision process.

The results discussed above demonstrate the ability of the applied modelling infrastructure to capture the uniqueness of the contractionary episodes in both considered economies by adding a variable z_t, drawn from a random distribution, to the average quarter-on-quarter growth rate across all contractionary episodes in the observed sample (α_1). In this sense, the quarter-on-quarter real GDP growth rates during each particular contraction in the respective economy are either weaker or stronger compared the constant mean (α_1), thereby not allowing for stronger and more persistent contractions to have a dominant role.

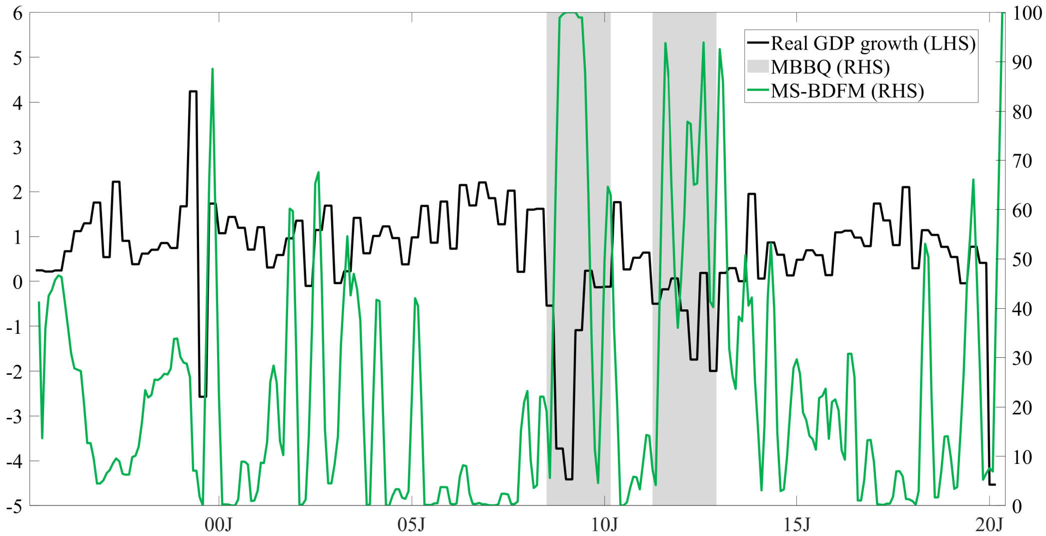
[35] Whenever the values of smoothed probabilities are above 50%, the quarter-on-quarter real GDP growth rates are perceived to be low or become negative (i.e. below the potential growth).

[36] Pseudo real-time refers to the recursive exercise with revised data and ragged edge pattern prevailing at the time of the obtaining the full-sample estimates.

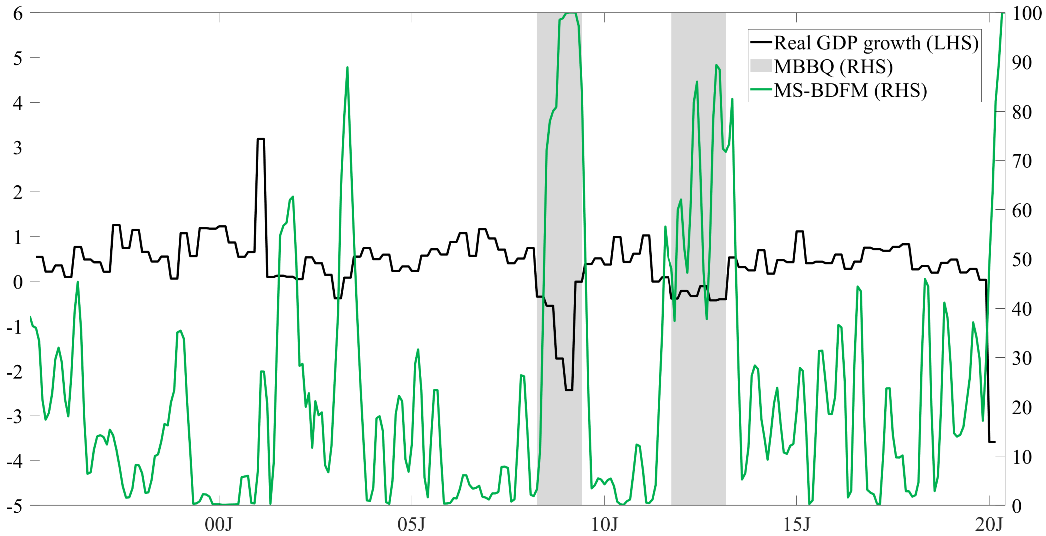
[37] In our case these are related to the global financial crisis, the sovereign debt crisis, and the COVID-19 pandemic.

Figure 5: MBBQ algorithm vs. MS-BDFM estimation results for Slovenia and the euro area (current quarter negative growth) – full-sample estimates

(a) Slovenia (q-o-q real GDP growth in %, probability of contraction in %, probability of current quarter negative growth in %)



(b) Euro area (q-o-q real GDP growth in %, probability of contraction in %, probability of current quarter negative growth in %)



Source: *Own calculations.*

Lastly, based on the results in Figures 3 and 4, we can consider how comparable the low growth rate episodes in Slovenia and the euro area are. As mentioned above, the severity of these phases depends on the realization of $\alpha_1 + z_t$ for every low growth rate period, which implies that the comparison of such episodes even within a single country is questionable. In order to avoid confusion regarding this issue, Figure 5 presents the smoothed probabilities of the current quarter negative quarter-on-quarter real GDP growth rate realization, which corresponds to some sort of nowcasting exercise. It can be clearly seen that despite the decline in probabilities of a low growth rate regime at the end of the observed sample in Figures 3 and 4 for both studied economies, the probability of observing a negative quarter-on-quarter real GDP growth rate in 2020Q2 remains close to 100% in Slovenia as well as in the euro area. Such a unified setting facilitates the interpretation as well as the comparison between heterogeneous contractionary episodes between the studied economies.

6 Conclusion

The current paper analyses the business cycle fluctuations in Slovenia and the euro area by exploring and discussing several modelling methodologies. The first section investigates the characteristics of contractions and expansions and examines the question of business cycle synchronization between the two observed economies. By utilizing the most common non-parametric approach (i.e. the MBBQ algorithm), we detect two periods of contraction and three periods of expansion in the real economic activity. This is observed in both of the studied economies, where the timings of all the identified phases largely coincide. Looking further at some other relevant business cycle characteristics, the analysis reveals greater heterogeneity between the studied economies, especially when considering differences in the average amplitude, the average cumulative movements, and the average degree of steepness in each phase as well as the asymmetries in the shapes of the trajectories of the logarithm of real GDP captured by the average excess movements in each phase. With respect to the degree of synchronization in the business cycle fluctuations between both studied economies, the results of the analysis confirm the strong integration of the small open Slovenian economy in the euro area.

Against this background, the subsequent analyses in Section 3 closely consider the most relevant parametric modelling alternatives in order to challenge the findings coming from the MBBQ algorithm. The results pertaining to the MS modelling methodology show that the smoothed probabilities of contraction, for both the basic as well as the extended versions of the model, are able to identify the double-dip recession related to the unfavourable developments in economic activity caused by the global financial crisis and the sovereign debt crisis. Due to the difference in characteristics between the MBBQ algorithm and the univariate MS modelling concept, the latter modelling infrastructure detects additional relevant periods of low growth rates in the case of both studied economies. The findings from the time series logit model specifications show that the predicted response probabilities struggle to full identify the sovereign debt crisis as one of the episodes of subdued economic growth.

The second section examines the presence of the duration dependence in a particular phase of the business cycle. The utilization of the most relevant weak- and strong-form testing procedures shows some evidence of positive duration dependence of contractions in the case of Slovenia, with the outcomes of the W, Z, Z^*, and SB test statistics indicating significant departures from the corresponding distribution under the null hypothesis. Results remain robust once the assumed minimum duration D_0 is taken into account in the $W(D_0)$, $Z(D_0)$, MT, and χ^2 test statistics. When considering the analysis of expansions, the results remain largely inconclusive for positive duration dependence in the case of the euro area, as only the results based on the W and Z test statistics indicate slight departures from the corresponding distributions under the null hypothesis.

Lastly, building on the empirical results obtained in Section 3, the last Section investigates the potential usefulness of combining the most up-to-date mixed-frequency DFM concept with the idea of MS in Bayesian setting to measure the degree of real-time weakness in economic activity. In the case of both observed economies, the applied modelling methodology accurately identifies the maximum probability of the low growth rate regime in the periods of marked slowdown in economic activity related to the global financial crisis, the sovereign debt crisis, and the COVID-19 pandemic. This shows the usefulness of the demonstrated approach in capturing different types of contractions, thereby not allowing for stronger and more persistent ones to have a dominant role. In addition, the MS-BDFM also detects some other relevant episodes where the quarter-on-quarter real GDP growth rate is perceived to be low or negative, which closely correspond to the periods identified in Section 3. The reliability of the full-sample estimates is also confirmed by the pseudo real-time out-of-sample recursive exercise. At the end, smoothed probabilities of current quarter negative quarter-on-quarter real GDP growth rate realization are presented in order to facilitate the interpretation as well as the comparison of heterogeneous low growth rate episodes among (or even within) the studied economies.

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Appendices

A Appendix to Section 3

A.1 Hamilton filter

Estimates of $\mathbb{P}\left(s_t = j|\Omega_{t-1}\right)$ are in the context of the iterative algorithm obtained in the following way (Hamilton, 1989, 1990, 1994 and Di Persio & Frigo, 2015):

- Setting the initial values at $t = 0$ for $\mathbb{P}\left(s_0 = j|\Omega_0\right)$ for $j = \{0, 1\} \Rightarrow$ by applying a naive guess:

$$\mathbb{P}\left(s_0 = j|\Omega_0\right) = 0.5 \tag{A.1}$$

  or using the steady-state probabilities:

$$\begin{aligned} \mathbb{P}\left(s_0 = 0|\Omega_0\right) &= \frac{1 - p_{00}}{2 - p_{00} - p_{11}} \\ \mathbb{P}\left(s_0 = 1|\Omega_0\right) &= \frac{1 - p_{11}}{2 - p_{11} - p_{00}}. \end{aligned} \tag{A.2}$$

- Performing the following steps for $t = 1, \ldots, T \Rightarrow$

  - Computing the prediction probabilities for $j = \{0, 1\}$, conditional on the available information in time $t - 1$:

$$\mathbb{P}\left(s_t = j|\Omega_{t-1}\right) = \sum_{i=0}^{1} p_{ij}\mathbb{P}\left(s_{t-1} = i|\Omega_{t-1}\right) \tag{A.3}$$

    where p_{ij} are the transition probabilities from Equation 8.

  - Given the prediction probabilities, the probability density function of y_t, conditional on observed information in time $t - 1$ and the extended parameter vector Θ^* is defined as:

$$f\left(y_t|\Omega_{t-1};\Theta^*\right) = \sum_{j=0}^{1} f\left(y_t|s_t = j, \Omega_{t-1};\Theta^*\right)\mathbb{P}\left(s_t = j|\Omega_{t-1}\right); \tag{A.4}$$

– Updating the filtering probabilities for $j = \{0, 1\}$ by applying the Bayes' theorem upon receiving new information:

$$\mathbb{P}(s_t = j|\Omega_t) = \frac{f(y_t|s_t = j, \Omega_{t-1}; \Theta^*)\,\mathbb{P}(s_t = j|\Omega_{t-1})}{f(y_t|\Omega_{t-1}; \Theta^*)} \tag{A.5}$$

Once the filtered probabilities are obtained and the filtering probability from the final iteration in the previous step is set as the initial value, it is possible to apply the backward filtering procedure (Hamilton, 1989 and Kim, 1994) to get the smoothed probabilities $\mathbb{P}(s_t = j|\Omega_T)$ for $t = T-1, \ldots, 1$. These values are also used in the empirical analysis of Section 3.

A.2 Mathematical formulation of business cycle characteristics

The duration of each phase measures the length of the time spent in a particular phase. The average duration of each phase is therefore denoted in the following way:

$$\overline{D}_p = \frac{1}{N_p} \sum_{n=1}^{N_p} D_p^n \tag{A.6}$$

where $p = \{0, 1\}$ refers to the two phases of the business cycle (i.e. contraction or expansion), $n = 1, \ldots, N_p$, and D_p^n denotes the duration of each phase. The amplitude of each phase refers to the decline (increase) in economic activity in contraction (expansion):

$$A_p^n = \left(y_{L,p}^n - y_{F,p}^n\right) \cdot 100 \tag{A.7}$$

where $y_{L,p}$ ($y_{F,p}$) represents the logarithm of real GDP at the end (the beginning) of the n^{th} phase. The difference between the two thus provides an approximation of the percentage growth over the course of each phase. Given the expression in Equation A.7, the average amplitude of each phase is defined as:

$$\overline{A}_p = \frac{1}{N_p} \sum_{n=1}^{N_p} A_p^n \tag{A.8}$$

The cumulative movements in each phase are obtained by accumulating differences between the logarithm of real GDP at the beginning of the n^{th} phase and each subsequent point until the value of the logarithm of real GDP at the end of the n^{th} phase has been reached.[38] This area can be approximated by adding together the areas of rectangles of the unit base and height equal to $\left(y^n_{d,p} - y^n_{F,p}\right)$ as:

$$C^n_p = \sum_{d=1}^{D^n_p} \left(y^n_{d,p} - y^n_{F,p}\right) \cdot 100 \tag{A.9}$$

where $d = 1, \ldots, D^n_p$ refers to observations between the first and last dates in each phase. Given the expression in Equation A.9, the average cumulative movements in each phase are given by:

$$\overline{C}_p = \frac{1}{N_p} \sum_{n=1}^{N_p} C^n_p \tag{A.10}$$

The excess movements in each phase (defined as a % of the right-angled triangle area) are best described by first considering the case where the decline (increase) of economic activity in each phase is directly proportional to the phase's duration, meaning that the logarithm of real GDP follows the hypotenuse of the right-angled triangle perfectly. In this case, the cumulative movements in each phase are defined by the following area:

$$\Delta^n_p = \frac{D^n_p A^n_p}{2} \tag{A.11}$$

and the difference between the calculations obtained from Equations A.9 (corrected for the $A^n_p/2$[39]) and A.11 is equal to 0. In any other situation the trajectory of the logarithm of real GDP either overshoots (positive difference) or undershoots (negative difference) the hypotenuse of the right-angled triangle, which defines the excess area of each phase. Divining the excess area of each phase by the expression in Equation A.11 at the end defines the measure as a % of the right-angled triangle area:

$$E^n_p = \left(\frac{C^n_p - \Delta^n_p - \frac{A^n_p}{2}}{\Delta^n_p}\right) \cdot 100 \tag{A.12}$$

Given the expression in Equation A.12, the average excess movements in each phase are obtained in the following way:

[38] In other words, the accumulated differences correspond to the total triangle area above the base level of $y^n_{F,p}$, and below the trajectory of the logarithm of real GDP.

[39] The correction is considered due to the fact that the approximation in Equation A.9 is too large, as each rectangle overshoots or undershoots the actual area by approximately half of the amplitude in each phase.

$$\overline{E_p} = \frac{1}{N_p} \sum_{n=1}^{N_p} E_p^n \tag{A.13}$$

The calculations in Table 2, pertaining to the CVs for all the measures discussed above, are conducted as follows:

$$CV_{m_p} = \frac{\sqrt{\frac{1}{N_p} \sum_{n=1}^{N_p} \left(m_p^n - \overline{m_p}\right)}}{\overline{m_p}} \tag{A.14}$$

where $m = \{D, A, E\}$ refer to the duration, amplitude and excess movements. Higher values of coefficients indicate greater variety in each phase. Lastly, the average degree of the steepness of each phase is determined in the following way:

$$\overline{S_p} = \frac{\overline{A}_p}{\overline{D}_p} \tag{A.15}$$

A.3 Additional Tables and Figures

Table A.1: Estimated parameters of the additional extended univariate MS models for Slovenia and the euro area

Parameter	Estimated value (SE)	
	Slovenia	Euro area
α_0	0.79*** (0.08)	0.44*** (0.02)
α_1	-0.44 (0.29)	-0.13** (0.06)
σ_0^2	0.19 (0.12)	0.00 (0.01)
σ_1^2	1.75 (1.38)	0.03 (0.05)
κ	0.65** (0.18)	1.18*** (0.31)
β_1	0.40*** (0.10)	0.67*** (0.03)
p_{00}	0.94*** (0.11)	0.93*** (0.39)
p_{11}	0.82 (NA)	0.74 (NA)
Exp. duration of contractions	5.65	3.78
Exp. duration of expansions	15.40	13.51
Observations	89	100

***Note**: Standard errors in parenthesis. *** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$. In this specification the shape or tail-thickness parameter (κ) is not allowed to change between the two states of the world.*
***Source**: Own calculations.*

Table A.2: Lag length selection criteria of time series logit model specifications for Slovenia and the euro area

Model variant	Logit (real GDP)				Logit (IP (BCD))			
	Slovenia		Euro area		Slovenia		Euro area	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Variant 1	48.93	56.72	**37.62**	**45.40**	71.80	79.23	53.69	**61.51**
Variant 2	**45.62**	**55.96**	39.34	49.68	70.69	80.55	**51.99**	62.37
Variant 3	45.95	58.82	41.19	54.07	65.19	77.46	53.78	66.71
Variant 4	47.71	63.10	43.11	58.49	**58.81**	**73.47**	55.68	71.12

***Note**: Variant 1 includes the current value and the first lag of regressor, while each subsequent variant incorporates one more lag (up to the maximum number of considered lags). AIC and BIC refer to Akaike and Bayesian information criteria. Numbers in bold indicate the lowest value for particular information criterion among all the model variants.*
***Source**: Own calculations.*

Figure A.1: Graphical visualization of business cycle characteristics

(a) Duration, amplitude, and cumulative movements in contractions (LHS) and expansions (RHS)

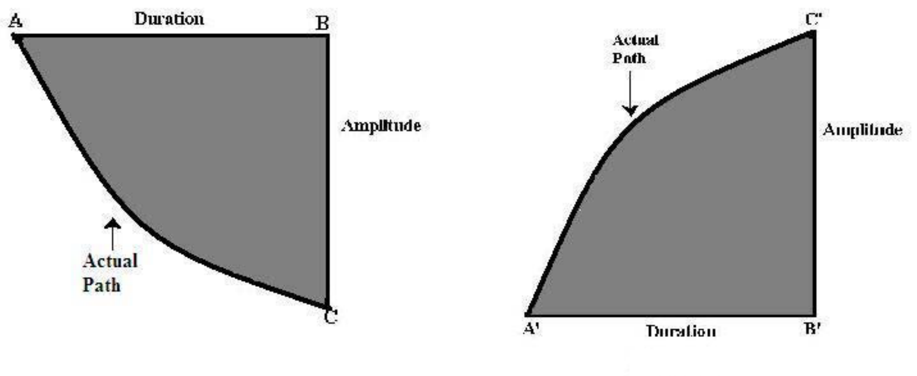
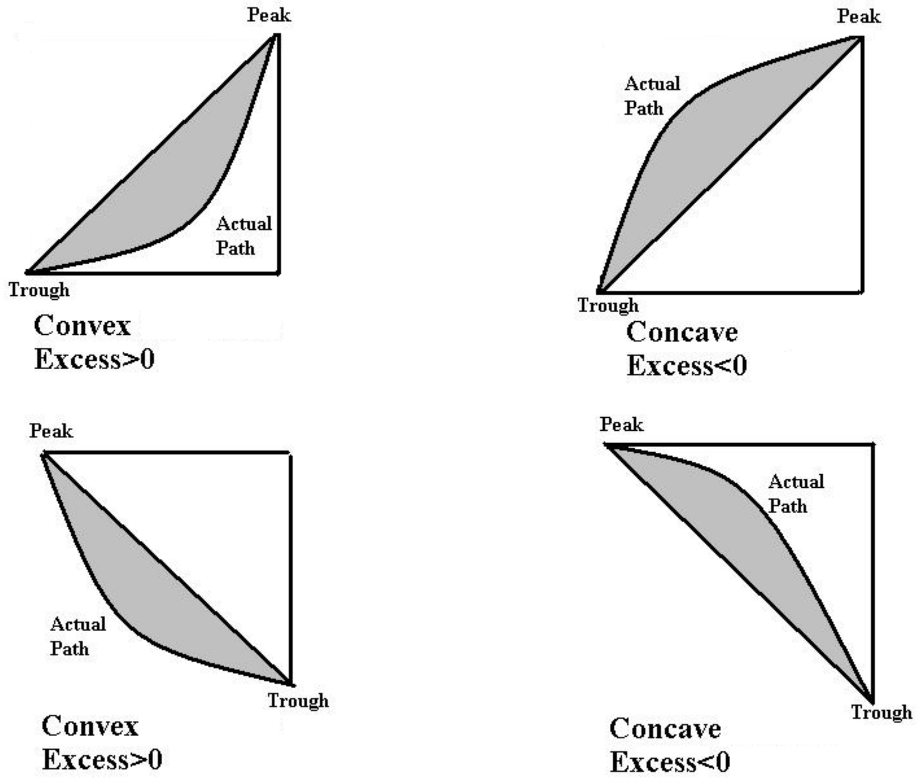


Figure A.1: Graphical visualization of business cycle characteristics (contd.)

(b) Shape of excess cumulative movements in contractions (second row) and expansions (first row)



Source: *Camacho et al. (2008).*

B Appendix to Section 4

B.1 Weak-form tests

The current exercise includes the following weak-form tests:

- The Shapiro-Wilk test $(W) \Rightarrow$ represents the extended version of their normality test, which accounts for the exponential probability density function under the null hypothesis (Shapiro & Wilk, 1972). The initial step in performing the test includes rearrangement of the durations in each phase in ascending order so that $D_1 \leq D_2 \leq \cdots \leq D_N$. The W test statistic is then defined by Equation 19, where N denotes the number of contractions (expansions). The test statistics corresponds to the scaled ratio of the squared difference between the mean and the shortest duration to the sample variance. It follows an exponential distribution with the exact finite sample critical values being tabulated by Shapiro and Wilk for N ranging from 3 to 100. Hence, for the samples of less than three contractions (expansions), the test is not applicable.

- The Stephens test ($W^*(D_0)$) $\Rightarrow$ a modified version of the W test statistic which relies on the exponential distribution under the null hypothesis and is conditioned on an assumed known minimum duration D_0 of a particular phase (Stephens, 1978). The $W^*(D_0)$ test statistic is given by Equation 20 and follows the same distribution as W, but due to the imposed minimum duration, the sample considered to obtain the critical values is $N+1$ instead of N. Similar to the W test statistic, the $W(D_0)$ can be implemented only for samples including at least three contractions (expansions).

- The Brain-Shapiro test (Z) $\Rightarrow$ based on a linear regression of normalized spacings between the ordered durations Y_n on the order of durations n, as the plot of Y_n versus n provides a mirror image of the plot of the hazard function[40] (Stephnes, 1978). The exponential distribution under the null hypothesis then implies that the estimated coefficient associated with the order of durations is zero. Brain and Shapiro (1983) use this result to compute a test statistic, denoted by Z. The testing procedure first defines the normalized spacings between the ordered durations for $n = 2, \ldots, N$ by Equation 22. With $\tilde{n} = n - N/2$ and $\hat{Y}_n = Y_n - \overline{Y}$ denoting the de-meaned variables, the test statistic is defined by Equation 21 and follows the asymptotically standard normal distribution to which it quickly approaches also in quite small samples.

- The modified Brain-Shapiro test with minimum duration ($Z(D_0)$) $\Rightarrow$ an extension of the Brain-Shapiro test that employs the assumed known minimum duration D_0 in a similar way as the Stephens test. This new observation is included as the first normalized spacing in Equation 22 such that $Y_1 = N(D_1 - D_0)$ and the summation in Equation 21 adjusts accordingly (i.e. from 1 to N). The resulting test statistic, denoted by $Z(D_0)$, follows the same distribution as Z.

- The modified Brain-Shapiro test for duration distributions associated with non-linear hazard functions (Z^*) $\Rightarrow$ another extension of the Brain-Shapiro test that accounts for the duration distributions associated with non-linear hazard functions. The test statistic in Equation 23, denoted by Z^*, is constructed by a linear regression and a quadratic regression, Z and Z_q, respectively, of normalized spacings between the ordered durations Y_n on the order of durations n. Z_q is computed in the same way as Z from Equation 21, where $\tilde{n}$ is defined as $\tilde{n} = (n - N/2)^2 - [N(N-2)/12]$. The test statistic follows a χ^2 distribution with two degrees of freedom, which also holds in small samples.

[40] Increasing of the spacings corresponds to the decreasing hazard function.

- The Mudambi-Taylor test (MT) $\Rightarrow$ proposed by Mudambi and Taylor (1991) and considers a discrete case, in which a test statistic is generated from the mean and the variance of the geometric distribution under the null hypothesis. It employs an assumed known minimum duration of the particular phase D_0 and is based on the mean and standard deviation of the transformed durations (i.e. deviations of durations from the assumed minimum duration $Q_n = D_n - D_0$). The test statistics is defined by Equation 24, where $\bar{Q}$ and S_Q are the mean and standard deviation of the transformed durations. It follows asymptotically a standard normal distribution, hence, in finite samples, Monte Carlo simulations are needed in order to obtain the critical values.

- The GMD test (GMD) $\Rightarrow$ analogous to the MT test, Mudambi and Taylor (1995) employ Tauchen's (1985) generalized method of moments (GMM) procedure to define the GMD test that is based on the moment condition implied by the geometric distribution under the null hypothesis.[41] The test statistic is defined by Equation 25. Once standardized, the distribution of the GMD test statistic is asymptotically standard normal. However, as it is highly skewed in finite samples, Monte Carlo simulations are necessary to obtain finite sample critical values.

- The state-based test (SB) $\Rightarrow$ proposed by Pagan (1998) and based on a simple linear regression, given by Equation 26, of the state variable S_t^*, as defined in the previous Section, on the lag of the duration variable of contractions (expansions) D_{t-1}, where $t = 1, \dots, T$ and $\mathbb{E}_{t-1}(\varepsilon_t) = 0$. In order to assure that the error term in Equation 26 is homoscedastic, the regressions for contractions and expansions have to be considered separately. This further requires using $(1 - S_t^*)$ instead of S_t^* in the regression for expansions, and to drop all the observations from the sample where $D_{t-1} = 0$ in both considered cases. Under the null hypothesis, which states that durations follow a geometric distribution, the test requires the estimated coefficient associated with the lagged duration variable of phases to be zero (i.e. $\beta_1 = 0$). This value of the coefficient implies that the exit from one phase does not depend upon the duration of the phase. Ohn et al. (2004) and Cardinal and Taylor (2009) show that the standard t test used in statistical inference for linear regression is appropriate for testing aforementioned hypothesis.

B.2 Strong-form tests

In order to be able to apply a representative of the strong-form tests, we proceed in the following manner. For any given set of contraction (expansion)

[41] Moment condition implied by the geometric distribution reads as $\mathrm{Var}(D) - [\mathbb{E}(D)]^2 - \mathbb{E}(D) - \gamma = 0$, where we can test whether $\gamma = 0$ using the GMM estimation of γ from the moment condition.

durations $(D_1, D_2, \ldots, D_N)$, the time spent in the n^{th} phase D_n is first transformed by taking into account an assumed known minimum duration D_0 as:

$$D_n^* = D_n - (D_0 - 1) \tag{B.1}$$

The shifted durations now entail the length of the shortest possible regime. Next, the minimum number of bins required for an optimal histogram is defined, according to Terrell and Scott (1985), as:

$$K^* = \lceil \sqrt[3]{2N} \rceil \tag{B.2}$$

where $\lceil \cdot \rceil$ indicate rounding up to the nearest integer. Given the optimal number of histogram bins K^*, the bin width BW is determined as:

$$BW = \frac{D_{max}^* - D_{min}^*}{K^*} \tag{B.3}$$

where D_{max}^* and D_{min}^* are the largest and smallest elements of the observed transformed duration sequence $(D_1^*, D_2^*, \ldots, D_N^*)$. Given such a setting, the element D_n^* is grouped in bin b if the following condition holds:

$$[D_{min}^* + (b-1)\,BW] \leq D_n^* < [D_{min}^* + bBW] \tag{B.4}$$

Taking into account all the above calculations, the χ^2 goodness-of-fit test entails observed and expected frequency in the particular bin and is calculated by Equation 27, where O_b denotes the observed number of elements in bin b and E_b is the expected number of elements in bin b given the geometric distribution under the null hypothesis.[42] The distribution of the χ^2 test statistic follows the asymptotic χ^2 distribution, hence, for finite samples, Monte Carlo simulations are necessary to obtain critical values.

[42] The expected number of elements in bin b is obtained by averaging across 1,000,000 Monte Carlo simulated samples from the geometric distribution.

B.3 Additional Tables and Figures

Table B.1: Business cycle characteristics of Slovenia and the euro area based on the monthly version of the MBBQ algorithm

	Slovenia		Euro area	
	Contractions	Expansions	Contractions	Expansions
Avg. duration ($\overline{D_p}$)	10.14	33.00	16.00	27.57
Avg. amplitude ($\overline{A_p}$)	-9.98	18.13	-7.96	9.57
Avg. cumulative movements ($\overline{C_p}$)	-52.55	456.25	-66.29	167.84
Avg. excess movements ($\overline{E_p}$)	16.78	5.38	-8.25	-5.13
CV of duration (CV_{D_p})	0.37	0.92	0.68	0.63
CV of amplitude (CV_{A_p})	-1.06	0.70	-1.11	0.57
CV of excess movements (CV_{E_p})	3.33	1.83	-2.69	-2.71
Avg. degree of steepness ($\overline{S_p}$)	-0.98	0.55	-0.50	0.35

Note: *Business cycle characteristics are obtained from analysing production in the aggregate industrials sector (excl. construction) in the case of both considered economies.*
Source: *Own calculations.*

C Appendix to Section 5

C.1 Bayesian estimation procedure

$Y = \{y_t\}_{t=1}^{T}$ denotes the quarter-on-quarter and month-on-month growth rates of the observable variables, $X = \{x_t\}_{t=1}^{T}$ is the collection of common factor and idiosyncratic components, $S = \{s_t\}_{t=1}^{T}$ corresponds to the unobservable variable, following a two-state Markov chain process, $Z = \{z_t\}_{t=1}^{T}$ contains the unobserved adjustments of the mean growth rate during the contractionary episodes, and $\Theta = \left[\alpha_0, \alpha_1, \sigma_f^2, \sigma_e^2, \{\sigma_1^2, \ldots, \sigma_5^2\}, \{\lambda_1, \ldots, \lambda_5\}, , \{\psi_{1,1}, \ldots, \psi_{1,5}\}, \{\psi_{2,1}, \psi_{2,2}, \ldots, \psi_{5,1}, \psi_{5,2}\}, p_{00}, p_{11}\right]$ denotes the model parameters that have to be estimated. The estimation of the model, defined by Equations 28 to 33, therefore proceeds in the following way, where $n = 1, \ldots, N$ denotes the number of iterations of the described procedure:[43]

- Given Y, S^n, Z^n, and Θ^n, generate $X^{n+1} \Rightarrow$ by utilizing Equations 28, 30 and 32 and the Carter and Kohn (1994) algorithm we perform the following steps:
  - First, the starting values of the Kalman filter are set in a way that $x_{1|0}$ defines a vector with the first element equal to $\alpha_0\left(1 - s_1^n\right) + \alpha_1 s_1^n + s_1^n z_1^n$ and the remaining elements equal to 0, and the covariance matrix is set to $P_{1|0} = I$.

[43]Further technical details regarding the described estimation procedure are available in the appendix of the Leiva-León et al. (2020).

- Accounting for the missing observations in y_t, $\left\{x_{t|t}, P_{t|t}\right\}_{t=1}^{T}$ is computed on the basis of the prediction and the update steps of the Kalman filter for $t = 1, \ldots, T$. The initial values of the prediction error and the prediction error variance are obtained from the starting values defined above (i.e. $x_{1|0}$ and $P_{1|0}$).
- Based on the output of the Kalman filter, the Kalman smoother is applied backwards for $t = T-1, \ldots, 1$, in order to obtain $\left\{x_{t|T}, P_{t|T}\right\}_{t=1}^{T}$. For the starting point of the smoothing procedure at $t = T$, $x_{t|T}$ and $P_{t|T}$ are equal to the final iteration of the Kalman filter.

- Given X^{n+1}, Z^n, and Θ^n, generate $S^{n+1} \Rightarrow$ by utilizing Equation 28 and the Carter and Kohn (1994) algorithm we perform the following steps:

  - For $t = 1, \ldots, T$ the $\mathbb{P}_t\left(s_t = 0\right)$ is computed, where the initial unconditional probability of the normal state can follow the steady-state probability, as defined in Equation A.2. The rest of the procedure resembles the logic of the Hamilton filter (1989) described in Appendix A.1.
  - By going from $t = T - 1, \ldots, 1$ and computing $\mathbb{P}_{t+1}\left(s_t = 0\right) = \mathbb{P}_T\left(s_t = 0\right)$, based on the backward filtering procedure (Hamilton, 1989 and Kim, 1994), obtain the unobservable state indicator s_t^{n+1} which is based on the smoothed probabilities. For the starting point of the smoothing procedure at $T = t$, $\mathbb{P}_T\left(s_T = 0\right)$ is equal to the final iteration in the previous step.

- Given S^{n+1}, X^{n+1}, and Θ^n, generate $Z^{n+1} \Rightarrow$ we utilize the same procedure as in the first bullet point above and adopt the approach of Durbin and Koopman (2002), except that Equation 28 is in this case treated as the signal (i.e. measurement) equation and Equation 29 is considered as the state equation. Furthermore, the procedure assumes that all of the elements except for z_t in Equation 28 are fixed.

- Given Y, Z^{n+1}, S^{n+1}, and X^{n+1}, generate $\Theta^{n+1} \Rightarrow$ we use the standard prior distributions (Table 10) in the Gibbs sampling procedure in order to simulate the posterior distributions of the parameters in Θ.

C.2 Additional Tables and Figures

Table C.1: Pairwise correlations with the quarter-on-quarter real GDP growth rate for Slovenia and the euro area

	Slovenia	Euro area
Industrial production index (excl. construction)	**0.58**	**0.86**
Volume of production in construction	0.43	0.37
Volume of production in (wholesale and retail) trade and services activities	0.56	0.50
Total imports of goods	**0.76**	**0.72**
Index of deflated turnover in retail trade	0.55	0.69
Total exports of goods	**0.66**	**0.75**
Main trading partner's industrial production index (excl. construction)	0.55	0.62
Economic sentiment indicator	0.32	0.54
Industrial confidence indicator	0.30	0.61
Assessment of export order-book levels	0.29	0.69
Assessment of order-book levels	**0.34**	**0.70**
Production expectations for the months ahead	0.28	0.47
Production trend observed in recent months	0.28	0.60
Assessment of stock of finished products	0.07	-0.41
Employment expectations for the months ahead	0.28	0.68
Harmonized unemployment rate (LFS)	-0.22	-0.43

***Note**: In order to apply selection criterion based on pairwise correlations, utilized monthly variables are aggregated to their quarterly counterparts. Furthermore, soft (i.e. survey) indicators are transformed to quarter-on-quarter differences, while other variables are cast in quarter-on-quarter growth rates.*
***Source**: Own calculations.*