Radiol Oncol 2009; 43(2): 132-135. doi:10.2478/v10019-009-0016-x

technical note

How well are clinical gross tumor volume DVHs approximated by an analytical function?

Pavel Stavrev[2], Colleen Schinkel[1,2], Nadia Stavreva[2] and B. Gino Fallone[1-3]

[1]Department of Physics, University of Alberta, [2]Department of Medical Physics, Cross Cancer Institute, [3]Department of Oncology, University of Alberta, Edmonton, Alberta, Canada

The dose heterogeneity in the tumor is often described as being normally distributed. Besides the normal distribution we propose the Fermi function describing Fermi statistics as a possible dose heterogeneity descriptor. In order to demonstrate the adequacy of the proposed functions as dose distribution descriptors 30 clinical gross tumor volume (GTV) dose-volume histograms (DVHs) are gathered and fit with the examined functions.

Key words: dose-volume histograms; gross tumor volume; Gaussian and Fermi statistics

In order to theoretically investigate a given tumor control probability (TCP) model for the case of heterogeneous irradiation, it is often necessary to simulate tumor dose-volume histograms (DVHs) that closely resemble clinical ones. Some authors[1-3] have assumed that tumor dose inhomogeneities are normally distributed around the target dose. In this case the integral DVH, *iDVH*, is represented by the erfc function:

$$iDVH:\ v(d\mid\mu,\theta)=0.5erfc\left(\frac{1}{\theta\sqrt{2\pi}}(d-\mu)\right) \quad [1]$$

where ν is the relative tumor volume irradiated to a maximum dose d, μ is a parameter corresponding to the mean (target) dose delivered to the tumor, and θ is a parameter related to the slope of the erfc function. However, no investigations of how well this function describes the clinical tumor DVHs are reported in the literature.

We propose the parallel use of the *Fermi* statistics function for the description of clinical tumor DVHs:

$$iDVH:\ v(d\mid\mu,\theta)=\frac{1}{1+\exp\left(\frac{d-\mu}{\theta}\right)} \quad [2]$$

This function describes the filling up of free energy levels in a Fermi system. The parameters d, μ and θ have the same meaning as in eq. [1].

To investigate this problem, we gathered 30 clinical gross tumor volume (GTV) DVHs for different treatment sites – lung, head & neck, prostate, etc., that were either

Received 14 January 2009
Accepted 15 February 2009

Correspondence to: Pavel Stavrev, Department of Medical Physics, Cross Cancer Institute, 11560 University Ave, Edmonton, Alberta, Canada. Phone: + 1 780 989 4334; Fax: +1 780 432-8615; E-mail: pavel.stavrev@gmail.com

obtained in the treatment planning process at the Cross Cancer Institute (CCI) or reported in the literature.[4-14] They were fit with the erfc [Eq. 1] and Fermi [Eq. 2] functions.

The fit was performed using the χ^2 criterion for goodness of fit, presuming a log-log-normal distribution for the integral DVHs. Correspondingly, the function to be minimized is:

The log-log form of the χ^2 criterion was used to account for the fact that an integral DVH is defined in the interval [0,1], while the standard χ^2 criterion deals with normally distributed random variables defined in $(-\infty, +\infty)$.[15] The experimental error is $\sigma_{\text{experimental}}$, which unfortunately is not reported in the literature. Therefore, we substituted $\sigma_{\text{experimental}}$ with a percentage band

$$\chi^2 = \sum_i \left[\frac{-\ln\left(-\ln\left(v^i_{theoretical}\right)\right) + \ln\left(-\ln\left(v^i_{\text{experimental}}\right)\right)}{\sigma^i_{\log\,\text{experimental}}} \right]^2 ; \sigma^i_{\log\,\text{experimental}} = \frac{-\sigma^i_{\text{experimental}}}{v^i_{\text{experimental}} \ln\left(v^i_{\text{experimental}}\right)}$$

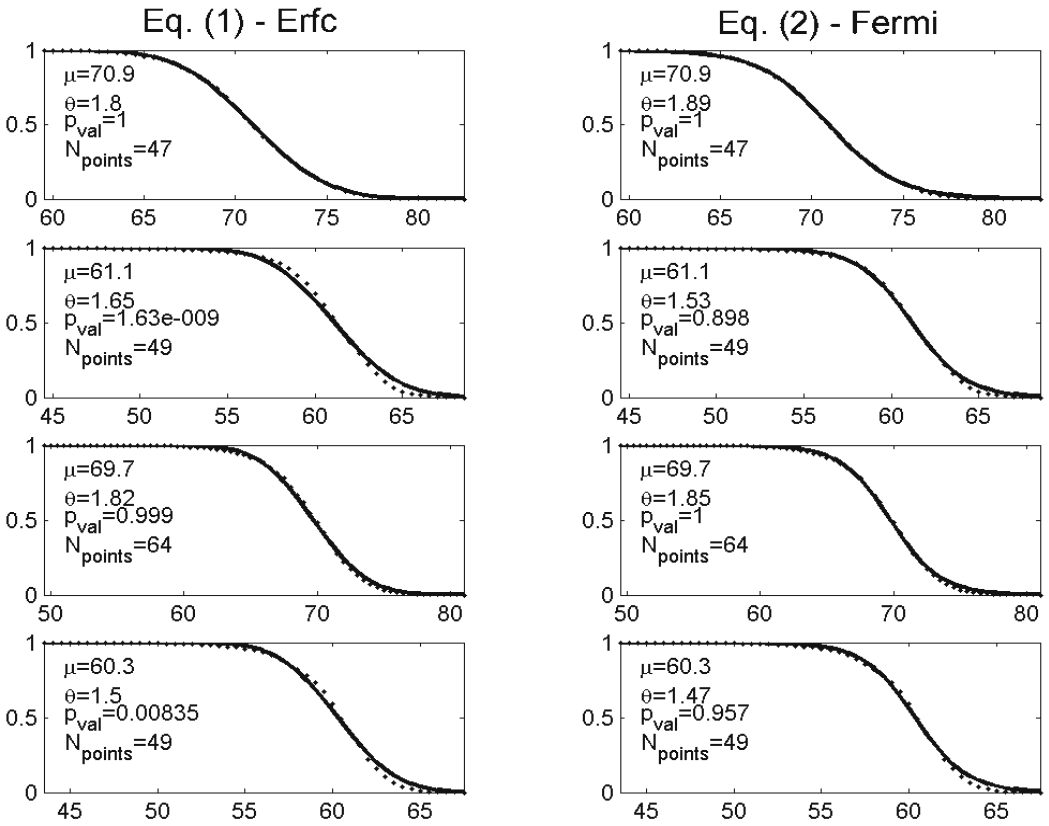

Figure 1. Fits to four clinical (head & neck) DVHs from CCI with the erfc function – a) and with the Fermi function – b) for the case of a 2% error band ($\sigma_{\text{experimental}} = 2\%$). On each subplot the p-value of the fit are shown, along with the statistics (number of data points, N_{points}) and the corresponding best fit values of the model parameters (μ and θ).

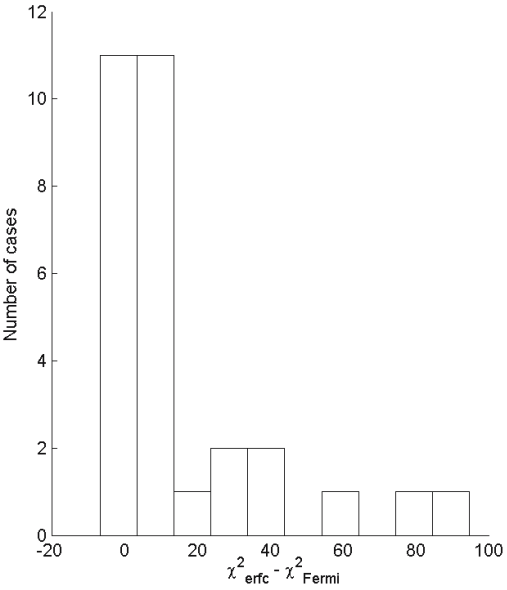

Figure 2. Distribution of the difference of χ^2 values for the fits obtained with the erfc and the Fermi functions, $\left(\chi^2_{erfc} - \chi^2_{Fermi}\right)$, built on the bases of fits to 30 clinical DVHs.

in which a statistically acceptable fit could be obtained. We initially presumed a 2% error band and found that in 24 out of 30 cases Eq. [2] produced acceptable fits, while Eq. [1] produced acceptable fits in 19 out of 30 cases. We also investigated a 4% error band and found out that Eq. [2] produced statistically acceptable fits in all considered cases, while Eq. [1] produced poor fits in 2 of all cases. As an illustration, fits of four head & neck DVHs obtained at CCI with the erfc function [Eq. 1] and with the Fermi function [Eq. (2] are shown in Figure 1a and Figure 1b, respectively. The fits correspond to the case of $\sigma_{\text{experimental}}$ =2%. As can be seen from the obtained χ^2 and p-values (shown in each subplot of Figure 1), the Fermi function produces excellent fits in all four cases, while the erfc function produces a poor fit in one case. It can be therefore concluded that the Fermi function describes clinical integral DVHs better than the erfc function. To further illustrate this conclusion a distribution of the difference of the χ^2 values of the fits obtained with the two proposed functions is shown in Figure 2. This distribution is constructed on the bases of the 30 clinical DVHs used in this study. As can be seen from Figure 2, the average of the distribution of $\left(\chi^2_{erfc} - \chi^2_{Fermi}\right)$ is greater than zero, showing that in most cases the *Fermi* function indeed produces better fits to clinical DVHs than the *erfc* function.

References

1. Ebert MA. Viability of the EUD and TCP concepts as reliable dose indicators. *Phys Med Biol* 2000; **45:** 441-57.

2. Stavrev P, Schinkel C, Stavreva N, Warkentin B, Fallone B. Population TCP estimators in case of heterogeneous irradiation. *Radiother Oncol* 2007; **84:** S279-S80.

3. Nahum AE, Sanchez-Nieto B. Tumour control probability modelling: basic principles and a**p**plications in treatment planning. *Physica Medica* 2001; **XVII(Suppl 2):** 13-23.

4. Wu Q, Mohan R, Niemierko A, Schmidt-Ullrich R. Optimization of intensity-modulated radiotherapy plans based on the equivalent uniform dose. *Int J Radiat Oncol Biol Phys* 2002; **52:** 224-35.

5. De Gersem WR, Derycke S, Colle CO, De Wagter C, De Neve WJ. Inhomogeneous target-dose distributions: a dimension more for optimization? *Int J Radiat Oncol Biol Phys* 1999; **44:** 461-8.

6. De Gersem WR, Derycke S, De Wagter C, De Neve WC. Optimization of beam weights in conformal radiotherapy planning of stage III non-small cell lung cancer: effects on therapeutic ratio. *Int J Radiat Oncol Biol Phys* 2000; **47:** 255-60.

7. Mohan R, Mageras GS, Baldwin B, Brewster LJ, Kutcher GJ, Leibel S, et al. Clinically relevant optimization of 3-D conformal treatments. *Med Phys* 1992; **19:** 933-44.

8. Teh BS, Mai WY, Uhl BM, Augspurger ME, Grant WH 3rd, Lu HH, et al. Intensity-modulated radiation therapy (IMRT) for prostate cancer with the use of a rectal balloon for prostate immobilization: acute toxicity and dose-volume analysis. *Int J Radiat Oncol Biol Phys* 2001; **49:** 705-12.

9. Reinstein LE, Wang XH, Burman CM, Chen Z, Mohan R, Kutcher G, et al. A feasibility study of automated inverse treatment planning for cancer of the prostate. *Int J Radiat Oncol Biol Phys* 1998; **40:** 207-14.

10. Stanescu T, Hans-Sonke J, Stavrev P, Fallone B. 3T MR-based treatment planning for radiotherapy of brain lesions. *Radiol Oncol* 2006; **40:** 125-32.

11. Bos LJ, Damen EMF, DeBoer RW, Mijnheer BJ, McShan DL, Fraass BA, et al. Reduction of rectal dose by integration of the boost in the large-field treatment plan for prostate irradiation. *Int J Radiat Oncol Biol Phys* 2002; **52:** 254-65.

12. Nederveen AJ, VanderHeide UA, Hofman P, Welleweerd H, Lagendijk JJW. Partial boosting of prostate tumours. *Radiother Oncol* 2001; **61:** 117-26.

13. Nutting CM, Rowbottom CG, Cosgrove VP, Henk JM, Dearnaley DP, Robinson MH, et al. Optimisation of radiotherapy for carcinoma of the parotid gland: a comparison of conventional, three-dimensional conformal, and intensity-modulated techniques. *Radiother Oncol* 2001; **60:** 157-63.

14. 14. Ólafsson A, Jeraj R, Wright SJ. Optimization of intensity-modulated radiation therapy with biological objectives. *Phys Med Biol* 2005; **50:** 5357-79.

15. Stavrev P, Stavreva N, Schinkel C, Fallone B. An objective function for TCP/NTCP curve fitting. *Med Phys* 2007; **34:** 2417-8.