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The Teaching of Initial Multiplication Concepts and Skills in Croatian Textbooks

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The goal of this paper is to describe the teaching of initial multiplication concepts and skills, up to the multiplication table, in the Croatian educational system. As Stiegler and Hiebert (1999) concluded, teaching is a complex system rooted in a cultural script of a given society. To describe it without ignoring certain features of it that appear to be self-evident to an insider, it is necessary to step out of this cultural frame. For that reason, we study the teaching of initial multiplication in Croatia by comparing Croatian mathematical textbooks with textbooks from Singapore, Japan, and England. For the textbook analysis, we adapt the framework of Charalambous, Delaney, Hsu, and Mesa that examines a textbook as an environment for the construction of knowledge of a single mathematical concept. The analysis provides evidence that practice and automation are at the centre of the initial learning of multiplication in Croatia. The meaning of multiplication usually is not clear, and pupils are not provided additional tools for developing understanding, nor they are encouraged to use different calculation strategies in a flexible manner. The study also indicates that Croatian textbooks present mathematics as a practice that is closed and pre-given, restricted to the one and the only right way through it.

Keywords: concept construction, multiplication, textbook analysis

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Pouk začetnih pojmov in spretnosti množenja v hrvaških učbenikih

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Cilj tega prispevka je opisati, kako se poučevanje začetnih pojmov in spretnosti množenja, vključujoče vse do poštevanke, udejanja znotraj hrvaškega vzgojno-izobraževalnega sistema. Kot sta sklenila Stiegler in Hiebert (1999), je poučevanje zapleten sistem, ukoreninjen v kulturnem zapisu dane družbe. Zato da ga opišemo, ne da bi zoperstavili določene značilnosti, ki se sicer njegovim notranjim članom zdijo samoumevne, moramo stopiti izven kulturnega okvirja. Iz tega razloga proučujemo začetno poučevanje množenja na Hrvaškem s primerjanjem hrvaških matematičnih učbenikov z učbeniki iz Singapura, Japonske in Anglije. Za analizo učbenika smo prevzeli ogrodje, ki ga predlagajo Charalambous, Delaney, Hsu in Mesa, ki presoja učbenik kot okolje za stvaritev znanja enega samega matematičnega koncepta.

Analiza priskrbi z dokazi, da sta praksa in avtomatizem v samem središču začetnega poučevanje množenja na Hrvaškem. Pomen množenja običajno ni jasen, pri čemer učencem niso zagotovljena dodatna orodja za razvoj razumevanja, niti niso spodbujeni, da bi uporabili različne računske strategije na fleksibilen način. Raziskava kaže tudi na to, da hrvaški učbeniki prikažejo matematiko kot prakso, ki je zaprta in vnaprej dana, omejena na en sam in edino pravilen način obravnave.

Ključne besede: konstrukcija konceptov, množenje, analiza učbenika

Introduction

In 2014, Croatia started a process of curriculum reform for primary and secondary education. New curricula for all school subjects were finished in 2016 and were sent for an expert review. In the new curriculum for mathematics, learning of multiplication and multiplication tables should start in Grade 2, but full mastery was expected in Grade 3, since it requires time and a great deal of practice. In their commentaries on the first draft of the curriculum, many primary teachers called for the expectation of mastery of the multiplication table by the end of Grade 2 from the old curriculum (MZOŠ, 2007) to be retained. This proposition was adopted in the revised version of the new mathematics curriculum (MZO, 2019).

The many recommendations concerning the mastery of the multiplication table lead us to consider the initial teaching and learning of multiplication as an example of what Stiegler and Hiebert (1999) described when they characterise teaching as a cultural activity. These kinds of activities are rooted in the cultural script: they reflect attitudes and beliefs of a culture and are manifested in the patterns of behaviour. These attitudes, beliefs, and behaviours are learned implicitly through observation and participation inside the culture. Since they are widely shared, they are difficult to see. This makes them highly stable over time and not easily changed.

The old Croatian mathematics curriculum reflects this: 'In the existing elementary school, mathematics is a subject with a long tradition and well-defined content, and no major interventions are required in current programmes. Therefore, the existing programmes formed the starting basis in the design of the mathematics curriculum' (MZOŠ, 2006, p. 238).

To investigate these kinds of practices, one needs to step out of one's cultural circle. In this way, one can become aware of the scripts they are using and, by comparing different cultural scripts, one can see possibilities, not just what is there but also what is not (Stiegler & Hiebert, 1999). One of the ways of doing this for teaching practices is through the comparative international textbook analysis (Haggarty & Pepin, 2002).

Multiplicative reasoning

That 'multiplicative thinking appears early and develops slowly' was the conclusion of Clark and Kamii (1996) based on their analysis involving children in Grades 1 to 5. There are multiple reasons for that.

In studying children's solutions to multiplication problems, Bell et al. (1984) found that the solutions varied according to the numbers involved, the structures involved, and the context of the problem. They saw this as another manifestation of 'meaning blindness'. Although traditionally, the teaching of multiplication and division was the teaching of procedures, this showed the necessity of also teaching different kinds of multiplicative structures. Or, in the words of Kaput (1985, as cited in Greer, 1992), mathematics and its 'applications' should be taught as being of a piece from the very beginning.

In their search for the origins of children's mistakes in multiplication problems, Fischbein et al. (1985) hypothesised that children link every arithmetic operation to an implicit, unconscious, primitive model that mediates the meaning of operation when solving problems. The model, in turn, imposes its own constraints on the understanding of operations. For multiplication, they argue, children use repeated addition as an implicit model. This hypothesis of repeated addition as the origin of multiplicative thinking is reflected in curriculum documents of different countries (Park & Nunes, 2001; MZOŠ, 2006).

When considering multiplicative situations or structures from the point of the underlying process or procedure, different kinds of these situations can be identified: equal grouping, rate, array, measure conversion, multiplicative comparison, Cartesian product (Anghileri, 1989; Bell et al., 1984). Greer (1992) synthesises classifications of multiplicative situations into four major classes: equal groups (with equal measure and rate; *Ann, Ivy, and Kathy have 5 apples each. How many apples do they have altogether?*), multiplicative comparison (*Mary has 3 times as many chocolates as Steve. If Steve has 5 chocolates, how many chocolates does Mary have?*), rectangular array (and area; *Chocolates are sorted in an array of 3 rows and 5 columns. How many chocolates are there?*) and Cartesian product (*John has 3 pants and 5 shirts. In how many ways can John dress?*).

Vergnaud (1988), in contrast, considers the mathematical relations between quantities in these situations. In this way, he finds two main types of situations. The first he calls 'an isomorphism of measures', which involves two pairs of different quantities. There are two types of relationships in this situation: one between the quantities of a different kind (ratio) and one between values of the same kind of quantities (scalar factor). Another type of situation concerns a product of measures, which involves a third quantity, a factor quantity, connecting two other kinds of quantities.

Different researchers considered children's solution strategies for multiplication problems. From the point of the degree of abstraction, these strategies were classified as modelling (and counting), counting (by 1, rhythmic, skip), repeated addition, and multiplicative operations (Anghileri, 1989; Kouba, 1989;

Mulligan & Mitchellmore, 1997). Kouba (1989) also noted that when children used physical objects for solving problems, they either represented individual elements in each set, or they represented only the tallies. For the same multiplication expression, children's solution strategies can change if the meaning of multiplication or its representation changes (Fosnot & Dolk, 2001; Kouba, 1989). This phenomenon is not exclusive to multiplication but has been observed in other topics (Kolar et al., 2018; Tirosh et al., 2018). Fosnot and Dolk (2001) elaborated multiplicative operations strategies by linking them to the properties of multiplication.

The research into children's solution strategies for multiplication problems put forward a theory of repeated addition as an origin of multiplication into question (Kouba, 1989). Although counting and adding are used to obtain the numerical answer, there is something that goes on before the counting (Nunes & Bryant, 1996; Steffe, 1994).

These considerations lead to an alternative hypothesis that views the concept of multiplication as defined by an invariant relation between two quantities, called 'ratio' or 'rate' (Piaget, 1965; Vergnaud, 1988). Thus, the development of a one-to-many correspondence scheme lies at the origin of the understanding of multiplicative situations (Nunes & Bryant, 1996). The works of Piaget (1965) and others (see Nunes & Bryant, 1996) show that children as early as five years of age are able to represent and use one-to-many correspondences. Furthermore, arguments for the repeated addition theory can result from children's previous school experience (Nunes et al., 2010). An intervention study by Park and Nunes (2001) showed that pupils who were taught multiplication through correspondence made significantly more progress than pupils who were taught multiplication through repeated addition.

This shows that multiplication is not simply a new arithmetic operation to be learned after addition but that there are also new meanings and new situations to be learned. Moreover, although there is a procedural connection between multiplication and addition, there is also a conceptual discontinuity between the two (Nunes & Bryant, 1996).

Research on Mathematical Textbooks

Mathematics textbook research is a rich and growing area of investigation. In their overview of the literature, Fan et al. (2013) conceptualised four categories of mathematics textbook research: the role of textbooks in mathematics teaching and learning, textbook analysis and comparison, textbook use, and other areas of textbook research.

Studies from different countries have shown that textbooks have a considerable impact on the teaching and learning taking place in the classrooms (Fan & Kaeley, 2000; Haggarty & Pepin, 2002; Pepin & Haggarty, 2001). Haggarty and Pepin (2001, 2002) analysed textbook structure and content, their usage by the teachers, and the access to the textbooks for the pupils. They found that textbooks from different countries differ both in the structure and in the complexities and learning opportunities they offer for the pupils. Furthermore, the way teachers used textbooks varied between different countries, but also between different educational strands in one country. Authors argued that some of the factors that influenced these issues come from the cultural tradition, consisting of the organisation of the educational system, educational traditions, values and epistemic beliefs, and the socio-economic conditions of both the pupils and teachers. Studies in Croatia also confirmed the impact that textbooks have on teaching and learning (Glasnović Gracin & Domović, 2009; Glasnović Gracin & Jukić Matić, 2016; Jukić Matić, 2019). These studies also revealed social issues that influence textbook utilisation.

A comprehensive TIMSS study of mathematics and science textbooks from 48 countries (Valverde et al., 2002) assumes a four-part model of the curriculum; besides the intended, implemented, and attained curriculum, they add the potentially implemented curriculum as the fourth part, and view textbooks as parts of it, as mediators between intended and implemented curriculum. Their description of textbooks considers their macrostructure (size, length, etc.) but also their microstructure for which textbooks are sequenced into blocks of analysis, which are then coded by block type, mathematical content, presentation expectations, and a wider perspective on the subject. This analysis shows variations in textbook structures across different school systems.

The study by Jones and Fujita (2013), which uses the adapted TIMSS framework, showed that mathematical textbooks in England and Japan clearly reflect the geometrical component of a national curriculum. Charalambous et al. (2010) developed a framework for investigating learning opportunities and pupils' expectations in the textbooks, particularly with respect to the presentation of specific content. Their study considered the treatment of the addition and subtraction of fractions in textbooks from Cyprus, Ireland, and Taiwan and found greater variations between textbooks across cultures than within one country, which is in accordance with the findings of the TIMSS video study (Stigler & Hiebert, 1999). This framework was later used in similar comparative studies in different countries and for different mathematical topics (Hong & Choi, 2014; Yang & Sianturi, 2017). Boonlerts and Inprasitha (2013) analysed the presentation of multiplication in elementary textbooks from Japan, Singapore, and Thailand through content analysis (sequencing of topics and meaning of multiplication). The meaning given to multiplication was conceptualised using Greer's (1992) categorisation of classes of situations involving multiplication, and the analysis showed differences in the textbooks from the three countries from this perspective. Analysis of the sequencing of topics showed that textbooks from Singapore and Japan offer their pupils increasingly sophisticated strategies to solve multiplication problems.

This study aims to describe important features of the treatment of initial multiplication in mathematics textbooks in Croatia. This kind of description could yield important information for the future development of the national curriculum and curricular materials. Also, the findings of this study can be used for informing and changing the teaching practice in primary schools in Croatia.

To describe the treatment of initial multiplication in Croatian textbooks, we compare them with textbooks from Singapore, Japan, and England. The primary reasons for choosing these countries are the good results that students from these countries achieve in international comparative studies and the availability of textbooks in the English language. Furthermore, since these countries are geographically distant, and their tradition is historically and culturally different from that of Croatia, one can expect that insight into culturally hidden assumptions and practices of the Croatian teaching tradition could be obtained by comparison with these countries.

We attempt to describe features of Croatian textbooks by answering the following research questions:

1. What is the structure of a typical lesson in Croatian textbooks?
2. What possible multiplication constructs and representations are used?
3. What multiplication strategies are promoted?
4. Do Croatian textbooks promote the usage of tools and manipulatives for learning?
5. What features of a mathematics classroom are implicated by Croatian textbooks?
6. To what extent do Croatian textbooks reflect the national curriculum?

General assumptions and initial multiplication in the Mathematics Curricula of Croatia, Singapore, Japan, and England

Before the textbook analysis, an overview of national mathematics curricula was made so that it could be used for the interpretation of the results.

The Croatian 2006 curriculum is divided into two parts: the introductory part and the list of themes for each year of study, together with pupils' learning achievements. Themes of beginning multiplication are placed in Grade 2.

The Singapore Mathematics Syllabus (MOE, 2012) is organised through a list of mathematical contents and appropriate learning experiences for each year of study are given at the end of the syllabus. The learning of multiplication and multiplication tables takes place in the first three years.

The 2009 Japanese curriculum standards (Isoda, 2010) operationalises its objectives for each grade through a list of learning outcomes, a list of relevant terms and symbols, a list of mathematical activities to be used, and some specific remarks on the ways of handling the content. The learning of multiplication and multiplication tables takes place in Grade 2.

The 2013 English Mathematics Curriculum (DfE, 2013) for each year gives statutory requirements but also some notes and guidance. Pupils learn multiplication during the first four years.

Table 1 summarises different aspects of the mathematics curriculum of these four countries. One can observe that curricula have much more in common regarding their main emphasis. However, when it comes to the elaboration of the content of initial multiplication and description of the ways of working, the Croatian curriculum is rather concise on that matter. It will be interesting to see whether this will also be reflected in the textbooks.

Table 1
Comparison of mathematics curricula of Croatia, Singapore, Japan, and England regarding multiplication

	Croatia	Singapore	Japan	England
Emphasis of the Curriculum	basic mathematical knowledge, development of skills, problem-solving	problem-solving, conceptual understanding, skills proficiency, mathematical process, attitudes, metacognition	basic knowledge and skills, ability to think and express, attitudes	fluency, mathematical reasoning, problem-solving
Grades	2	1, 2, 3	2	1, 2, 3, 4
Multiplication constructs	addition	addition, equal groups, scaling	equal groups	addition, equal groups, array, scaling, Cartesian product
Multiplication strategies	commutativity	counting, concretisation, patterns, heuristics	counting, properties of multiplication, commutativity, adding next, patterns	counting, concretisation, doubling, patterns, connections, commutativity, associativity, distributivity
Representations and manipulatives		pictorial, concrete objects	pictorial, concrete objects	pictorial, concrete objects
Forms of work		group work, sharing ideas		

Method

Based on the data of the Croatian Ministry of Science and Education (MZOS, 2014), the three most commonly used Croatian textbook series were chosen for the analysis: *Matematika*, *Moj sretni broj*, *Nove matematičke priče*. In accordance with the Croatian curriculum, all these textbooks are used in the second year of study. Together they comprised a market share of 78.33% of mathematics textbooks in the second grade in Croatia. In addition, the following textbook series were analysed: *Primary Mathematics* and *My Pals are Here* from Singapore, *Sansu Math* from Japan, and *Power Maths* from England, used from Grades 1–3, 2–3, and 1–4, respectively. Throughout this paper, the names of these textbooks will be abbreviated as CT1, CT2, CT3, ST1, ST2, JT, and ET, respectively.

An analytical framework was based on the framework of Charalambous, Delaney, Hsu, and Mesa (2010) for textbook analysis, which views a textbook as an environment for the construction of knowledge of a single mathematical concept. All lessons of a textbook concerning initial multiplication learning upon the multiplication table were analysed. Each lesson was divided into smaller blocks that were analysed from several aspects (categories). Based on the results of the pilot study (Baković et al., 2019.), the analytical framework was adapted to include the following categories: block type, social form of work, context, use of concrete materials, images, characters, representations, construct and multiplication strategies. The coding list for each category was created by using the grounded theory approach (Glaser & Strauss, 1967); the initial list was created on the basis of the literature review and was later revised on the basis of the observed data. The full list of codes is given in Table 2.

The category *block type* concerns its function in the text with the way that the block is communicated to pupils.

The category *context* deals with the context of the mathematical problem posed in a block. Authentic context refers to a pupil's personal experience while realistic to a possible experience.

The category *construct* refers to the meaning of multiplication. For this purpose, Greer's (1992) classification was revised so that multiplicative comparison, rate, and equal measure problems were included in a joint class that was named 'scaling'. This change was motivated by children's different ways of representing grouping and matching problems in Kouba's study (1989).

Three categories were concerned with the visual features of blocks. The first one, the *use of pictures* category registers the presence of the picture and whether it is mathematically relevant or not.

The *representations* category refers to the way that multiplication is represented. The representation usually corresponds to the construct, but sometimes the two can differ. For instance, the text can be about four groups of three, while the picture shows 12 objects sorted in a 4×3 array.

Finally, the category *other graphical objects* refers to the function of graphical objects that were used. These were mostly images of pupils or some other characters that communicate some information to pupils, but there were also diagrams, drawings, and images of objects to use.

The category *use of manipulatives* assesses if pupils are instructed to use concrete, manipulative materials and, if so, which ones. Physical manipulatives are divided into standardised, which are used throughout the textbook, and sporadic, which are used only once.

The category *multiplication strategy* assesses both which strategies for carrying out multiplication are promoted in the block, as well as problem-solving and general learning strategies.

The category *social form of work* characterises organisational ways pupils are supposed to work. Only blocks for which this is explicitly stated were categorised, other blocks were categorised as 'unclear'.

Table 2
Coding list

Block Type
Situation
Recap
Worked example
Definition/rule
Exercise
Activity/game
Self-assessment

Context
Authentic
Realistic
Intra-mathematical

Construct
Addition
Equal groups
Array
Scaling/Comparison
Counting
Cartesian product
Without construct

Use of pictures
Absent
Mathematically relevant
Context related

Representation
Equal groups
Linear
Array
Number line
Multiplications table
Counting
Bar/Scaling
10×10 table
Cartesian product
Without Representation

Use of manipulatives
Standardised manipulative
Sporadic manipulative
10x10 table
Games
Multiplication cards
Hands

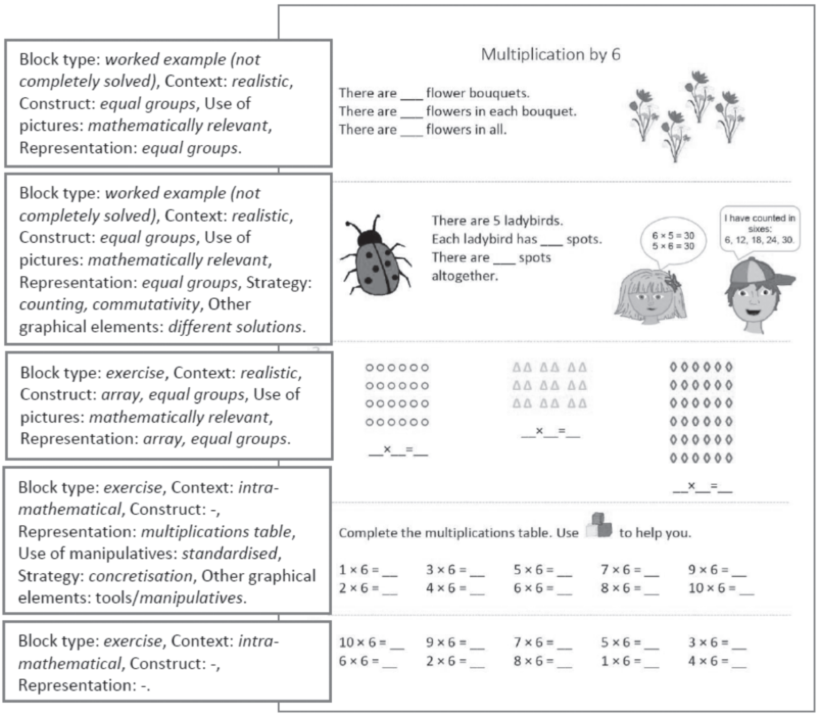
Strategy
Commutativity
Multiplications table
Multiplication by 1 and 0
Counting
Adding/Subt next
Distributivity
Doubling
Bar-model
Modelling/Concretisation/Drawing
Metacognition
Estimation
Mnemonic
Associativity

Other graphical elements
Instruct/describe
Solution
Explanation
Guidance
Different solutions
Additional questions
Terminology
Rule
Drawing/diagram
Tools/manipulatives

Social form of work
Pair
Group
Family
Unclear

We demonstrate the coding procedure on the sample page given in Figure 1.

Figure 1
Example of coding a textbook page



Results and discussion

The total number of blocks that were identified and analysed in the textbooks is given in Table 3.

Table 3
Number of analysed blocks in the textbooks

CT1	CT2	CT3	ST1	ST2	ET	JT
139	117	171	197	136	420	101

The results of the analysis of blocks are given below, for each category, in terms of percentages of the blocks in the textbooks.

Block type

The distribution of codes inside this category is shown in Table 4. Recaps appear more frequently in Croatian textbooks than in non-Croatian textbooks. In non-Croatian textbooks, they appear only at the beginning of chapters, while in Croatian textbooks each lesson begins with a recap. This can be considered an indicator of a compartmentalised and unconnected view of mathematical knowledge. Also, Croatian textbooks had neither activities/games nor self-assessment blocks.

Table 4
Distribution of block type in the textbooks

Block Type	CT1	CT2	CT3	ST1	ST2	ET	JT
Situation	1%	1%		1%			2%
Recap	10%	18%	15%	1%	1%		
Worked example	24%	9%	26%	24%	29%	8%	30%
Definition/rule	6%	15%	6%	8%	4%		9%
Exercise	59%	57%	53%	57%	48%	89%	53%
Activity/game			1%	9%	15%		6%
Self-assessment					3%	2%	

Context and Construct

Table 5
Distribution of Context in the textbooks

Context	CT1	CT2	CT3	ST1	ST2	ET	JT
Authentic				13%	16%	6%	3%
Realistic	32%	24%	30%	63%	71%	62%	47%
Intra-mathematical	68%	76%	70%	23%	13%	33%	50%

The context of the majority of the blocks in Croatian textbooks is intra-mathematical, and it is never authentic (see Table 5). This is different from non-Croatian textbooks in which there is a prevalence of realistic content, and all of them include blocks with authentic context.

Table 6

Distribution of Construct in the textbooks

Construct	CT1	CT2	CT3	ST1	ST2	ET	JT
Addition	17%	27%	19%	8%	7%	14%	4%
Equal groups	23%	15%	11%	65%	60%	41%	23%
Array	2%	9%	3%	21%	13%	17%	17%
Scaling/Comparison	17%	5%	24%	6%	7%	20%	33%
Counting				20%	7%	2%	1%
Cartesian product						2%	
Without construct	55%	64%	54%	17%	19%	28%	49%

The majority of blocks of Croatian textbooks do not have a construct specified (see Table 6). In non-Croatian textbooks, a construct is evident in almost all blocks. The high rate of blocks without evident constructs in JT textbooks is a consequence of the fact that the focus of the last 20% of lessons is to encourage pupils to use different multiplication strategies and to investigate multiplication properties and patterns; thus, a construct in these lessons is not evident.

Some non-Croatian textbooks systematically introduce different constructs and sometimes explicitly ask pupils to make connections between different constructs. This is not the case in Croatian textbooks in which new constructs usually appear 'out of nowhere', with the occasional observation that 'this is also multiplication'.

It can be noted that in Croatian textbooks repeated addition is promoted at a higher rate of blocks than in non-Croatian textbooks. Also, non-Croatian textbooks promote counting as a construct, which is not the case in Croatian textbooks.

An explanation for these findings could be rooted in differences between curricula. Unlike other curricula, the Croatian curriculum does not point out different aspects of multiplication and defines multiplication only as repeated addition.

Representations, use of pictures and manipulatives

Table 7
Use of pictures in the textbooks

Use of pictures	CT1	CT2	CT3	ST1	ST2	ET	JT
Absent	76%	75%	74%	37%	17%	28%	38%
Mathematically relevant	24%	25%	24%	60%	79%	66%	55%
Context related			2%	3%	1%	5%	7%

Almost 75% of blocks in Croatian textbooks are without representation, which is significantly lower than in non-Croatian textbooks (see Table 7). They are usually present in worked examples at the beginning of a lesson, while in exercises a representation is usually absent.

Table 8
Distribution of Representation in the textbooks

Representation	CT1	CT2	CT3	ST1	ST2	ET	JT
Equal groups	15%	11%	8%	46%	32%	36%	28%
Linear	5%	2%	4%	3%	15%	3%	7%
Array	1%	10%	8%	20%	29%	20%	27%
Number line	3%	12%	8%			14%	
Multiplications table	7%	11%	6%	6%	13%	3%	20%
Counting	1%	7%		20%	10%	2%	2%
Bar/Scaling				4%	2%	11%	10%
10×10 table				2%	2%	1%	
Cartesian product						2%	
Without Representation	70%	58%	75%	21%	18%	35%	39%

Besides pictorial representation, some non-pictorial representations also appear; in Croatian textbooks, these are a multiplication table and counting, while in non-Croatian textbooks, we additionally registered a 100-table (see Table 8). There is also a difference in the usage of multiplication tables; in Croatian textbooks, they are simply given without any further instructions. In non-Croatian textbooks, pupils are encouraged to use these tables to observe patterns and properties of multiplication.

Table 9

Use of manipulatives in the textbooks

Use of manipulatives	CT1	CT2	CT3	ST1	ST2	ET	JT
Standardised manipulative				15%	10%	5%	3%
Sporadic manipulative				2%	1%		
10×10 table				2%	1%	1%	8%
Games					1%	1%	3%
Multiplication cards				2%	2%		2%
Hands				8%	1%		

As shown in Table 9, Croatian textbooks do not promote the use of manipulatives. In non-Croatian textbooks, one can observe the intention to standardise the use of manipulatives: the same manipulatives are used throughout different lessons, giving pupils the opportunity to make connections between mathematical content and its representation with those manipulatives.

Strategies

Table 10

Distribution of Strategy in the textbooks

Strategy	CT1	CT2	CT3	ST1	ST2	ET	JT
Commutativity	6%	15%	13%	23%	8%	10%	16%
Multiplications table	7%	10%	12%	5%	13%	8%	24%
Multiplication by 1 and 0	7%	6%	8%	2%	1%	3%	4%
Counting	1%	7%		21%	10%	6%	1%
Adding/Subt next	1%			3%	3%	6%	24%
Distributivity	2%			9%	5%	4%	14%
Doubling				2%		9%	3%
Bar-model				5%	2%	4%	8%
Modelling/Concretisation/Drawing			3%	12%	4%	12%	15%
Metacognition	1%			5%	6%	15%	22%
Estimation						3%	
Mnemonic		1%		4%	1%		1%
Associativity							1%

Croatian textbooks promote only multiplication tables, multiplication by 1 and 0 and commutativity as strategies for carrying out multiplication (see Table 10). Multiplication by 0 and 1 are mostly present inside corresponding lessons listed in the curriculum. In non-Croatian textbooks, this strategy is far less present. Commutativity is also differently treated: in non-Croatian textbooks it is a strategy that pupils can use in their calculations, or when looking for patterns or global properties of multiplication. Unlike this, Croatian textbooks usually present commutativity as a property that pupils are just supposed to check.

Modelling, concretisation, and drawing as strategies that enable pupils to organise their own thinking around multiplication problems are almost never present in Croatian textbooks. This is in accordance with the Croatian curriculum which, unlike non-Croatian curricula, pays no attention to these strategies. The same holds for counting as a strategy.

Strategies that enable pupils to calculate flexibly, such as doubling, adding/subtracting one and distributivity, are not present at all in Croatian textbooks, while non-Croatian textbooks systematically encourage pupils to use these strategies.

Non-Croatian textbooks systematically introduce the bar model as a pre-algebraic strategy for solving word problems. This is not present in Croatian textbooks where pupils are not given additional tools to help them with modelling or solving problems.

Unlike Croatian textbooks, non-Croatian textbooks pay attention to the development of different metacognitive strategies: recognition of boundaries of certain concepts, self-assessment, scaffolding in problem-solving, making connections between different parts of knowledge, and similar.

Other graphical elements

Table 11

Use of other graphical elements in the textbooks

Other graphical elements	CT1	CT2	CT3	ST1	ST2	ET	JT
Instruct/describe		2%	14%	2%	5%	1%	5%
Solution		5%	1%		4%	2%	3%
Explaining			2%	16%			2%
Guidance				11%	8%	11%	14%
Different solutions				2%	1%	5%	11%
Additional questions				2%	7%	6%	9%
Terminology		1%		1%	7%		
Rule		1%			4%	1%	2%
Drawing/diagram				2%			5%
Tools/manipulatives				13%	15%		

The majority of *other graphical elements* refer to certain characters in textbooks. These are mostly pupils, but some abstract characters also appear (e.g., a magician in CT3). It is interesting to note the difference between the functions of these characters in textbooks (see Table 11). Characters in Croatian textbooks give instructions, solutions, and explanations. The magician in CT3 leads pupils through the story. Besides these functions, characters in non-Croatian textbooks give guidance for pupils' solutions, they ask additional questions and give and seek more than one solution. We believe that the functions of these characters give insight into the way teaching is supposed to unfold in the classroom. Croatian textbooks present a classroom in which the teacher is the central figure, 'the sage on the stage'. Pupils in this classroom are expected to give solutions to problems in only one, correct way. The mathematics that this classroom presents is closed and given, restricted to the one and only right way through it.

Unlike this, non-Croatian textbooks present classrooms that are pupil-centred. Pupils are the ones that solve problems, guide each other, discuss different solutions, and open new questions. Mathematics in this case is an open and connected domain of flexible thinking and discussions among practitioners.

Social form of work

Table 12
Social forms of work in the textbooks

Social form of work	CT1	CT2	CT3	ST1	ST2	ET	JT
Pair				3%	3%	2%	4%
Group				7%	2%		4%
Family				1%			

Unlike non-Croatian textbooks, Croatian textbooks do not call for work in pairs or in a group (see Table 12). This additionally supports the idea of a mathematical activity as a solitary endeavour. Furthermore, it gives fewer educational opportunities for pupils (Boaler, 2016).

Conclusion

This analysis provides evidence that there are only minor variations of teaching multiplication within different Croatian textbooks and at the same time all Croatian textbooks are noticeably different from the observed textbooks from England, Japan, and Singapore. The initial learning of multiplication in Croatia puts practice and automation in focus. The majority of content is intra-mathematical, not connected to the real world or authentic problems, and pupils are not encouraged to use manipulatives to help them model realistic problems or represent abstract mathematical problems. The meaning of multiplication is usually not clear nor is it visually represented. It is expected that the pupils solve exercises, but they are not encouraged to use different strategies and approaches.

The study also indicates some general features of teaching and learning mathematics in Croatia. Unlike non-Croatian textbooks, Croatian textbooks do not promote the development of metacognitive skills and strategies. Pupils are not encouraged to find different solutions and to make connections between them, to discuss their solutions, or to pose problems.

These findings suggest that the teaching practice should strive to compensate for these deficits. This means helping pupils in making sense of multiplication by situating problems in a familiar context, giving them opportunities to explore multiplication by using pictures and manipulatives, and giving them opportunities to develop different strategies for calculating.

Focusing solely on the textbooks is a limitation of this study. To obtain a fuller picture, one should see what is going on inside the actual classrooms. In fact, to get insight into this, an observational study had been started in March 2020 but was closed soon because of a lockdown due to the Covid-19 pandemic. Since the implementation of the new curriculum in Grade 2 started in September 2020, it was no longer possible to continue it.

All these features are in accordance with the Croatian curriculum from 2006. The new curriculum, which puts more stress on the use of manipulatives and pictorial representations, on problem-solving and formative assessment, will be fully implemented by 2022. It would be interesting to conduct a follow-up study in the future and see to what extent the changes made in the curriculum will be reflected in the textbooks and the teaching practice.

References

Anghileri, J. (1989). An investigation of young children's understanding of multiplication. *Educational Studies in Mathematics*, *20*(4), 367–385. https://doi.org/10.1007/BF00315607

Baković, T., Trupčević, G., & Valent, A. (2019). Treatment of initial multiplication in textbooks from Croatia and Singapore. In Z. Kolar-Begović, R. Kolar-Šuper, & Lj. Jukić Matić (Eds.), *Towards new perspectives on mathematics education* (pp. 216–228). Element.

Bell, A., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: The effects of number size, problem structure and context. *Educational Studies in Mathematics*, *15*(2), 129–147. https://doi.org/10.1007/BF00305893

Boaler, J. (2016). *Mathematical mindsets: unleashing students potential through creative math, inspiring messages. and innovative teaching*. Jossey-Bass.

Boonlerts, S., & Inprasitha, M. (2013). The Textbook Analysis on Multiplication: The Case of Japan, Singapore and Thailand. *Creative Education*, *4*(4), 259–262. https://doi.org/10.4236/ce.2013.44038

Charalambous, C.Y., Delaney, S., Hsu, H.-Y., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning, 12*(2), 117–151. https://doi.org/10.1080/10986060903460070

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1-5. *Journal for Research in Mathematics Education*, *27*(1), 41–51. https://doi.org/10.2307/749196

Department for Education [DfE]. (2013). *Mathematics programmes of study: Key stages 1 and 2. National curriculum in England*. DfE. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf

Fan, L., & Kaeley, G. S. (2000). The influence of textbooks on teaching strategies: An empirical study. *Mid-Western Educational Researcher*, *13*(4), 2–9.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: Development status and directions. *ZDM: The International Journal on Mathematics Education*, *45*(5), 633–646. https://doi.org/10.1007/s11858-013-0539-x

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, *16*(1), 3–17. https://doi.org/10.2307/748969

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work. Constructing multiplication and division.* Heinemann.

Foong, P. Y., Chang S.H., Lim L. G. P., & Wong O.H. (2014). *Primary mathematics 1B*. Shinglee.

Foong, P. Y., Chang S.H., Lim L. G. P., & Wong O.H. (2015). *Primary mathematics 3A*. Shinglee.

Foong, P. Y., Lim L. G. P., & Wong O. H. (2014). *Primary mathematics 2A*. Shinglee.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Aldine.

Glasnović Gracin, D. & Domović, V. (2009). Upotreba matematičkih udžbenika u nastavi viših razreda osnovne škole [The use of mathematics textbooks in teaching of upper primary grades]. *Odgojne znanosti*, *18*(2), 45–65. https://hrcak.srce.hr/48441

Glasnović Gracin, D., & Jukić Matić, Lj. (2016). The role of mathematics textbooks in lower secondary education in Croatia: An empirical study. *The Mathematics Educator 16*(2), 31–58.

Greer, B. (1992). Multiplication and division as models of situations. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 276–295). Macmillan.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what? *British Educational Research Journal*, *28*(4), 567–590. https://doi.org/10.1080/0141192022000005832

Hong, D. S., & Choi, K. M. (2014). A comparison of Korean and American secondary school textbooks: the case of quadratic equations. *Educational Studies in Mathematics, 85*(2), 241–263. https://doi.org/10.1007/s10649-013-9512-4

Isoda, M. (Ed.) (2010). *Elementary school teaching guide for the Japanese course of study mathematics (Grade 1-6)*. CRICED, University of Tsukuba. https://www.criced.tsukuba.ac.jp/math/apec/ICME12/Lesson_Study_set/Elementary_School_Teaching_Guide-Mathematics-EN.pdf

Jones, K., & Fujita, T. (2013). Interpretations of national curricula the case of geometry in textbooks from England and Japan. *ZDM Mathematics Education*, *45*(5), 671–683. https://doi.org/10.1007/s11858-013-0515-5

Jukić Matić, Lj. (2019). The teacher as a lesson designer. *Center for Educational Policy Studies Journal*, *9*(2), 139–160. Ttps://doi.org/10.26529/cepsj.722

Kheong, F. H., Ramakrishnan, C., & Choo, M. (2014). *My pals are here maths 2A*. MC Education.

Kheong, F. H., Ramakrishnan, C., & Choo, M. (2015). *My pals are here maths 3A*. MC Education

Kheong, F. H., Ramakrishnan, C., & Wah, B. L. P. (2013). *My pals are here maths 1B*. MC Education.

Kolar, V. M., Hodnik Čadež, T., & Vula, E. (2018). Primary teacher students' understanding of fraction representational knowledge in Slovenia and Kosovo. *Center for Educational Policy Studies Journal*, *8*(2), 71–96. https://doi.org/10.26529/cepsj.342

Kouba, V. L. (1989). Children's solution strategies for equivalent set multiplication and division word problems. *Journal for Research in Mathematics Education*, *20*(2), 147–158. https://doi.org/10.2307/749279

Markovac, J. (2015). *Matematika 2* [Mathematics 2]. Alfa.

Miklec, D., Prtajin, G., & Jakovljević Rogić, S. (2015). *Moj sretni broj 2 [My lucky number 2*]. Školska knjiga.

Ministarstvo znanosti obrazovanja i športa [MZOŠ]. (2006). *Nastavni plan i program za osnovnu školu. [Course of study for primary school].* MZOŠ.

Ministry of Education Singapore [MOE]. (2012). *Mathematics syllabus: Primary one to Six*. MOE. https://www.moe.gov.sg/-/media/files/primary/mathematics_syllabus_primary_1_to_6.pdf

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education*, *28*(3), 309. https://doi.org/10.2307/749783

Ministarstvo znanosti i obrazovanja [MZO]. (2019). Odluka o donošenju kurikuluma za nastavni predmet Matematike za osnovne škole i gimnazije u Republici Hrvatskoj [Decision of declaration of the curriculum of Mathematics for primary and secondary school]. *Narodne novine* 7/2019. https://narodne-novine.nn.hr/clanci/sluzbeni/2019_01_7_146.html

Ministarstvo znanosti obrazovanja i sporta [MZOS]. (2014). Konačne liste odabranih udžbenika i pripadajućih dopunskih nastavnih sredstava prema odabiru stručnih aktiva u osnovnim i srednjim školama [Final lists of selected textbooks and related supplementary teaching aids according to the selection of professional boards in primary and secondary schools]. https://mzo.gov.hr/vijesti/konacne-liste-odabranih-udzbenika-i-pripadajucih-dopunskih-nastavnih-sredstava-prema-odabiru-strucnih-aktiva-u-osnovnim-i-srednjim-skolama/1031

Nunes, T., & Bryant, P. (1996). *Children doing mathematics*. Blackwell.

Nunes, T., Bryant, P., Evans, D., & Bell, D. (2010). The scheme of correspondence and its role in children's mathematics. *British Journal of Educational Psychology*, *2*(7), 83–99. https://doi.org/10.1348/97818543370009X12583699332537

Park, J., & Nunes, T. (2001). The development of the concept of multiplication. *Cognitive Development*, *16*(3), 1–11. https://doi.org/10.1016/S0885-2014(01)00058-2

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *ZDM: The International Journal on Mathematics Education*, 33(5), 158–175. https://doi.org/10.1007/BF02656616

Piaget, J. (1965). *The child's conception of number*. Norton.

Polak, S., Cindrić, D., & Duvnjak, S. (2014). *Nove matematičke price 2 [New mathematical tales 2].* Profil.

Sansu Math 2. (2015). Koyo Publishing.

Sansu Math 3. (2015). Koyo Publishing.

Staneff, T. (Ed.) (2017). *Power maths 1C*. Pearson.

Staneff, T. (Ed.) (2017). *Power maths 2A*. Pearson.

Staneff, T. (Ed.) (2018). *Power maths 3A*. Pearson.

Staneff, T. (Ed.) (2018). *Power maths 3B*. Pearson.

Staneff, T. (Ed.) (2018). *Power maths 4A*. Pearson.

Steffe, L. (1994). Children's multiplying schemes. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 3–40). SUNY Press.

Stiegler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. The Free Press.

Stručna radna skupina [SRS]. (2016). *Nacionalni kurikulum nastavnog predmeta Matematika, Prijedlog* [The national curriculum of Mathematics, A proposal]. http://www.kurikulum.hr/wp-content/uploads/2016/03/Matematika.pdf

Stručna radna skupina [SRS]. (2016). *Odgovori na pristigle priloge stručnoj raspravi Nacionalni kurikulum nastavnoga predmeta Matematika* [Answers to the received contributions to the expert discussion of the national curriculum of Mathematics]. http://www.kurikulum.hr/wp-content/uploads/2016/06/komentari-stručna-rasprava-matematika.pdf

Tirosh, D., Tsamir, P., Barkai, R., & Levenson, E. (2018). Engaging young children with mathematical activities involving different representations: Triangles, patterns, and counting objects. *Center for Educational Policy Studies Journal*, *8*(2), 9–30. https://doi.org/10.26529/cepsj.271

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Kluwer.

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in middle grades* (pp.141–161). NCTM.

Yang, D. C., & Sianturi, I. A. (2017). An analysis of Singaporean versus Indonesian textbooks based on trigonometry content. *EURASIA Journal of Mathematics Science and Technology Education*, *13*(7), 3829–3848. https://doi.org/10.12973/eurasia.2017.00760a

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