

The covering lemma and q -analogues of extremal set theory problems

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Abstract

We prove a general lemma (inspired by a lemma of Holroyd and Talbot) about the connection of the largest cardinalities (or weight) of structures satisfying some hereditary property and substructures satisfying the same hereditary property. We use it to show how results concerning forbidden subposet problems in the Boolean poset imply analogous results in the poset of subspaces of a finite vector space. We also study generalized forbidden subposet problems in the poset of subspaces.

Keywords: Subspace lattice, forbidden subposet, covering, profile polytope.

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Krovna lema in q -analogije ekstremalnih problemov v teoriji množic

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Povzetek

Dokažemo splošno lemo (po vzoru leme Holroyda in Talbota) o zvezi med največjimi kardinalnostmi (ali utežmi) struktur, ki zadoščajo nekaterim hereditarnim lastnostim, in podstruktur z isto hereditarno lastnostjo. S pomočjo te leme pokažemo, da rezultati v zvezi s problemi prepovedanih delno urejenih podmnožic v Booleovi delno urejeni množici implicirajo analogne rezultate v delno urejeni množici podprostorov končnega vektorskega prostora. Raziskujemo tudi probleme posplošenih prepovedanih delno urejenih podmnožic v delno urejeni množici podprostorov.

Ključne besede: Mreža podprostorov, prepovedana delno urejena množica, pokritje, profilni politopi.

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