UNDRAINED SHEAR STRENGTH OF SATURATED COHESIVE SOILS DEPENDING ON CONSOLIDATION PRESSURE AND MINERALOGICAL PROPERTIES

BOJANA DOLINAR

About the author

Bojana Dolinar
University of Maribor,
Faculty of Civil Engineering,
Smetanova ulica 17, 2000 Maribor,
SI-Slovenia,
E-mail: bojana.dolinar@uni-mb.si

Abstract

It is well known that the relationship between the water content and the undrained shear strength of finely grained soils can be described with a non-linear function in which the type of soil is determined by two parameters. These parameters depend primarily on the size of clay minerals, their quantity in soil composition and the interlayer water quantity in expanding clay minerals. This article asserts that there exists the exactly defined relationship also between the water content and consolidation pressure. In the function describing this relationship, the type of soil is determined by two parameters. They can be expressed depending on the same mineralogical properties of soils as the values of parameters in the function showing the relationship between the water content and the undrained shear strength. These findings allow us to express the ratio between undrained shear strength and consolidation pressure depending on mineralogical properties of soils.

Keywords

clay, specific surface, undrained shear strength, compressibility

1   INTRODUCTION

Mechanical properties of cohesive soils depend on the quantity of contained water which, in turn, mostly depends on mineral compositions of soils. Few investigations have been undertaken about the above dependencies. The results of individual investigations are only known.

Koumoto [5] and Koumoto and Houlsby [6] found that the relationship between the undrained shear strength c_u and the water content w in soils could be expressed by the function

$$w = a\, c_u^{-b} \qquad (1)$$

where a and b are soil-dependent parameters. a [%] is the water content in soils at the undrained shear strength $c_u = 1$ kPa, and b is the slope of the linear function which represents the ratio between the logarithm of the water content w [%] and the logarithm of undrained shear strength c_u [kPa]. Parameters a and b can be obtained experimentally with a few measurements of water content at different undrained shear strength.

Dolinar [2] investigated the influence of mineralogical properties of soils on the values of both parameters in Eq. (1), water content and undrained shear strength. On the basis of theoretical and experimental tests, she established that the undrained shear strength only depends on the quantity of intergrain water, whilst interlayer water in expanding clay minerals, which is strongly bound between the layers of clay particles, can not influence it. To take into account this important fact, the equation (1) has been rewritten as follows

$$w_e = a_e\, c_u^{-b_e} \qquad (2)$$

where suffix e means that parameters a_e and b_e are determined from the intergrain water content w_e only. It was found that soil-dependent parameters could be calculated in this case by Eqs. (3) and (4), from the known portion p and specific surface A_S [m²/g] of soils

$$a_e = p\,\alpha + \beta\,A_S \qquad (3)$$

$$b_e = \gamma \cdot A_{SC}^{\lambda} \qquad (4)$$

where $\alpha = 33.70$, $\beta = 0.99$, $\gamma = 0.05$ and $\lambda = 0.27$. Note that α, β, γ and λ are equal for all cohesive soils. A_{SC} [m²/g] is the external surface per one gram of clay minerals, while A_S [m²/g] is the external surface per one gram of soil ($A_S = p\,A_{SC}$).

This article presents the results of the research based on the above findings whilst being their logical continuation. The experimentally determined inter-dependence of the consolidation pressure and intergrain water quantity of saturated soils is studied. It was established that this dependence could be expressed by the non-linear function in which the type of the soil was described by two parameters. It was also established that the values of these parameters could be calculated from the same mineralogical properties of clays as are the values of parameters in Eq. (2). These findings allow us to express the ratio between undrained shear strength of saturated clays and consolidation pressure depending on mineralogical properties of soils.

2 TESTED SOIL SAMPLES DATA

Three monomineral samples (Table 1) were used in tests: a well crystallized kaolinite (Sample 1), a poorly crystallized kaolinite (Sample 2) and Ca-montmorillonite (Sample 3). The investigated clays originate from the United States regions. Being intended for fundamental studies, these clays have a known mineral composition, structural formula of a unit cell and specific surface A_S (Tab. 1). The below properties of tested samples are taken from literature [8].

expanding mineral thus needed to be reduced.

3.1 THE CALCULATION OF INTERGRAIN WATER IN AN EXPANDING MINERAL

The montmorillonite structure consists of an octahedral sheet sandwiched between two silica sheets. The layers thus formed are continuous in the a and b directions and stacked one above the other in the c direction. Bonds between layers are weak, thus water or other polar molecules can enter the space between the unit layers causing the lattice to expand in the c direction. The basal spacing (spacing between the center of two neighboring layers) in the c direction which is $d_1 = 0.96$ nm for dry calcium montmorillonite (dried at 105^0 C) rises to $d_2 = 1.54$ nm after the adsorption of water. In case of a calcium exchangeable cation in montmorillonite, the adsorption of water between layers is completed already at very low water content and the basal spacing is then practically constant. The volume of interlayer water V_{wi} can thus be calculated in accordance with [3] using Eq. (5)

$$V_{wi} = \frac{A_{Si}(d_2 - d_1)}{2} = 0.1817 \text{ cm}^3/\text{g} \qquad (5)$$

where A_{Si} [m²/g] is the internal specific surface of montmorillonite. It was determined from the total specific

Table 1. Mineral composition of clays, specific surface and structural formula of the unit cell.

Mineral composition / portion of clay in sample	A_S [m²/g]	Structural formula of the unit cell
1 – Kaolinite / $p = 1$	10.05	$(Mg_{0.2}Ca_{0.1}Na_{0.1}K_{0.1})[Al_{3.86}Fe(III)_{0.2}Mn_{sl.}Ti_{0.11}][Si_{3.83}Al_{0.17}]O_{10}(OH)_8$
2 – Kaolinite / $p = 1$	23.50	$(Ca_{sl.}K_{sl.})[Al_{3.66}Fe(III)_{0.07}Mn_{sl.}Mg_{sl.}Ti_{0.16}][Si_{4.0}]O_{10}(OH)_8$
3 – Montmorillonite / $p = 1$	97.42	$(Ca_{0.39}Na_{0.36}K_{0.2})[Al_{2.71}Mg_{1.11}Fe(III)_{0.12}Mn_{0.1}Ti_{0.03}][Si_{8.0}]O_{20}(OH)_4$

3 METHODS AND RESULTS OF LABORATORY TESTS

The water content w, which is determined by drying soils at the temperature of 105° C, equals the intergrain water content w_e of non-expanding clay minerals (Samples 1 and 2). With the montmorillonite clay, the interlayer water w_i, which is bonded to internal grain surfaces, also appears besides the intergrain water. It is known that the interlayer water cannot be drained from an expanding mineral at usual stresses, which leads to the conclusion that the intergrain water quantity and consolidation pressure can only be compared. The total water content for the interlayer water portion in the

surface of one gram of clay A_{St} reduced for the value of the measured external specific surface A_S (Eq.(6)).

$$A_{Si} = A_{St} - A_S = 626.80 \text{ m}^2/\text{g} \qquad (6)$$

The total specific surface A_{St} was calculated in accordance with [9] by Eq. (7)

$$A_{St} = \frac{1}{m} N_A \, 2 \, s_w \, s_l = 724.22 \text{ m}^2/\text{g} \qquad (7)$$

where $m = 762$ is the calculated relative molecular weight of the unit cell, $N_A = 6.022 \times 10^{23}$/mol is Avogadro's number, $s_w = 0.515$ nm is the width of the unit cell and $s_l = 0.89$ nm is the length of the unit cell.

3.2 THE RELATIONSHIP BETWEEN THE INTERGRAIN WATER CONTENT AND CONSOLIDATION PRESSURE

The quantity of intergrain water in saturated clays was measured at different effective stresses by an oedometer consolidation test in accordance with BS standard [1]. The initial moisture content in the samples was near the liquid limit. The void ratio e and the corresponding water content w after consolidation of the clays were determined at axial stress $\sigma' = 50$ kPa, 100 kPa and 200 kPa. The total water content w was decreased for the portion of interlayer water w_i ($w_e = w - w_i$) in order to determine the intergrain water content w_e in the expanding mineral (Table 2).

It is established that the relationship between the void ratio e and vertical stress σ' is approximately linear for normally consolidated saturated soils when using a semi-logarithmic scale (e, $\log \sigma'$). In this case, the
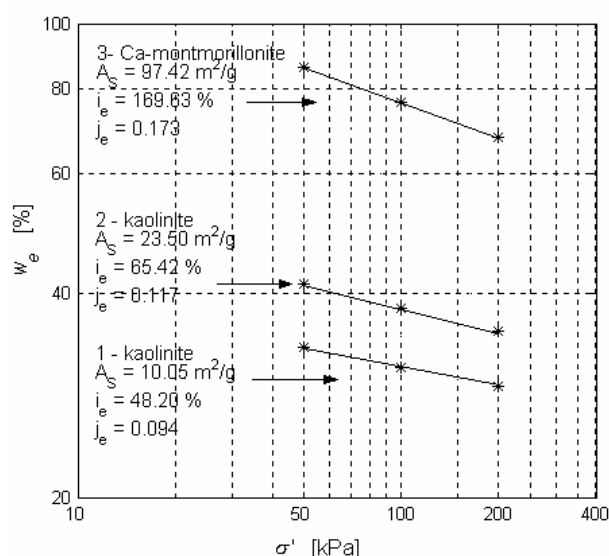

Figure 1. Intergrain water content w_e as a function of effective stress σ'.

Table 2. The relative water content w before the test, specific weight γ_s , void ratio e and intergrain water content w_e after consolidation at different axial stresses σ', and soil dependent parameters i_e and j_e .

	Sample								
	1 - kaolinite			2 – kaolinite			3 – montmorillonite		
w [%]	39.60			54.56			120.00		
γ_s	2.58			2.50			2.30		
σ' [kPa]	50	100	200	50	100	200	50	100	200
w_e [%]	33.29	31.15	29.19	41.32	38.09	35.12	86.25	76.62	67.88
e	0.859	0.804	0.753	1.033	0.952	0.878	1.984	1.762	1.561
i_e [%]	48.20			65.42			169.63		
j_e	0.094			0.117			0.172		

slope of the straight line is the compression index C_c. A complete linear relationship, however, was only established when both variables were shown on a double logarithmic scale (Fig.1). The inter-dependence of the quantity of intergrain water $w_e = (e/\gamma_s)$ [%] and the effective stress σ' [kPa] can thus be expressed using the function

$$\log w_e = \log i_e - j_e \log \sigma' \implies w_e = i_e \, \sigma'^{-j_e} \qquad (8)$$

where i_e (the water content in the soil at $\sigma' = 1$ kPa) and j_e (the slope of the linear function) are soil dependent parameters (Table 2). It is evident from Fig. 1 that parameters i_e and j_e increase with the increase of the specific surface A_S of clays.

3.3 THE RELATIONSHIP BETWEEN PARAMETERS i_e AND j_e AND MINERALOGICAL PROPERTIES OF CLAYS

Parameters i_e and j_e vary with the type of soil, yet they can also be influenced by temperature, the structure of the soil, organic additions and chemical composition of the pore water. To adjust test conditions as much as possible, laboratory tests were performed at the temperature of 20° C using distilled water on samples without organic matter having a parallel arrangement of clay particles.

In accordance with previous findings [2], it was expected that parameters i_e and j_e depended on the size and quantity of clay grains in samples.

It is evident from Fig. 2 that the relationship between parameter i_e [%] and the specific surface A_S [m²/g] is linear.

This can be shown by the expression

$$i_e = p\,\eta + \kappa A_S \qquad (9)$$

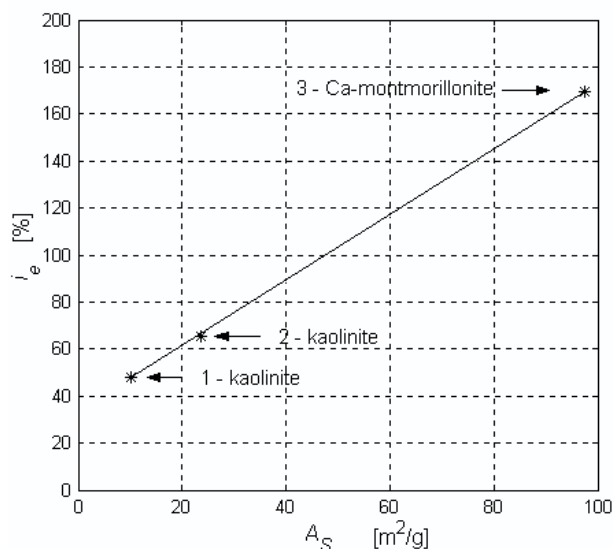where $\eta = 33.46$ and $\kappa = 1.39$. Note that the weight portion of clay minerals in tested samples was $p = 1$.

Figure 2. Parameter i_e as a function of external specific surface A_S .

It was established that parameter j_e totally equaled parameter b_e in Eq. (4) and could therefore be expressed by the function

$$j_e = b_e = \gamma \cdot A_{SC}{}^{\lambda} \qquad (10)$$

where $\gamma = 0.05$ and $\lambda = 0.27$.

3.4 APPLICATION OF THE RESULTS TO NATURAL SOILS

The calculation of parameters i_e and j_e from mineralogical properties was based on test results obtained on monomineral clays. To adopt this criterion to practical applications, it was however necessary to investigate the validity of the test results for heterogeneous soils. Five samples of natural soils were chosen for this purpose.

The quantity of intergrain water in saturated clays was measured at different effective stresses by an oedometer consolidation test in accordance with BS standard [1]. The mineralogical composition of the soils was analyzed using an X-ray diffractometer (Philips 1820) with Cu-Kα radiation. Powdered samples were used to determine the bulk sample composition (Table 3). Clay minerals were characterized with the help of oriented clay mineral aggregates (Tables 4 and 5). The results of chemical analyses were used to determine the quantity of individual minerals in the soils. The external surface area was measured by the five-point BET method with N_2 (Table 3). The obtained results allowed a comparison between the experimentally determined parameters i_e and j_e and those that were calculated from the mineralogical properties of samples by Eqs. (9) and (10).

Table 3. Mineral composition of the powder bulk samples.

Mineral composition [%]	Sample				
	1	2	3	4	5
Illite + MLC illite/montmorillonite	25	35	28	35	34
Chlorite + MLC chlorite/montmorillonite	8	14	16	0	18
Kaolinite	5	0	0	12	0
Ca-montmorillonite	14	0	0	34	0
Quartz	34	25	42	18	43
Plagioclase	9	3	9	0	3
Microcline	5	0	4	0	3
Calcite	0	22	0	0	0
Goethite/hematite	0	1	0	1	0
Specific surface A_S [m²/g]	30.1	28.5	16.7	54.1	32.6

Note: MLC – mixed layer clays

Table 4. Mineral composition of the clay fraction < 0.002 mm.

Sample	Illite, MLC illite / Ca-montmorillonite, Ca-montmorillonite [%]								
		R1	R1	R3/1	R1	R0	R0	R0	R0	
	ILL	I/ΣI1	I/ΣI2	I/ΣI3	I/ΣI4	I/ΣI5	I/ΣI6	I/ΣI7	I/ΣI8	Ca-M
1	4	0	0	0	60/2	0	0	23/2	0	6
2	7	0	82/6	0	0	55/5	0	0	0	2
3	2	0	0	78/1	0	0	42/1	0	0	1
4	6	0	0	78/4	0	0	42/3	0	0	16
5	7	84/5	0	78/4	0	0	42/3	0	15/2	0

LEGEND

R0, R1, R3/1	ordering of layers in MLC	I	percent of illite in MLC illite/Ca-montmorillonite
ILL	Illite		
Ca-M	Ca-montmorillonite	ΣI1-ΣI8	percent of MLC illite/Ca-montmorillonite in sample

Table 5. Mineral composition of the clay fraction < 0.002mm.

Sample	Kaolinite, MLC kaolinite/Ca-montmorillonite, MLC chlorite/Ca-montmorillonite [%]						
			R1	R1	R1	R1	
	Ka	Ch	K/ΣKα1	C/ΣC1	C/ΣC2	C/ΣC3	< 0.002 mm
1	1	0	0	0	0	0	14.54
2	3	2	0	0	68/1	0	26.21
3	0	3	0	76/2	0	63/2	12.84
4	2	0	90/2	0	0	0	33.02
5	4	0	0	0	0	0	24.96

LEGEND

R1	ordering of layers in MLC	C	percent of chlorite in MLC chlorite/Ca-montmorillonite
Ka	Kaolinite		
Ch	Chlorite	ΣKα1	percent of MLC kaolinite/Ca-montmorillonite in fraction < 0.002 mm
K	percent of kaolinite in MLC kaolinite/Ca-montmorillonite	ΣC1-ΣC3	percent of MLC chlorite/Ca-montmorillonite in sample

In determining parameters i_e and j_e experimentally, it was necessary to consider only the intergrain water content in the soil. The interlayer water in the tested samples, however, resulted from the presence of Ca-montmorillonite. Its quantity was calculated by Eq. (5) considering the internal specific surface of montmorillonite grains $A_{Si} = 626.80$ m^2/g and adequate mass portions of this mineral in individual soils.

The research results are given in Table 6. It is evident that parameters i_e and j_e and the quantity of water after consolidation at different vertical stresses calculated from the results of a mineralogical analysis are almost equal to the experimentally obtained values. Minor differences probably appear due to the presence of organic substances in the soils (1–3 %).

3.5 UNDRAINED SHEAR STRENGTH TO CONSOLIDATION PRESSURE RATIO

Substituting the quantity of intergrain water w_e in Eq. (2) by Eq. (8), which determines the relationship between the quantity of intergrain water w_e and effective stress σ', and considering that parameter $j_e = b_e$, we obtain Eq. (11).

$$c_u = {}^{b_e}\!\sqrt{\frac{a_e}{i_e}}\ \sigma' \qquad (11)$$

Table 6. Specific surface A_S and portion p of clay minerals in soil composition, water content w before and after the test, calculated interlayer water quantity w_i, measured and calculated intergrain water quantity w_e after consolidation at different axial stresses σ', and calculated and experimentally determined soil dependent parameters i_e and j_e.

	Sample				
	1	2	3	4	5
A_S [m²/g]	30.1	28.5	16.7	54.1	32.6
p	0.52	0.49	0.44	0.89	0.52
w [%] before the test					
$\sigma' = 50$ kPa	49.36	45.92	31.20	70.13	37.77
$\sigma' = 100$ kPa	49.36	44.43	30.38	69.35	37.70
$\sigma' = 200$ kPa	44.99	44.05	27.95	66.22	35.25
w [%] after the test					
$\sigma' = 50$ kPa	37.47	36.76	25.82	66.04	34.72
$\sigma' = 100$ kPa	33.67	34.23	22.56	60.12	31.51
$\sigma' = 200$ kPa	30.85	32.68	23.42	55.16	27.87
w_i [%]	4.05	1.02	0.55	8.14	0.93
$w_e = w - w_i$ [%] measured					
$\sigma' = 50$ kPa	33.42	35.74	25.27	57.90	33.79
$\sigma' = 100$ kPa	29.62	33.21	22.01	51.98	30.58
$\sigma' = 200$ kPa	26.80	31.66	22.87	47.02	26.94
$w_e = i_e\, \sigma'^{-j_e}$ [%] calculated					
$\sigma' = 50$ kPa	33.06	31.26	22.54	57.92	34.60
$\sigma' = 100$ kPa	29.82	28.20	20.50	52.13	31.14
$\sigma' = 200$ kPa	26.89	25.43	18.74	46.91	28.02
i_e [%] calculated	59.23	56.01	37.93	104.98	62.71
i_e [%] measured	62.10	55.14	39.60	104.04	64.26
j_e calculated	0.149	0.149	0.133	0.152	0.152
j_e measured	0.159	0.111	0.115	0.150	0.163

Due to the known relationships between the soil-dependent parameters and the mineralogical properties of clays, the relationship c_u/σ' can be expressed by Eq. (12).

$$\frac{c_u}{\sigma'} = {}^{(0.05\,A_{SC}{}^{0.27})}\sqrt{\frac{33.70 + 0.99 A_S}{33.46 + 1.39 A_S}} \qquad (12)$$

4 CONCLUSION

The article illustrates the interdependence of the quantity of intergrain water in saturated clays and consolidation pressure. It has been established that this relationship can be described by the non-linear function

(8), where i_e and j_e are soil-dependent parameters. According to previous findings, the parameters can be expressed depending on the quantity and size of clay minerals. These relations are shown by Eqs. (9) and (10).

The known relationships between the intergrain water content, undrained shear strength, consolidation pressure and the mineralogical properties of soils allow us to express the c_u/σ' ratio depending on the specific surface and the quantity of clay minerals in soil composition as shown by Eq. (12).

REFERENCES

[1] British Standards Institution (1990). *Methods of test for soils for civil engineering purposes.* BS 1377, London.

[2] Dolinar, B. (2004). *Physical properties of saturated cohesive soils depending on their mineralogical composition.* PhD Thesis, University of Maribor, Faculty of Civil Engineering.

[3] Fink, D.H. and Nakayama, F.S. (1972). Equation for describing the free swelling of montmorillonite in water. *Soil Science*, 114, No. 5, 355-358.

[4] Fripiat, J.J., Letellier, M. and Levitz, P. (1984). Interaction of water with clay surfaces. *Philosophical Transactions of the Royal Society of London*, A311, 287-299.

[5] Koumoto, T. (1990). Determination of the both liquid and plastic limits of clay by the fall cone test. *J. Jpn Soc. Irrigation, Drainage and Reclamation Engng*, 146, 95-100.

[6] Koumoto, T. and Houlsby, G.T. (2001). Theory and practice of the fall cone test. *Geotechnique*, LI, No. 8, 701-712.

[7] Mitchell, J.K. (1993). *Fundamentals of Soil Behaviour.* John Wiley & Sons, New York.

[8] Van Olphen, H. and Fripiat, J.J. (1979). *Data handbook for clay minerals and other non-metallic materials.* Pergamon press.

[9] Van Olphen, H. (1977). *An Introduction to Clay Colloid Chemistry*. 2nd ed., Wiley Interscience, New York.