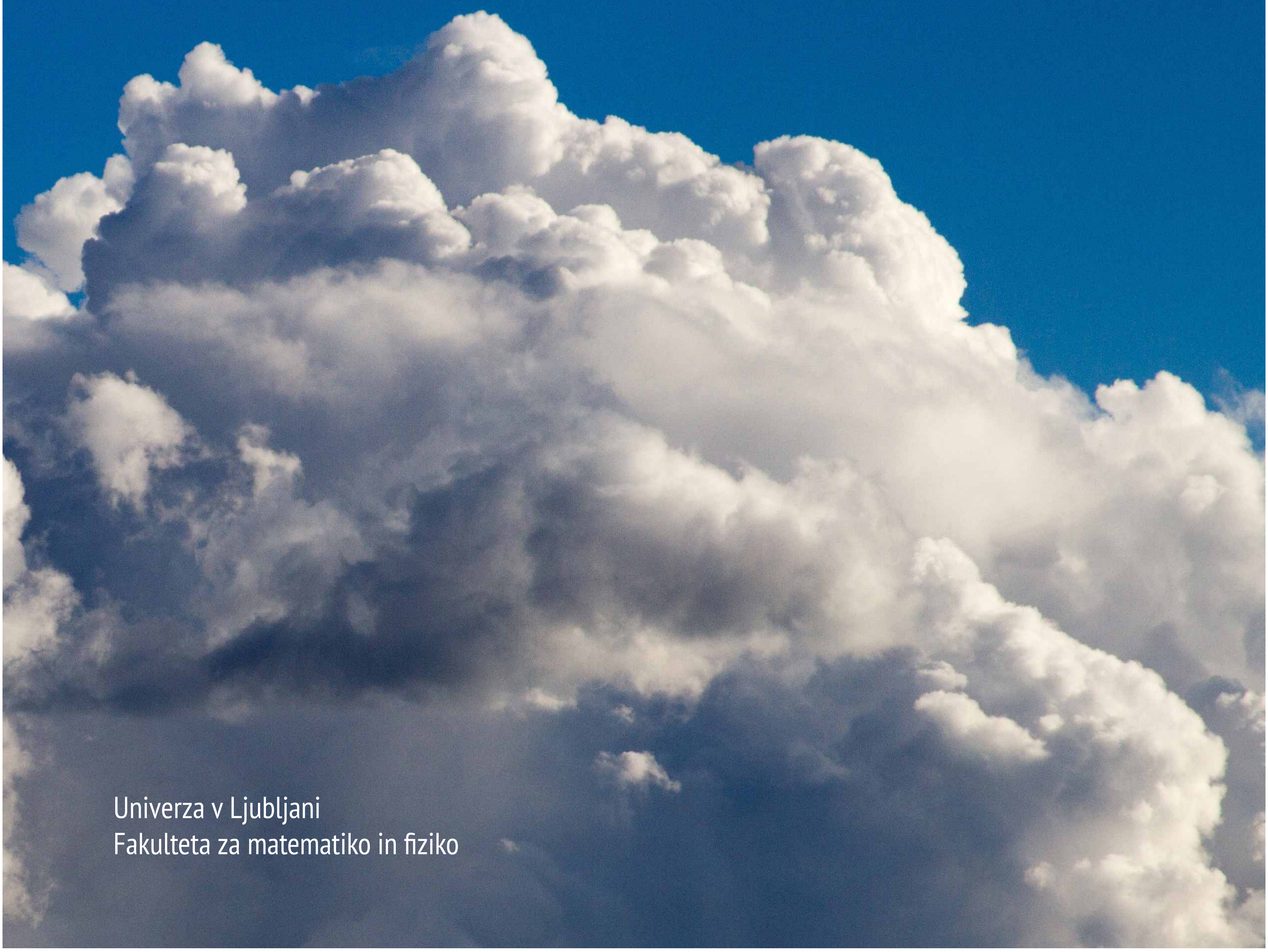
Gregor Skok

Introduction to meteorology



Univerza v Ljubljani
Fakulteta za matematiko in fiziko

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INTRODUCTION TO METEOROLOGY

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Contents

1 Introduction

What happens in the atmosphere has a great impact on the Earth's surface, life on it, and human society. The geophysical science that deals with processes and phenomena in the atmosphere, and the phenomena that depend on them, is called ***meteorology*** or ***atmospheric science***. The processes that occur in the atmosphere faithfully follow the basic laws of physics, which makes understanding them essential for coming to grips with important weather and climate phenomena like clouds, winds, thunderstorms, and climate change.

A ***meteorologist*** is an expert who holds special knowledge about the atmosphere and weather. A meteorologist uses scientific approaches to observe, understand, explain, and predict weather phenomena in the Earth's atmosphere and their impact on Earth. In Slovenia, a person can acquire the knowledge needed to become a meteorologist by studying at the Faculty of Mathematics and Physics of the University of Ljubljana. The study of meteorology is closely related to the study of physics, with which it shares the fundamental courses in physics and mathematics. In the higher years, the knowledge acquired is used to describe what is happening in the atmosphere and to deal with more complex topics like the numerical modeling and forecasting of atmospheric processes.

Not only meteorology students at the Faculty of Mathematics and Physics, but students at some other faculties, for whom basic knowledge of meteorology is also important, have introductory courses in meteorology. This textbook is primarily intended for all of these students, and its content enables a basic understanding of the most important processes in the atmosphere and how weather forecasts are prepared. The content and sequence of the sections in the textbook correspond well with the lectures in the mentioned courses. The textbook also contains many practically-themed problem-solving exercises with solutions provided, making the theoretical concepts more understandable and practically useful. Additional problem-solving exercises are in the booklet *Introduction to meteorology: solved problems* by Saša Gaberšek, Gregor Skok, and Rahela Žabkar [1], which is available online for free.

A high-school level of knowledge of physics and mathematics is generally sufficient to understand the textbook's content, while for those a little more curious some of the more complex derivations are provided in the Appendixes found at the end of the textbook. The treatment of meteorology at a slightly higher physics-mathematical level is available in the excellent textbook in Slovenian *Osnove meteorologije za naravoslovce in tehnike* written by Prof. Jože Rakovec and Assist. Prof. Tomaž Vrhovec [2]. Their textbook's content is also wider and addresses topics not mentioned here (e.g., electrical and optical phenomena, processes related to snow cover, and typical weather in Slovenia). There is another textbook on meteorology in Slovenian that is a little older, *Meteorologija: osnove in nekatere aplikacije* written by Prof. Andrej Hočevar and Prof. Zdravko Petkovšek [3], last reprinted in 1995. The textbook in English *Atmospheric Science: An Introductory Survey* by Wallace and Hobbs [4] provides even more extensive and in-depth introductory knowledge of meteorology.

This is an English version of the textbook in Slovenian *Uvod v meteorologijo* [5] first published in 2020. The English version is chiefly intended for international students who are attending the courses mentioned above but do not understand the Slovenian language. It is also aimed for readers from the general public who are curious about our atmosphere and the weather and may find the content interesting or useful.

The author wishes to thank Prof. Jože Rakovec and Dr. Žiga Zaplotnik for their thorough review of the Slovenian version of this book and the many helpful suggestions that helped improve it in both professional and pedagogical terms.

2 Composition of the Atmosphere

The ***atmosphere*** is a gaseous envelope that surrounds the Earth. Compared to the size of the planet, it is very thin – the Earth's radius measures about 6400 km, while 99% of the atmosphere's mass is found in the lowest 30 km.

The atmosphere is made up of a mixture of gases. Its composition has changed during the planet's history, with the current state being shown in Table 1. By far most of the atmosphere is composed of ***nitrogen*** (N_2), which together with ***oxygen*** (O_2) and ***argon*** (Ar) account for around 99.6% of its mass. Nitrogen, oxygen, and argon are examples of permanent gases that receive their name because they are well mixed in the atmosphere and their mass fraction is not altered significantly by location and time.

Yet, the same is not true for variable gases, whose amount can vary enormously. Of these, ***water vapor*** (H_2O) is very important, and its mass fraction can vary spatially and temporally considerably. For example, above the surface of a warm tropical ocean, its mass fraction may be 3% whereas in cold regions of the atmosphere it can reach

Gas	Symbol	Mass fraction
	Permanent gases	
Nitrogen	N_2	75.3%
Oxygen	O_2	23.0%
Argon	Ar	1.3%
	Variable gases	
Water vapor	H_2O	0.33%
Carbon dioxide	CO_2	0.05%
Methane	CH_4	0.0001%
Nitrous oxide	N_2O	0.000 05%
Ozone	O_3	0.000 06%
	Particles	
Hydrometeors	H_2O	
Aerosol	various	

Table 1: List and mass fractions of important gases and particles in the atmosphere

just 0.0006%. In general, water vapor is most abundant closest to the surface since its biggest source is the oceans, from which water evaporates. Water vapor is also the only gas in the atmosphere that frequently exists in other states of matter (liquid and solid) and is therefore able to form clouds and precipitation. In addition, water vapor is the strongest greenhouse gas. Greenhouse gases are able to absorb part of the infrared radiation emanating from the ground and cause temperatures closer to the surface of the Earth to be higher than they otherwise would be (for more on greenhouse gases and the greenhouse effect, see Section 33). ***Carbon dioxide*** (CO_2) is also a greenhouse gas and its concentration also varies significantly according to the time of year and location. Still, the amount of it in the atmosphere has greatly increased in the last century due to human use of fossil fuels. The result is a more intense greenhouse effect that is responsible for climate change and global warming (for more on this, see Section 34). The same is true for ***methane*** (CH_4), but since there is considerably less of it in the atmosphere its impact on global warming is less than that of carbon dioxide. The same applies to ***nitrous oxide*** (N_2O), which has an even smaller impact on global warming.

A special role is played by ***ozone*** (O_3) that possesses the crucial ability to intercept some of the Sun's harmful ultraviolet radiation that would otherwise reach the ground. Most of the ozone is located in the stratosphere (for more on the characteristic layers in the atmosphere, see Section 3) with the highest concentrations found at altitudes around 20-25 km. Ozone in the stratosphere is informally known as the ***ozone shield***. In the past, mainly due to human emissions of CFCs (chlorofluorocarbons), the amount of ozone in the atmosphere started to decrease. The quantity of ozone has decreased especially in the areas above Antarctica, what we colloquially call the ***ozone hole***. In 1987, the ***Montreal Agreement*** was signed, which banned the use of ozone-depleting substances. Although the agreement has led to the slow recovery of the ozone layer, it will take several more decades before the amount of ozone reaches its original level. Ozone is also formed near the ground, especially in summer, due to photochemical reactions associated with pollution emitted by traffic and industry. While stratospheric ozone is very beneficial for life on Earth, ground-level ozone is detrimental to the health of humans, plants, and animals.

In addition to gases, particles exist in the atmosphere. These can be divided into two main types. The first are liquid or solid particles mostly composed of water called ***hydrometeors***. These can be liquid water droplets, solid ice particles, or a mix of the two (e.g., melting snow). These particles form clouds, fog, and precipitation. All other liquid and solid particles in the atmosphere are usually called ***aerosol***. These tend to be microscopic particles that cannot be seen by the naked eye and include small crystals of sea salt, mineral dust, smog, and particles of biological origin. They come into the atmosphere with volcanic eruptions, wildfires, and human activities, or the wind lifts them off the ground. These particles are so small that they fall very slowly through the air and can stay in the atmosphere for a remarkably long time. Their concentration depends on location and time, although they are present everywhere in the atmosphere and play an essential role in the formation of clouds and precipitation (for more on this, see Section 16) and also absorb some of the Sun's and Earth's radiation.

3 Characteristic Layers of the Atmosphere

Depending on how temperature changes with altitude, the atmosphere can be divided into several characteristic layers, as shown in Figure 1. Closest to the ground is the ***troposphere***, which contains about 80% of the mass of the atmosphere and in which the vast majority of weather phenomena occur. In the troposphere, the temperature typically decreases with altitude, with the average surface air temperature being about 15 °C and at the top of the troposphere about −60 °C. The troposphere is about 10 km thick in midlatitudes. In the polar regions it is somewhat thinner, while in the tropics it is significantly thicker (≈ 17 km). The thickness of the troposphere also varies with the seasons and is generally bigger in warmer periods.

Above the troposphere lies the ***stratosphere***, with the boundary between the troposphere and the stratosphere being called the ***tropopause***. The stratosphere is a very stable layer and acts as a "lid" over the troposphere, preventing weather phenomena penetrating from the troposphere into the atmosphere's higher layers. In the lower part of the stratosphere, immediately above the tropopause, there is an isothermal layer in which the temperature does not change significantly with altitude. Above this layer, however, the temperature begins to increase with the altitude, and at the top of the stratosphere, at an altitude of about 50 km, the temperature is about 0 °C. There is ozone in the stratosphere. Since ozone and oxygen can absorb ultraviolet solar radiation, the energy of this radiation is used to heat the air – this is also why the temperature increases with the altitude in the stratosphere.

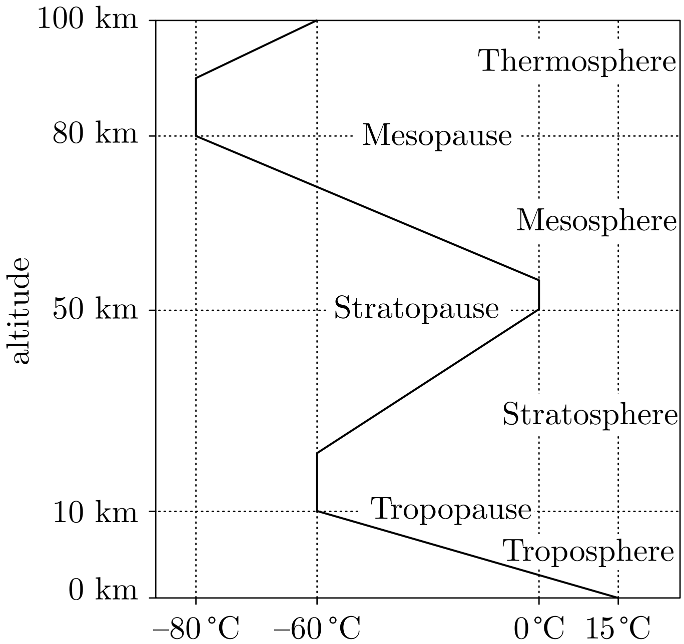


Figure 1: An idealized air temperature profile above the Earth's surface showing the characteristic layers of the atmosphere

Even higher are ***mesosphere*** and ***thermosphere***, with the boundary between the stratosphere and mesosphere called ***stratopause*** and the boundary between the mesosphere and thermosphere called ***mesopause***. In the mesosphere, the temperature decreases with the altitude and reaches its lowest value of approximately −80 °C at the mesopause (altitude around 80 km). In the thermosphere, the temperature again rises due to nitrogen and oxygen absorbing solar radiation of shorter wavelengths.

4 International Standard Atmosphere

The International Standard Atmosphere (ISA) is an international standard (ISO 2533: 1975) that defines a simplified state of the atmosphere. The ISA assumes that at sea level the atmospheric pressure is 1013.25 hPa, the temperature 15 °C, and the air density 1.225 kg/m^3. The ISA also assumes a simple linear change of temperature with the altitude. In the troposphere, defined by the ISA as the layer of air from the surface of the Earth to an altitude of 11 km, the temperature decreases with the altitude by 6.5 K/km. In the lower part of the stratosphere between 11 and 20 km, the ISA assumes a constant temperature of −56.5 °C. The ISA temperature profile is displayed in Figure 2.

The ISA reflects the typical state of the atmosphere in midlatitudes, yet the actual state of the atmosphere almost always deviates from the ISA. Atmospheric pressure at sea level can deviate considerably from the value defined by the ISA, while the atmosphere can also be much warmer or colder, and the change of temperature with the altitude is almost never completely linear.

The ISA is often used as a reference while testing the performance of various equipment and machinery at different altitudes and is also used in aviation. Namely, aircraft altimeters estimate the altitude based on the atmospheric pressure measured on the aircraft. When

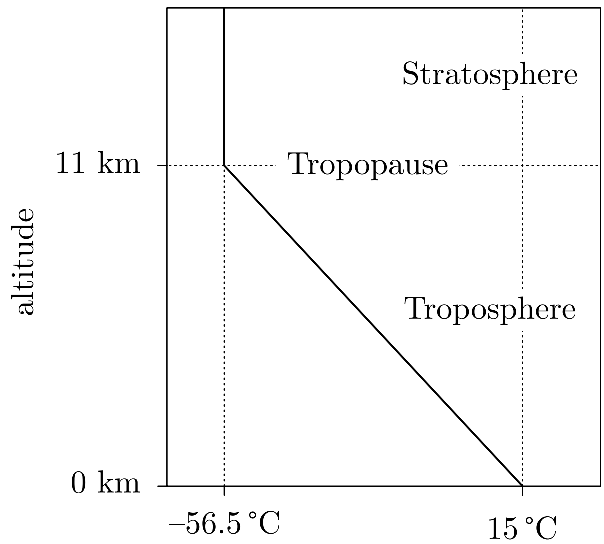


Figure 2: Vertical temperature profile defined by the International Standard Atmosphere.

converting pressure to the appropriate altitude, altimeters must assume certain atmospheric conditions (e.g., pressure at sea level and the vertical temperature profile), and aircraft altimeters assume conditions defined by the ISA (for more on estimating the altitude from the atmospheric pressure measurements, see Section 9).

5 Meteorological Variables, Weather, and Climate

Meteorological variables are physical quantities that describe the state of the air. Air is a mixture of gases whose state can be defined quite well with only a few quantities. The most important meteorological variables are atmospheric pressure, temperature, air density, and wind. Humidity is also very important, but will be described in greater detail in Section 7. If the values of all meteorological variables are known for a location, then the state of the atmosphere there is known. A weather forecast is essentially a forecast of the values of meteorological variables for the future. Knowing the values of the meteorological variables for tomorrow means that we can determine what the weather will be like tomorrow.

Meteorological variables describe the state of the air on a macroscopic scale (e.g., 1 m or more). For better understanding, it is good to highlight the connection between the variables and what is happening on a molecular scale. Air is made up of molecules of different gases. In these gases, individual molecules move freely around in space, colliding, and bouncing with each other (Figure 3). Individual molecules are really small and very numerous. Near the ground, where the air density is about 1.2 kg/m^3, there are about $2.5 \cdot 10^{19}$ molecules in every cubic centimeter of air.

The ***density of air*** (ρ) defines the total mass of molecules located within a certain volume. The basic unit for density is kg/m^3.

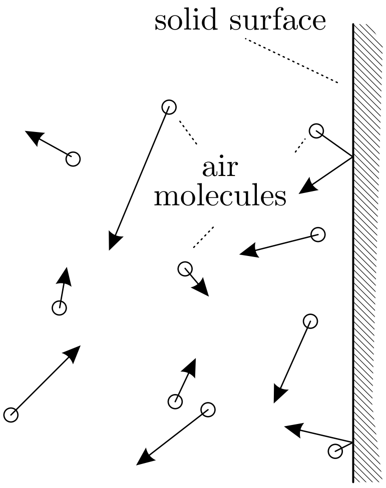


Figure 3: The motion of air molecules. The arrows show the direction of movement of individual molecules. These move freely around, colliding, and bouncing with each other and bouncing off the surfaces of solids and liquids.

Temperature (T) is related to the speed at which molecules move. The higher the temperature, the faster the movement. The speed of individual molecules is very high (e.g., the average speed of nitrogen molecules at 15 °C is approximately 450 m/s), but not all molecules move at the same speed. The speed of individual molecules can be quite different and change very quickly. If a molecule has an above-average speed, this may change the next time it collides with another molecule. Despite the relatively large variability of the speed of individual molecules, the average speed of the molecules is well defined and does not alter if the temperature is constant. A higher temperature means a higher average speed. The basic unit for temperature is Kelvin (K), with degrees Celsius (°C) also being used frequently.

In air that is still, molecules move in all directions with equal frequency. ***Wind*** ($\vec{v}$) occurs when one direction of molecular motion is more frequent than others. This appears on a macroscopic scale as the movement or flow of air in a certain direction, which we call wind. The wind speed in the atmosphere is always much lower than the speed of individual molecules. The highest wind speeds are found along the boundary between the troposphere and stratosphere, where jet streams frequently occur. There, the wind reaches speeds of up to about 100 m/s, which is still much less than the speed of individual molecules. Wind can be described by a three-dimensional velocity vector pointing in the direction of the movement of air, and its length represents the speed of motion. The basic unit for wind velocity is m/s. Usually, only horizontal wind is reported and measured, as there is much less vertical movement of air in the atmosphere than horizontal (although the movement in the vertical direction is essential for some weather phenomena like clouds, precipitation, and thunderstorms). For horizontal winds, the direction and speed are typically given separately. The speed is usually stated in units of m/s, while the wind direction always refers to the direction from which the wind blows. Generally, the cardinal points of the compass are used to denote the direction (e.g., northerly wind (N), northeasterly wind (NE), easterly wind (E), etc.). Thus, a westerly wind blows from west to east and a southeasterly one from southeast to northwest. The wind direction can also be given via an azimuth in angular degrees (e.g., wind with an azimuth of 90° blows from east to west).

Not only do they collide with each other, but molecules also collide and rebound off the surfaces of solids and liquids. Each collision of an individual molecule with a surface means the molecule exerts a small force on the surface. So many collisions with the surface appear on the macroscopic scale as a constant force that acts perpendicular to the surface, assuming the air is not moving. ***Atmospheric pressure*** (p) is defined as the magnitude of this force per unit area of surface, where the surface can also be imaginary (pressure is also defined when there is no real surface with which the molecules can collide). Atmospheric pressure has a basic unit of pascal (Pa), with hectopascals (hPa) and millibars (mbar) also often being used in meteorology. One hectopascal equals to one millibar, with 1 hPa = 100 Pa = 1 mbar. If the volume of air does not change, then an increase in density or temperature leads to an increase in atmospheric pressure. As the density rises, the number of molecules colliding against the surface rises, increasing the force felt by the surface (which can also be imaginary). As the temperature grows, the velocity of the molecules increases, which therefore hit the surface more forcefully, which also leads to an increase in the force on the surface.

Weather is the state of the atmosphere above a certain geographical area at a particular time. It can be described by the set of values of meteorological variables an area. ***Climate*** represents the characteristics of the weather over a longer time period (usually 30 years). Time-averaged or maximum/minimum values of meteorological variables and their variability are typically used. These characteristics define different types of climates, such as desert, alpine, continental, and Mediterranean.

6 The Ideal Gas Law

Gases in the atmosphere approximate ideal gases quite well. The ideal gas law applies to each individual gas, yet it can also be used for a mixture of ideal gases. For air, we can write the ideal gas law as

$$pV = \frac{m}{M_\mathrm{z}} R^* T, \tag{1}$$

where m is the mass of air in volume V, M_z is the molar mass of air (approximately 29 kg/kmol), and R^* is the universal gas constant with the value 8314 J/(kmol K). The derivation of Equation 1 is shown in Appendix A.1. A slightly different form of the ideal gas equation is often used in meteorology:

$$p = \rho R T. \tag{2}$$

The equation 2 is obtained from the equation 1 via division by V, using the relation $\rho = m/V$ and defining a gas constant for dry air of $R = R^*/M_\mathrm{z} = 287$ J/(kg K). The ideal gas equation connects the three basic meteorological variables (p, ρ, and T). If we know the values of two variables, we can always calculate the value of the third one.

Problem 1: Determine the mass of air in a classroom with dimensions 10 m $\times$ 8 m $\times$ 4 m, if the temperature and atmospheric pressure are 20 °C and 1000 hPa.

Solution: One needs to use the ideal gas law and express the density

$$\rho = \frac{p}{RT} = \frac{10^5 \text{ Pa}}{287 \text{ J/(kg K)} \cdot 293 \text{ K}} = 1.18 \text{ kg/m}^3.$$

The mass of air can be calculated from the density

$$m = \rho V = 1.18 \text{ kg/m}^3 \cdot 320 \text{ m}^3 = 381 \text{ kg}.$$

7 Humidity

Water vapor is a gaseous state of water. Water plays a special role in the atmosphere since it is the only gas that exists in larger quantities in other states of matter (liquid and solid) and thus forms clouds and precipitation. Phase changes of water are very common in the atmosphere. Any phase change is associated with a significant amount of energy that is either consumed or released, which can substantially affect the development of certain weather phenomena. For example, without water vapor, there would be no thunderstorms. Figure 4 shows the phase transitions of water and its phase diagram.

Different physical quantities can describe the amount of water vapor in the air. One of them is ***water vapor density*** (ρ_v), also called ***absolute humidity***, which represents the total mass of all water vapor molecules in a given volume. Often, ***partial pressure of water vapor*** or ***vapor pressure*** (p_v or e) is used. Vapor pressure can be understood similarly to atmospheric pressure, except that only collisions by water vapor molecules on an (imaginary) surface are considered. The total atmospheric pressure is therefore always higher than the vapor pressure as it takes the collisions of molecules of all gases into account. Water vapor also approximates an ideal gas well and we can write the ideal gas law as

$$p_\mathrm{v} = e = \rho_\mathrm{v} R_\mathrm{v} T, \tag{3}$$

where R_v is the gas constant for water vapor with the value $R_\mathrm{v} = R^*/M_{\mathrm{H_2O}} = 461\ \mathrm{J/(kg\,K)}$.

In addition to absolute humidity and vapor pressure, the amount of water vapor in the air can be expressed by specific humidity or the mixing ratio. ***Specific humidity***

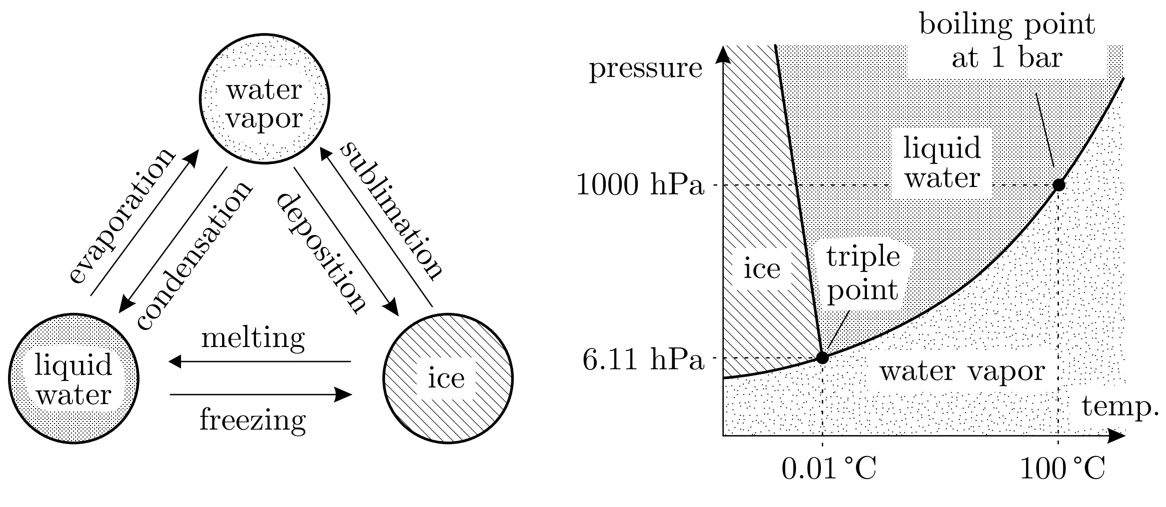


Figure 4: Left: states of water displaying the names of all phase transitions. Right: a simplified sketch of a water phase diagram showing at which temperatures and pressures different states of water can exist. The triple point shows the temperature and pressure (0.01 °C and 6.11 hPa) at which the three phases (gas, liquid, and solid) of water can coexist in a thermodynamic equilibrium. The boiling point of water (100 °C) at a pressure of 1000 hPa is also shown.

Problem 2: Determine the mass of water vapor in a classroom with the dimensions 10 m × 8 m × 4 m, if the temperature is 20 °C, the atmospheric pressure is 1000 hPa, and the vapor pressure is 22 hPa.

Solution: One needs to use the ideal gas law for water vapor and express the water vapor density

$$\rho_\mathrm{v} = \frac{e}{R_\mathrm{v} T} = \frac{2200 \text{ Pa}}{461 \text{ J/(kg K)} \cdot 293 \text{ K}} = 1.62 \cdot 10^{-2} \text{ kg/m}^3.$$

The mass of water vapor can be calculated from the water vapor density

$$m = \rho_\mathrm{v} V = 1.62 \cdot 10^{-2} \text{ kg/m}^3 \cdot 320 \text{ m}^3 = 5.21 \text{ kg}.$$

(q) is defined as the ratio between the density of water vapor and the density of air

$$q = \rho_\mathrm{v}/\rho, \tag{4}$$

and is usually given in units of g/kg. ***Mixing ratio*** (r) is defined similarly as the ratio between the density of water vapor and the density of the dry part of air without water vapor ($\rho_\mathrm{s} = \rho - \rho_\mathrm{v}$))

$$r = \rho_\mathrm{v}/(\rho - \rho_\mathrm{v}) = \rho_\mathrm{v}/\rho_\mathrm{s}, \tag{5}$$

which, like specific humidity, is typically given in units of g/kg.

It turns out that the amount of water vapor is constrained by the saturation limit. When the amount of water vapor reaches this limit, we say the air is saturated with moisture. The saturation limit can be expressed by one of the variables describing the amount of water vapor by adding the subscript "s" (e.g., e_s is the ***saturated vapor pressure***, while r_s is the saturated mixing ratio). If the amount of water vapor exceeds the saturation limit, the excess water vapor molecules are deposited into hydrometeors. In this case, if there are no hydrometeors in the air, they will form. If hydrometeors are already present, they will increase in size due to the deposition of additional water molecules. This process continues until the amount of water vapor is reduced to the saturation limit.

It also emerges that the value of the saturated vapor pressure depends only on the temperature. This is best illustrated by the example of a water droplet enclosed in an airtight container (Figure 5). A droplet is made up of a large number of water molecules that move relatively freely within the volume of the droplet. At the same time, the molecules cannot easily escape from the droplet as the attractive forces of the surrounding molecules appear on the droplet's surface. Similar to gas, water molecules in a liquid droplet have different energies. Some have enough energy to overcome the attractive forces of the surface and thereby escape from the droplet and become water vapor molecules. These move freely inside the closed container, colliding and bouncing with each other and the walls of the container, eventually colliding with the droplet and becoming trapped

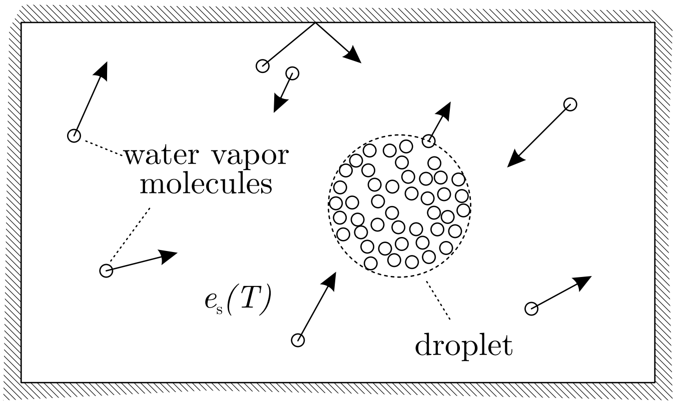


Figure 5: Schematic representation of water molecules in a droplet and the surrounding area inside an airtight container. The arrows show the direction of movement of individual water vapor molecules. An equilibrium state is quickly established with free molecules causing saturated vapor pressure $e_s(T)$.

again. The water molecules constantly escape from the droplet and are recaptured. An equilibrium state is quickly established where the rate of escaped molecules is the same as the number of recaptured molecules, and the number of free molecules in the container is approximately constant. The equilibrium pressure caused by the free molecules is called saturated vapor pressure. If, at some point, additional water vapor molecules were to be introduced into the container, the vapor pressure would momentarily rise above the saturated value. Then, within a relatively short time, the additional molecules would collide with the droplet and become trapped, and the vapor pressure would again decrease to the saturated value (the droplet would however increase slightly due to the additional molecules). If the temperature in the container increases, the energy of the molecules in the droplet also increases, making escape more likely, and increasing the equilibrium vapor pressure. A higher temperature thus corresponds to higher saturated vapor pressure, meaning that warmer air can contain more water vapor than colder air. The dependence of saturated vapor pressure on the temperature is described by the Claussius-Clapeyron equation

$$e_s(T) = e_{s0} e^{\frac{h_v}{R_v}\left(\frac{1}{T_{s0}} - \frac{1}{T}\right)}, \tag{6}$$

where h_v is the latent heat of vaporization, which at 0 °C has a value of $2.50 \cdot 10^6$ J/kg. $e_{s0} = 6.1$ hPa and $T_{s0} = 273$ K are the pressure and temperature of the triple point of water. The value of h_i depends on the temperature at which the phase change occurs. In the atmosphere, phase changes of water almost always occur at temperatures well below the boiling point of water in standard conditions (100 °C). The value of h_v at 100 °C is about 10% smaller than the value at 0 °C. Saturated vapor pressure for evaporation/condensation is different from sublimation/deposition. However, equation 6 can be used to calculate the values in both cases, except that in the case of sublimation/deposition, the latent heat of vaporization h_v needs to be replaced by the latent heat of sublimation $h_s = 2.83 \cdot 10^6$ J/kg (at 0 °C).

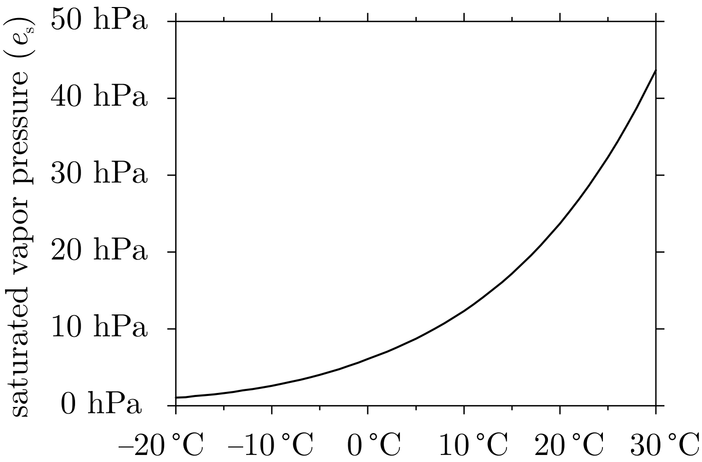


Figure 6: Dependence of saturated vapor pressure e_s on temperature. The values are calculated using Equation 6, taking account of the latent heat of vaporization for temperatures above freezing point and the latent heat of sublimation for temperatures below freezing point.

The dependence of saturated vapor pressure on temperature is shown in Figure 6, with the numerical values being provided in Table 2. The dependence is exponential. For example, the saturated vapor pressure is approximately 1 hPa at −20 °C and approximately 43 hPa at 30 °C. This means that air at 30 °C can contain 43 times more water vapor than air at −20 °C.

Saturated vapor pressure can be used to define ***relative humidity*** (f). This is the ratio of the actual vapor pressure to the saturated vapor pressure at the current temperature

$$f = \frac{e}{e_\mathrm{s}(T)}, \tag{7}$$

and is most commonly expressed as a percentage. Thus, 50% relative humidity means we can double the amount of water vapor before the air becomes saturated. If the relative humidity reaches 100%, the air is saturated, and hydrometeors begin to form.

The ***dewpoint temperature*** (T_d) can also be defined. This is the temperature to which the air must be cooled in order for saturation to occur. The dewpoint temperature is always lower than the actual temperature, except when the air is already saturated – in this case, the temperature equals the dewpoint temperature. The lower the T_d, the drier the air (it contains less water vapor). The equation for dewpoint temperature can be derived from Equation 6 by expressing the temperature under the assumption of $e_\mathrm{s} = e$:

$$T_\mathrm{d} = \left(\frac{1}{T_\mathrm{s0}} - \frac{R_\mathrm{v}}{h_\mathrm{v}} \ln \frac{e}{e_\mathrm{s0}} \right)^{-1}. \tag{8}$$

Water vapor is the only gas in the Earth's atmosphere that exists in large quantities in other physical states (liquid and solid) and hence forms clouds and precipitation. This is because a large amount of water is available on the Earth's surface, especially in the

	+0.0 °C	+0.1 °C	+0.2 °C	+0.3 °C	+0.4 °C	+0.5 °C	+0.6 °C	+0.7 °C	+0.8 °C	+0.9 °C
−20 °C	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.13
−19 °C	1.14	1.15	1.16	1.17	1.18	1.19	1.20	1.21	1.23	1.24
−18 °C	1.25	1.26	1.27	1.29	1.30	1.31	1.32	1.33	1.35	1.36
−17 °C	1.37	1.39	1.40	1.41	1.42	1.44	1.45	1.47	1.48	1.49
−16 °C	1.51	1.52	1.54	1.55	1.56	1.58	1.59	1.61	1.62	1.64
−15 °C	1.65	1.67	1.68	1.70	1.71	1.73	1.75	1.76	1.78	1.80
−14 °C	1.81	1.83	1.85	1.86	1.88	1.90	1.91	1.93	1.95	1.97
−13 °C	1.98	2.00	2.02	2.04	2.06	2.08	2.10	2.11	2.13	2.15
−12 °C	2.17	2.19	2.21	2.23	2.25	2.27	2.29	2.31	2.33	2.36
−11 °C	2.38	2.40	2.42	2.44	2.46	2.49	2.51	2.53	2.55	2.58
−10 °C	2.60	2.62	2.64	2.67	2.69	2.72	2.74	2.76	2.79	2.81
−9 °C	2.84	2.86	2.89	2.91	2.94	2.97	2.99	3.02	3.05	3.07
−8 °C	3.10	3.13	3.15	3.18	3.21	3.24	3.27	3.29	3.32	3.35
−7 °C	3.38	3.41	3.44	3.47	3.50	3.53	3.56	3.59	3.62	3.65
−6 °C	3.69	3.72	3.75	3.78	3.82	3.85	3.88	3.91	3.95	3.98
−5 °C	4.02	4.05	4.09	4.12	4.16	4.19	4.23	4.26	4.30	4.34
−4 °C	4.37	4.41	4.45	4.49	4.52	4.56	4.60	4.64	4.68	4.72
−3 °C	4.76	4.80	4.84	4.88	4.92	4.96	5.01	5.05	5.09	5.13
−2 °C	5.18	5.22	5.26	5.31	5.35	5.40	5.44	5.49	5.53	5.58
−1 °C	5.63	5.67	5.72	5.77	5.81	5.86	5.91	5.96	6.01	6.06
0 °C	6.11	6.15	6.20	6.24	6.29	6.34	6.38	6.43	6.48	6.52
1 °C	6.57	6.62	6.66	6.71	6.76	6.81	6.86	6.91	6.96	7.01
2 °C	7.06	7.11	7.16	7.21	7.26	7.32	7.37	7.42	7.48	7.53
3 °C	7.58	7.64	7.69	7.75	7.80	7.86	7.91	7.97	8.03	8.08
4 °C	8.14	8.20	8.26	8.31	8.37	8.43	8.49	8.55	8.61	8.67
5 °C	8.73	8.80	8.86	8.92	8.98	9.05	9.11	9.17	9.24	9.30
6 °C	9.37	9.43	9.50	9.56	9.63	9.70	9.77	9.83	9.90	9.97
7 °C	10.04	10.11	10.18	10.25	10.32	10.39	10.46	10.54	10.61	10.68
8 °C	10.76	10.83	10.90	10.98	11.06	11.13	11.21	11.28	11.36	11.44
9 °C	11.52	11.60	11.68	11.76	11.84	11.92	12.00	12.08	12.16	12.24
10 °C	12.33	12.41	12.50	12.58	12.67	12.75	12.84	12.92	13.01	13.10
11 °C	13.19	13.28	13.37	13.46	13.55	13.64	13.73	13.82	13.91	14.01
12 °C	14.10	14.20	14.29	14.39	14.48	14.58	14.68	14.78	14.87	14.97
13 °C	15.07	15.17	15.27	15.37	15.48	15.58	15.68	15.79	15.89	16.00
14 °C	16.10	16.21	16.31	16.42	16.53	16.64	16.75	16.86	16.97	17.08
15 °C	17.19	17.31	17.42	17.53	17.65	17.76	17.88	18.00	18.11	18.23
16 °C	18.35	18.47	18.59	18.71	18.83	18.96	19.08	19.20	19.33	19.45
17 °C	19.58	19.70	19.83	19.96	20.09	20.22	20.35	20.48	20.61	20.74
18 °C	20.88	21.01	21.15	21.28	21.42	21.56	21.69	21.83	21.97	22.11
19 °C	22.25	22.39	22.54	22.68	22.83	22.97	23.12	23.26	23.41	23.56
20 °C	23.71	23.86	24.01	24.16	24.32	24.47	24.62	24.78	24.93	25.09
21 °C	25.25	25.41	25.57	25.73	25.89	26.05	26.22	26.38	26.55	26.71
22 °C	26.88	27.05	27.22	27.39	27.56	27.73	27.90	28.07	28.25	28.42
23 °C	28.60	28.78	28.96	29.14	29.32	29.50	29.68	29.86	30.05	30.23
24 °C	30.42	30.61	30.80	30.99	31.18	31.37	31.56	31.76	31.95	32.15
25 °C	32.34	32.54	32.74	32.94	33.14	33.34	33.55	33.75	33.96	34.17
26 °C	34.37	34.58	34.79	35.00	35.22	35.43	35.64	35.86	36.08	36.30
27 °C	36.52	36.74	36.96	37.18	37.40	37.63	37.86	38.08	38.31	38.54
28 °C	38.78	39.01	39.24	39.48	39.71	39.95	40.19	40.43	40.67	40.91
29 °C	41.16	41.40	41.65	41.90	42.15	42.40	42.65	42.90	43.16	43.41
30 °C	43.67	43.93	44.19	44.45	44.71	44.98	45.24	45.51	45.78	46.05

Table 2: Tabulated values of saturated vapor pressure e_s in hPa as a function of temperature. The values are calculated using equation 6, taking into account the latent heat of vaporization for temperatures above freezing point and the latent heat of sublimation for temperatures below freezing point. The left column represents the temperature given in whole °C, and the first row shows the temperature in tenths of °C, which must be added to the temperature in the left column. For example, e_s at temperature 10.3 °C is 12.58 hPa since one needs to look at the line at 10 °C and the column at 0.3 °C.

oceans. The oceans are a constant source of water vapor, which continually evaporates from their surface. There is much more water in the oceans than in the atmosphere (the total mass of water in the oceans is about 10^5 - times greater than the total mass of water vapor in the atmosphere). In suitable conditions, water vapor in the atmosphere condenses and falls to the ground as precipitation. Namely, it can return to the oceans and complete the hydrological cycle. The temperature conditions on Earth (temperatures not too far from the temperature of the triple point of water) also allow the simultaneous existence of water vapor, liquid water, and ice.

The situation is different for other gases in the atmosphere. Nitrogen, for example, is much more abundant in the atmosphere than water vapor. Still, there is not nearly enough of it for its partial pressure to reach the saturation limit (at least at temperatures that exist on Earth – nitrogen has a melting point of -210 °C and boiling point of -195 °C). Nitrogen also does not have a large source, such as that provided by the oceans for water vapor and its amount in the atmosphere accordingly remains limited and does not change significantly. Thus, the conditions in the Earth's atmosphere are unsuitable for nitrogen to exist in liquid or solid form (the same applies to the other gases that make up the atmosphere). This means we do not have clouds and precipitation made of liquid or solid nitrogen on Earth. However, this does not necessarily apply to the atmosphere of some other planet where the temperatures would be much lower or there would be much greater nitrogen in the atmosphere. An interesting example from our solar system is Saturn's moon Titan, where there is a lot of methane in the atmosphere. At the same time, the temperatures are low enough to cause methane condensation in the atmosphere, creating methane clouds and precipitation, while seas and lakes of liquid methane cover part of the moon's surface.

Problem 3: Temperature 15 °C, relative humidity 75%, and atmospheric pressure 1010 hPa were measured at the weather station in Koper. Calculate the vapor pressure, absolute humidity, specific humidity, mixing ratio, and dewpoint temperature.

Solution: First, the saturated vapor pressure at 15 °C must be determined. Its value can be read from the table 2 or calculated from the equation 6. Its value is $e_s = 17.19$ hPa. To calculate the vapor pressure, e can be expressed using Equation 7

$$e = f \cdot e_s(15\ ^\circ\mathrm{C}) = 0.75 \cdot 17.19\ \mathrm{hPa} = 12.89\ \mathrm{hPa}.$$

For absolute humidity, the density of water vapor can be expressed from the ideal gas law for water vapor

$$\rho_v = \frac{e}{R_v T} = \frac{1289\ \mathrm{Pa}}{461\ \mathrm{J/(kg\,K)} \cdot 288\ \mathrm{K}} = 9.7 \cdot 10^{-3}\ \mathrm{kg/m^3}.$$

To determine the specific humidity and the mixing ratio, it is first necessary to identify the air density from the ideal gas law for air

$$\rho = \frac{p}{RT} = \frac{101\,000\ \mathrm{Pa}}{287\ \mathrm{J/(kg\,K)} \cdot 288\ \mathrm{K}} = 1.22\ \mathrm{kg/m^3}.$$

Knowing the density of air and water vapor, the specific humidity and the mixing ratio can then be calculated

$$q = \frac{\rho_v}{\rho} = \frac{9.7 \cdot 10^{-3}\ \mathrm{kg/m^3}}{1.22\ \mathrm{kg/m^3}} = 7.94 \cdot 10^{-3} = 7.94\ \mathrm{g/kg},$$

$$r = \frac{\rho_v}{\rho - \rho_v} = \frac{9.7 \cdot 10^{-3}\ \mathrm{kg/m^3}}{1.22\ \mathrm{kg/m^3} - 9.7 \cdot 10^{-3}\ \mathrm{kg/m^3}} = 8.01 \cdot 10^{-3} = 8.01\ \mathrm{g/kg}.$$

The approximate value of the dewpoint temperature can be obtained from Table 2 by identifying the temperature at which the saturated vapor pressure is as close as possible to the value of 12.89 hPa – this approximately corresponds to the temperature 10.7 °C. A more accurate value can be calculated from the equation 8

$$\begin{aligned} T_d &= \left(\frac{1}{T_{s0}} - \frac{R_v}{h_v} \ln \frac{e}{e_{s0}} \right)^{-1} \\ &= \left(\frac{1}{273\ \mathrm{K}} - \frac{461\ \mathrm{J/(kg\,K)}}{2.50 \cdot 10^6\ \mathrm{J/kg}} \ln \frac{1289\ \mathrm{Pa}}{610\ \mathrm{Pa}} \right)^{-1} \\ &= 283.66\ \mathrm{K}. \end{aligned}$$

Problem 4: In the late afternoon, the temperature in Ljubljana is 23.8 °C and the relative humidity is 57%. The night is calm and clear, with the temperature falling during the night by 10 °C. Will Ljubljana see fog in the morning?

Solution: Radiation fog usually develops during a clear, calm night when the air near the ground cools enough to become saturated and hydrometeors start forming (for more on radiation fog, see Section 18 and Figure 28).

There are several ways to arrive at an answer. One can calculate the dewpoint temperature and compare it to the morning temperature. Since the night is calm, we may assume that the air near to the ground remains still, meaning that the vapor pressure stays approximately the same since the number of water molecules in the air does not change.

It is first necessary to determine the saturated vapor pressure at the temperature of 23.8 °C. The value can be read from the table 2 or calculated from the equation 6. The value is $e_s = 30.05$ hPa.

To calculate the vapor pressure, e can be expressed using Equation 7

$$e = f \cdot e_s(23.8\ ^\circ\mathrm{C}) = 0.57 \cdot 30.05\ \mathrm{hPa} = 17.13\ \mathrm{hPa}.$$

The approximate value of the dewpoint temperature can be obtained from Table 2 by identifying the temperature at which the saturated vapor pressure is as close as possible to the value of 17.13 hPa – this approximately corresponds to the temperature 15.0 °C.

In the morning, the temperature will be 13.8 °C, which is less than the dewpoint temperature. This means the air will become saturated during the night and hydrometeors will start forming, resulting in fog.

8 Meteorological Measurements

Meteorological measurements are measurements of meteorological variables. It is essential to know the current values of meteorological variables in the atmosphere to make a weather forecast (for more on this, see Section 36). Moreover, studying climate change is impossible without a long time series of meteorological measurements. It is also important that the measurements are as accurate as possible and that the measurement error is as small as possible. Meteorological measurements are conducted in many different ways: near to the ground at weather stations, ships, and buoys and higher up with sounding balloons, aircraft, radars, and satellites. The measurements are performed using various instruments. Some are classic instruments that have been relied on for over 100 years, while others are relatively new. The following is an overview of some commonly used classical and modern meteorological instruments and measurement methods.

An instrument for measuring temperature is called a ***thermometer***. A classic example of a thermometer is the ***mercury thermometer***, which relies on the volumetric temperature expansion of mercury for its operation (other liquids like alcohol can also be used). Most of the mercury is stored in a bulb to which a glass tube with a narrow diameter is attached. As the temperature rises, the mercury expands into the glass tube (Figure 7). A measuring scale is drawn next to the tube, allowing the temperature value to be read. A well-calibrated mercury thermometer is a stable instrument whose accuracy can be 0.1 °C. A widespread modern type of instrument is the ***resistance thermometer***, which exploits the fact that the electrical resistance of substances varies with temperature. An excellent example of such a metal is platinum whose resistance is almost linearly dependent on temperature. Due to their good properties and the possibility of miniaturization, the use of resistance thermometers in meteorology is common.

An instrument measuring atmospheric pressure is called a ***barometer***. A classic example of a barometer is the ***mercury barometer***. It usually consists of a glass tube that is closed at one end, filled with mercury, and inverted with the open end immersed in the mercury (Figure 8). The surface of the mercury in the cistern is exposed to atmospheric pressure, while the surface of the mercury inside the tube is exposed to the vacuum. Consequently, the mercury inside the tube rises, and its height varies along with the atmospheric pressure. If the height of the mercury column is measured, the

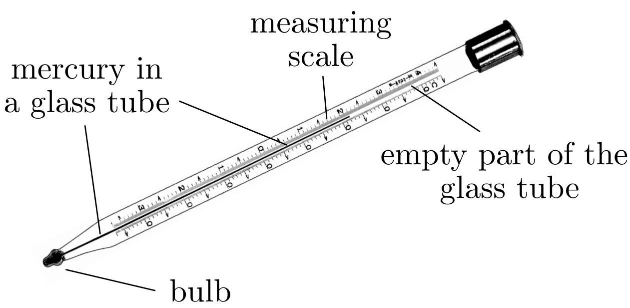


Figure 7: A mercury thermometer.

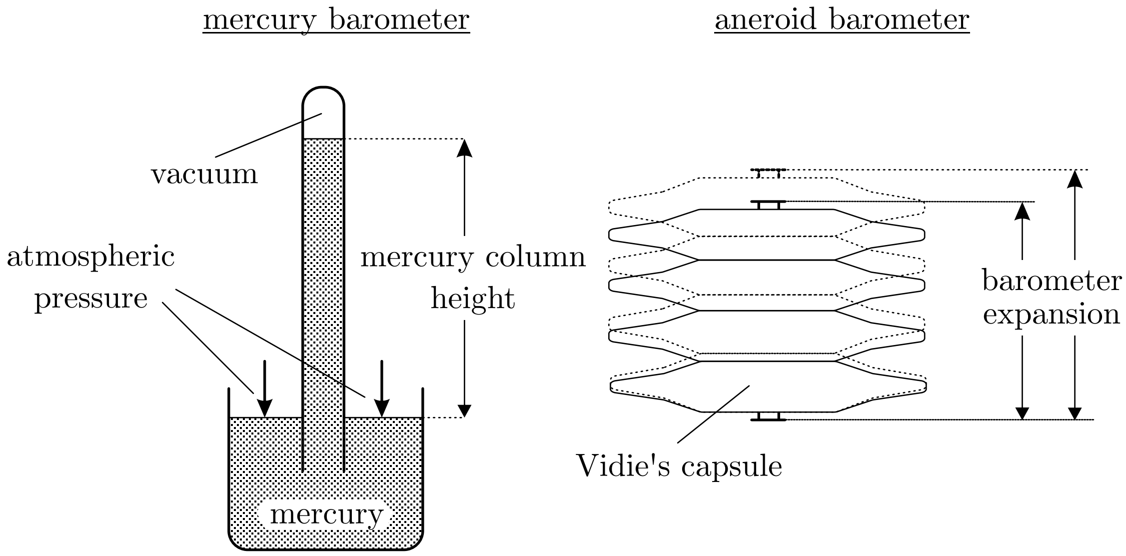


Figure 8: A mercury barometer and a classical aneroid barometer consisting of four Vidie capsules.

value of the atmospheric pressure can be determined using a simple equation. Nowadays, the most commonly used barometer is the ***aneroid barometer***, which measures the deformation of an aneroid capsule to determine the atmospheric pressure. The classic version of this barometer uses a Vidie capsule. This is an airtight, hollow, small, and flexible metal box that deforms according to the surrounding atmospheric pressure. If the ambient atmospheric pressure changes, the capsule expands or contracts by a small amount. With the traditional mechanical version of the instrument, the size change is measured mechanically, with several Vidie capsules being linked together to make the change easier to measure (Figure 8). A modern version of this barometer can be very small and light. Typically, a small flexible diaphragm is used, with the separation between the diaphragm and a fixed plate measured electronically.

One of the basic meteorological variables is the density of air, which proves difficult to measure directly. In practice, density is usually determined by measuring the temperature and atmospheric pressure, then using the ideal gas law (Equation 2) to calculate the density.

The wind is a somewhat special variable because it is a vector. Only the horizontal wind is generally measured, where the wind direction and speed are given separately. Instruments designed to measure wind-related parameters are called anemometers. Classical instruments are the cup anemometer and wind vane. A ***cup anemometer*** measures the wind speed and consists of a rotating cross-shaped frame made of three or four rods with hemispherical or conical cups mounted at the ends (Figure 9). The asymmetrical shape of the cups causes the frame to start rotating should there be wind. The higher the wind speed, the faster it spins. ***Wind vane*** comprises an elongated object, usually in the shape of an arrow with a large vertically oriented tail fin attached to a vertical pole (Figure 9). The object can freely rotate around the vertical axis. Due to the asymmetrical

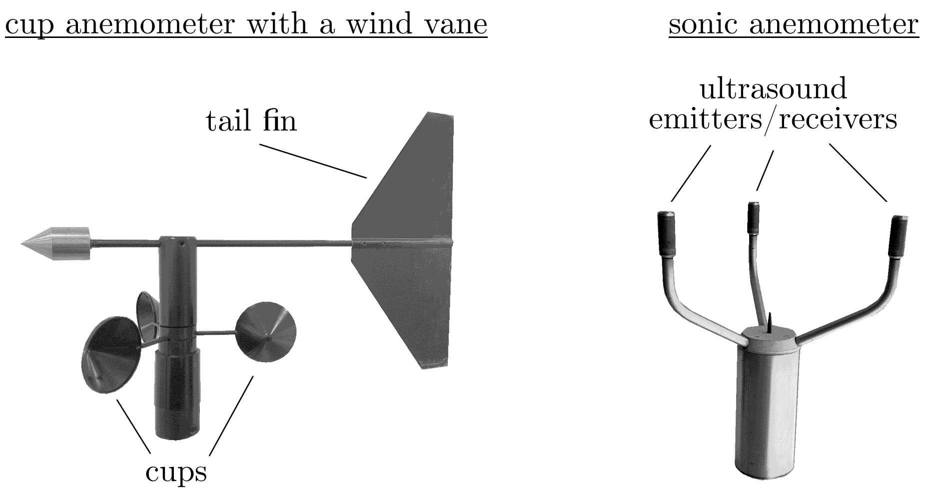


Figure 9: A cup anemometer with a wind vane and a sonic anemometer.

shape, the airflow resistance forces the object to turn toward the direction of the wind. A modern version of the instrument is the ***sonic anemometer***. It typically consists of three or four ultrasound emitters/receivers measuring the transit times of acoustic pulses sent between emitter/receiver pairs. The transit times depend on the wind speed and direction and, once the transit times are measured, the wind velocity vector can be determined.

The amount of water vapor in the air is measured with hygrometers. Determining the precise amount of moisture in the air is quite difficult and so hygrometers tend to have a relatively large measurement error (often exceeding a few percent, even with more expensive hygrometers). A ***hair hygrometer*** is a simple yet inaccurate traditional instrument. It takes advantage of the fact that the length of hair changes with relative humidity. The change in hair length can be measured mechanically with a lever that moves the pointer on a scale. A more accurate traditional instrument is a ***psychrometer***. It has two matching thermometers (Figure 10). One is called a "dry-bulb" thermometer and measures the ambient temperature. The other one is called a "wet-bulb" thermometer and has the bulb covered with water-soaked cloth. Due to the water's evaporation from the cloth, the temperature measured by the wet-bulb thermometer is generally lower than the ambient temperature. The drier the ambient air, the more water evaporates from the cloth and the lower the temperature shown by a wet-bulb thermometer. The measured values of dry- and wet-bulb temperatures make it possible to determine the amount of water vapor in the air. No evaporation occurs if the ambient air is saturated and both thermometers show the same temperature. A small fan is typically used to achieve a constant airflow past the bulbs of both thermometers. A more modern type of hygrometer is a ***capacitive hygrometer***. It uses an electronic capacitor and a dielectric material inserted between the capacitor's plates (Figure 10). The inserted dielectric is porous and can absorb or release water vapor, which affects the capacitance (namely, a capacitor's ability to hold an electrical charge). The ambient humidity can be determined by measuring the capacitance of the capacitor.

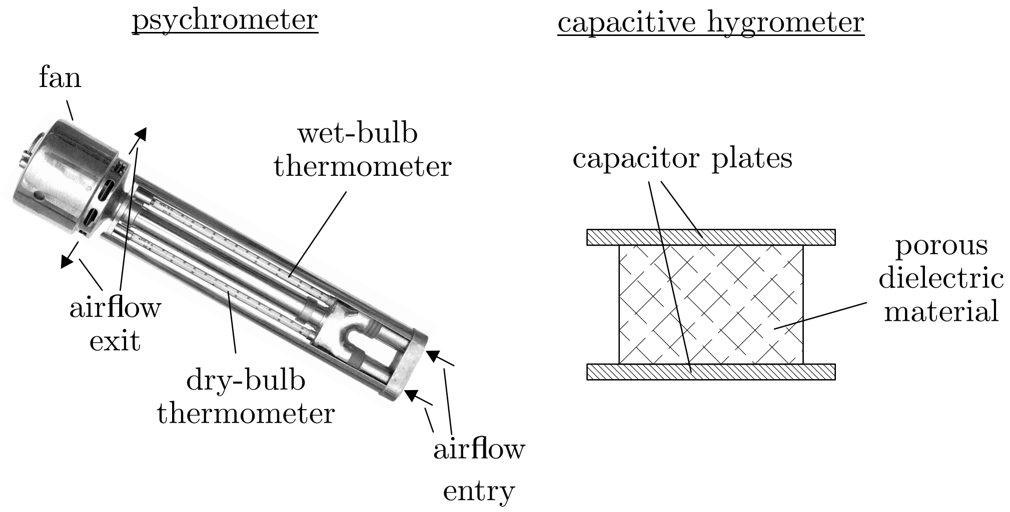


Figure 10: A psychrometer and a sketch of a capacitive hygrometer.

Another important meteorological variable is the amount of precipitation. It is measured with a ***precipitation gauge***, also known as an ***ombrometer***. The amount of precipitation is expressed by the liquid water depth (usually in millimeters) that would have accumulated on a horizontal surface over a specified period of time, assuming the water had not run off or evaporated. For example, on a day with heavy rain, the precipitation amount could exceed 100 mm, while the average annual precipitation in Slovenia is approximately 1400 mm. The liquid water depth expressed in millimeters is numerically equal to liters per square meter (e.g., rainfall of 30 mm is equal to 30 liters of water per square meter). If precipitation falls in solid form (e.g., snow), the depth represents the amount of water if the solid precipitation were to melt (e.g., the density of snow lying on the ground is about ten times lower than the density of liquid water, hence, snow depth of 10 cm approximately corresponds to a water column height of 10 mm). Several types of precipitation gauges are in use. In the past, the most commonly relied on was the manually operated ***ordinary precipitation gauge***. The gauge has an intercepting funnel at the top (Figure 11). Any precipitation that falls into the funnel flows down the funnel tube into a container placed inside the gauge. The gauge is operated manually by a meteorological observer. Measurements are usually performed once a day when the observer removes the container from the gauge and measures the precipitation amount that has accumulated in the past 24 hours. If precipitation is in solid form, the observer first takes the gauge into a warm room, waits for the snow to melt, and only then records the precipitation amount. Nowadays, automatic gauges are mainly employed, not requiring manual operation. A ***tipping-bucket gauge*** has an intercepting funnel at the top, similar to an ordinary gauge. The precipitation that falls into the funnel flows down the funnel tube onto a tipping-bucket balance (Figure 11). The balance consists of two symmetrical parts with buckets, with the water from the funnel only flowing into one at a time. When enough water has accumulated into one, the balance tips and the second empty bucked is positioned under the funnel, while the water from the first bucket

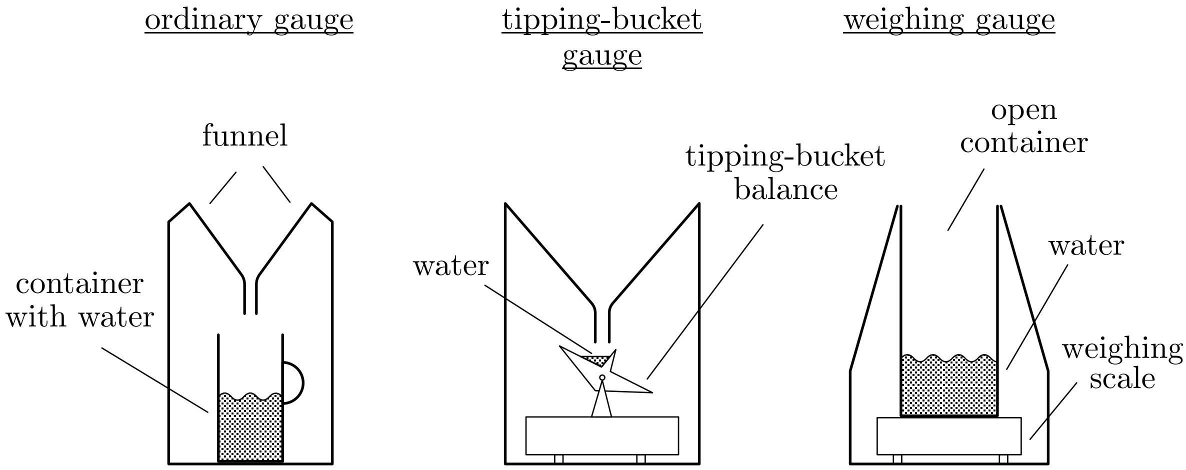
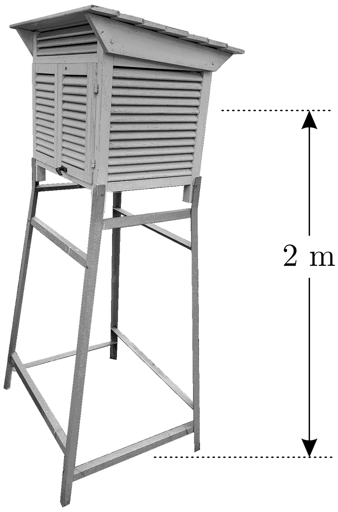


Figure 11: Various types of precipitation gauges.

is poured out. Each tip corresponds to a certain precipitation amount. In the case of solid precipitation, the funnel can become clogged and the gauge will not register any precipitation. This problem can be avoided if the funnel is heated at temperatures below freezing point so the snow is continuously melting. A ***weighing gauge*** determines the amount of precipitation by weighing the mass of water in a container with an open top (Figure 11). As precipitation falls, the mass of water in the container increases, but the fact that the water can evaporate must also be taken into account.

Using some of theinstruments mentioned above, regular measurements are performed at numerous weather stations in Slovenia and around the world. At these stations, the values of variables are measured close to the ground. Typically, temperature, atmospheric pressure, and humidity are measured at a height of 2 m inside an instrument shelter, also known as a Stevenson screen, that is mounted on a stand (Figure 12). The shelters are generally painted white and protect the instruments from precipitation and exposure to direct sunshine (which can cause errors in temperature measurements). The wind is usually measured at the top of a pole at a standard height of 10 m. In the past, measurements at stations were performed by meteorological observers, but these days automatic weather stations are becoming ever more widespread, with measurements being performed without human involvement.

Upper-air measurements of meteorological variables are important for weather forecasting. Namely, the weather on the Earth's surface is largely determined by what is happening higher up in the atmosphere (e.g., precipitation first forms in the clouds and only then falls to the the ground). Upper-air measurements are performed by aircraft and sounding balloons. Measurements with ***sounding balloons***, also called ***radiosonde measurements***, are conducted by attaching a light disposable measuring probe to a helium-filled balloon (Figure 12). This probe contains an instrument package that measures the values of important meteorological variables like temperature, atmospheric pressure, humidity, and horizontal wind. While rising, the probe measures and transmits the measured values to a ground station via a wireless connection. In about 1 hour, the



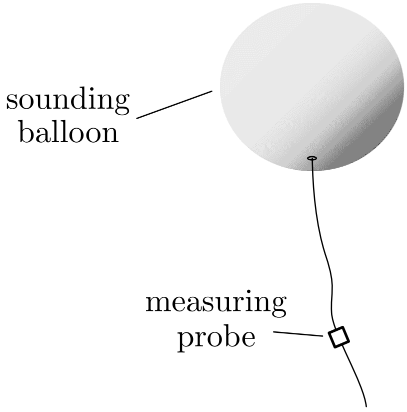


Figure 12: An instrument shelter with a stand and a radiosonde.

balloon rises from the ground to a height of about 20 km at which it bursts and then slowly falls back to the ground. Figure 13 shows an example of a radiosonde measurement.

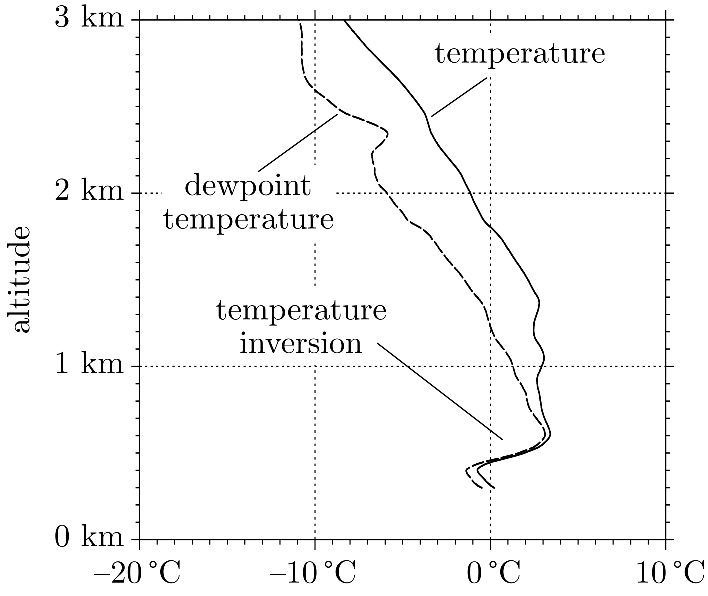


Figure 13: Vertical profiles of the temperature (solid line) and dewpoint temperature (dashed line) above Ljubljana as measured by a sounding balloon at 4 am on February 24, 2010. The graph shows altitudes ranging from 300 m (the altitude of Ljubljana) to 3 km. In the layer between 400 and 600 m, there is an upper-air temperature inversion where the temperature increases with the altitude by about 4 °C. In this layer, the temperature and dewpoint temperature are almost the same, which means the air is saturated and likely contains clouds.

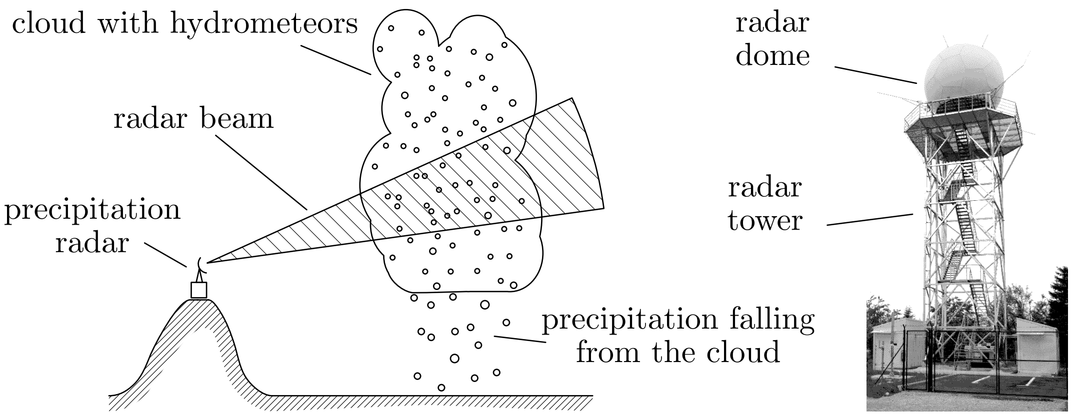


Figure 14: The operation of a precipitation radar and a photograph of the radar on the Pasja ravan mountain. The radar antenna is positioned at the top of the radar tower beneath the dome.

Meteorological measurements can also be performed via remote sensing. One example is precipitation measurements with a ***radar***. Unless the terrain is flat, the radar is typically located on a mountaintop to ensure that the view is as unobstructed as possible in all directions. There are currently two radars in Slovenia, one being located on the Lisca mountain in the eastern part of Slovenia and the other on Pasja ravan mountain in the central part. A precipitation radar emits short pulses of microwave electromagnetic radiation, which propagates away from the radar (Figure 14). The pulse travels in the direction the radar antenna is facing. If the beam's path intersects with hydrometeors (e.g., in a cloud or precipitation), some of the radiation is reflected from the hydrometeors back towards the radar, which measures the amount of reflected radiation. By measuring the transit time of the pulse (from radar to cloud and back), the radar can estimate the distance to the cloud and, based on the antenna's orientation, the direction in which the cloud is located. The amount of reflected radiation depends on the number of hydrometeors in the cloud and their size. This means it is possible to approximately determine the precipitation intensity from measurements of reflected radiation coming back to the radar. During operation, the radar antenna is usually rotating to allow the radar to scan the atmosphere in all directions.

Another example of remote sensing measurements is measurements with ***meteorological satellites***. Depending on their orbit type, they can be divided into Low Earth orbit and geostationary orbit satellites (Figure 15). ***Low Earth orbit satellites*** are typically positioned at altitudes of between 300 and 800 km and circle the Earth approximately once per hour. While circling, the instruments on the satellite constantly scan the atmosphere and the ground beneath them. With different instruments, it is possible to approximately determine some meteorological parameters such as temperature, humidity, wind, and the amount of aerosol at different altitudes below the satellite. Since these satellites are relatively close to the Earth's surface, they can only see and scan a relatively small portion of the atmosphere beneath them at any given time. It is different with ***geostationary***

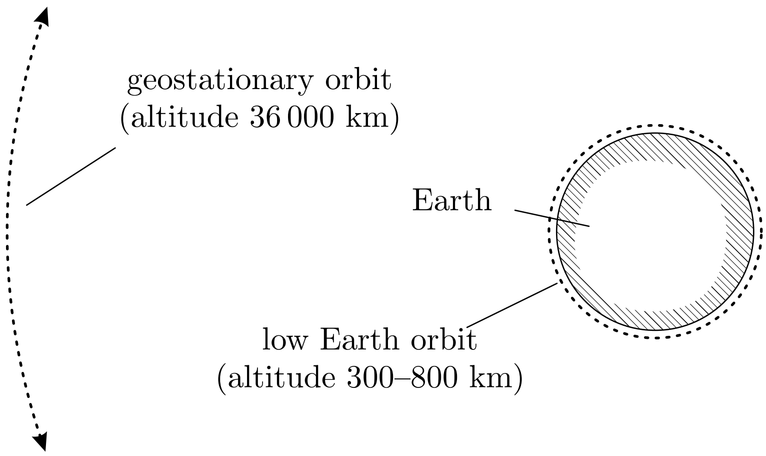


Figure 15: Comparison of geostationary and low Earth orbits.

orbit satellites, which are located above the equator and fly at an altitude of about 36 000 km. These satellites circle the Earth in exactly 1 day, which means that they circle with the same angular velocity as the Earth is rotating and hence always see the same area of the planet. Given that they are so far away, they can see almost half of the Earth's surface, which is one of their advantages over the low Earth orbit satellites. However, the advantage of low Earth orbit satellites is that they can scan the area below them in much more detail because they are much closer to the surface than geostationary orbit satellites.

9 Change of Atmospheric Pressure with Altitude

The change of atmospheric pressure with altitude is caused by the Earth's gravitational pull. The Earth attracts the mass of the atmosphere through the force of gravity (for more on the gravity force, see Section 23), which causes the air in the lower layers to compress under the weight of the air above (Figure 16). Thus, the density of air and atmospheric pressure are highest near the ground and always decrease with altitude.

Suppose the air is still or moving only horizontally. In this case, a hydrostatic equilibrium is rapidly established in the atmosphere in the vertical direction in which the gravity and buoyancy forces are in equilibrium (buoyancy force is the vertical component of the pressure gradient force described in more detail in Section 23). Here, assuming that the temperature does not change with the altitude (the atmosphere would then be isothermal), the change in atmospheric pressure with the altitude can be described by equation

$$p(z) = p(z_0) \cdot \mathrm{e}^{-\frac{g}{RT}(z-z_0)}, \tag{9}$$

where $g = 9.81\ \mathrm{m/s^2}$ is gravity, and $p(z_0)$ and $p(z)$ are the atmospheric pressures at altitudes z_0 and z (derivation the equation 9 is in Appendix A.2). The equation 9 assumes that the temperature in the layer between z and z_0 is everywhere equal to T. The equation takes the form of an exponential function, which means the atmospheric pressure decreases

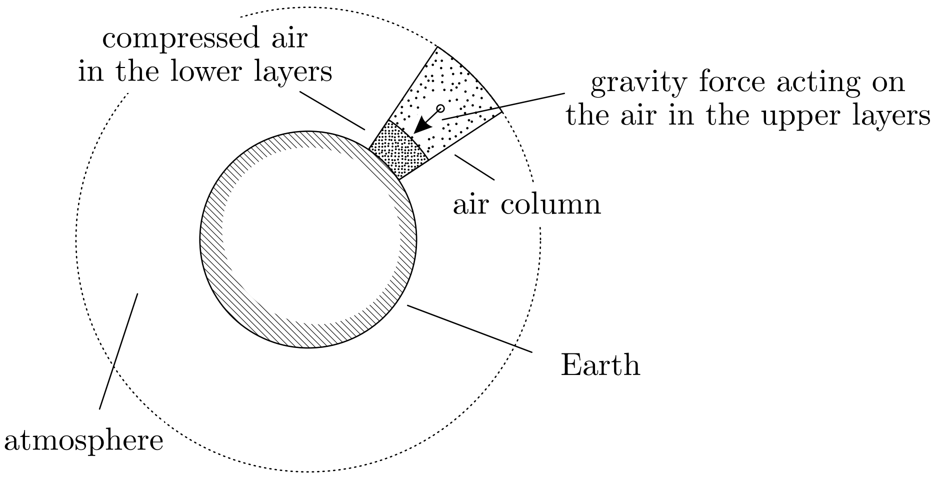


Figure 16: The Earth attracts the mass of the atmosphere through the force of gravity, causing the air in the lower layers to compress under the weight of the air above. The thickness of the atmosphere shown in the figure is unrealistic – in reality, the thickness is much smaller than the Earth's diameter.

exponentially with the altitude. Equation 9 can also be used to determine the altitude where the atmospheric pressure equals some prescribed value. In this case, the altitude z must be expressed from Equation 9

$$z = z_0 + \frac{RT}{g} \cdot \ln \frac{p(z_0)}{p}. \tag{10}$$

Equation 9 is also used for sea-level pressure reduction. The reduction is mostly performed in order to be able to compare the atmospheric pressure values measured at weather stations at different altitudes. Without this reduction, the measured values of atmospheric pressure would tend to always be lower at stations located at higher altitudes, regardless of pressure changes in the horizontal direction. After the reduction, the pressure at different stations can be compared to establish the locations and extent of phenomena such as cyclones and anticyclones. The temperature measured at a station is used in the reduction.

In reality, the temperature almost always varies in the vertical direction, whereas the decrease of atmospheric pressure with the altitude is only approximately exponential – which means that the equations 9 and 10 are only approximations. A slightly more precise equation can be derived by assuming that the temperature changes linearly with the altitude (see Appendix A.3). Aircraft altimeters, which assume that the vertical temperature profile is the same as that which the International Standard Atmosphere prescribes, use such an equation to determine altitude.

Problem 5: At the station in Rateče (altitude 864 m), the atmospheric pressure 920 hPa and the temperature 5 °C are measured. Determine the pressure at the top of Mount Triglav (altitude 2864 m), assuming that the atmosphere is isothermal. At what altitude is the atmospheric pressure equal to 800 hPa?

Solution: Since the atmosphere is assumed to be isotherm, Equation 9 can be used

$$\begin{aligned} p(2864\ \mathrm{m}) &= p(z_0) \cdot \mathrm{e}^{-\frac{g}{RT}(z-z_0)} \\ &= 92\,000\ \mathrm{Pa} \cdot \mathrm{e}^{-\frac{9.81\ \mathrm{m/s^2}}{287\ \mathrm{J/(kg\,K)} \cdot 278\ \mathrm{K}}(2864\ \mathrm{m} - 864\ \mathrm{m})} \\ &= 719\ \mathrm{hPa}. \end{aligned}$$

To determine the altitude at which the atmospheric pressure is equal to 800 hPa, Equation 10 can be used

$$\begin{aligned} z &= z_0 + \frac{RT}{g} \cdot \ln \frac{p(z_0)}{p} \\ &= 864\ \mathrm{m} + \frac{287\ \mathrm{J/(kg\,K)} \cdot 278\ \mathrm{K}}{9.81\ \mathrm{m/s^2}} \cdot \ln \frac{92\,000\ \mathrm{Pa}}{80\,000\ \mathrm{Pa}} \\ &= 2001\ \mathrm{m}. \end{aligned}$$

Problem 6: At the station in Ljubljana (altitude 300 m), the atmospheric pressure 980 hPa and the temperature −7 °C are measured. Determine the value of the atmospheric pressure after a sea-level pressure reduction.

Solution: For a sea-level pressure reduction, one needs to use Equation 9 in which the temperature measured at the station is used.

$$\begin{aligned} p(0\ \mathrm{m}) &= p(z_0) \cdot \mathrm{e}^{-\frac{g}{RT}(z-z_0)} \\ &= 98\,000\ \mathrm{Pa} \cdot \mathrm{e}^{-\frac{9.81\ \mathrm{m/s^2}}{287\ \mathrm{J/(kg\,K)} \cdot 266\ \mathrm{K}}(0\ \mathrm{m} - 300\ \mathrm{m})} \\ &= 1019\ \mathrm{hPa}. \end{aligned}$$

10 The Thermodynamic Energy Equation

The first law of thermodynamics can be used to describe the temperature change of an air parcel. The law states that the change of the internal energy of an air parcel can be influenced by the quantity of energy supplied to the parcel as heat and by the amount of thermodynamic work done by the parcel on its surroundings.

$$\mathrm{d}W_\mathrm{n} = \mathrm{d}Q - \mathrm{d}A, \tag{11}$$

where $\mathrm{d}W_\mathrm{n}$, $\mathrm{d}Q$, and $\mathrm{d}A$ are small changes in internal energy, heat, and the work the parcel does. The thermodynamic energy equation can be derived from Equation 11 (the derivation is shown in Appendix A.4). It describes the rate of change of the parcel's temperature over time, assuming there are no phase changes of water,

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{\rho c_\mathrm{p}}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{1}{m c_\mathrm{p}}\frac{\mathrm{d}Q}{\mathrm{d}t}, \tag{12}$$

where $c_\mathrm{p} = 1004$ J/(kg K) is the specific heat of air at a constant pressure. The term $\frac{\mathrm{d}T}{\mathrm{d}t}$ represents the rate of the air temperature's change over time and has units K/s (e.g., -2 K/h means that the parcel is cooling by 2 °C per hour). The terms that represent possible causes for the temperature change are on the right-hand side of the equation. The first is related to the change in atmospheric pressure, which causes the compression or expansion of the air. This term is responsible for the change in temperature during lifting, as the parcel moves to a higher altitude with lower atmospheric pressure ($\mathrm{d}p/\mathrm{d}t < 0$), which leads to the parcel's expansion and its cooling. The reverse is true if the parcel is descending (the air is compressed, and the parcel's temperature increases). The second term is associated with the energy supplied or taken from the parcel from the surrounding environment. An example is the absorption of incident infrared or ultraviolet radiation, whereby the parcel receives the energy of the intercepted radiation ($\mathrm{d}Q/\mathrm{d}t > 0$), which raises the parcel's temperature. The opposite happens if the parcel emits more radiation than it receives.

11 Temperature Change of Ascending or Descending Air

As mentioned in Section 10, air will cool when it ascends and become warmer when it descends. Assuming that the ascent or descent occurs without heat exchange with the surroundings (in this case, the process is adiabatic and therefore $\mathrm{d}Q = 0$), and there are no phase changes of water, it is possible to derive an expression for the adiabatic lapse rate of temperature (i.e., the rate of decrease of an air parcel's temperature with the altitude in the case of ascending) from Equation 11 (the derivation is shown in Appendix A.5).

$$\frac{\mathrm{d}T}{\mathrm{d}z} = -\Gamma_\mathrm{a}, \tag{13}$$

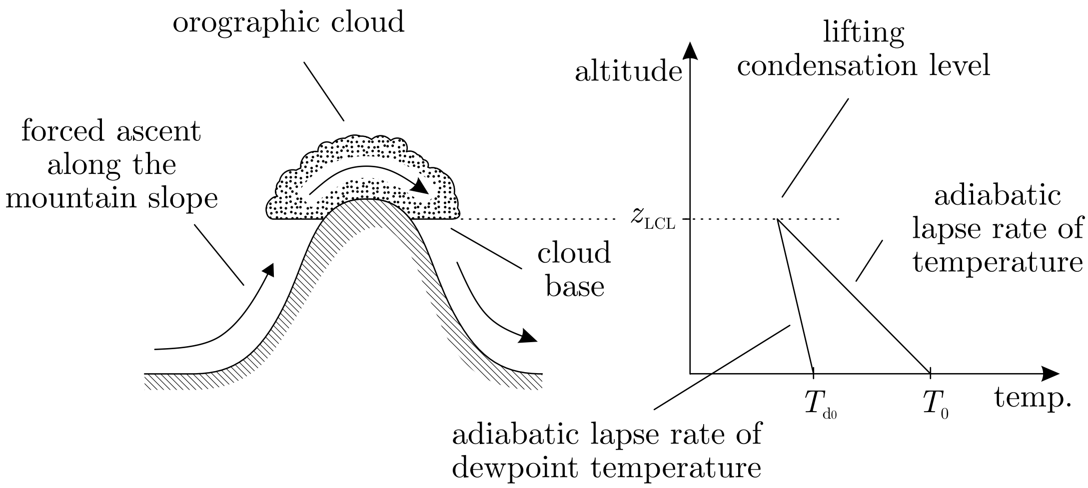


Figure 17: The formation of an orographic cloud. Before ascending, the air near the ground has temperature T_0 and dewpoint temperature T_{d0}. While the air rises along the mountain's slopes, the temperature decreases faster than the dewpoint temperature. The altitude at which both temperatures are the same and the air becomes saturated is called the lifting condensation level (z_{LCL}).

where $\mathit{\Gamma}_a = g/c_p \approx 10$ K/km. While ascending, the air cools by about 10 °C for each kilometer of ascent (the opposite is true for descent). Equation 13 is only valid so long as the air is dry (i.e., not saturated with moisture) as it does not take into account the energy released or consumed by the phase changes of the water. The index "a" in $\mathit{\Gamma}_a$ is used to highlight the assumption that the process is assumed to occur under adiabatic conditions (the parcel does not exchange energy with its surroundings).

While the parcel is ascending/descending, its dewpoint temperature also changes. Near the ground, the adiabatic lapse rate of dewpoint temperature is approximately (the derivation can be found in Appendix A.6):

$$\frac{\mathrm{d}T_d}{\mathrm{d}z} \approx -\frac{1}{6}\mathit{\Gamma}_a. \tag{14}$$

Equation 14 indicates that the dewpoint temperature changes about six times slower than the temperature. The two temperatures may be different before the ascent starts, but provided that the ascent does not stop the temperature will eventually "catch" the dewpoint temperature, and the air will become saturated. In this case, hydrometeors and a cloud will form. The altitude at which this happens is called the ***lifting condensation level*** (LCL) – the vast majority of clouds in the atmosphere form in this way.

LCL is a reasonable estimate of ***cloud base height*** when parcels experience a forced ascent, for example, if the air is forced to rise along a mountain slope. While rising, the air can become saturated and an ***orographic cloud*** can form (Figure 17) above the mountain to encompass the mountain top. On the downwind side, the air is descending and becoming warmer. Assuming no precipitation falls from the cloud, the air will become unsaturated at the same altitude as on the upwind side. Such clouds can often be observed in Slovenia as well as elsewhere. If the air closest to the surface is sufficiently dry while the air higher up is more humid, the cloud base can occur above the mountain top – in this case, the mountain top will not be in the cloud.

The altitude of the lifting condensation level can be determined easily from equations 13 and 14. The initial temperature and dewpoint temperature of the air near the ground can be denoted by T_0 and $T_{\mathrm{d}0}$. The decrease of both temperatures with the altitude can be approximately described by simple linear functions

$$\begin{aligned} T(z) &= T_0 - \mathit{\Gamma}_{\mathrm{a}} \cdot (z - z_0), \\ T_{\mathrm{d}}(z) &= T_{\mathrm{d}0} - \frac{1}{6}\mathit{\Gamma}_{\mathrm{a}} \cdot (z - z_0), \end{aligned} \tag{15}$$

where z_0 is the initial altitude. The altitude at which both temperatures will be the same and which determines the lifting condensation level (z_{LCL}) is obtained from the intersection of the two non-parallel lines defined by equations 15

$$z_{\mathrm{LCL}} = z_0 + \frac{T_0 - T_{\mathrm{d}0}}{5/6 \cdot \mathit{\Gamma}_{\mathrm{a}}}. \tag{16}$$

The situation is slightly different if the air is saturated since the energy released or consumed during the water's phase changes must also be considered. If the saturated air ascends, its temperature will also decrease, but slower than for unsaturated air because some energy is released due to the condensation or deposition of water vapor into a liquid or solid form.

The adiabatic lapse rate of temperature for saturated air can be expressed as

$$\frac{\mathrm{d}T}{\mathrm{d}z} = -\mathit{\Gamma}_{\mathrm{s}}. \tag{17}$$

The index "s" in $\mathit{\Gamma}_{\mathrm{s}}$ refers to the word saturation. The value of $\mathit{\Gamma}_{\mathrm{s}}$ depends on the values of atmospheric pressure and temperature (a detailed derivation and explanation are given in Appendix A.7). This means the temperature change will not be constant during ascent/descent since the pressure and the temperature will also change.

Given that very cold air can contain very little water vapor, the value of $\mathit{\Gamma}_{\mathrm{s}}$ at the top of the troposphere is approximately the same as the value of $\mathit{\Gamma}_{\mathrm{a}}$. The adiabatic lapse rate for saturated and unsaturated air is accordingly very similar in the upper troposphere (about 10 K/km). Near the ground, the situation is different and the values of $\mathit{\Gamma}_{\mathrm{s}}$ are about 5 K/km, which means that, if ascending, the saturated air will be cooling about 50% slower than the unsaturated air. Figure 18 displays the temperature change of air ascending from the ground to an altitude of 5 km, assuming that the temperature and the dewpoint temperature near the ground are 15 °C and 5 °C. Up to the lifting condensation level (altitude 1.2 km), the air is cooling by 10 K/km. From there up, the air is saturated and the cooling is slower, yet as the ascent continues the saturated lapse rate becomes more and more similar to 10 K/km.

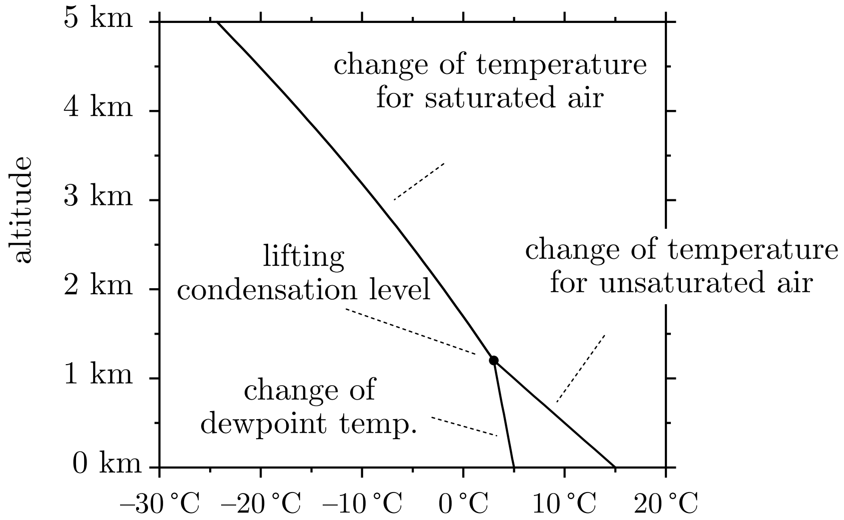


Figure 18: Change of temperature of ascending air assuming that the temperature and the dewpoint temperature near the ground are 15 °C and 5 °C.

Problem 7: At Podnanos, at an altitude of 110 m the temperature is 17 °C and the dewpoint temperature is 11 °C. There is a southwesterly wind, which forces the air to rise from Podnanos over the ridge of Mount Nanos at an altitude of 1100 m. Determine whether an orographic cloud will form over the ridge of Mount Nanos and the altitude of the cloud base.

Solution: First, the lifting condensation level must be determined using Equation 16

$$z_{\mathrm{LCL}} = z_0 + \frac{T_0 - T_{\mathrm{d}0}}{5/6 \cdot \mathit{\Gamma}_{\mathrm{a}}} = 110\ \mathrm{m} + \frac{290\ \mathrm{K} - 284\ \mathrm{K}}{5/6 \cdot 0.01\ \mathrm{K/m}} = 830\ \mathrm{m}.$$

Since the lifting condensation level altitude is lower than the altitude of the ridge of Mount Nanos, an orographic cloud will form. The cloud base will be at an altitude of 830 m.

12 Convection

In meteorology, the term convection (also called free, natural, gravitational, or buoyant convection) is used to refer to the vertical movement and mixing of air caused primarily by differences in temperature. Figure 19 depicts a situation where an air parcel has a different temperature than the surrounding environment. Two forces are acting on the parcel; the buoyancy force $\vec{F}_\mathrm{v}$ in the upward direction, and the gravity force $\vec{F}_\mathrm{g}$ in the downward direction. The size of both forces can be expressed as

$$\begin{aligned} F_\mathrm{v} &= m_\mathrm{e} \cdot g, \\ F_\mathrm{g} &= m \cdot g, \end{aligned} \tag{18}$$

where m is the mass of the air parcel with temperature T, and m_e is the weight of displaced air (the mass of the air from the surrounding environment that would fill the volume of the parcel) with temperature T_e. Both masses can be expressed using the ideal gas law (Equation 1), where

$$\begin{aligned} m_\mathrm{e} &= \frac{pV}{RT_\mathrm{e}}, \\ m &= \frac{pV}{RT}, \end{aligned} \tag{19}$$

and

$$\begin{aligned} F_\mathrm{v} &= g \cdot \frac{pV}{RT_\mathrm{e}}, \\ F_\mathrm{g} &= g \cdot \frac{pV}{RT}. \end{aligned} \tag{20}$$

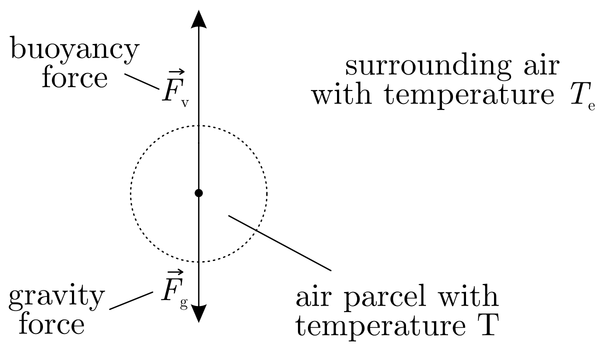


Figure 19: Forces acting on an air parcel with a different temperature than the surrounding air.

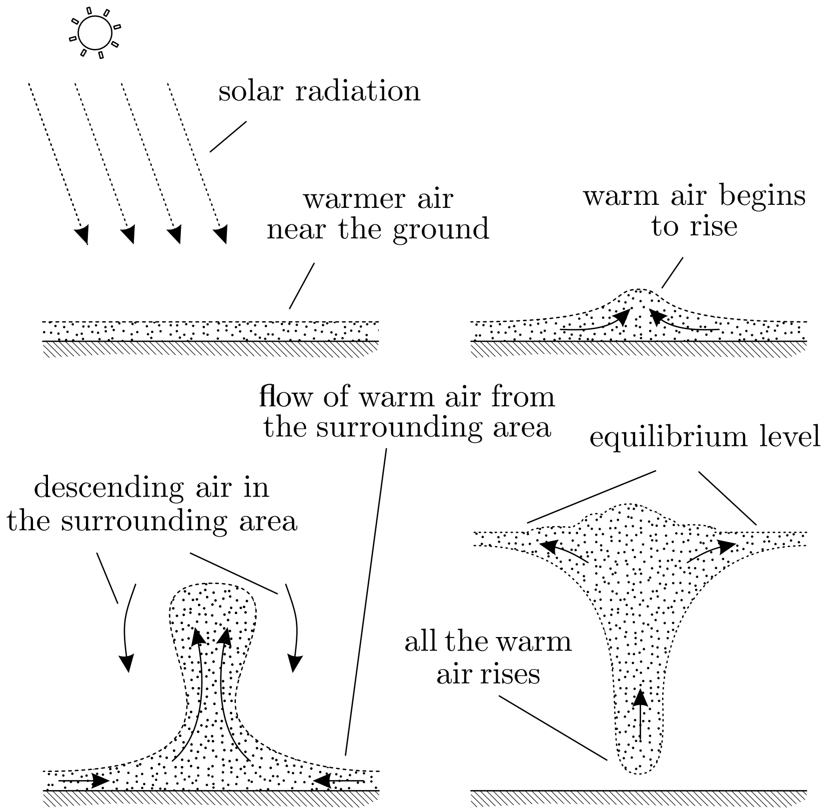


Figure 20: Typical formation and development of convection on a clear sunny day.

If the parcel is initially at rest, it will begin to move in the direction indicated by the sum of both forces. The sum of the forces in the vertical direction is determined as follows

$$\begin{aligned} F_\mathrm{v} - F_\mathrm{g} &= g \cdot \frac{pV}{RT_\mathrm{e}} - g \cdot \frac{pV}{RT} = g \cdot \frac{pV}{R} \left(\frac{1}{T_\mathrm{e}} - \frac{1}{T} \right) = \\ &= g \cdot \frac{pV}{R} \left(\frac{T - T_\mathrm{e}}{T_\mathrm{e} \cdot T} \right) = g \cdot \frac{pV}{RT} \left(\frac{T - T_\mathrm{e}}{T_\mathrm{e}} \right) = g \cdot m \left(\frac{T - T_\mathrm{e}}{T_\mathrm{e}} \right). \end{aligned} \tag{21}$$

If F_v is greater than F_g, the parcel will start to rise. This happens if it is warmer than the surrounding air ($T > T_\mathrm{e}$). The reverse is also true – the parcel will start to descend if it is colder than the surrounding air ($T < T_\mathrm{e}$). This occurs because warmer air is lighter (has a lower density) and colder air is heavier (denser) than the air surrounding it.

Convection is a very common phenomenon in the atmosphere. It most often occurs in summer during the day when the weather is clear and sunny. Due to the absorption of solar radiation, the temperature of the ground increases, causing the temperature of the air close to the ground to go up as well (Figure 20). When the temperature of the air near the ground increases, it begins to rise. While rising, the air is cooling such that the ascent continues as long as the air remains warmer than the air surrounding. The altitude at which the ascending air becomes colder than the surrounding air is called the ***equilibrium level***. The ascent can be short (e.g., only a few 100 m) or long (all the way up to the tropopause), mainly depending on the ascending air's initial temperature and the vertical temperature profile of the ambient air (for more on this, see Section 15 about the convective stability of the atmosphere). If the initial temperature is high enough and the temperature of the ambient air aloft low enough, clouds and thunderstorms can form. In thunderstorms, the air rises to high altitudes and can move with considerable vertical speed (up to a few 10 m/s). Near the ground, the convection "sucks" the warm air out from the surrounding area. When all the warm air has risen, the convection stops. The whole process generally takes a few minutes to 1 hour, depending on how high the air is rising and the amount of warm air.

13 Static Stability

Static stability can be defined for each individual layer in the atmosphere. Static stability is a property of a layer that indicates the likelihood of an air parcel returning to its original position if it became vertically displaced. If the parcel is likely to return, the layer is called stable, if it moves further away from its original position it is said to be unstable, and if it remains in the displaced position, the layer is called neutral.

Such stability is reliant on the temperatures of the displaced and ambient air at the displaced altitude. For example, if a parcel rises a little and becomes warmer than the surrounding air, the parcel will continue to rise due to convection ($T > T_\mathrm{e}$, Equation 21) and the layer is unstable. To determine static stability, one must know how the temperature in the ambient air changes with the altitude – this is expressed by the lapse rate of the temperature in the environment γ, defined as $\gamma \equiv -\frac{\partial T_\mathrm{e}}{\partial z}$. If γ is positive, the temperature in the environment is decreasing with the altitude and, vice versa, if γ is negative (e.g., if $\gamma = 8$ K/km the temperature in the environment will be 8 °C cooler at an altitude that is 1 km higher than the current altitude, while $\gamma = -8$ K/km means that the temperature would be 8 °C warmer). The γ value is usually obtained from a measurement with a sounding balloon.

Static stability can be ascertained by comparing the values of $\mathit{\Gamma}_\mathrm{a}$ or $\mathit{\Gamma}_\mathrm{s}$ with γ. $\mathit{\Gamma}_\mathrm{a}$ or $\mathit{\Gamma}_\mathrm{s}$ represent the temperature change of the air which is rising ($\mathit{\Gamma}_\mathrm{a}$ for unsaturated, $\mathit{\Gamma}_\mathrm{s}$ for saturated air), while γ represents the change in temperature with altitude in the air in the environment (which is assumed to be stationary). Five classes of static stability can be defined:

- If the ambient temperature in the layer is rapidly decreasing with the altitude and $\gamma > \mathit{\Gamma}_\mathrm{a}$, the ascending air will be warmer than the ambient air, and convection will occur (the situation with γ_1 in Figure 21). In this case, the layer is ***absolutely unstable***.

- If $\gamma = \mathit{\Gamma}_\mathrm{a}$, and assuming that the air is not saturated, the ascending air will have the same temperature as the ambient air. Here, the layer is ***unsaturated neutral***.

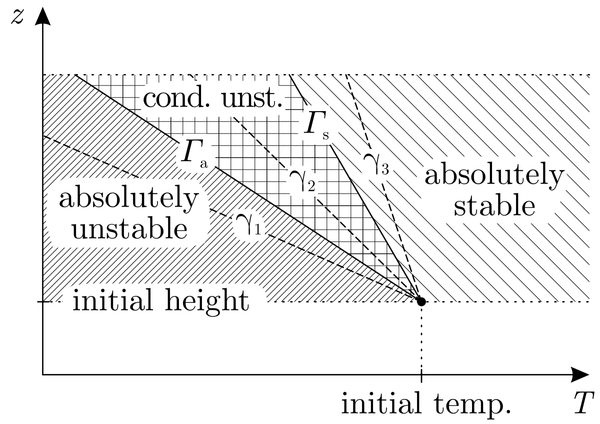


Figure 21: Determining the static stability of a layer. Lines denoted by Γ_a and Γ_s show how an unsaturated or saturated air parcel will cool when it starts to rise from its initial position. The lines with γ and the areas around them show three possibilities regarding how the ambient temperature changes with the altitude inside the layer. In the first case, $\gamma_1 > \Gamma_a$ and the layer will be absolutely unstable. In the second case, $\gamma_2 < \Gamma_a$ and at the same time $\gamma_2 > \Gamma_s$, and the layer will be conditionally unstable. In the third case, $\gamma_3 < \Gamma_s$ and the layer will be absolutely stable.

- If $\gamma < \Gamma_a$ and at the same time $\gamma > \Gamma_s$, the ascending air will be warmer than the ambient air, and there will be free convection, but only if the air is saturated (the situation with γ_2 in Figure 21). In this case, the layer is ***conditionally unstable***.
- If $\gamma = \Gamma_s$, and assuming the air is saturated, the ascending air will have the same temperature as the ambient air. Here, the layer is ***saturated neutral***.
- If $\gamma < \Gamma_s$, the ascending air will always be colder than the air in the environment (the situation with γ_3 in Figure 21). In this case, the layer is ***absolutely stable***. Some examples of very stable layers that often occur in the atmosphere are described in Section 14.

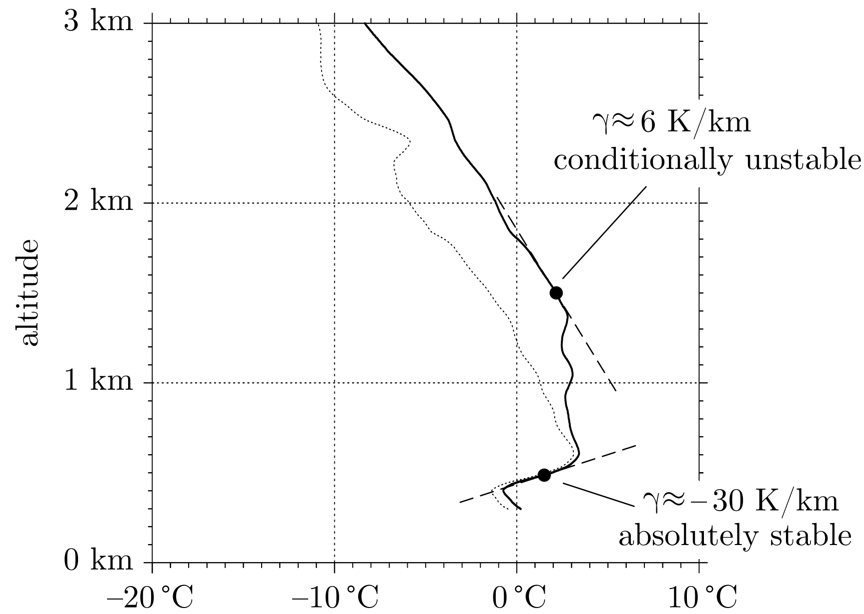
Most of the time, the layers in the atmosphere are absolutely stable or conditionally unstable. If instability does occur, convection is quickly triggered, which causes the air to mix in the vertical direction, after which a stable situation is re-established. In general, the instability will increase if the air up above becomes colder or the air below becomes warmer (in both cases, γ will increase).

Problem 8: Determine the static stability of the international standard atmosphere near the Earth's surface and at altitude of 11 km. Assume that the values of Γ_s near the surface and at 11 km are 4.9 K/km and 9.8 K/km.

Solution: In the ISA, the temperature decreases in the troposphere by $\gamma = 6.5$ K/km. A comparison of the values of Γ_a, Γ_s and γ shows that the ISA is conditionally unstable near the surface since $\Gamma_s < \gamma < \Gamma_a$. It is different at 11 km, where Γ_s is almost equal to Γ_a. There, the ISA is absolutely stable because $\gamma < \Gamma_s$.

Problem 9: Determine the static stability for the situation depicted in Figure 13. Determine the stability for layers at altitudes of 500 and 1500 m assuming that $\Gamma_s = 5$ K/km.

Solution: The easiest way to determine the static stability of an individual layer is to draw a short straight line at the relevant height, which approximates the tilt of the temperature curve as much as possible. Once the line is drawn, one can determine the value of γ by determining the tilt of the line. Then the value of γ can be compared to the values of Γ_a and Γ_s to identify the stability class.



At 500 m, there is a layer with a temperature inversion, where γ is about -30 K/km, which means that the layer is absolutely stable. At 1500 m, γ is about 6 K/km, which indicates that the layer is conditionally unstable.

14 Examples of Very Stable Layers in the Atmosphere

An example of a very stable layer is the stratosphere. This is an approximately 40 km thick layer above the troposphere in which the temperature is constant or increases with the altitude. Hence, γ is zero or negative, which means that γ is always much smaller than Γ_s. The stratosphere is, therefore, very stable, and because it is also very thick, it acts as a "lid" over the troposphere, preventing weather phenomena from penetrating from the troposphere into the layers above.

Another example of a very stable layer that frequently appears in the troposphere is a ***temperature inversion***. In an inversion, the temperature rises with the altitude – just the opposite of what usually occurs in the troposphere. Very common is the ***surface or ground temperature inversion***, which typically forms near the ground in the afternoon in clear and calm weather and can persist overnight. In clear weather, the ground begins to cool in the afternoon as the power of incoming solar radiation decreases, while the ground continues to emit infrared radiation. If the weather is calm (no wind), the air in the lowest layer in contact with the ground also cools. This layer is usually a few tens of meters thick, and the temperature near the ground can be a few degrees lower than at the top of the layer. In clear weather, such a layer generally disappears within a few hours after sunrise when the energy of solar radiation begins to heat the ground again. In such conditions, ***dew*** can occur. These are water droplets appearing on objects near to the ground formed by the condensation of water vapor from the air. It occurs when the surface temperature of objects is lower than the dewpoint temperature of the surrounding air. At night, the ground and objects close to the ground cool down the most (e.g., grass, low trees, or parked cars), so dew most often forms on them. If the temperature is below freezing, ***frost*** can form instead of dew. These are ice crystals that are created by the deposition of water vapor from the air. If the air contains enough water vapor, the radiation fog will also form in such a layer (see Section 18 for more details). On mountains, cold air begins to flow down the slopes in what is called mountain wind. If cold air flows into closed basins, cold-air pools can form (for more on mountain wind and cold-air pools, see Section 28).

Another example of a temperature inversion frequently observed in Slovenia and elsewhere is an ***upper-air temperature inversion***. Here, the vertical temperature profile is similar to the one shown in Figure 22.

In the lowest layer, just above the ground, the temperature first decreases with the altitude. The lower boundary of the inversion layer can be just a few 100 m above the ground, albeit it can also be much higher, and the temperature in the inversion can increase by more than 10 °C. Since the inversion layer is very stable, it acts as a "lid" over the layer below. The air below the inversion is thereby trapped, and while there may be strong winds above the inversion the air below the inversion can be stationary. Such a situation can persist for more than a week, which is a problem due to pollution. Namely, the pollution released by traffic, industry, and residential buildings remains trapped and accumulates in the lower layer. In Slovenia, this is especially a problem in winter when the heating of residential buildings releases a considerable amount of harmful

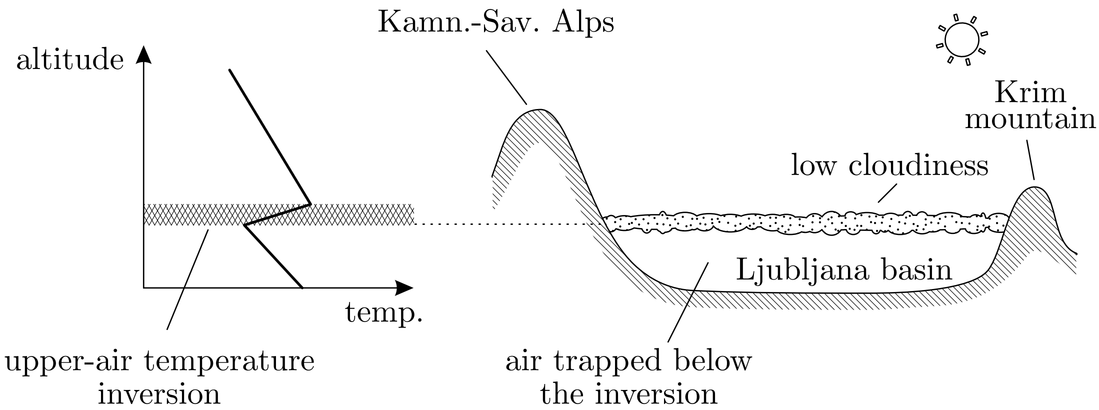


Figure 22: A typical situation and vertical temperature profile above the Ljubljana basin in the case of an upper-air temperature inversion.

particles (e.g., PM10 and PM2.5 particles). If the inversion is low and persists long enough, particle concentrations near ground level can exceed the guideline values deemed safe, which negatively affects health. This is most problematic in densely populated areas in narrow valleys (e.g., towns in the Zasavje region) where the available air volume below the inversion is small due to the proximity of the surrounding hills. Hence, concentrations increase faster than, for instance, in the Ljubljana Basin, which has a larger volume. At the bottom boundary of the inversion (where the temperature is the lowest), low cloudiness often forms (Figure 22). If the inversion occurs low enough, the nearby mountain tops may already be above the clouds. This explains why we can have gloomy, cold, and cloudy weather with increased pollution in the valleys and basins, whereas in the mountains the weather might be nice and sunny with a beautiful view of low clouds from above, looking like a sea made up of clouds.

An example of a real-world situation with an upper-air inversion is shown in Figure 13. In this case, the temperature inversion above Ljubljana started around 100 m above the ground and was about 200 m thick. In the inversion, the temperature increased by about 4 °C and the temperature at the top of the inversion was 3 °C, which was higher than on ground level. In the inversion layer, the temperature and dewpoint temperature were almost the same, indicating a very high probability of cloudiness. The clouds ended at an altitude of about 600 m and thus the top of the nearby Mount Saint Mary (Slovenian: Šmarna gora) with an altitude of 676 m was probably already above the clouds.

15 Convective Stability of the Atmosphere

As presented in Section 13, static stability determines the stability of individual layers of the atmosphere. Typically, however, we have several different layers in the troposphere, which are positioned one above the other and have varying stability. ***Convective stability*** represents the stability of the atmosphere (especially the troposphere) as a whole. In a convectively unstable atmosphere, there is a high probability that air (usually from the ground) will rise to a high altitude (perhaps even all the way up to the tropopause) by convection. A thunderstorm will form in the case of a fast and long-lasting ascent. If the atmosphere is convectively stable, the probability of a thunderstorm forming is low.

There are several different measures to express the magnitude of convective stability. ***Convective available potential energy*** (***CAPE***) is frequently used. CAPE has the unit J/kg and its value is typically determined from radiosonde measurements or the results of a numerical weather prediction model (for more on models, see Section 36). The exact definition and procedure for calculating the CAPE value is relatively complex and explained in finer detail in Appendix A.8. Fortunately, the CAPE value can also be illustrated in a simple graphical way.

Figure 23 on the left shows an example where the air on ground level has a temperature of 15 °C and a dewpoint temperature of 5 °C. The change in temperature of the air that would begin to rise from the ground is shown with solid lines, while the dashed line represents the ambient air temperature at different altitudes (this can be determined from radiosonde measurements). Up to the lifting condensation level, which is at an altitude of 1.2 km, the air would cool by 10 K/km. From there on, it would be saturated with moisture and the cooling would be slower, although it would come closer and closer to 10 K/km as the ascent continues. Initially, the rising air temperature would be lower

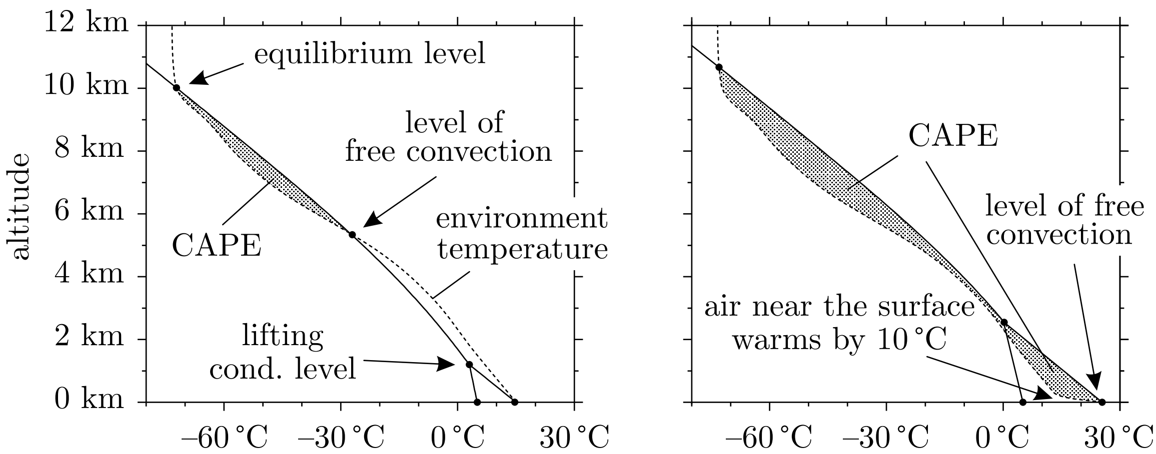


Figure 23: Change in the temperature of rising air (solid lines) and the ambient air temperature at different altitudes (dashed line). On the left is an example with a relatively small value of CAPE, and on the right is an identical example, except that the air on the ground warms by 10 °C and the CAPE is much larger. Details are described in the text.

than the surrounding temperature and the air could rise only if forced. This means that there would have to be some external cause for the lift. For example, the air would rise by being pushed up the side of the mountain. With enough ascent, the air would reach the ***level of free convection*** at approximately 5.3 km. There, it would become warmer than the air surrounding it. From here on, it would be free to rise by itself all the way to the equilibrium level. This level is at an altitude of approximately 10 km, where the air would again become cooler than the surrounding air, and the ascent would stop. The size of CAPE is proportional to the size of the area between the lines of the rising air temperature and the ambient temperature, but only where the condition for convection is present. The condition for convection (the air parcel temperature must be higher than the ambient temperature) is fulfilled between the level of free convection and the equilibrium level.

In Figure 23 on the left, the CAPE is relatively small, and the air would have to continue to be forced up to an altitude of 5.3 km before convection would occur. In the case shown on the right, the situation is precisely the same, except that the air near to the ground is warmer by 10 °C. Here, the CAPE increases considerably and the level of free convection drops almost to the ground. This means that forced lifting is no longer necessary and the air can rise by itself from ground level all the way up to the tropopause. Figure 23 illustrates a typical situation on a clear summer day. CAPE can be very small or even zero in the morning, yet during the day, due to incoming solar radiation, the ground and the air close to the ground warm up, increasing the CAPE. Namely, the atmosphere can be calm in the morning and thunderstorms can start forming in the afternoon, which only stop in the evening.

In Slovenia, on days with intense thunderstorms CAPE usually tends to be between 1000 and 2000 J/kg. In the USA, where thunderstorms can be more intense, CAPE can exceed 5000 J/kg in extreme cases. In general, CAPE is larger if the air on the lower layers is warmer and if the air in the upper layers is cooler. CAPE is also larger if the air near to the ground is more humid.

The ***Showalter index*** is also frequently used as a measure of convective stability. It is defined simply as $S_{\mathrm{ind}} = (T_{500} - T_{\mathrm{L500}})$. T_{500} is the air temperature at the altitude where the atmospheric pressure is equal to 500 hPa. T_{L500} is the temperature that the air would have if it rose adiabatically from an altitude where the atmospheric pressure is 850 hPa to an altitude where it is 500 hPa. Like CAPE, the Showalter index value is usually calculated from radiosonde measurements or numerical weather prediction model results. A negative value of the index (when the elevated air is warmer than the surrounding air) indicates the possibility of convection developing, while values smaller than −3 °C are mainly associated with the development of intense convection.

Generally, the colder it is above or the warmer or humid it is below, the greater the convective instability. The bigger the convective instability, the bigger the probability that thunderstorms will form and that they will be more intense. Still, large convective instability does not necessarily immediately lead to the formation of thunderstorms since convection must first be triggered. For convection to start, the air must rise at least a little in some other way. This is usually easy in Slovenia, which is quite mountainous, as the air can be forced up along the slopes of one of the many hills or mountains.

16 Cloud Formation

Clouds are formed when air rises and cools below the dewpoint temperature, and hydrometeors form. To a smaller extent, a cloud can also form from fog when it rises off the ground, albeit such a cloud cannot be very thick.

Hydrometeors in the atmosphere are always formed by water vapor molecules first accumulating on aerosol particles. These are microscopically small solid or liquid particles that cannot be seen with the naked eye (e.g., tiny crystals of sea salt, mineral dust, smog, and particles of biological origin). These particles fall very slowly through stationary air and thereby remain in it for a very long time. Their concentration varies considerably, but they are present everywhere in the atmosphere. Hydrometeors could also form in completely clean air in which there were no aerosol particles, but laboratory studies have shown that in this case the relative humidity should be very high (approximately 600%). This is due to the very small chance of a few molecules of water vapor accidentally colliding and sticking together long enough to form a hydrometeor. Wettable (hygrophilic) solid aerosol particles reduce the required relative humidity by allowing water vapor molecules to stick to their surfaces, which thereby more easily come into contact with other water vapor molecules. Figure 24 illustrates the formation of a droplet on a wettable solid particle. Some aerosols are made of substances that are soluble in water. In these, water vapor molecules are trapped and stored inside the particle, where the amount of water gradually increases until it becomes greater than the amount of aerosol material, forming a hydrometeor. Aerosol particles reduce the relative humidity required to form hydrometeors to about 100%, in some cases even to 80%. Since aerosols are present everywhere, hydrometeors always form on aerosols, and the relative humidity in the atmosphere never rises much above 100%.

Supercooled water can accumulate on aerosols at temperatures below freezing, forming ***supercooled droplets***. Supercooled water is water in liquid form that has a temperature lower than 0 °C. Droplets from supercooled water can exist at temperatures from 0 to −40 °C. Clouds at temperatures of up to a few degrees below freezing almost always consist of supercooled droplets, but at lower temperatures, there are fewer and fewer supercooled droplets as they often already freeze. The reason for the existence of supercooled droplets is related to what happens on a molecular scale and the fact that a solid aerosol particle's crystal structure often differs from the crystal structure of ice.

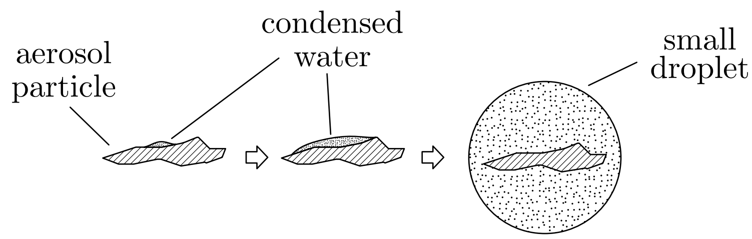


Figure 24: The formation of a droplet on a wettable solid particle

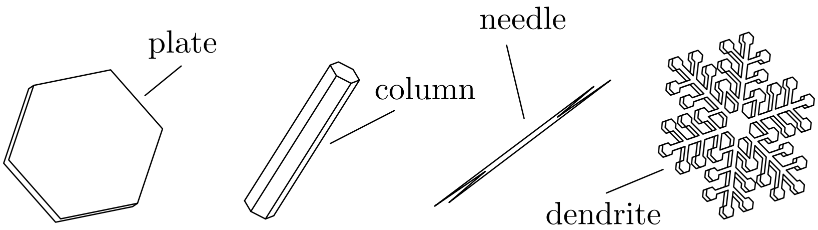


Figure 25: Examples of typical ice crystal shapes

Supercooled droplets are especially problematic for airplanes because they can freeze and stick to wings and fuselage as they collide with them. This effect is called aircraft icing and can lead to an increase in mass and deterioration of the aerodynamic shape of the aircraft. Particularly problematic are large supercooled droplets, which cause a relatively thick and heavy layer of ice that can quickly accumulate on the aircraft's body, so pilots must avoid areas where such droplets may be located. Supercooled drops can also freeze as they fall to the ground. They can also freeze on to tree branches, power lines, or other objects and buildings on the surface of the Earth. When enough ice accumulates, they can break branches, topple trees, or damage power lines. This situation occurred in Slovenia in February 2014 when damage caused by icing was estimated at around 430 million euros, with approximately half in forests and on forest roads.

Cloud droplets are very small and fall very slowly. The typical radius and falling speed of a cloud droplet are 10 µm and 1 cm/s. Given that drops fall so slowly, the clouds seems to maintain their shape and altitude, or even seem to rise, because when a cloud is formed the speed of the updraft is often greater than the speed of the drops falling. In a cloud exist many droplets – several hundred in every cubic centimeter. The fact the droplets are so numerous and so small that they cannot be detected with the naked eye makes us see the cloud as a semi-transparent solid object, even though it is actually composed of air and hydrometeors.

Similar to droplets, but at a temperature low enough that supercooled droplets are not created, ***ice crystals*** also form and grow. In this case, water is deposited on the aerosol particles in solid form, but the ice crystals are not spherical like droplets. The non-spherical shape is related to the crystal structure of ice in which the water molecules are partly arranged in hexagons. This is also reflected in the ice particle's shape, which usually displays certain hexagonal elements. The shapes of ice crystals can vary greatly and depend on the local conditions determining the crystal's growth. Figure 25 displays some typical ice crystal shapes.

17 Cloud Classification

Clouds can be distinguished by shape. Two shapes are the most common. The first is a layered (Latin *stratus*) form where the clouds consist of a relatively homogeneous layer of cloud cover without any distinct individual parts (Figure 26). The horizontal extent of such clouds is usually much larger than its vertical dimensions. The second is the heap or pile (Latin *cumulus*) form where the clouds generally have a flat lower edge, while the upper one is jagged in the form of towers or domes. In these clouds, the vertical extent is comparable to or greater than the horizontal one.

Clouds are also divided according to the altitude at which they are located. In the midlatitudes, low clouds are found at an altitude of 0.1 to 2 km, medium from 2 to 6 km, and high between 6 and 12 km. Another special distinction can be noted for vertically extensive clouds, which are usually at least several kilometers thick and tend to have their lower edge at the altitude of low clouds and their upper edge at the altitude of medium or high clouds.

Clouds are divided into ten main types or genera, as shown in Figure 27. High clouds are composed of ice particles and have the prefix "cirr" in their name. ***Cirrostratus*** is a thin, relatively homogeneous, semi-transparent cloud layer without distinct individual parts that covers all or part of the sky. If the Sun is behind a cloud, a halo is often observed – an optical effect on ice crystals in which a bright ring is visible around the Sun. ***Cirrus*** is a cloud in the form of white filaments, which often have a torn or fragmented appearance. ***Cirrocumulus*** is a cloud in the form of a thin semi-transparent patch or layer consisting of many smaller parts. These parts can be arranged in a more or less regular pattern.

Middle clouds have the prefix "alto" in their name. ***Altostratus*** is a fibrous or homogeneous cloud layer of white or gray appearance that covers the entire sky or just part of it and through which the Sun can be seen at least approximately, with no halo surrounding it. ***Altocumulus*** is a cloud that can take a range of shapes. In contrast to the altostratus, it is not fibrous or homogeneous but has a more pronounced shape. It can be a single larger cloud or consist of many smaller parts that can be arranged in a more or less regular pattern.

Low clouds do not have a special prefix in their name. ***Cumulus*** is a smaller, dense, dome-shaped cloud with a distinct border. It forms during convection and can grow very large in the event of strong convective instability. The parts of the cloud that are

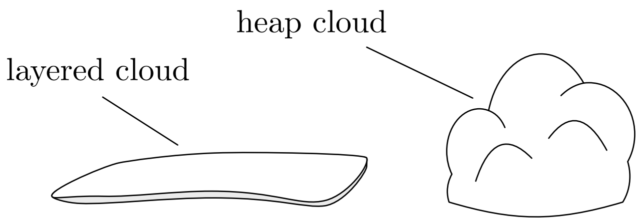


Figure 26: The two most common cloud shapes

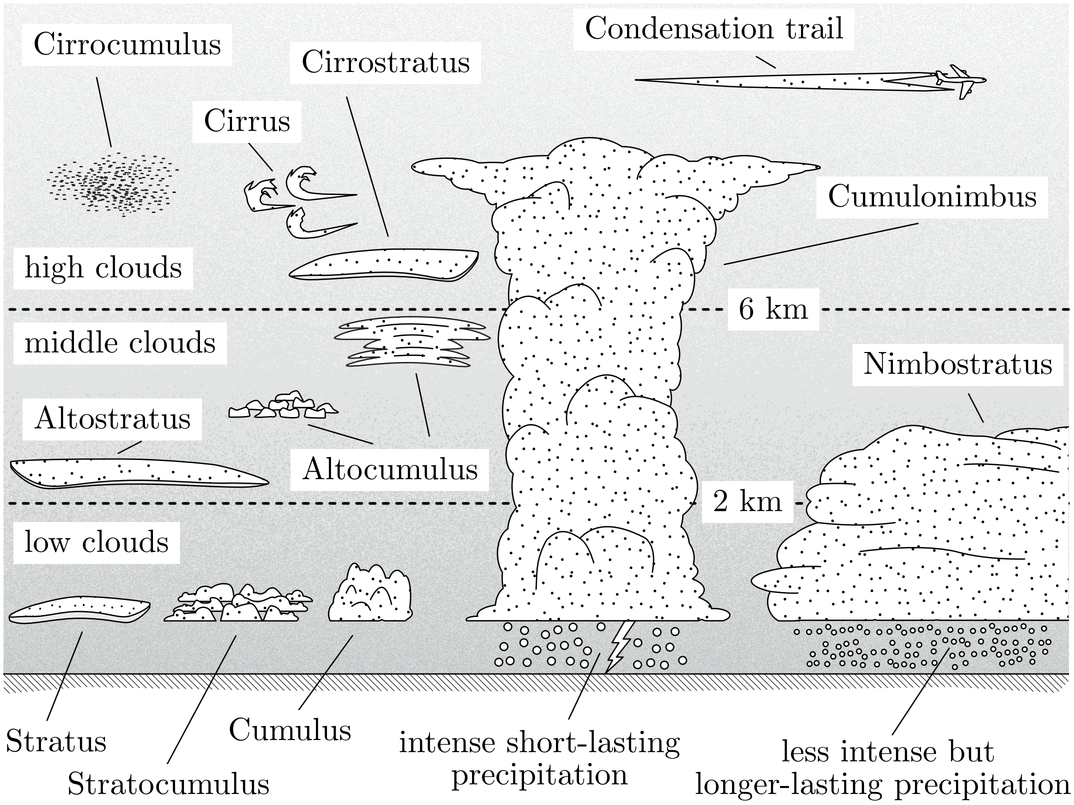


Figure 27: The main cloud types. Many photographs of clouds of all types are available on the World Meteorological Organization Cloud Atlas website (`https://cloudatlas.wmo.int/home.html`).

illuminated by the Sun are noticeably white, while the base of the cloud is darker and flat. ***Stratus*** is a cloud with a distinct edge and relatively homogeneous composition. While the Sun can be seen through it, there is no halo. ***Stratocumulus*** is a gray or whitish cloud layer, which, compared to stratus, is not homogeneous but consists of clearly distinguished smaller parts, which may or may not partly overlap and can be arranged in a more or less regular pattern.

Cumulonimbus and nimbostratus are thick clouds with precipitation falling from them. ***Cumulonimbus*** is a thunderstorm cloud that grows from a cumulus. It emerges in situations with large convective instability where the air rises very high by convection. If the air rises all the way up to the tropopause, the top of the cloud takes on a characteristic anvil shape. Precipitation from cumulonimbus comes in the form of showers – they can be very intense yet last only a short time. Such clouds are characterized by lightning and thunder, and hail may also fall from them. ***Nimbostratus*** is a thick cloud layer from which precipitation falls relatively steadily. It appears dark from below and is so thick that the Sun cannot be seen through it. Precipitation is less intense than with a cumulonimbus, although it is more uniform and tends to last much longer. Cumulonimbus and nimbostratus are the only clouds from which medium- to high-intensity precipitation can fall. Light precipitation may also fall from certain other clouds, albeit the amount of precipitation typically cannot be large.

A ***condensation trail***, or ***contrail*** for short, can also form behind airplanes while flying at high altitudes. These are special line-shaped clouds that form in the aircraft's exhaust following the release of water vapor during the combustion of kerosene. In the engine, the water vapor produced by combustion combines with the water vapor already present in the surrounding air. If the combined water vapor amount exceeds the saturation point, condensation and cloud formation will occur in the exhaust gases of the engine as they cool down behind the aircraft. Condensation trails usually form at high altitudes where the temperature is very low and it is easier to reach saturation point. If the air at the aircraft's altitude is very dry, condensation will not occur because there is not enough water vapor to attain saturation point. However, if the surrounding air is already almost saturated with moisture, the additional water vapor from the combustion causes a strong supersaturation, and a long white cloud trail being formed behind the aircraft, which can persist for several hours. In this case, if there are many airplanes the sky can become streaked with condensation trails.

18 Fog

Fog has a similar composition to a cloud, except that it is located very close to the ground. Depending on the formation, several types of fog may be distinguished. ***Radiation fog*** forms overnight in clear calm weather (Figure 28 on the right). On a clear night, the ground cools considerably due to the emission of heat by infrared radiation. If the night is calm (without wind), the air in contact with the ground also cools. A layer of cold air that is tens of meters thick forms near the surface of the Earth (usually, a ground

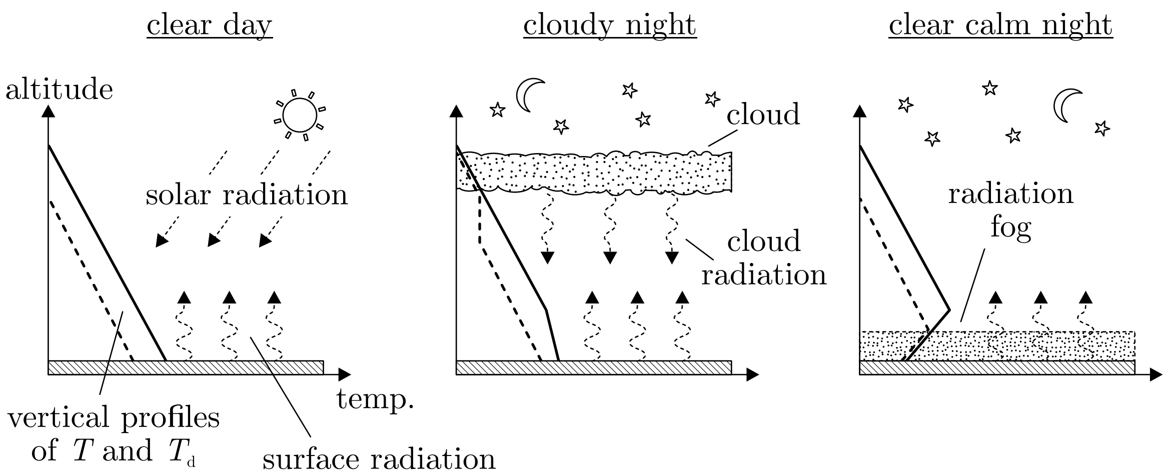


Figure 28: Vertical temperature (solid line) and dewpoint temperature (dashed line) profile near the ground during day and night in cloudy and clear weather. Radiation fog is formed on a clear, calm night when ground temperature inversion occurs.

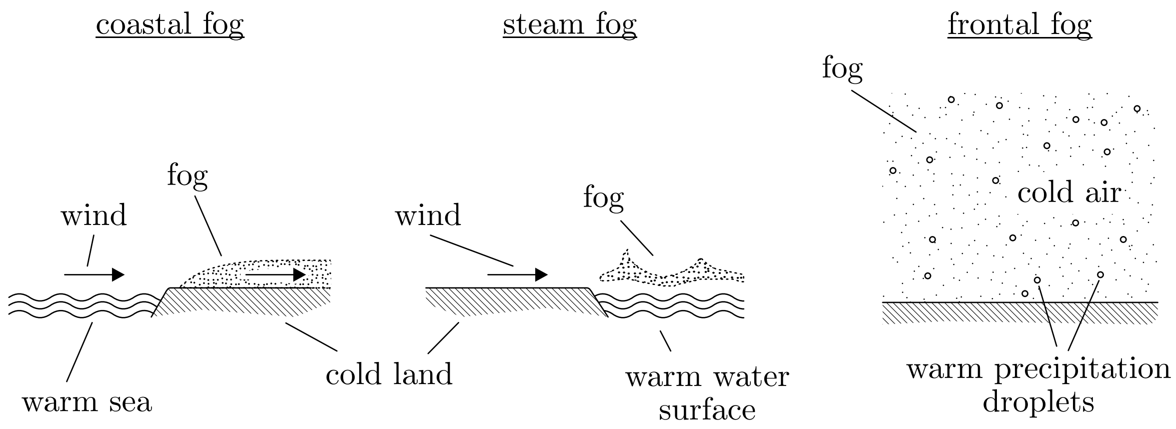


Figure 29: The formation of coastal, steam, and frontal fog.

temperature inversion occurs). Fog will form if the air cools sufficiently overnight and the temperature drops below the dewpoint. It generally disappears in the morning when solar radiation begins to heat the ground and the air temperature rises above the dewpoint temperature. Radiation fog only occurs in calm, clear weather. If the weather is cloudy, the ground will cool less overnight as it receives additional infrared radiation emitted by the clouds. In windy weather, the cold air near the ground mixes with the warmer air above, and the cooling is not as strong. This type of fog is very common in Slovenia. In closed basins, the cooling effect intensifies as cold air (which is heavier) also flows from the surrounding slopes to the bottom of the basin (see the description of mountain winds and cold-air pools in Section 28). Here, the temperature inversion is thicker and takes longer to dissolve (more solar radiation energy is needed to warm the air and ground sufficiently). In this way, the fog can stay for a long time – in some cases, even several days.

Advection fog is caused by wind that moves the air from one location to another where the ground temperature is different (Figure 29 – for more on advection, see Section 22). There are two types of advection fog. *Coastal fog* is created when warm and moist air is carried by wind over a cold surface. One example is the air over a warm sea being carried by the wind over cold land (it can also be the other way around if the sea is colder than the land). If the air in contact with the cold surface cools below its dewpoint temperature, fog will form. *Steam fog* is produced as cold air moves over warm and moist surfaces. One example is cold air carried by the wind over a warm lake or river. Given that the water is warm, the evaporation from it will be considerable, and condensation will occur in the cold air above. This can also be accompanied by small-scale convection, making it look as if steam is rising up from the surface.

Frontal fog is formed during rain when warm raindrops fall through cold air (this often happens in the case of weather fronts – for more on fronts, see Section 27). Like steam fog, water vapor evaporates from the warm droplets, and condensation occurs in the surrounding cold air, causing the fog.

Upslope fog is actually an orographic cloud (Figure 17) that forms low enough to touch the ground. It occurs when air is forced to rise along a mountain slope and cools below the dewpoint temperature, leading to condensation and the forming of fog.

19 Precipitation

In most cases, precipitation occurs when air rises sufficiently high and the hydrometeors in the clouds grow large enough to fall out of a cloud. As we mentioned in Section 16, cloud hydrometeors are small and fall slowly (their typical size is about 10 µm, and their fall speed is 1 cm/s). Precipitation hydrometeors are much bigger and therefore fall much faster. For example, the size of raindrops can be a few millimeters, with such drops falling through the air at a speed of a few m/s. Because they fall so fast, they can fall out of a cloud all the way down to the ground.

The initial growth of hydrometeors in a cloud occurs due to the continuous excess of water vapor as the saturated air cools due to rising. Excess water vapor molecules are deposited in hydrometeors during the ascent, which causes their growth (so-called diffusion growth). However, in this way alone, cloud hydrometeors cannot grow to a large enough size (i.e., a millimeter or more). Droplet coalescence also takes place in the cloud. Not all drops are precisely the same size. This happens since they are formed on aerosol particles of different shapes, sizes, and compositions. As the drops are not the same size, they fall through the air at slightly different speeds, which increases the likelihood of two drops touching or colliding. When this occurs, they merge (coalesce), and a new droplet is formed, which contains all the water from the initial two. Due to it being larger, this droplet falls even quicker and picks up more droplets (Figure 30). Droplets can in this way become the size of a few mm within a short time. When they grow to this size, they break up. The surface tension force maintains the droplet's spherical shape. Yet, as they fall through the air, the force of air drag causes the drops to flatten in a horizontal direction, further adding to the drag force. The high falling speed means the drag force of the biggest drops increases to the extent that the drop flattens so much that the surface tension cannot hold the droplet together, and it breaks up into several smaller parts (Figure 30). These continue to fall, and they can quickly grow back again and break up again as long as they remain in the cloud and can collide with other droplets.

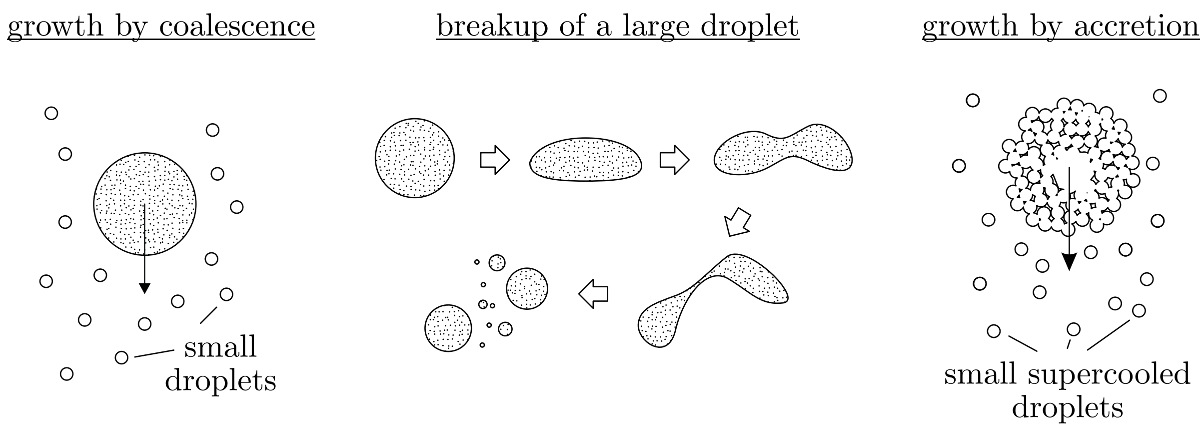


Figure 30: Growth of a bigger droplet due to coalescence, the breakup of a large droplet, and the growth of a larger ice particle due to the accretion of supercooled droplets.

A similar process occurs in the case of ice crystals, except they can stick together instead of coalescing. Crystals can fuse together when they collide (this is called sintering). If they have branches (e.g., a dendritic shape – Figure 25), they can also stick together if their branches interlock. When several ice crystals stick together, a larger snowflake is formed. The probability of two colliding ice crystals sticking together is greater at higher temperatures. Consequently, large snowflakes tend to form at near-freezing temperatures, while smaller ice particles tend to fall at lower temperatures.

The freezing of supercooled droplets is also important for the growth of ice particles. This happens when an ice particle drifts into an area containing small supercooled droplets. The process is similar to the coalescence of droplets with the difference being that the supercooled droplets freeze on the descending ice particle (Figure 30). When a small supercooled droplet touches the surface of an ice particle, it freezes to it very quickly. This is called growth by accretion or riming. Given that, unlike a droplet, an ice particle is solid, it does not break up easily and thus has no size limit, and is able to grow to a much bigger size. This is how large ice particles like ice pellets, snow pellets, and hail are formed.

When precipitation falls below the cloud base, it can begin to evaporate since the relative humidity there is typically less than 100%. If the air below the cloud is very dry, the precipitation can completely vaporize before it reaches the ground. In this case, ***virga*** can be seen under the cloud, which looks like a "band" or "curtain" of precipitation that comes out of the cloud yet does not reach the Earth's surface.

Precipitation can be divided by the type of hydrometeors that fall to the ground. In Slovenia, observers at meteorological stations classify precipitation in some of the following classes:

- ***drizzle*** – very small droplets with a diameter of less than 0.5 mm. Such precipitation usually falls from fog or very low clouds.
- ***rain*** – droplets with a diameter larger than 0.5 mm.
- ***snow*** – precipitation in the form of ice particles, which can be individual ice crystals of various shapes or crystals combined into clumps.
- ***rain with snow*** – rain and snow fall at the same time.
- ***snow pellets*** – also called graupel. Larger white, opaque grains with a diameter of up to 5 mm that resemble snow in structure, but are round or conical in shape and consist of a large number of small frozen droplets. They form by the accretion of supercooled droplets. If they drop onto a hard surface, they can break apart. They generally fall in the form of showers, together with snow or rain, and at temperatures of around 0 °C.
- ***ice pellets*** – larger transparent or semi-transparent ice particles, spherical or irregular in shape. Their diameter is up to 5 mm. They bounce on a hard surface and the impact can be heard. They may be frozen raindrops, melted snowflakes that have refrozen, or snow crystals covered with a thin crust of ice.

- ***hail*** – large ice particles with a diameter of over 5 mm. The grains can be transparent, semi-transparent, or opaque. Hail usually only forms in strong thunderstorms.

As mentioned, in the vast majority of cases, precipitation occurs with a long enough ascent, whereby the hydrometeors in the clouds must grow enough to fall out of a cloud. Depending on the intensity of the lifting, precipitation is divided into convective and stratiform.

Convective precipitation is precipitation from cumulonimbus clouds. In the core of the cloud lies a region of strong uplift, where air from lower layers of the troposphere rises very high very quickly. Convective precipitation falls in the form of showers. A ***shower*** can include very intense precipitation, but its duration is only short. Convective precipitation can include thunder, lightning, or even hail. In Slovenia, convective precipitation is very common and can lead to floods if several cumulonimbus clouds form over an area in a short period. One example is the flooding in the town of Železniki on September 18, 2007 when a large number of storms with heavy rainfall occurred in the hilly part of western Slovenia, with an especially huge large amount of rainfall falling in the upper part of the Selca Valley (Slovenian: Selška dolina). This led the Selca Sora river and its tributaries to flood extensively, causing substantial destruction on their way through the Selca Valley.

Stratiform precipitation is precipitation from nimbostratus clouds. In this case, the precipitation is not so intense but can last much longer. Depending on the cause of the ascent, stratiform precipitation can be divided into two main types. In ***frontal precipitation***, the air rises along a frontal plane (for more on weather fronts, see Section 27). The tilt of the frontal surface is relatively small, and the ascent is slow. ***Orographic precipitation*** is a result of air moving against the slope of a hill, causing it to rise. It can occur directly next to an orographic barrier on the windward side. Air rising to a higher plateau can also form it. A common example in Slovenia is precipitation in the case of a southwesterly wind when relatively moist air from the Adriatic/Mediterranean Sea rises over the Alpine–Dinaric barrier. While the air ascends by only a few hundred meters, this is enough to create a relatively thick cloud in which larger hydrometeors are gradually formed through droplet coalescence or ice particle fusion, which fall out as precipitation. Here, the precipitation is not only limited to the area directly along the plateau's edge on the windward side but can extend along the airflow far into the interior of Slovenia. Such precipitation is less intense than convective, but is more uniform and lasts for as long as the weather situation that causes it persists – which may be several days. That is why floods can occur in Slovenia following long-term stratiform precipitation. An example is the flooding between September 17 and 21 in 2010. With the prevailing southwesterly airflow, most of Slovenia experienced relatively strong and large-scale stratiform precipitation, leading to flooding across a considerable part of Slovenia.

It is important to stress that convective and stratiform precipitation often fall simultaneously. For example, stratiform precipitation is common along weather fronts, but at the same time convective instability means that individual thunderstorms can also form.

20 Thunderstorms

Thunderstorms emerge when there is sufficiently large convective instability, and air from the lower layers of the troposphere rises very high. This establishes a convective cumulonimbus cloud from which intense precipitation falls, accompanied by thunder. If the ascent continues all the way up to the tropopause, the cloud at the top takes the shape of an anvil. In a thunderstorm, there is a core area of strong ***updraft*** (an upward movement of air), while in the surrounding area the air is descending. In a severe thunderstorm, the updraft speed can be several tens of meters per second. A strong updraft is a precondition for larger ice particles like hail to form. The falling speed of larger ice particles is more than ten meters per second. If the speed of the updraft is low, the particles in it fall quicker than the air is rising, and they fall out of the cloud before they can grow to larger dimensions. However, if the updraft is strong, the particles can remain in the cloud for a longer time and thus become larger following the accretion of supercooled droplets. When they grow too large, or if they fall out of the updraft, or if the updraft weakens, they descend from the cloud as precipitation.

The least intense are ***single-cell thunderstorms***. Figure 31 shows the three phases of a single-cell thunderstorm. In the initial phase, the ascent starts similarly to what is shown in Figure 20. Free convection can be triggered when the air near the Earth's surface is relatively warm (e.g., due to incoming solar radiation, which heats the ground) while the air above is cold. As the warm air begins to rise, the warm air from the surrounding regions is also sucked toward the updraft region near the surface. A cumulus cloud forms, which grows. In the mature phase, the cumulus cloud reaches high and already contains large

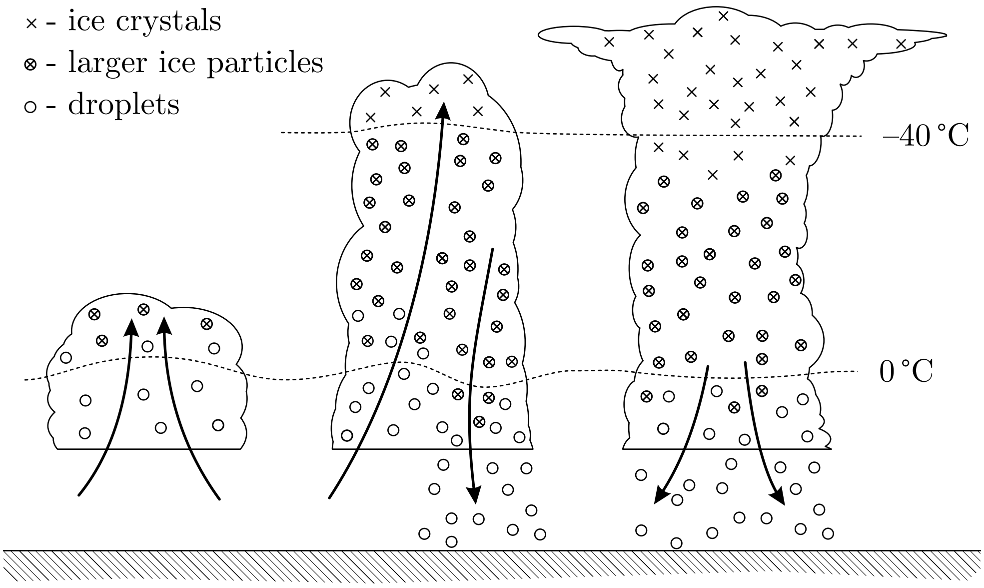


Figure 31: The three stages of a single-cell thunderstorm extending to the tropopause. From left to right: the initial phase with cumulus cloud formation, the mature phase with intense precipitation, and the dissipating phase.

precipitation particles, which cause intense precipitation on the ground. Different types of hydrometeors are typically found one above the other in the cloud. The droplets are the lowest; above the freezing level, there are supercooled droplets, snowflakes, and other larger ice particles. At the very top one finds smaller ice crystals. In the cloud, regions with ascending warm air and descending cooler air, from which most of the precipitation falls, are established. Electric discharges within different cloud regions and between the cloud and the ground occur, causing lightning and thunder. In the dissipating phase, when all the warm air near the ground has risen, the updraft ceases, and all remaining large hydrometeors fall out of the cloud. The whole process, from start to finish, usually takes 0.5 hour to 1 hour, depending on how high the air rises and the amount of warm air near the surface.

Multicell and supercell thunderstorms exists in addition to single-cell thunderstorms. They form when the horizontal wind changes significantly with altitude (called vertical wind shear). A ***multicell thunderstorm*** is characterized by the development of several single-cell storms in quick succession, with the preceding storm triggering the next. Successive storms occur so quickly and close to each other that it is difficult to separate them from each other, leading to the whole system being called a multicell storm. They generally move and last longer than single-cell storms and produce hailstones more often. Vertical wind shear is also required for the formation of ***supercell thunderstorms***. They are denoted by rotation in the horizontal direction, which occurs in specific wind shear conditions. Supercell storms last longer than single-cell storms, up to several hours, and often move and produce large hailstones. Although tornadoes can form near them, they occur much more often in the USA than in Europe. Supercell thunderstorms can also develop in Slovenia, but they are relatively rare. Such a storm was seen on June 8, 2018 near the town of Črnomelj in SE part of Slovenia. The storm produced very large hailstones with a diameter of over 8 cm, causing considerable destruction and damage exceeding 18 million euros. Many buildings were damaged, together with vehicles that were parked outdoors, as well as crops, fruit trees, and vineyards.

21 Scalar Fields, Isolines, and Gradients

This section is intended to provide a basic understanding of the mathematical concepts of scalar variables and fields, isolines, and gradients. Understanding these concepts is necessary for some of the content in the rest of the textbook. Since these are not necessarily part of the mathematics courses in secondary schools, they are briefly explained here.

An example of a ***scalar variable*** is temperature, which is also one of the basic meteorological variables. At any given time, we can define one temperature value at each location. This is also a general property of scalar variables – namely, they have only one value defined at a given time at a given location. Most basic meteorological variables are scalar; an exception is, e.g., wind, which is a three-dimensional vector and therefore defined by three values instead of one.

The distribution of scalar variable values in space is called a scalar field. The value of the temperature field at the point given by coordinates x, y, z at time t can be expressed as

$$T = T(x, y, z, t). \tag{22}$$

The temperature value thus depends on the location given by x, y, z and time t. ***Isolines*** can be defined for scalar fields. These lines connect locations with the same variable value at a particular moment in time. Isolines of different variables have different names (e.g., ***isotherms*** for temperature and ***isobars*** for pressure). Isolines allow us to visually illustrate the appearance of a scalar field in a simple way. Figure 32 shows an example of a simple scalar field for temperature over Slovenia, where the temperature increases uniformly towards the south.

A ***gradient*** can also be defined for a scalar field. It is denoted by the sign nabla (∇). A gradient is a vector mathematically defined as a variable's partial derivative over all

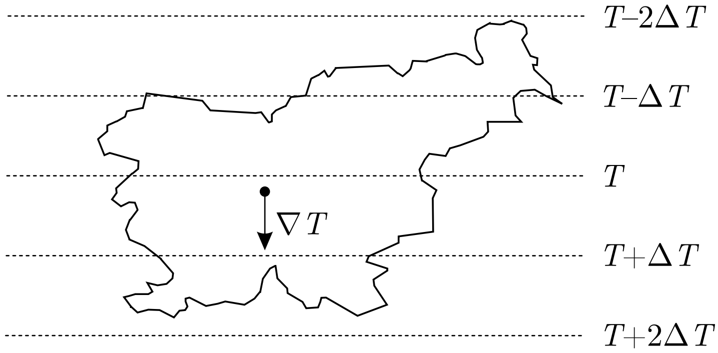


Figure 32: An example of a simple temperature field over Slovenia in which the temperature increases towards the south. The dashed lines show the isotherms, whereas the temperature gradient direction in the middle of Slovenia is indicated by the vector denoted by ∇T; in this case, the gradient has the same direction and magnitude everywhere.

the spatial coordinates. For example, the temperature gradient is defined as

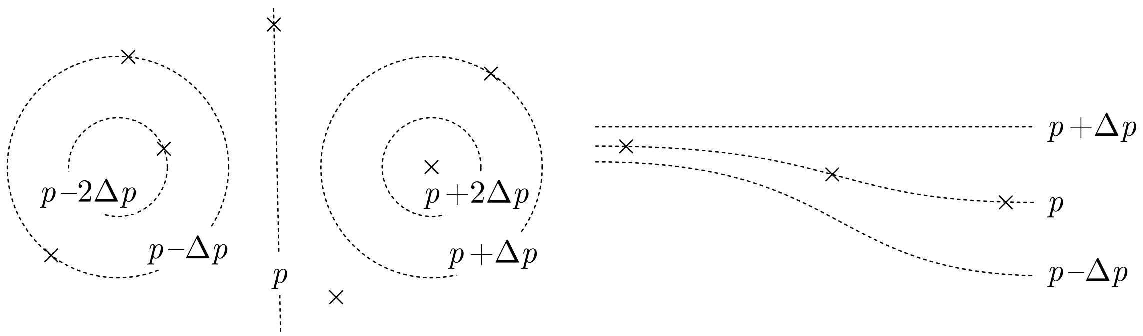
$$\nabla T = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z} \right), \tag{23}$$

where $\frac{\partial T}{\partial x}$, $\frac{\partial T}{\partial y}$ and $\frac{\partial T}{\partial z}$ are partial derivatives of temperature according to coordinates x, y and z. It can be proven mathematically (not shown here) that the gradient has some useful properties:

- the gradient vector always points in the direction of the maximum increase in the value of the variable;
- the gradient vector is always perpendicular to the isolines of the variable;
- the length of the gradient vector is proportional to the magnitude of the increase in the variable's value (in the direction of the gradient).

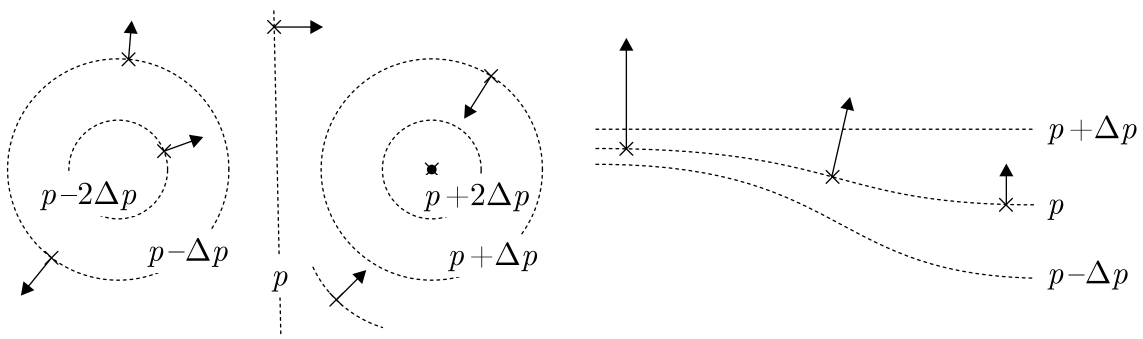
In Figure 32, the temperature gradient vector is also shown alongside the isotherms. Since the temperature increases the most towards the south, the gradient also points there. At the same time, it is also perpendicular to the isotherms, which run in an east–west direction. In the example shown, the gradient is the same everywhere over Slovenia, although this is not the case if, for instance, the isotherms were curved. The gradient vector can vary at different locations and change with time. Since the magnitude of the gradient vector is proportional to the magnitude of the variable value change in space, the gradient will be larger when the isolines are closer together.

Problem 10: The figure below shows the horizontal atmospheric pressure field. Draw the pressure gradient vectors at the locations indicated by the crosses. For the field on the left side, indicate only the direction of the gradient vector, while for the field on the right, also try to indicate the proportional magnitude of the gradient.



Solution: The left field shows the situation with a cyclone (an enclosed area of low atmospheric pressure) and with an anticyclone (an enclosed area of high atmospheric pressure). The gradient is always perpendicular to the isolines of the variable, which means the gradient vectors are always perpendicular to the isobars and point in the direction of increasing atmospheric pressure. A special case is the location in the center of the anticyclone, where the local maximum of atmospheric pressure is located. In this particular case, the gradient is zero (the vector has all components equal to 0) – in the figure showing the solution, this is indicated by a dot instead of a vector. Another slightly special case is the cross on the lower left below the anticyclone, which does not lie on any isobar. Here, we can start by first drawing what the isobar passing through the cross is most likely to look like and then draw the gradient perpendicular to that isobar.

In the field on the right, we determine the gradient directions in a similar way. At the same time, the relative magnitude of the gradient at different locations can be roughly determined from the distances between the isobars. Since the isobars on the right cross are approximately three times farther apart than on the left cross, the gradient vector on the left cross will be approximately three times longer.



Problem 11: Suppose that the horizontal temperature field over Slovenia can be described by the equation $T(x, y) = T_0 + a \cdot x + b \cdot y$, where the coordinate x points to the east, and y to the north. The parameters $T_0 = 10$ °C, a, and b have values $T_0 = 10$ °C, $a = -2$ °C/ $SI100km$, and $b = -1$ °C/100 km. The center of the coordinate system ($x = 0, y = 0$) is in Ljubljana. Determine the temperatures in Ljubljana, Maribor ($x = 85$ km$, y = 55$ km), and Koper ($x = -65$ km$, y = -55$ km). Calculate the temperature gradient vector. Draw a figure showing an outline of Slovenia, marking the direction of the isotherms and the temperature gradient.

Solution: The temperatures in Ljubljana and the other two cities are obtained by inserting the coordinates into the expression representing the field, which yields $T_{\mathrm{Lj}} = 10$ °C, $T_{\mathrm{Mb}} = 7.8$ °C and $T_{\mathrm{Kp}} = 11.9$ °C. The gradient vector is calculated according to Equation 23, taking only the first two coordinates since we simply have a horizontal field (without the z coordinate). The partial derivative in terms of x equals a, and in terms of y to b. We thus obtain

$$\nabla T = (\partial T/\partial x, \partial T/\partial y) = (a, b) = (-2\ ^\circ\mathrm{C}/100\ \mathrm{km}, -1\ ^\circ\mathrm{C}/100\ \mathrm{km}).$$

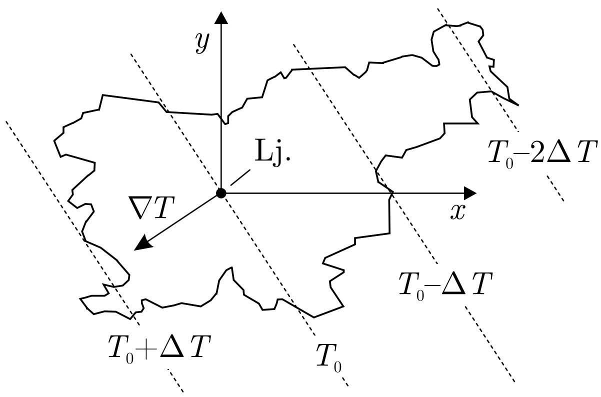
We can also express the equation for individual isotherms from the expression for the field. For example, for an isotherm with temperature T_0 (10 °C), we place T_0 on the left side of the expression and obtain

$$T_0 = T_0 + a \cdot x + b \cdot y.$$

Then we express the y coordinate as a function of x

$$y = -a/b \cdot x = -\frac{-2\ ^\circ\mathrm{C}/100\ \mathrm{km}}{-1\ ^\circ\mathrm{C}/100\ \mathrm{km}} \cdot x = -2x.$$

This is the equation for a line that passes through the origin of the coordinate system and has a direction coefficient of -2. Similarly, we can also calculate the equation for the isotherm with a slightly higher temperature $T_0 + \Delta T$, where we obtain $y = -a/b \cdot x + \Delta T/b$, which is the same line as for the isotherm T_0, except that it is shifted slightly to the south because $\Delta T/b$ is negative. Based on these results, we can visualize the temperature field over Slovenia where we mark the direction of the isotherms and the gradient perpendicular to the isotherms.



22 Advection

Let us imagine the situation shown in Figure 33. The temperature over Slovenia increases towards the south, and we consider three possible winds with the same speed: south wind ($\vec{v}_1$), west wind ($\vec{v}_2$), and southwest wind ($\vec{v}_3$). We are interested in the expected temperature change at Ljubljana. If a southerly wind blows, it will bring air from the area south of Ljubljana. Warmer air is located there, meaning the temperature at Ljubljana will rise. A westerly wind will bring in air from the west, which has the same temperature as the current air in Ljubljana. The temperature will hence not change in this case. If a southwesterly wind blows, the temperature will rise similarly as for the southerly wind, except the temperature increase will be slower because the wind does not blow perpendicular to the isotherms.

Such changes in the value of meteorological variables caused by the wind are called advective changes. The value of the variable changes since wind brings air with different properties to a specific location. It should be emphasized that the properties of individual air parcels do not necessarily change. In the example presented in Figure 33, the temperature of individual air parcels does not change. Still, a temperature change occurs at a particular location simply because the wind brings in air with a different temperature.

The rate of temperature change at a specific location can be described mathematically with the advection equation (the derivation of the equation is provided in Appendix A.9)

$$\left(\frac{\partial T}{\partial t}\right) = -|\vec{v}| \cdot |\nabla T| \cdot \cos\varphi + \left(\frac{\mathrm{d}T}{\mathrm{d}t}\right). \tag{24}$$

The term $\left(\frac{\partial T}{\partial t}\right)$ represents the temperature change rate at a certain location, the term $\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right)$ the temperature change rate of individual air parcels (which move with the wind), which is given by the equation 12. The term $-|\vec{v}| \cdot |\nabla T| \cdot \cos\varphi$ represents the advection effect, where $|\vec{v}|$ is the wind speed, $|\nabla T|$ the magnitude of the temperature gradient, and φ the angle between the wind and temperature gradient vectors.

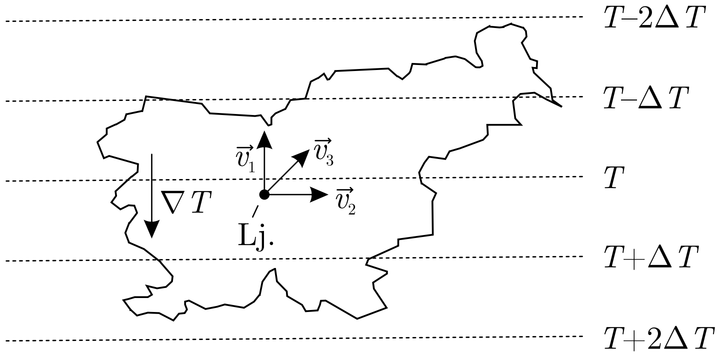
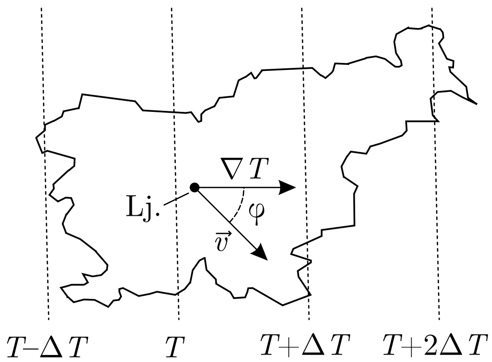


Figure 33: An example of temperature advection over Slovenia

Problem 12: Over Slovenia, the temperature decreases from east to west by 2 °C/100 km. The northwesterly wind is blowing at a speed of 10 m/s. At 10 am, the temperature in Ljubljana is 10 °C. Sketch the temperature field over Slovenia, and indicate the direction of the temperature gradient and wind vectors. Calculate what the temperature in Ljubljana will be at 1 pm if: a) the weather is cloudy and the temperature of air parcels is constant; and b) the weather is sunny, and the temperature of the air parcels increases by 1 °C/h due to strong solar radiation.

Solution: In the situation described, the isotherms run in a north–south direction, the temperature gradient points to the east, and the wind vector to the southeast:



We use the advection equation (Equation 24) to calculate the temperature. The angle between the temperature gradient and the wind vectors is $\varphi = 45°$. In case: a) the air does not receive or emit energy from the surroundings, the air parcel temperature is constant, and the term $(\mathrm{d}T/\mathrm{d}t)$ is equal to zero. The temperature change rate at a specific location is therefore

$$\left(\frac{\partial T}{\partial t}\right) = -|\vec{v}| \cdot |\nabla T| \cdot \cos\varphi = -10 \text{ m/s} \cdot 2 \text{ °C}/(10^5 \text{ m}) \cdot \cos 45°$$
$$= -1.41 \cdot 10^{-4} \text{ °C/s} = -0.51 \text{ °C/h}.$$

The temperature, therefore, decreases by about half a degree Celsius per hour. Given that it is 10 °C at 10 am in Ljubljana, the temperature 3 hours later will be

$$T_{\text{Lj 1pm}} = T_{\text{Lj 10am}} + \left(\frac{\partial T}{\partial t}\right) \cdot \Delta t = 10 \text{ °C} - 0.51 \text{ °C/h} \cdot 3 \text{ h} = 8.47 \text{ °C}.$$

In case b), in addition to the advection effect, the air parcel temperature increases due to solar radiation by $(\mathrm{d}T/\mathrm{d}t) = 1$ °C/h and the rate of the temperature change it is

$$\left(\frac{\partial T}{\partial t}\right) = -|\vec{v}| \cdot |\nabla T| \cdot \cos\varphi + \left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = -0.51 \text{ °C/h} + 1 \text{ °C/h} = 0.49 \text{ °C/h}.$$

In this case, the temperature in Ljubljana after 3 hours will be 11.47 °C.

23 Forces and Momentum Equation

Although the previous sections already stated something about what causes the movement of air in the vertical direction, mainly due to convection, nothing was mentioned about the causes of movement in the horizontal direction. The wind is caused by differences in atmospheric pressure, primarily caused by temperature differences between different regions, mainly due to the uneven distribution of solar radiation energy on Earth. For example, differences in the incidence angle of solar radiation mean that tropical regions receive the most radiation energy, and polar regions the least (for more on the energy of incident radiation, see Section 32). As a result, there is a significant temperature difference between tropical and polar regions, which also causes a difference in atmospheric pressure, with one of the consequences being winds on a global scale (for more on the general circulation of the atmosphere, see Section 35). In order to have a good understanding of the phenomenon of wind, it is first necessary to comprehend and describe the causes that affect the movement of each air parcel.

The wind is the movement of air and Newton's second law can be used to describe the movement of an air parcel

$$\sum \vec{F}_i = m \cdot \vec{a}, \tag{25}$$

where m represents the mass of the air parcel, $\vec{a}$ its acceleration, and $\sum \vec{F}_i$ the sum of all forces acting on it. Expressing the acceleration using the above equation yields

$$\vec{a} = \sum \vec{F}_i / m = \sum \vec{f}_i, \tag{26}$$

where $\vec{f}_i$ are the specific forces (forces divided by the mass of the air parcel) acting on the parcel. Acceleration is defined as the change in velocity over time ($\vec{a} = \mathrm{d}\vec{v}/\mathrm{d}t$), thus

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \sum \vec{f}_i. \tag{27}$$

The most important forces for describing the movement of air are the pressure gradient force $\vec{f}_{\mathrm{grad}}$, the Coriolis force $\vec{f}_{\mathrm{Cor}}$, the gravity force $\vec{f}_{\mathrm{g}}$, and the frictional force $\vec{f}_{\mathrm{fr}}$. Equation 27 can therefore be rewritten as

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \vec{f}_{\mathrm{grad}} + \vec{f}_{\mathrm{Cor}} + \vec{f}_{\mathrm{g}} + \vec{f}_{\mathrm{fr}}. \tag{28}$$

Equation 28 is one of the basic meteorological equations and is called a ***momentum equation*** because it describes the movement of air. A more detailed description of each force follows, while Table 3 presents an overview of the forces and some of their properties.

Force	Notation	Direction	Magnitude
Pressure grad. force	$\vec{f}_{\mathrm{grad}}$	towards lower atmospheric pressure	$\frac{1}{\rho}\cdot\lvert\nabla p\rvert$
Coriolis force	$\vec{f}_{\mathrm{Cor}}$	to the right in the N Hemisphere to the left in S Hemisphere	$f\cdot v$
Frictional force	$\vec{f}_{\mathrm{fr}}$	backwards	$k_{\mathrm{fr}}\cdot v$
Gravity force	$\vec{f}_{\mathrm{g}}$	downward	g
Centrifugal force	$\vec{f}_{\mathrm{cent}}$	outwards	$\frac{v^2}{R_{\mathrm{u}}}$

Table 3: List of specific forces acting on air and some of their properties. A more detailed description of the forces appears in the text. Centrifugal force is explained in more detail in Section 24 in the description of a gradient wind.

Pressure gradient force is caused by differences in atmospheric pressure. The force always points in the direction of the fastest decrease in pressure, i.e., opposite to the direction of the atmospheric pressure gradient. The magnitude of the force is equal to $1/\rho\cdot\lvert\nabla p\rvert$, where $\lvert\nabla p\rvert$ is the magnitude of the atmospheric pressure gradient. A detailed presentation of this force lies beyond the scope of this book, although the force's existence can be explained by the situation displayed in Figure 34. The small square in the picture represents an air parcel. The atmospheric pressure increases to the left and is greater on the left edge of the parcel than on the right, i.e., the left side of the air parcel feels a larger pressure force than the right. The imbalance of forces between the left and right sides manifests as a gradient force directed towards lower atmospheric pressure.

The ***gravity force*** on the rotating Earth is the sum of the gravitational force $\vec{f}_{\mathrm{grav}}$ and the centrifugal force caused by the rotation of the Earth $\vec{f}_{\mathrm{cent.rot.Earth}}$.

$$\vec{f}_{\mathrm{g}} = \vec{f}_{\mathrm{grav}} + \vec{f}_{\mathrm{cent.rot.Earth}}. \tag{29}$$

The gravitational force always points towards the center of the Earth, while the centrifugal force is perpendicular to the axis of the Earth's rotation (Figure 35). On Earth, the

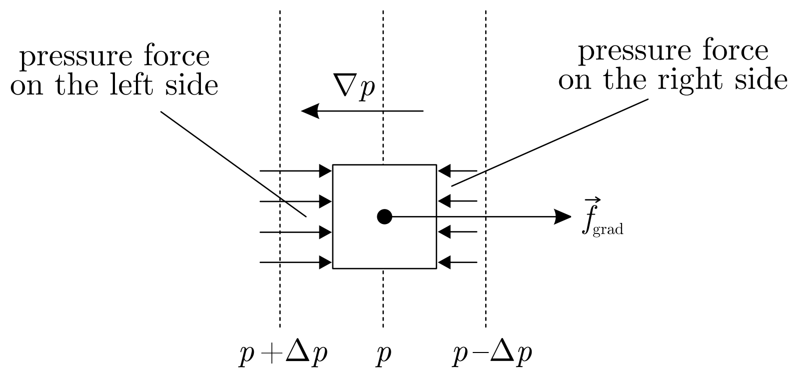


Figure 34: The pressure gradient force exhibited on an air parcel (see the text).

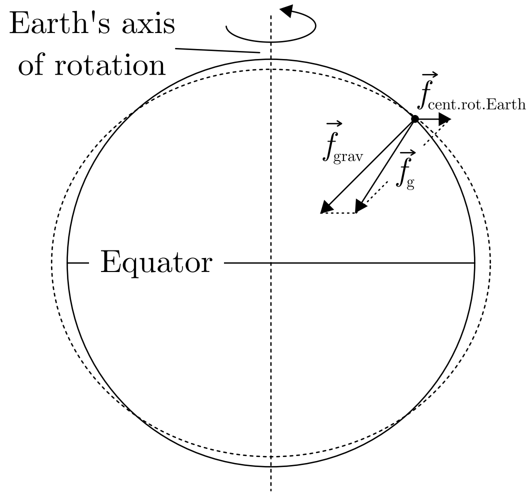


Figure 35: The direction of the gravity force ($\vec{f}_{\text{g}}$) in midlatitudes. The dashed ellipse shows the Earth's flattened shape. The magnitude of the flattened shape and magnitude of the centrifugal force (with respect to gravitational force) are exaggerated in the figure.

centrifugal force is always much weaker than the gravitational force, yet it still causes the force of gravity not to point precisely toward the center of the Earth (except at the poles and the equator). Due to the centrifugal force, the Earth is also somewhat flattened and has an ellipsoidal shape instead of a perfectly spherical one. The flattening effect is not very large, but still noticeable – the radius of the Earth in the equatorial direction is about 0.3% larger than the radius in the polar direction. Namely, the gravity force is biggest at the poles (where there is no centrifugal force) and smallest at the equator (where the centrifugal force is the largest and points opposite to the gravitational force). The force always points downwards toward the ground. The force's magnitude is denoted by g, for which we adopt the standard value 9.81 m/s^2 (which roughly corresponds to the value in midlatitudes at the Earth's surface). At the equator, the value of g is smaller, while at the poles, it is larger (g also decreases with altitude).

Coriolis force is one of the apparent forces that develop due to the rotation of the Earth. A detailed explanation and the derivation of the corresponding expression for the force is quite complicated and will be omitted here. We focus only on the horizontal component of this force, which significantly influences horizontal winds. The magnitude of the horizontal component of the force can be expressed as $f \cdot v$, where v is the wind speed, and f is the Coriolis parameter defined as

$$f = 2\omega \sin \varphi, \tag{30}$$

where ω is the angular velocity of the Earth's rotation ($\omega = 2\pi/24\ \text{h} = 7.27 \cdot 10^{-5}\ \text{s}^{-1}$) and φ is the latitude. In the northern hemisphere, the Coriolis force always points 90° to the right of the direction of movement (wind – Figure 36), and in the southern hemisphere by 90° to the left of the direction of movement.

The force is nonzero only if the air is moving with respect to the Earth's surface. At the equator ($\varphi = 0°$), there is no Coriolis force because the Coriolis parameter is zero

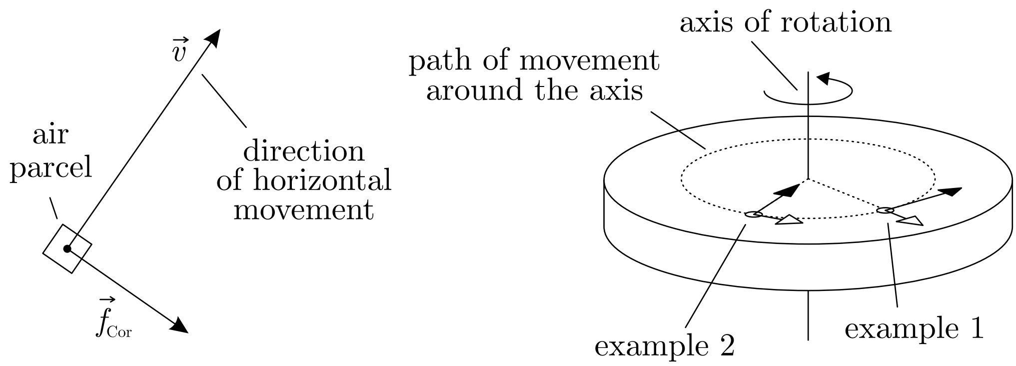


Figure 36: Left: the direction of the Coriolis force ($\vec{f}_{\mathrm{Cor}}$) in the northern hemisphere, pointing 90° to the right of the direction of motion. In the southern hemisphere, the force points in the opposite direction (90° to the left of the direction of motion). Right: Illustration of the formation of the Coriolis force on a flat carousel (for an explanation, see the description in the text).

there. For a fixed wind speed, the magnitude of the force increases with the latitude and is greatest at the poles.

The appearance of the Coriolis force can be roughly explained with the help of a flat carousel. Figure 36 shows a flat carousel with two examples of movement. In both examples, the carousel is rotating and thus all the bodies on it rotate around the rotation axis and feel the centrifugal force (not shown in the picture), which always points away from the axis of rotation. In the first example, as well as the rotation, there is an additional movement in the direction of the rotation (presented with a solid arrow). Such movement seemingly increases the circumferential rotation speed around the axis and causes an apparent increase in the magnitude of the centrifugal force. This increase in centrifugal force can be understood as the Coriolis force, which here points away from the axis of rotation (shown by an empty arrow), i.e., 90° to the right of the direction of motion on the carousel. The second example shows a movement toward the carousel's center. A body moving in this way comes from a region of higher rotational velocity to an area of lower velocity. There it is "too fast" with respect to the speed of the rotating floor, and it is swept towards the right. The effect is similar, as if some additional force appeared that tried to move the body to the right. The existence of Coriolis force may be explained similarly for the Earth, except that the planet is spherical while the carousel in the example is flat.

Frictional force occurs primarily near the ground. In the case of horizontal wind, turbulent eddies form in the layer close to the ground, which inhibits the free movement of air and reduces its speed (for more on this, see Section 26). The force always points in the direction opposite to the direction of the movement (wind). In its simplest form, the magnitude of the frictional force can be expressed as $k_{\mathrm{fr}} \cdot v$, where k_{fr} is the friction coefficient. k_{fr} tends to be the largest near the Earth's surface and decreases with height. The size of k_{fr} also largely depends on the static stability and the surface roughness (e.g., k_{fr} will be larger for forests than for grasslands).

24 Balanced Flow Winds

Winds in the atmosphere form due to forces acting on individual air parcels. It is helpful to introduce the balanced flow concept to understand the horizontal winds that frequently occur at mid and high latitudes. Balanced flow winds are horizontal winds that would appear at a specific location if the sum of all the forces acting on individual air parcels was equal to zero. Given that in this case the forces would be in balance, such winds are called balanced flow winds. Although the equilibrium assumption does not necessarily hold true and real-world winds may be different, such winds well approximate the large-scale winds that appear at mid and high latitudes.

Since we assume that the winds are horizontal, we can ignore the gravity force, which only has a component in the vertical direction and thereby cannot directly affect the movement in the horizontal direction. For now, we also limit ourselves to altitudes high enough above the ground that mean the frictional force can be neglected. Depending on which forces are balanced, several different types of balanced flow winds exist.

The wind that corresponds to the balance of the pressure gradient and Coriolis force is called ***geostrophic wind***. Here, Equation 28 may be simplified to

$$\vec{f}_{\text{grad}} + \vec{f}_{\text{Cor}} = 0. \tag{31}$$

The pressure gradient force always points perpendicularly to the isobars in the direction of decreasing pressure. Given that the sum of all forces in Equation 31 is zero, the Coriolis force must be equal in magnitude and opposite in direction to the pressure gradient force. Since the Coriolis force is always perpendicular to the direction of motion, the wind must blow along the isobars. An additional condition is that the isobars are straight because, in the event of a curved path, the centrifugal force would also be present (more on this force in the remainder of the section).

Figure 37 shows an example of such a situation in the Northern Hemisphere. The Coriolis force points to the right of the wind direction, meaning that the wind must blow in such a way that the lower pressure is on the left side (in the southern hemisphere, it is the other way around). For the sum of the forces to be zero, the magnitudes of the pressure gradient force and the Coriolis force must be the same:

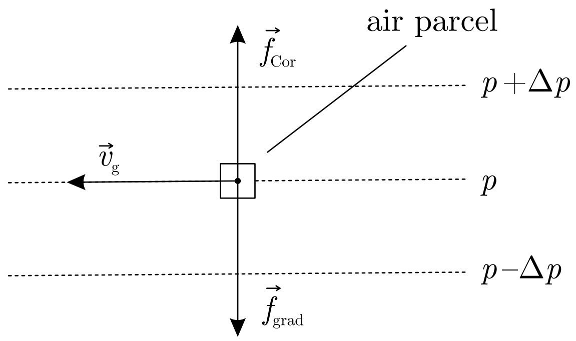


Figure 37: The situation in which a geostrophic wind blows in the Northern Hemisphere.

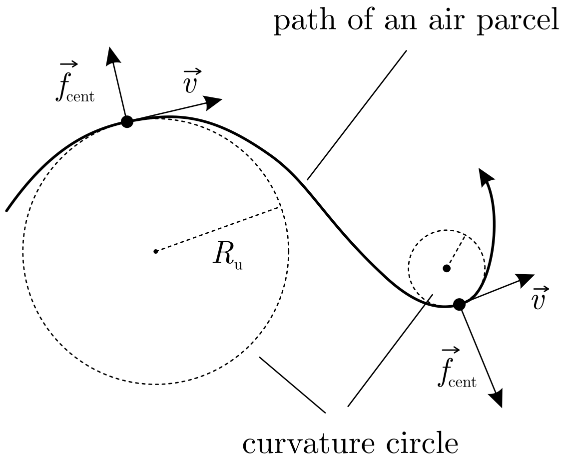


Figure 38: An example of the movement of an air parcel with a curved trajectory with the centrifugal force indicated ($\vec{f}_{\text{cent}}$).

$$\begin{aligned} \left|\vec{f}_{\text{grad}}\right| &= \left|\vec{f}_{\text{Cor}}\right|, \\ \frac{1}{\rho}|\nabla p| &= f \cdot v_{\text{g}}, \end{aligned} \tag{32}$$

where v_{g} represents the speed of the geostrophic wind, which can be expressed from Equations 32 as

$$v_{\text{g}} = \frac{1}{\rho \cdot f}|\nabla p|. \tag{33}$$

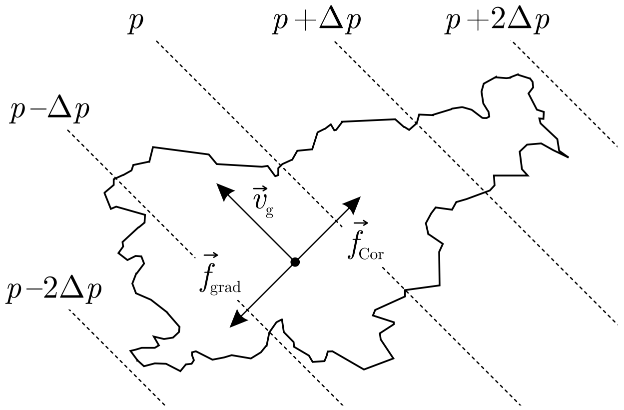
The speed of the geostrophic wind therefore depends on the magnitude of the atmospheric pressure gradient (the larger it is, the greater the speed), the air density (the smaller it is, the greater the speed), and the latitude, which is represented in the Coriolis parameter. Geostrophic wind cannot exist near the equator because the Coriolis force there is zero. The geostrophic wind thus exists only in mid and high latitudes in regions where the isobars are straight. Equation 33 also indicates why winds at high altitudes tend to be faster than those closer to the ground – the air is less dense higher up than near the ground. In addition, for reasons that will not be discussed here, relatively large pressure gradients often occur near the boundary between the troposphere and stratosphere. ***Jet streams*** often occur there, which are very strong, mostly westerly winds whose speed frequently exceeds 200 km/h.

Another type of balanced flow wind is the ***gradient wind*** that blows along curved isobars. A complete derivation of this wind is quite complex and will not be shown here. The derivation requires the introduction of a natural coordinate system in which the direction of the coordinates changes in time and place, and one of the coordinates is always pointing in the direction of motion, and the other perpendicular to the motion (a detailed derivation is available in [6]).

If the path of the air parcel is curved, an additional apparent force is present – the ***centrifugal force***. Figure 38 displays a situation where the trajectory of an air parcel

Problem 13: Over Slovenia (latitude of 45°), atmospheric pressure decreases in the southwest direction by 2 hPa/100 km. At a height of 5 km, the temperature is −20 °C and the atmospheric pressure is 500 hPa. Sketch the pressure field above Slovenia, indicate the forces acting on an air parcel, and determine the direction and speed of the geostrophic wind.

Solution: Since the atmospheric pressure decreases in the southwest direction, the isobars will run perpendicular to this direction, i.e., in the northwest–southeast direction. The gradient force points towards lower pressure (toward the southwest). Since the forces need to be balanced, the Coriolis force must point in the opposite direction, towards the northeast. Since Slovenia is located in the northern hemisphere, the Coriolis force points to the right of the wind direction, which means the wind must blow to the northwest.



The speed of the geostrophic wind is calculated with Equation 33. However, the air density and the Coriolis parameter must be calculated before this equation can be used. The latter is calculated according to Equation 30

$$f = 2\omega \sin\varphi = 2 \cdot 7.27 \cdot 10^{-5}\ \mathrm{s}^{-1} \cdot \sin 45^\circ = 1.03 \cdot 10^{-4}\ \mathrm{s}^{-1}.$$

Air density is calculated from the ideal gas law (Equation 2), from which the density can be expressed

$$\rho = \frac{p}{RT} = \frac{5 \cdot 10^4\ \mathrm{Pa}}{287\ \mathrm{J/(kg\,K)} \cdot 253\ \mathrm{K}} = 0.69\ \mathrm{kg/m^3}.$$

The geostrophic wind speed is

$$v_{\mathrm{g}} = \frac{1}{\rho \cdot f}|\nabla p| = \frac{200\ \mathrm{Pa}/10^5\ \mathrm{m}}{0.69\ \mathrm{kg/m^3} \cdot 1.03 \cdot 10^{-4}\ \mathrm{s}^{-1}} = 28.1\ \mathrm{m/s}.$$

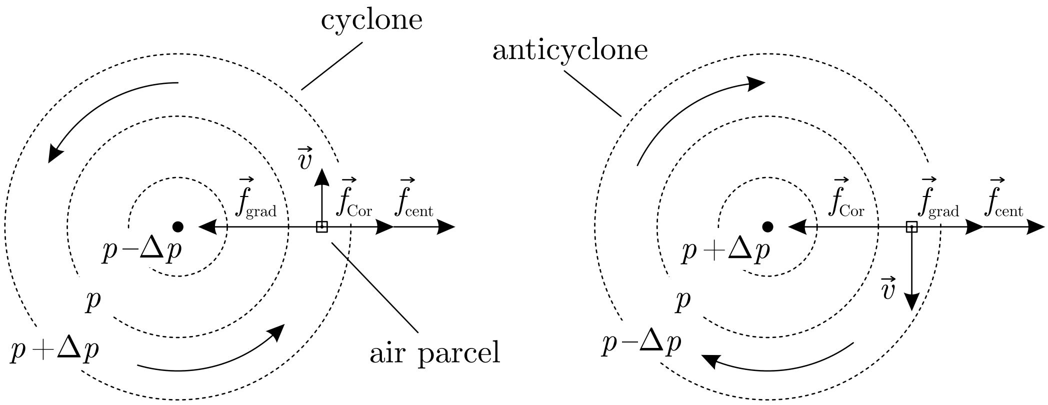


Figure 39: Isobars and the direction of the circulating air (curved arrows) in a cyclone and anticyclone in the Northern Hemisphere. Also shown are the forces acting on an air parcel and its velocity vector.

first turns to the right and later to the left. Centrifugal force is always perpendicular to the movement (wind) and points outwards –"out of a bend" (a similar force is felt in a car as it goes around a bend in the road). The magnitude of the centrifugal force is v^2/R_{u}, where R_{u} is the radius of curvature, which can be visualized by a curvature circle (Figure 38). The smaller the radius of the curvature, the more curved the path is, and the greater the centrifugal force. In the case of a straight path, the radius is infinitely large, and there is no centrifugal force.

A gradient wind blows when the pressure gradient, Coriolis, and centrifugal forces are in balance – all three must also be nonzero. The centrifugal force is nonzero only when the path is curved while the Coriolis force is nonzero only in regions not close to the equator. Namely, a gradient wind can be present in mid and high latitudes when the isobars are curved. Like the geostrophic wind, it always blows along isobars, except they are curved. Typical examples of a gradient wind are midlatitude cyclones and anticyclones.

A midlatitude ***cyclone*** is a large area of low atmospheric pressure that often occurs in midlatitudes. The isobars in the cyclone have a more or less circular shape and are closed in on themselves, while the atmospheric pressure drops towards the cyclone's center (Figure 39). The horizontal dimension of the cyclone ranges from a few 100 to a few 1000 km. The atmospheric pressure in the cyclone's center is usually lower than 990 hPa, whereas in strong cyclones, especially in the southern hemisphere, it can even be below 950 hPa (for more on typical weather in cyclones, see Sections 25 and 27). A detailed explanation of the causes of the formation of cyclones and anticyclones is beyond the scope of this textbook. Nevertheless, the gradient wind approximates the winds in them quite well. In a cyclone, the pressure gradient force points toward the inside of the cyclone, while the Coriolis and centrifugal forces point outwards (Figure 39). Since the Coriolis force in the northern hemisphere points to the right of the wind direction, the air in the cyclone circulates in a counterclockwise direction (in the southern hemisphere, it is the other way around). Similarly to the geostrophic wind, the gradient wind speed in the cyclone can be determined from the balance of forces:

$$\left|\vec{f}_{\text{grad}}\right| = \left|\vec{f}_{\text{Cor}}\right| + \left|\vec{f}_{\text{cent}}\right|, \tag{34}$$

$$\frac{1}{\rho}|\nabla p| = f \cdot v + \frac{v^2}{R_{\text{u}}}. \tag{35}$$

Equation 35 is a quadratic equation with respect to wind speed where only a solution with a positive value of the root is physically meaningful

$$v = \frac{1}{2}\left(-fR_{\text{u}} + \sqrt{f^2R_{\text{u}}^2 + 4R_{\text{u}}|\nabla p|/\rho}\right). \tag{36}$$

Like with the geostrophic wind, the speed of the gradient wind depends on the magnitude of the atmospheric pressure gradient (the larger it is, the greater the speed) and the air density (the smaller it is, the greater the speed).

The situation is somewhat different in a midlatitude ***anticyclone***; namely, a large area of high atmospheric pressure. The isobars in the anticyclone have a roughly circular shape and are closed in on themselves, and the pressure rises towards the center of the anticyclone (Figure 39). The horizontal dimension of an anticyclone is usually larger than that of a cyclone (for more on typical weather in anticyclones, see Section 25). Like in cyclones, the winds in it can be well approximated by gradient winds. The pressure gradient and centrifugal forces point toward the outside of the anticyclone, while the Coriolis force points inward. Given that the Coriolis force in the Northern Hemisphere points to the right of the wind direction, the air in an anticyclone circulates clockwise (in the Southern Hemisphere, it is the other way around). Similar to a cyclone, the gradient wind speed in an anticyclone can be determined through the balance of forces

$$\left|\vec{f}_{\text{Cor}}\right| = \left|\vec{f}_{\text{grad}}\right| + \left|\vec{f}_{\text{cent}}\right|, \tag{37}$$

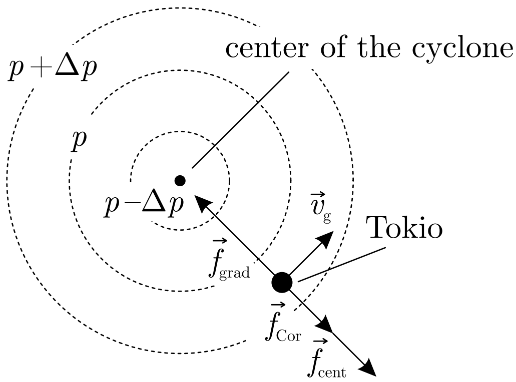
$$f \cdot v = \frac{1}{\rho}|\nabla p| + \frac{v^2}{R_{\text{u}}}. \tag{38}$$

Equation 38 is again a quadratic equation with respect to wind speed. Both solutions are positive, but only a solution with a negative root value appears in nature

$$v = \frac{1}{2}\left(fR_{\text{u}} - \sqrt{f^2R_{\text{u}}^2 - 4R_{\text{u}}|\nabla p|/\rho}\right). \tag{39}$$

Problem 14: Japan is experiencing a circular cyclone in which the magnitude of the atmospheric pressure gradient is equal to 3 hPa/200 km. The cyclone's center is located 200 km northwest of Tokyo. At an altitude of 3 km above Tokyo, the temperature is 0 °C and the atmospheric pressure is 700 hPa. Sketch the isobars above Tokyo, indicate the forces acting on an air parcel, and determine the direction and speed of the gradient wind.

Solution: Since the center of the cyclone is in the northwest direction and noting that the wind in a northern hemisphere cyclone blows in a counterclockwise direction, a southwesterly wind will be present above Tokyo. Although the isobars are circular, they run in the southwest–northeast direction above Tokyo. The pressure gradient force points to the center of the cyclone, while the Coriolis force and the centrifugal force point in the opposite direction.



The wind speed is calculated using Equation 36. Still, before this equation can be used, the air density and the Coriolis parameter must be calculated. The latter is calculated according to Equation 30

$$f = 2\omega \sin\varphi = 2 \cdot 7.27 \cdot 10^{-5}\ \mathrm{s}^{-1} \cdot \sin 36^\circ = 8.55 \cdot 10^{-5}\ \mathrm{s}^{-1}.$$

Air density is calculated from the ideal gas law (Equation 2) as

$$\rho = \frac{p}{RT} = \frac{7 \cdot 10^4\ \mathrm{Pa}}{287\ \mathrm{J/(kg\,K)} \cdot 273\ \mathrm{K}} = 0.89\ \mathrm{kg/m^3}.$$

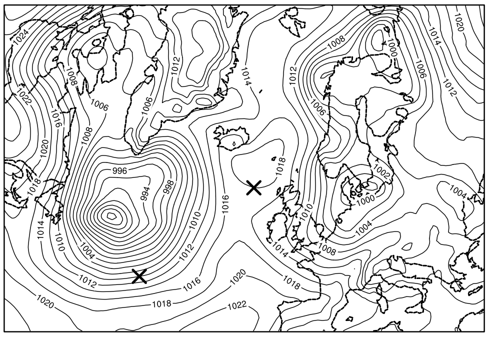
Since the cyclone is circular and the winds blow along the circular isobars, the radius of curvature is equal to the distance from the center of the cyclone ($R_\mathrm{u} = 200$ km), and the wind speed is

$$v = \frac{1}{2}\left(-fR_\mathrm{u} + \sqrt{f^2R_\mathrm{u}^2 + 4R_\mathrm{u}|\nabla p|/\rho}\right) = \frac{1}{2}\Bigg(-8.55 \cdot 10^{-5}\ \mathrm{s}^{-1} \cdot 2 \cdot 10^5\ \mathrm{m} +$$

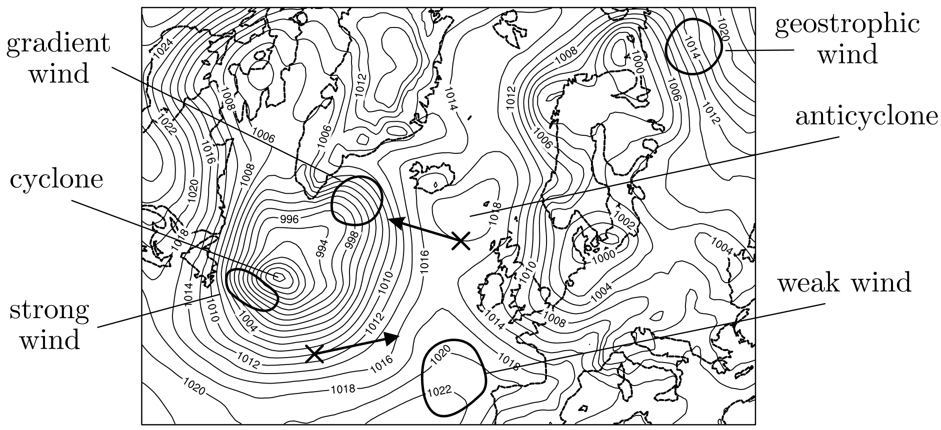
$$+ \sqrt{\left(8.55 \cdot 10^{-5}\ \mathrm{s}^{-1}\right)^2 \cdot \left(2 \cdot 10^5\ \mathrm{m}\right)^2 + \frac{4 \cdot 2 \cdot 10^5\ \mathrm{m} \cdot 300\ \mathrm{Pa}/(2 \cdot 10^5\ \mathrm{m})}{0.89\ \mathrm{kg/m^3}}}\ \Bigg)$$

$$= 11.7\ \mathrm{m/s}.$$

Problem 15: The figure below shows the sea-level atmospheric pressure over the North Atlantic and Europe. Isobars shown in units of hPa are plotted every 2 hPa. On the figure, mark one example of a cyclone and an anticyclone. At the two locations marked with crosses indicate the direction of the balanced flow wind that would blow if the frictional force is ignored. Further, mark an example of regions where the geostrophic or gradient wind blow and another example with strong and weak winds.



Solution: The solution is shown in the figure below. A cyclone is an enclosed area of low atmospheric pressure. Several cyclones are present in the figure, although the most prominent one is located over the Atlantic to the east of Newfoundland. In the center of this cyclone, the pressure is slightly lower than 985 hPa. An anticyclone is an enclosed area of high pressure. Several anticyclones are present in the figure – one is located between Great Britain and Iceland, with atmospheric pressure exceeding 1018 hPa in the center. Bold arrows show the wind direction at locations marked with crosses. In the absence of the frictional force, the balanced flow winds always blow along the isobars. In a cyclone, the air circulates counterclockwise, while the opposite is true for an anticyclone (this is the case in the Northern Hemisphere). Geostrophic winds are present where the isobars are straight – an example is part of Russia in the upper right corner. Gradient winds are present where the isobars are curved – an example of strongly curved isobars is in the northeastern part of the cyclone mentioned above. The strongest winds are where the atmospheric pressure gradient is largest (where the isobars are close together) – for example, in the southwestern part of the aforementioned cyclone. Conversely, the wind will be weak where the isobars are far apart – for example, in the northwest of the Iberian Peninsula.



As for a cyclone, the higher the atmospheric pressure gradient and the lower the air density, the higher the wind speed will be. In contrast to a cyclone, an additional condition must be satisfied that the expression under the root in Equation 39 is positive. Thus

$$f^2 R_\mathrm{u}^2 - 4R_\mathrm{u} |\nabla p| / \rho > 0, \tag{40}$$

which can be rearranged into

$$|\nabla p| < \frac{f^2 R_\mathrm{u} \rho}{4}. \tag{41}$$

This means that the atmospheric pressure gradient in the anticyclone cannot be too large. Cyclones do not have this condition, which explains why the pressure gradients in anticyclones are smaller and hence anticyclones are usually larger than cyclones. The fact that pressure gradients are smaller, especially near the center where R_u is small, means the horizontal wind speeds in anticyclones are typically lower than in cyclones.

25 Influence of Friction on Balanced Flow Winds

In Section 24, we ignored the influence of friction on balanced flow winds. The frictional force mainly occurs near the ground, meaning that balanced winds blow slightly differently there than at high altitudes.

It is easiest to describe how friction affects the geostrophic wind. As before, the geostrophic wind blows in the case of straight isobars. In addition to the pressure gradient and Coriolis forces, a frictional force is present close to the ground. It points in the opposite direction of the wind and inhibits motion, resulting in a slower wind speed. This affects the Coriolis force because its magnitude depends on the wind speed and, as the speed decreases, the magnitude of the Coriolis force also decreases. This causes the wind direction to deviate from the direction parallel to the isobars towards the lower pressure (Figure 40). All three forces need to be balanced and can be split into components in two directions. It is easiest to orient the coordinate system such that the x axis points in the direction of the frictional force and the y axis in the direction of the Coriolis force. If the angle by which the wind deviates from the isobars is denoted by β, we can express the two components of the pressure gradient force (Figure 40 right) as

$$\begin{aligned} \left|\vec{f}_\mathrm{grad.x}\right| &= \left|\vec{f}_\mathrm{grad}\right| \cdot \sin\beta, \\ \left|\vec{f}_\mathrm{grad.y}\right| &= \left|\vec{f}_\mathrm{grad}\right| \cdot \cos\beta. \end{aligned} \tag{42}$$

For the wind to be balanced, the sum of the forces in both directions must be zero, thus

$$\begin{aligned} \left|\vec{f}_\mathrm{fr}\right| &= \left|\vec{f}_\mathrm{grad.x}\right|, \\ \left|\vec{f}_\mathrm{Cor}\right| &= \left|\vec{f}_\mathrm{grad.y}\right|, \end{aligned} \tag{43}$$

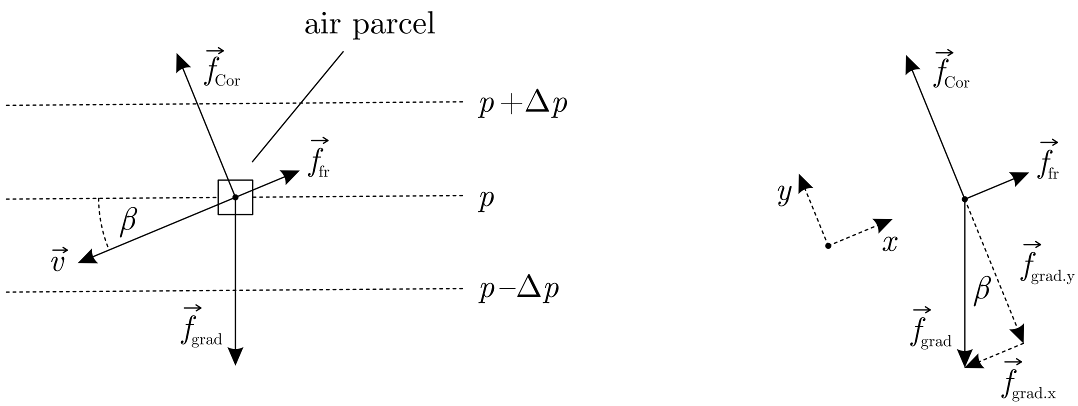


Figure 40: The influence of friction on the geostrophic wind. The forces, wind, and isobars are shown on the left. On the right, the gradient force is split into two components in the direction of the x and y axes.

through which we obtain a system of two equations for two unknown variables (v and β)

$$k_{\mathrm{fr}} \cdot v = \frac{1}{\rho}|\nabla p| \cdot \sin\beta, \tag{44}$$

$$f \cdot v = \frac{1}{\rho}|\nabla p| \cdot \cos\beta. \tag{45}$$

Dividing the equations 44 and 45 and taking into account the identity $\tan\beta = \frac{\sin\beta}{\cos\beta}$ gives us

$$\tan\beta = k_{\mathrm{fr}}/f, \tag{46}$$

and

$$\beta = \arctan\left(k_{\mathrm{fr}}/f\right). \tag{47}$$

It can be seen from Equation 47 that the larger the friction coefficient, the larger the deflection of the wind from the isobars. Since the friction coefficient tends to decrease with height, the wind will be most deflected near the ground and will become more parallel to the isobars as the height increases.

Expressing v from equation 45, yields

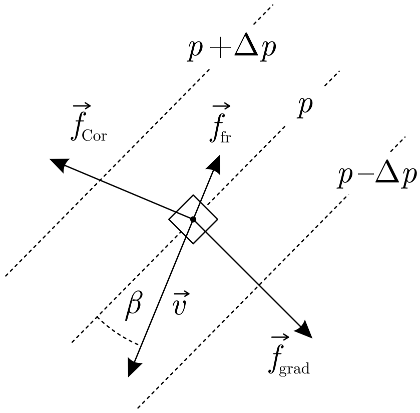
$$v = \frac{1}{\rho \cdot f}|\nabla p| \cdot \cos\beta = v_{\mathrm{g}} \cdot \cos\beta, \tag{48}$$

where v_{g} is the frictionless geostrophic wind speed (Equation 33). Since $\cos\beta$ can only be less than or equal to 1, the speed of the geostrophic wind in the case of friction cannot be greater than v_{g}. This means the frictional force does indeed reduce the speed of the geostrophic wind.

Like the geostrophic wind, the gradient wind is also affected by friction. In the case of friction, the gradient wind slows down as well and deviates from the isobars towards low pressure (for a more detailed explanation, please refer to [6]), which significantly affects the airflow in cyclones and anticyclones. Near the ground, friction causes the air

Problem 16: Over Iceland, atmospheric pressure increases in the northwest direction by 2 hPa/100 km. In Reykjavík (at a latitude of 64°), the temperature is 5 °C and the atmospheric pressure 1008 hPa. Draw a field of isobars above Iceland, indicate the forces acting on an air parcel located near the ground, and indicate the direction of the geostrophic wind. In addition, calculate the geostrophic wind's speed and angle of deviation from the isobars. Assume that the friction coefficient is 10^{-4} s^{-1}.

Solution: As the atmospheric pressure increases towards the northwest, the isobars will run perpendicular to this direction, i.e., southwest–northeast. The pressure gradient force points to lower pressure, i.e., towards the southeast. Due to friction, the wind deviates from the isobar towards the lower pressure by an angle of β. The Coriolis force points to the right of the wind, and the friction force points backwards.



The deviation of the geostrophic wind from the isobars and its speed can be calculated from equations 47 and 48. Yet, before these equations can be used, the air density and the Coriolis parameter values must be calculated. The latter is calculated with equation 30

$$f = 2\omega \sin\varphi = 2 \cdot 7.27 \cdot 10^{-5}\ \text{s}^{-1} \cdot \sin 64° = 1.30 \cdot 10^{-4}\ \text{s}^{-1}.$$

Air density is calculated from the ideal-gas law (Equation 2) as

$$\rho = \frac{p}{RT} = \frac{100\,800\ \text{Pa}}{287\ \text{J/(kg K)} \cdot 278\ \text{K}} = 1.26\ \text{kg/m}^3.$$

The deviation of the geostrophic wind from the isobars and its speed are

$$\beta = \arctan\left(k_{\text{fr}}/f\right) = \arctan\left(\frac{10^{-4}\ \text{s}^{-1}}{1.30 \cdot 10^{-4}\ \text{s}^{-1}}\right) = 37.6°$$

$$v = \frac{1}{\rho \cdot f}|\nabla p| \cdot \cos\beta = \frac{200\ \text{Pa}/10^5\ \text{m}}{1.26\ \text{kg/m}^3 \cdot 1.30 \cdot 10^{-4}\ \text{s}^{-1}} \cdot \cos 37.6° = 9.67\ \text{m/s}.$$

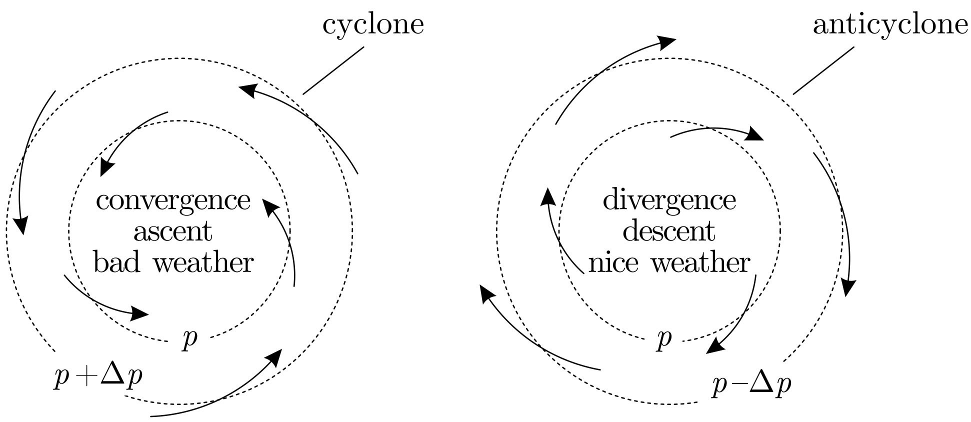


Figure 41: Airflow near the ground in cyclones and anticyclones

in cyclones to flow toward the cyclone's center (Figure 41). The air already present in the central part of the cyclone rises to make room for the inflow of air from the sides. As it rises, it cools, which can lead to the forming of clouds and precipitation, resulting in bad weather. The exact opposite happens in an anticyclone, where the air near the ground flows from the center outwards. In the central part of the anticyclone, the air is descending from up above as it has to replace the air that is flowing away near the ground. As the air descends, it warms up and thus an anticyclone is usually associated with nice weather.

26 The Surface Layer and Turbulence

The ***atmospheric boundary layer***, or ***boundary layer*** for short, is the lowest layer of the atmosphere closest to the ground where the influence of the surface is the greatest – an example is the influence of friction on balanced flow winds as presented in Section 25. Above the boundary layer is the ***free atmosphere*** where the influence of the Earth's surface is small (e.g., here, geostrophic and gradient winds blow along isobars). The thickness of the boundary layer can vary greatly. It is generally about 1 or 2 km, yet in some situations the thickness can be considerably smaller or larger (from only a few tens of meters to more than 4 km).

The influence of the surface is the greatest in the ***surface layer***, which usually comprises the lower 10% of the boundary layer. The thickness of the surface layer can also vary a lot and largely depends on the weather situation, especially the static stability of the air in the surface layer and the speed of the winds up above – its thickness can range from a few to several hundred meters.

The wind speed is significantly reduced in the surface layer. This is primarily a consequence of viscosity that causes the air in direct contact with the surface of objects (e.g., the ground, vegetation, buildings) to remain completely still. The layer where the air is still is very thin (only a fraction of a millimeter), albeit the effect causes strong wind shear. The expression ***wind shear*** is used to denote a situation where the wind

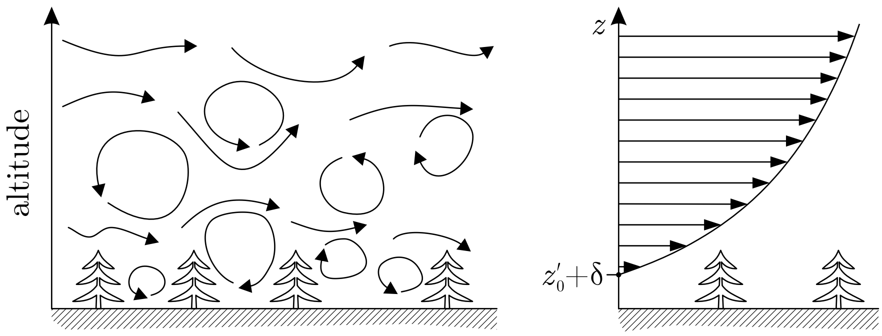


Figure 42: Left: An example of a turbulent airflow over a forest in the surface layer. Turbulent eddies are formed in the flow due to wind shear. Right: the logarithmic wind profile given by equation 49.

changes considerably over a short distance (e.g., the air directly next to a surface is at rest, while the air a little farther away can already be moving). Wind shear causes turbulence. ***Turbulence*** is the irregular and unpredictable mixing of air, represented by short-lived eddies of various sizes, lasting from a fraction of a second to a minute (Figure 42). In a turbulent flow, eddies are constantly forming, changing, dissipating, and forming anew. Turbulence in the surface layer is generally caused by the wind up above that tries to force the air below to move as well, causing wind shear near the surface. Eddies obstruct the airflow, which manifests as a frictional force present in the boundary layer, which tends to decrease with height. Turbulence causes the mean horizontal wind speed in the surface layer to increase approximately logarithmically with the height (the so-called 'logarithmic wind profile').

The turbulence will be stronger if the surface is more rough/uneven or the air must flow around many obstacles (e.g., high vegetation and buildings, and tall waves at sea). If the wind speed above the surface layer is low, there will also tend to be less turbulence because there will be less wind shear. Turbulence is also significantly affected by the static stability in the surface layer. In the case of high stability, the turbulence will be weaker since the vertical movements will be damped, making it harder for eddies to form. On a clear day, due to heating by solar radiation, the surface layer can become unstable, while a very stable temperature inversion can form near the ground during a clear night.

Strong wind shear can additionally occur at high altitudes, for instance, near thunderstorms or jet streams; thus, turbulence is not limited to the boundary layer. For example, in thunderstorms, there is a core region with a strong updraft, while in the immediate vicinity, the air can be stationary or even descending (Figure 31). The term ***clear air turbulence*** (CAT) is often used for turbulence that happens at high altitudes in clear weather, particularly near jet streams.

In the surface layer, the change of the mean wind speed with height can be approximated by a logarithmic wind profile defined as

$$v(z) = \frac{v_*}{k} \ln\left(\frac{z-\delta}{z_0}\right), \tag{49}$$

where k is a constant with a value of approximately 0.4, z_0 is a surface roughness parameter depending on the roughness of the surface (from approximately 1 mm for flat sandy ground to 1 m for forests), and δ is the zero plane displacement (it is usually approximated by multiplying the height of obstacles on the ground, e.g., vegetation or buildings, by about 0.7. v_* is a parameter called friction velocity, which is mostly determined from wind measurements. The size of v_* determines how fast the wind will increase with the height, which depends on the current weather situation, with the wind speed in the layer above the surface layer and static stability exerting a significant influence.

The ***logarithmic wind profile*** represented by Equation 49 refers to the mean horizontal wind speed. An example of a logarithmic profile is shown in Figure 42. From the surface to the height $z = z_0 + \delta$, the wind speed is assumed to be zero. In fact, the air is not still given that turbulent eddies also form here, but movement in all directions is equally frequent, which causes the average velocity to be zero. Above the height $z = z_0 + \delta$, eddies also form, but the airflow is somewhat more ordered, and one direction of motion is more frequent than the others, meaning the average velocity is different from zero and is assumed to increase logarithmically with altitude.

Problem 17: Wind speed of 3 m/s is measured with an anemometer over a flat meadow at a height of 10 m above the ground. The grass is short and therefore its height can be neglected ($\delta = 0$ m), and the roughness parameter for flat grass-covered ground is 1 cm. Assuming a logarithmic wind profile, determine the wind speeds at heights of 3 and 30 m above the ground.

Solution: The change in wind with height can be determined using Equation 49, whereby the friction velocity v_* must first be determined from the wind measurement. We determine this by expressing and calculating v_* from Equation 49 where we take into account that at the height $z = 10$ m the wind is equal to $v(10\ \text{m}) = 3$ m/s

$$v_* = v(z) \cdot k \cdot \left[\ln\left(\frac{z-\delta}{z_0}\right)\right]^{-1} = 3\ \text{m/s} \cdot 0.4 \cdot \left[\ln\left(\frac{10\ \text{m}}{0.01\ \text{m}}\right)\right]^{-1} = 0.174\ \text{m/s}.$$

Wind speeds at heights of 3 and 30 m are

$$v(3\ \text{m}) = \frac{v_*}{k}\ln\left(\frac{z-\delta}{z_0}\right) = \frac{0.174\ \text{m/s}}{0.4} \cdot \ln\left(\frac{3\ \text{m}}{0.01\ \text{m}}\right) = 2.48\ \text{m/s},$$

$$v(30\ \text{m}) = \frac{0.174\ \text{m/s}}{0.4} \cdot \ln\left(\frac{30\ \text{m}}{0.01\ \text{m}}\right) = 3.48\ \text{m/s}.$$

27 Weather Fronts

The atmosphere frequently contains two large air masses with different properties (e.g., different temperatures or humidity) located next to each other. A ***weather front*** represents the transition zone at the boundary between different air masses. A nice example is the ***polar front*** – the boundary between the colder polar and warmer subtropical air masses, usually located near a latitude of 60°.

Fronts are also very common in midlatitude cyclones. While we do not provide a detailed explanation of what causes cyclones and their fronts to form, we can mention that midlatitude cyclones often consist of two large air masses of comparable size, one being warmer than the other. (Figure 43). A front is present at the boundary between the two air masses, which often goes through the cyclone's center. As the air in the cyclone circulates, the front moves in the same direction. On one part of the boundary, the colder air is progressing while the warmer air is receding – this part of the front is called a ***cold front***. On the other part of the boundary, the opposite is true – this part of the front is called a ***warm front***. On weather maps, warm and cold fronts are indicated by continuous lines marked with either semi-circles (warm front) or triangles (cold front) drawn on the side towards which the front is advancing. It turns out that a cold front usually moves faster than a warm one, and the cold front catches the warm front, creating an ***occluded front***, or ***occlusion*** for short. It is marked on the weather map with alternating semi-circles and triangles (Figure 43).

Cold and warm fronts are generally associated with bad weather and precipitation. In midlatitudes, precipitation associated with fronts accounts for about 2/3 of all precipitation. This is because a large amount of air rises in the frontal zones. The boundary between warm and cold air is not vertical but is always inclined over the colder, heavier air, which

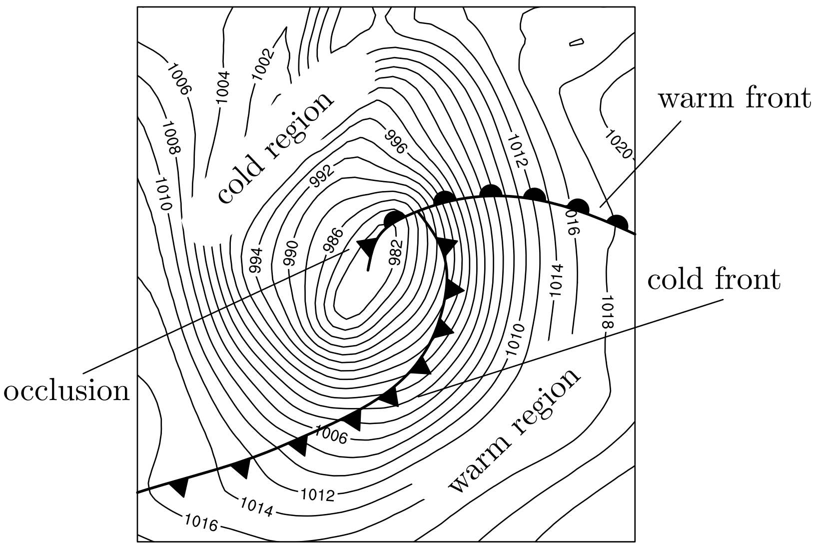


Figure 43: An example of a well-developed cyclone with cold and warms fronts and an occlusion. The lines represent isobars of sea-level atmospheric pressure in units of hPa and are drawn every 2 hPa.

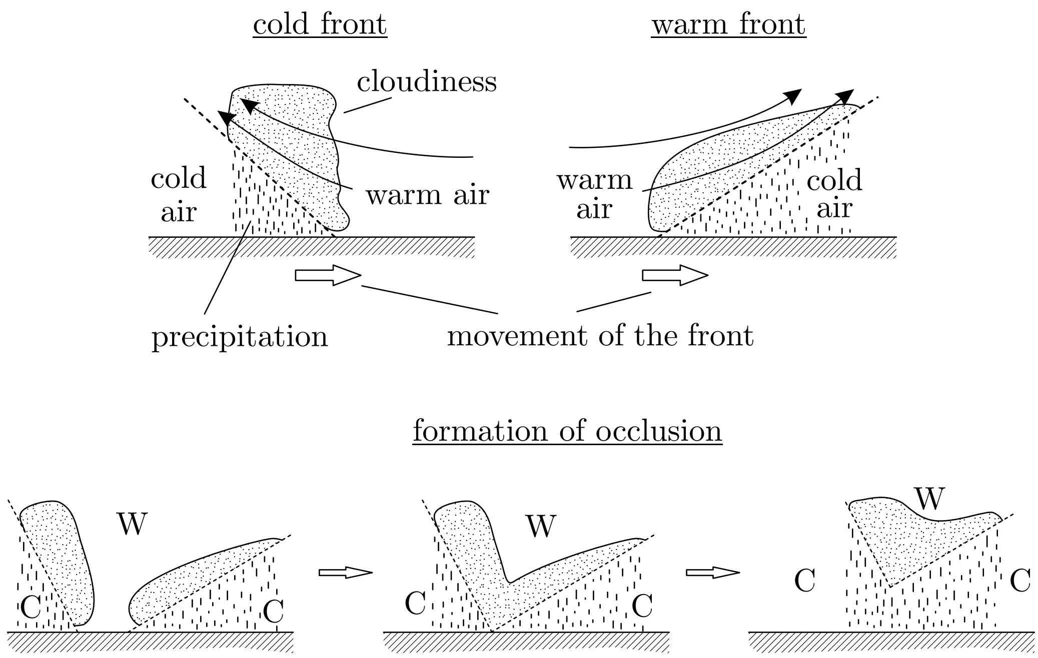


Figure 44: The vertical cross-sections of cold and warm fronts are shown on the top. In the figure, the tilt of both fronts is at several tens of degrees, which is unrealistic. In reality, the tilt of a front is much smaller – only a few tenths of a degree – which means that the boundary between the warm and cold air is almost horizontal. Below is the formation of an occlusion where the cold front catches the warm front, causing the warm air to rise. The letters C and W denote cold and warm air masses.

forms a kind of wedge below the warmer air. The tilt of the boundary is small, only about 1:200, which means that the boundary runs almost horizontally. Thus, if the lower part of the boundary is located over Slovenia, its upper part (e.g., 8 km high) can be as far as 1000 to 2000 km away – e.g., above France. Warm air rises along the "slope" represented by the cold air wedge. As the front advances, the cold air tries to move under the warm air, causing it to rise. Figure 44 shows a typical cross-section of cold and warm fronts. In both cases, rising warm air causes clouds and precipitation to form, the precipitation in a cold front typically being more convective and intense. The tilt of a cold front is usually larger than that of a warm front (the boundary is less horizontal).

For an observer on the ground, in the case of a cold front passing over, the bulk of the precipitation falls simultaneously with the most significant temperature change or lags slightly behind. Yet it is different in the case of a warm front as the bulk of the precipitation tends to fall before the most significant temperature change. A cold front advances faster than a warm front, while an occlusion occurs when the former catches up with the latter. This first happens near the cyclone's center where the two fronts are initially the closest to each other. When the occlusion occurs, the cold air from both fronts merges (Figure 44). At the same time, a large amount of warm air rises in a relatively short time, which can trigger intense precipitation. The precipitation lasts as long as the warm air rises and then gradually weakens.

28 Local Winds

The term local winds refers to winds near the ground that blow over relatively short distances but frequently occur in certain geographical regions. A ***local coastal wind*** is a weak wind (speeds are up to a few m/s) that blows from the sea towards the land during the day (the so-called ***sea breeze***), and vice versa at night (the so-called ***land breeze***). It is formed in good weather when the land heats up more than the sea during the day, and the warmer air above the land rises by free convection. A characteristic air circulation forms in the coastal zone where near to the ground the wind blows from the sea towards the land (Figure 45). In the eastern Adriatic, such a wind is called ***maestral***. On a clear night, the land surface cools more than the sea surface, and the situation is reversed, with the near-surface wind blowing from the land towards the sea. Such a wind is called ***burin*** along the eastern Adriatic.

Slope winds form in a rugged relief in clear calm weather. During the day, the air near slopes facing the sun warms and rises. Similarly to a sea breeze, air circulation forms whereby the wind rises along the slopes (Figure 46). Such a wind is called a ***valley breeze***. The situation is somewhat different on a clear, calm night when the slopes and the air near them cool down considerably. Due to the greater density, the cold air begins to flow down the slopes, which is felt as a light cool breeze. Such a wind is called a ***mountain breeze*** and is typically only a few meters thick. Cold air can also flow to the bottom of basins and smaller closed valleys, where a ***cold-air pool*** forms. The temperatures there can reach much lower values than in the immediate surroundings outside the basin (the difference can be greater than 10 °C). There are many locations around Slovenia where cold-air pools frequently form. This is a result of the rugged karst relief with innumerous sinkholes, karst fields, closed valleys, etc. At some locations, the cooling is so intense and frequent that there is a pronounced effect on the vegetation. A good example is the vegetation inversion in the Smrekova Draga basin located on the Trnovo Forest Plateau, where a large sinkhole (about 200 m deep) is located. At the top are broad-leaved trees, lower down are trees with needle-like leaves, while shrub vegetation covers the bottom of the sinkhole (usually, the inverse sequence with respect to altitude is observed).

Foehn is a warm and dry local wind that blows on the leeward side of mountain ridges. A Foehn will form when a large-scale wind forces the air to flow over a sufficiently high mountain barrier. There are two types of foehn winds. *Foehn with precipitation* occurs

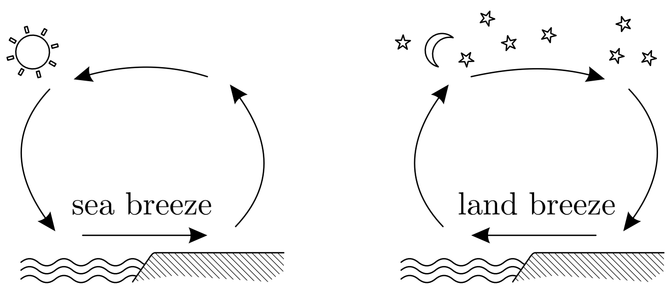


Figure 45: Local coastal winds during the day and night

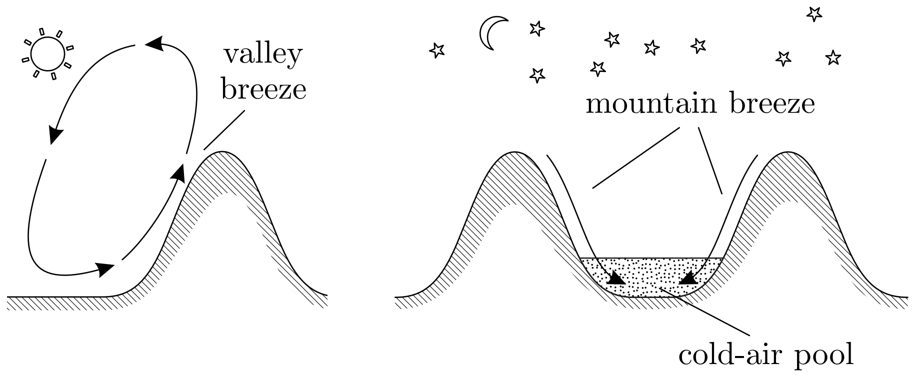


Figure 46: Slope winds during the day and night. The figure on the right also shows the formation of a cold air pool in a closed basin.

when the air on the windward side of a mountain barrier cools and becomes saturated with moisture. In the case of a sufficiently high mountain barrier, the rising continues until the hydrometeors in the cloud grow to such a size that they fall out as precipitation (Figure 47). On the other side of the barrier, the air descends but due to precipitation it is less moist than the air on the windward side. Therefore, the cloud base is higher on the leeward side than on the windward side. At the same time, the air on the leeward side is warmer than on the windward side since precipitation means there are fewer remaining hydrometeors in the air, and less energy is expended for their evaporation. It is different with the *blocking Foehn*, where only air higher up near mountain-top level can pass over the mountain barrier, while the air below is blocked. Behind the barrier, as the air from above descends to lower levels, it heats up and becomes drier (Figure 47). In Slovenia, a northern Foehn often occurs when air coming from the north flows over the Alps from the Austrian side to Slovenia. An example of such a wind in the United States is Chinook, which generally occurs when air flows over the Rocky Mountains from west to east.

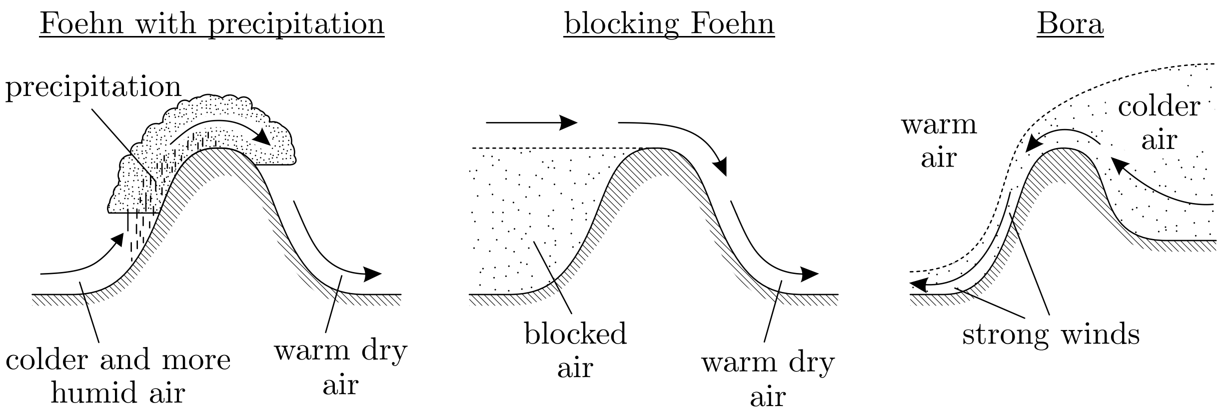


Figure 47: The situations in the case of Foehn and Bora winds.

Bora is a relatively strong, cold, dry, and gusty local wind that in Slovenia frequently occurs in the Primorska region. It blows mainly in winter when sufficiently cold air moves from the inland over the mountain barriers (e.g., the Trnovska forest plateau and the ridge of Mount Nanos in Slovenia, and the Velebit mountain range in Croatia). It can persist longer when significantly warmer air is present up above on the leeward side. The heavier colder air flows over the ridges and accelerates downward on the leeward slopes (Figure 47). In Slovenia, a Bora usually lasts from 10 to 20 hours, yet can also last more than 4 days in certain cases. Strong turbulent wind gusts can form in a Bora, which can reach speeds of up to 200 km/h near the Earth's surface.

29 The Continuity Equation

The continuity equation is a basic meteorological equation. It is built on the law of conservation of mass and describes how mass in a specific volume (density) changes depending on how much air flows in or out through the volume boundaries. In mathematical form, the law can be expressed as

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\vec{v}. \tag{50}$$

We omit a detailed derivation of the continuity equation here (the derivation is found in Appendix A.10). Still, we can mention some of its basic properties. The expression $\nabla\cdot\vec{v}$ is called wind divergence, or divergence for short, and is mathematically defined as $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$. Wind divergence will be less than zero if the wind field is such that more air flows into the volume than out (this situation is also called convergence – Figure 48 right). In this case, the mass of the air in the volume would increase and therefore the density would also increase – meaning the term $\frac{\mathrm{d}\rho}{\mathrm{d}t}$ would be greater than zero. If the divergence were greater than zero, the density would decrease, and the term $\frac{\mathrm{d}\rho}{\mathrm{d}t}$ would be less than zero.

The air in the atmosphere considerably compresses or expands only while descending or ascending. Accordingly, excluding ascent or descent, the air density does not tend to change significantly, and the airflow in the atmosphere tends to be non-divergent (namely, it satisfies the condition $\frac{\mathrm{d}\rho}{\mathrm{d}t} = 0$, and due to Equation 50 it must also hold that $\nabla\cdot\vec{v} = 0$). In this case, air inflow from some directions is compensated by outflow in others. For example, if the air near the ground converges the air at the center will start to rise (as already mentioned in the case of cyclones in Section 25). An example of a non-divergent wind field is shown in Figure 48 on the right. The airflow from the top and bottom directions converges, but simultaneously, in the left and right directions, there is an outflow, which causes the divergence to be zero and the density to be constant.

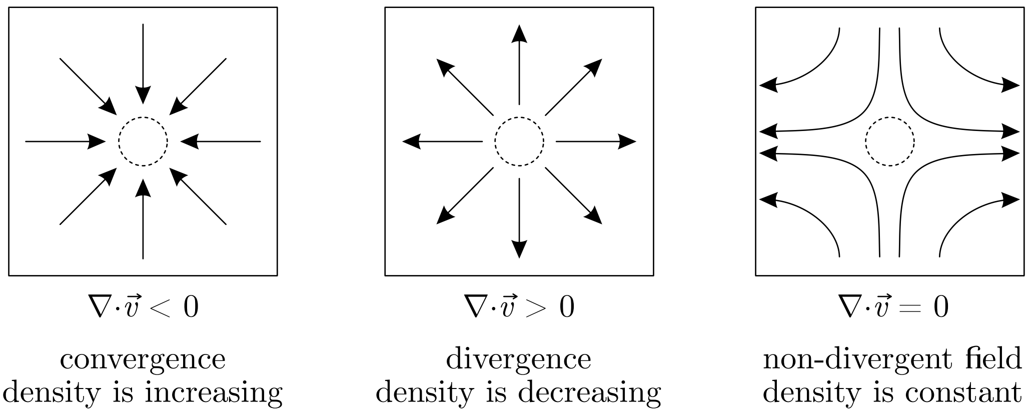


Figure 48: Examples of wind fields for situations with different wind divergence. The indicated sign of divergence applies to the volume at the center, shown by the dashed circle.

30 Solar and Terrestrial Radiation

The ***Sun*** is the closest star to our planet, which is about 270 000 times closer to Earth than the next closest star. ***Solar radiation*** is by far the biggest energy source the Earth's surface receives. All other energy sources, such as geothermal heat flow from the planet's hot interior or radiation reaching the Earth from outside the solar system, are a few thousand times smaller than the energy the Earth's surface receives from the Sun. Solar radiation is why the temperatures on Earth are not very low, and at the same time the energy of the solar radiation "drives" the weather. If no energy came from the Sun, the Earth would be a planet with a completely frozen surface where the temperature would be only a few tens of Kelvin above absolute zero. At such low temperatures, most gases in the atmosphere would condense and freeze, meaning there would be no atmosphere and no weather as we know it.

Inside the Sun, a nuclear fusion reaction is constantly underway in which four hydrogen atoms combine to form a helium atom, releasing a large amount of energy in the process. Nuclear reactions occur in the Sun's core, where the temperature is several million Kelvin. The temperature decreases away from the center and on the surface of the Sun, in a layer called the photosphere, from which most of the radiation is emitted into space, it is approximately 5800 K (Figure 49). The Sun emits several types of radiation (electromagnetic radiation and various particles such as protons and electrons). However, by far the most energy it emits is in the form of electromagnetic radiation and accordingly, from here on, the term radiation is used to refer only to electromagnetic radiation.

The term ***electromagnetic radiation*** refers to waves in the electric and magnetic fields that propagate through space and carry electromagnetic energy with them. For electromagnetic radiation, its energy and wavelength are important. With respect to its wavelength, electromagnetic radiation is divided into various parts of the electromagnetic spectrum (Table 4). For radiation affecting the atmosphere, the most important are ultraviolet (UV), visible, and infrared (IR) radiation.

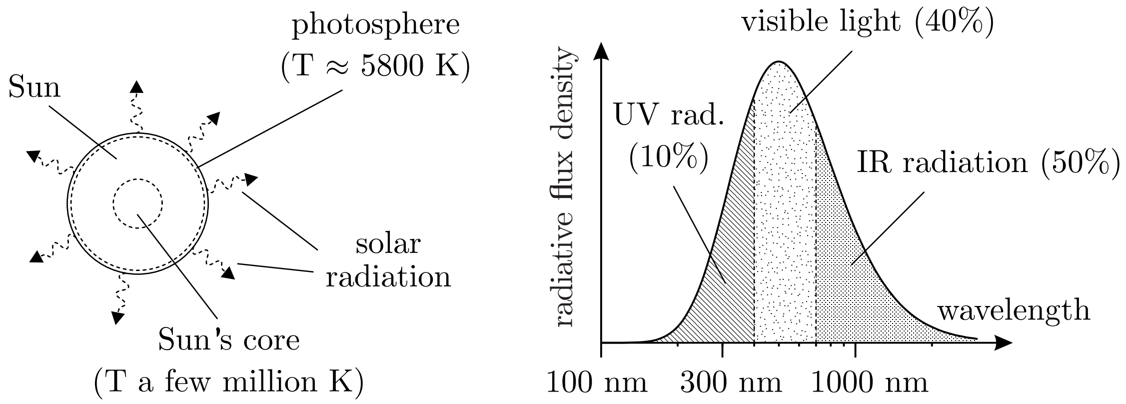


Figure 49: The Sun and the spectrum of the radiation it emits on the assumption that it radiates as a black body with a temperature of 5800 K. The percentages shown in parentheses indicate the approximate energy fractions of the different types of radiation.

The energy of the radiation can be expressed by ***radiative flux density*** j, which has units of W/m^2. The Stefan-Boltzmann law describes the radiative flux density emitted by the surface of a black body (an imaginary body composed of a material that would absorb all incident radiation). A surface that is not entirely black radiates less, which is explained by the emissivity parameter of the material (ε). Using the Stefan-Boltzmann law and emissivity, the emitted radiative flux density can be expressed as

$$j = \varepsilon\sigma T^4, \tag{51}$$

where $\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W/(m^2\,K^4)}$ is the Stefan-Boltzmann constant, and T is the temperature of the emitting surface (given in units of K). The amount of emitted radiation thus depends on the temperature and emissivity and, due to the fourth power, the surface will emit much more radiation at higher temperatures than at lower ones. Emissivity is a parameter between 0 and 1 that tells how well a material radiates compared to a black body. A black body represents some imaginary substance with an emissivity equal to 1, while real materials have a lower emissivity. The emissivities of the surface of the Sun,

Radiation type	Typical wavelength (m)
X or gamma rays	10^{-9}
Ultraviolet radiation	10^{-7}
Visible light	$5 \cdot 10^{-7}$ (400 – 700 nm)
Infrared radiation	10^{-6}
Microwave radiation	10^{-3}
Radio waves	$> 10^{-3}$

Table 4: The electromagnetic spectrum.

Problem 18: Determine the area size of the Sun's surface that produces the same amount of energy in terms of emitted radiation as produced by the Krško Nuclear Power Plant (the plant's nominal power is 696 MW). Assume that the Sun's surface radiates as a black body with a temperature of 5800 K.

Solution: The flux density of radiation emitted by the Sun's surface can be expressed by the Stefan-Boltzmann law (Equation 51). The power of the emitted radiation can be expressed as the product of the radiative flux density and the area size of the surface

$$P = S \cdot j = S \cdot \varepsilon \sigma T^4 .$$

This power must be equal to the output power of the power plant, whereby we can express the corresponding area size of the Sun's surface as

$$S = \frac{P_{\text{NEK}}}{\varepsilon \sigma T^4} = \frac{696 \cdot 10^6\ \text{W}}{1 \cdot 5.67 \cdot 10^{-8}\ \text{W}/(\text{m}^2\,\text{K}^4) \cdot (5800\ \text{K})^4} = 10.9\ \text{m}^2 .$$

The necessary area size is only 10 square meters, which is negligible compared to the entire surface of the Sun. This highlights the enormous amount of energy emitted by the Sun.

land, sea, and sufficiently thick clouds on Earth are almost 1 – so they radiate almost like a black body. At the same time, this does not apply to a cloudless atmosphere since its emissivity tends to be considerably lower – approximately 0.7 (for more on this, see Section 31).

Planck's law defines the spectrum of the radiation emitted by a black body. The spectrum depends on the temperature and can be described by the Planck curve. A detailed discussion of Planck's law and curve is omitted here. In general, at higher temperatures, the spectrum of emitted radiation shifts towards shorter wavelengths. The Planck curve for solar radiation is shown in Figure 49. The Sun emits most of its radiation at wavelengths representing visible and IR radiation. About 10% of the radiation's energy lies in the UV part of the spectrum, about 40% in visible light, and the rest in the near-IR part of the spectrum, close to visible light (the entire IR spectrum extends to much longer wavelengths).

Since solar radiation spreads into the surrounding space, its flux density decreases with the distance from the Sun, reaching a value of approximately 1366 W/m^2 near Earth. This value is called the ***solar constant*** and denoted by j_0.

The path of the Earth around the Sun is not perfectly circular, with the orbit taking the shape of an ellipse. The Sun is also not quite in the center of this ellipse (Figure 50). This means that the distance of the Earth from the Sun also changes slightly during the year. The two points on the orbit where the Earth is either closest or farthest from it are called perihelion and aphelion. Perihelion is a few days after the New Year, and aphelion

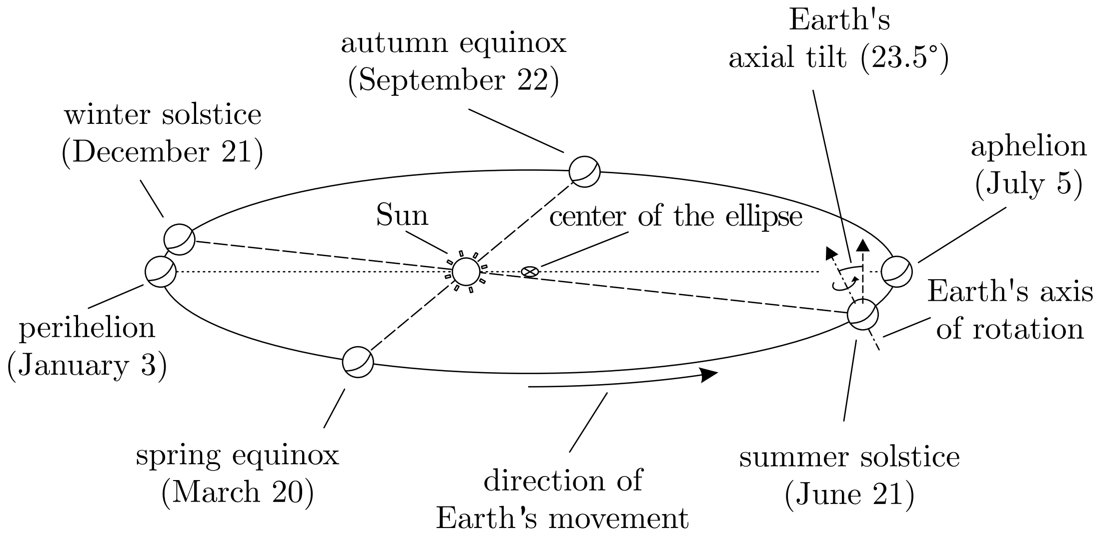


Figure 50: The Earth's elliptical orbit around the Sun. Approximate dates of important points along the orbit are marked in parentheses.

is in early July. The average distance of the Earth from the Sun is about $150 \cdot 10^6$ km and varies by ±2% depending on whether the Earth is near perihelion or aphelion. Thus, during the year, the flux density of solar radiation that reaches the Earth also varies (by approximately ±4%). The flux density also changes slightly due to the 11-year sunspot cycle (±0.1%) and other short-term phenomena on the Sun (e.g., solar flares).

The seasons on Earth are caused by the tilt of the Earth's axis of rotation and are not related to the ellipticity of the orbit. The Earth's axis of rotation is tilted by 23.5° relative to the plane of the Earth's orbit around the Sun. In summer, the northern hemisphere is more exposed to solar radiation, making it warmer there. It is most exposed to the Sun at the summer solstice, around June 21. The opposite is true half a year later, at the winter solstice when the southern hemisphere receives the most solar radiation. The winter solstice roughly coincides with the time of perihelion (Figure 50), which means that in this period the solar radiation is stronger than at the summer solstice. This leads to the Southern hemisphere receiving slightly more solar radiation than the Northern Hemisphere over the course of a full year. Although this is the current situation, it might change slowly over longer periods (e.g., ten millennia or more) due to the gradual changes in the orientation and tilt of the Earth's rotational axis and changes in its orbit around the Sun.

Like the Sun, the Earth emits radiation. The radiation emitted by the Earth is called ***terrestrial radiation*** – in the sense that all the radiation that comes from the Earth (from its surface and the atmosphere). The Earth has a much lower temperature than the Sun and the radiation it emits is thus at much longer wavelengths and entirely in the infrared part of the spectrum (Figure 51). Due to this, solar and terrestrial radiation are also frequently referred to as the ***shortwave*** and ***longwave radiation***. The spectrums of these two types of radiation overlap very little, while the border between them is at approximately 5 microm.

31 Passage of Radiation Through the Atmosphere

When radiation is traversing a medium (e.g., air), it can interact with the matter that constitutes it. One option is that some of the radiation's energy is transferred to the matter, which usually heats up as a result. This process is called ***absorption***. Some of the radiation may pass through the matter unimpeded, some may be absorbed, and some may be redirected. ***Albedo*** is a parameter with values between 0 and 1 that represents the fraction of radiation reflected or redirected in the backward direction. Various substances have different albedo values, which also depend on the wavelength of the incident radiation. For example, for incident solar radiation, fresh snow or a sufficiently thick cloud has an albedo of about 0.8 (meaning that they reflect most of the incoming radiation), while grass-covered land has a value of about 0.2 and the sea a value of about 0.1 (meaning they absorb most of the incoming radiation).

Absorption also occurs when radiation travels through the atmosphere, yet the situation is slightly different compared to liquids and solids. The gases usually have a distinct ***banded absorption spectrum***. They absorb significantly only in very narrow intervals of wavelengths, called absorption lines, while at other wavelengths, they absorb very little. A detailed explanation for this behavior of gases lies beyond the scope of this textbook and requires a description of what is happening on the microscopic level using quantum mechanics. Various gases absorb differently and have absorption lines at different wavelengths. It is also common for gas to have many partially-overlapping absorption lines in a limited wavelength interval, called an absorption band.

Figure 51 shows the absorption spectra of some of the most important gases in the atmosphere. The fraction of radiation that is absorbed while passing through the atmosphere's entire thickness in the vertical direction is shown, assuming there are no clouds. Although nitrogen N_2 is the most abundant gas in the atmosphere, it is not shown in the figure as it absorbs almost nothing in either the visible or infrared parts of the spectrum and, in this sense, is an almost completely inert gas. The situation is somewhat different for methane CH_4 and nitrous oxide N_2O, which already have some isolated but fairly distinct absorption bands in the infrared part of the spectrum. Next are oxygen O_2 and ozone O_3, which otherwise absorb only a little in the visible and infrared parts of the spectrum, although much more important is the almost 100% absorption of ultraviolet radiation for wavelengths, shorter than 300 nm. This radiation is hazardous to living things, and it is essential that it does not reach the ground. Oxygen absorbs UV radiation up to about 200 nm, while ozone absorbs radiation between 200 and 300 nm. The next important gas is carbon dioxide CO_2, which has some quite broad and distinct absorption bands in the infrared part of the spectrum. Water vapor absorbs the most, with many broad absorption bands being found in the infrared part of the spectrum.

The combined effect of all the gases together is shown at the bottom of Figure 51. The atmosphere is principally transparent for visible light, while most of the UV radiation is absorbed. For infrared radiation, there are distinct interchanging intervals of strong absorption and transparency. Wavelength intervals in which the cloud-free atmosphere is predominantly transparent are called ***atmospheric windows***. The most distinct

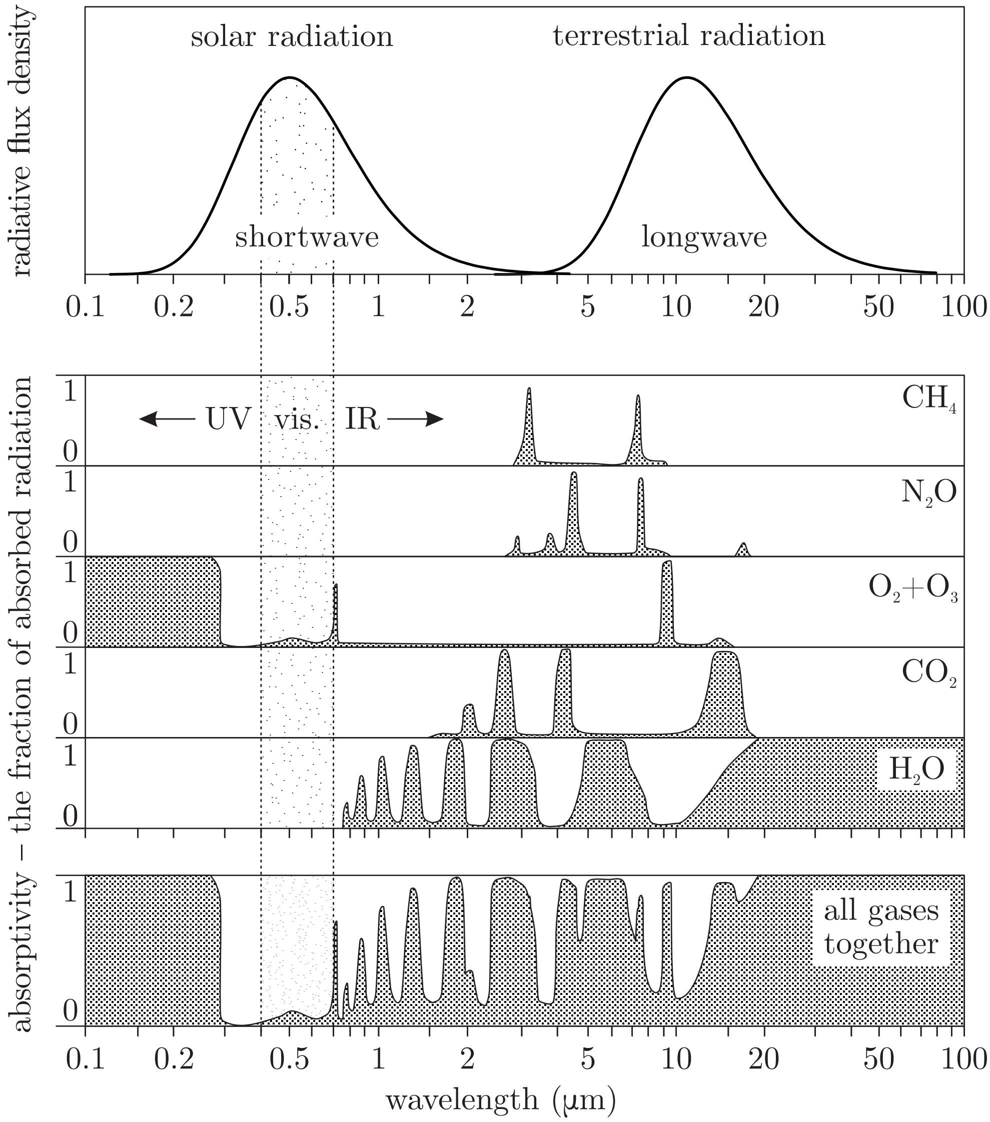


Figure 51: Top: the spectra of solar and terrestrial radiation, assuming the Sun and Earth radiate as black bodies with temperatures of 5800 K and 255 K, respectively. In the figure, the spectra are normalized to have the same maximum power, yet in reality the power of solar radiation is much greater than the power of terrestrial radiation. Bottom: the fraction of radiation absorbed by various gases vertically passing through the entire cloud-free atmosphere. Values close to 0 represent wavelengths at which there is almost no absorption. Values close to 1 represent wavelengths where the radiation is almost completely absorbed as it travels through the atmosphere. Adapted from [7].

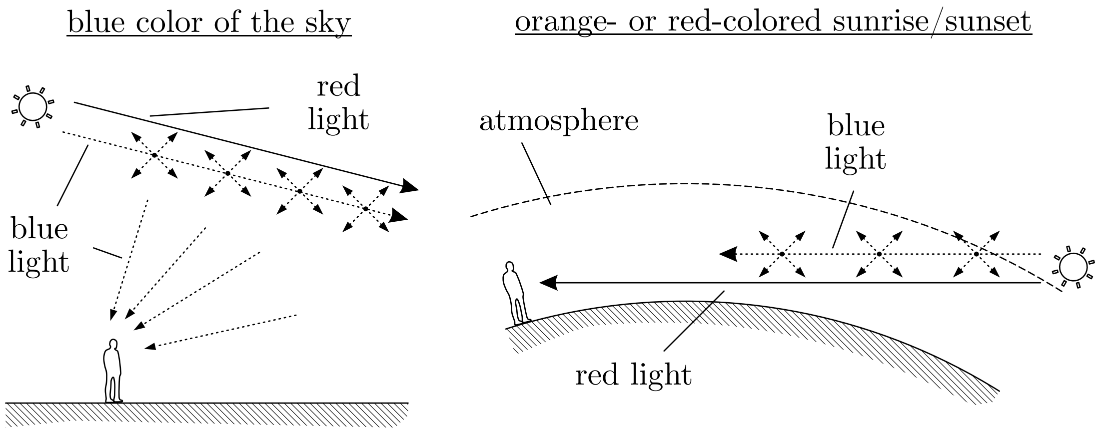


Figure 52: Explanation of the blue-colored sky and the orange- or red-colored Sun at sunrise or sunset. Both phenomena are caused by the scattering of visible light, which is stronger for cold-colored light than warm-colored light.

atmospheric window is in the visible part of the spectrum, yet there are also others in the infrared part.

In addition to being absorbed, radiation can scatter as it passes through a medium. Radiation ***scattering*** is a process in which incoming radiation is redirected, often in a wide range of directions, upon contact with the molecules or particles of a medium (e.g., air or hydrometeors). While here we omit a detailed discussion of scattering, we note that in general the radiation of shorter wavelengths is scattered more intensively. Since the Sun emits most of its energy at shorter wavelengths than the Earth, it is primarily solar radiation that scatters on air molecules. Solar radiation with wavelengths corresponding to warm colors (e.g., red, orange) passes relatively unhindered through the cloudless atmosphere, unlike radiation associated with cold colors, which is strongly scattered. This also explains why the sky is blue (Figure 52). Out of visible light, the scattering is the strongest for violet light, which has the shortest wavelength. However, solar radiation contains more blue than violet light, and the photoreceptors in human eyes are also more sensitive to blue light – that is why the sky appears blue to us instead of violet. The situation is different for scattering caused by clouds, where solar radiation is refracted on hydrometeors. In this case, all wavelengths of visible light are refracted with approximately the same intensity, causing the apparent color of the cloud to be the same as the color of sunlight (white color).

The intense scattering of cold-colored light is also responsible for the Sun appearing red or orange at sunrise and sunset. When the Sun is very low above the horizon, the sunlight's path through the atmosphere is very long (Figure 52). On this path, most of the cold-colored light is scattered to the sides, and only the warm-colored light reaches the observer. When the Sun is low, the warm-colored light also frequently falls on the nearby clouds, hills, and objects, covering them with beautiful and unusual colors.

32 Energy of Solar Radiation at the Earth's Surface

When solar radiation passes through the atmosphere, part of it is reflected back into space (mainly due to scattering in clouds and on air molecules), while some is also absorbed by air molecules, clouds, or aerosol particles. Therefore, only a portion of solar radiation reaches the surface of the Earth.

Solar radiation is divided into two types according to the direction from which it comes. ***Direct solar radiation*** represents that part of solar radiation that does not scatter on its way through the atmosphere and reaches the surface only from the direction of the Sun. ***Diffuse solar radiation*** represents the scattered solar radiation that is usually coming to the surface from all directions. In clear weather, much more direct than diffuse radiation reaches the ground. In cloudy weather, it is the other way around, with mostly diffuse radiation reaching the surface given that solar radiation is scattered very efficiently by clouds (Figure 53). In cloudy weather, a considerable portion of the radiation is scattered back into space, and much less radiation reaches the Earth's surface compared to in clear weather – this is also why clear days are generally warmer than cloudy days.

In clear weather and when the Sun is at its zenith (90° above the horizon), approximately 80% of direct solar radiation reaches the surface. The Lambert-Beer law can be used to calculate the attenuation of direct radiation on the vertical path through the entire atmosphere (the derivation is shown in Appendix A.11)

$$j_\mathrm{s} = j_0 \mathrm{e}^{-\tau_\mathrm{zen}}. \tag{52}$$

τ_zen is the atmosphere's optical thickness when the Sun is at its zenith. In clear weather, τ_zen is approximately 0.22. j_0 and j_s represent the flux density of direct solar radiation above the atmosphere (solar constant) and at the surface.

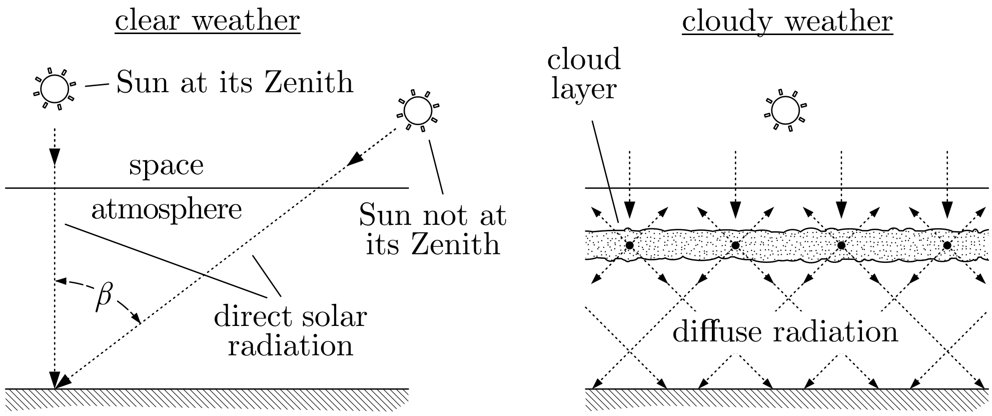


Figure 53: The path of solar radiation to the Earth's surface in clear and cloudy weather. In clear weather, much more direct than diffuse radiation reaches the ground. In cloudy weather, it is the other way around.

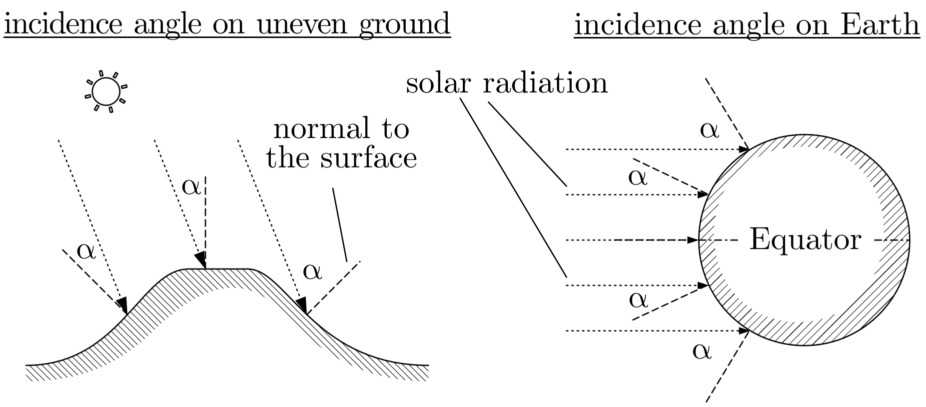


Figure 54: The incidence angle of direct solar radiation on uneven ground and on Earth at different latitudes at the solar equinox. α denotes the incidence angle – the angle between the direction of radiation and the normal to the surface (which always points perpendicular to the surface).

If the Sun is not at its zenith, the radiation's path through the atmosphere is longer by a factor of $1/\cos\beta$, where the angle β represents the zenith angle (the angle between the direction of radiation and the zenith – Figure 53). Therefore, the optical thickness increases from τ_{zen} to $\tau_{\mathrm{zen}}/\cos\beta$ and the radiation flux density on the surface can be expressed as

$$j_{\mathrm{s}} = j_0 \mathrm{e}^{-\frac{\tau_{\mathrm{zen}}}{\cos\beta}}. \tag{53}$$

The power of solar radiation absorbed by the ground also depends on the angle at which the radiation hits the surface. Its power will be greatest if the radiation reaches the ground perpendicularly. If the radiation is not arriving perpendicularly, the absorbed power will be lower since the same amount of radiation energy is distributed over a bigger surface area. The power of solar radiation absorbed by the surface can be expressed as

$$P = (1-a) j_{\mathrm{s}} S \cdot \cos\alpha, \tag{54}$$

where a is the albedo of the ground, S is the area size, j_{s} is the flux density of the solar radiation on the surface, and α is the incidence angle (the angle between the direction of the incident radiation and the normal to the ground, which always points perpendicular to the surface, Figure 54). For surfaces oriented horizontally, α is the same as the zenith angle β, yet this is generally not the case. Equation 54 is only valid if radiation is coming from a single direction. For example, in the case of direct solar radiation, which comes only from the direction of the Sun but does not apply to diffuse solar radiation or radiation emitted by the atmosphere, which comes from different directions in the sky).

The amount of energy the ground receives through direct solar radiation therefore depends strongly on the albedo and the incidence angle at which the radiation hits the ground. The dependence on the incidence angle also helps explain why in the northern hemisphere south-facing slopes receive more energy from solar radiation than north-facing ones (α is smaller for south-facing slopes – Figure 54). Depending on the Sun's position

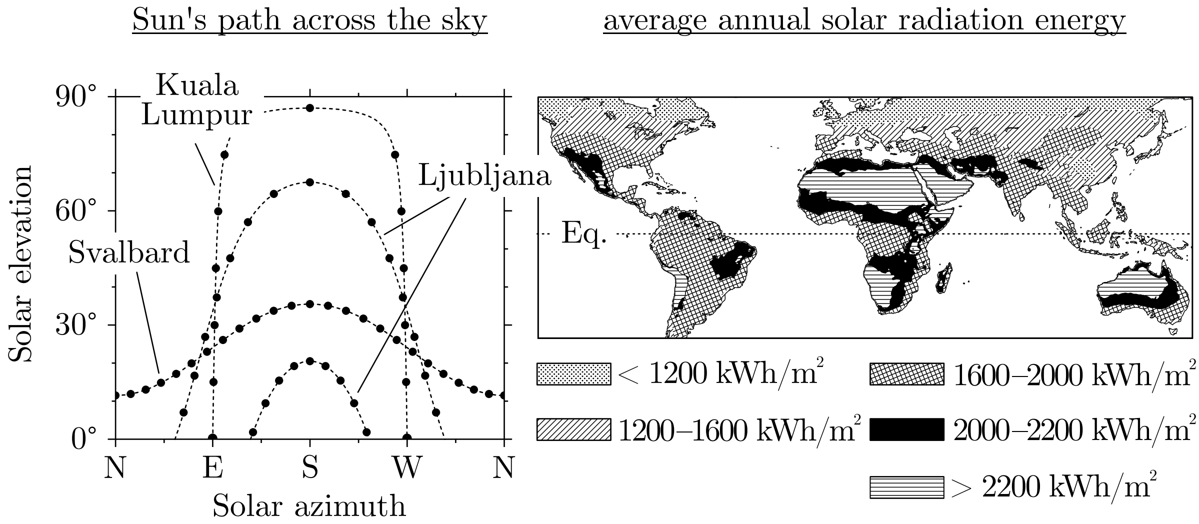


Figure 55: Left: The Sun's path across the sky for Ljubljana on the winter and summer solstices, Kuala Lumpur (3°S) at the equinox, and the Svalbard archipelago (78°S) on the summer solstice. Black dots represent hours according to local solar time defined such that the Sun is always highest at noon. Right: The average annual solar radiation energy received by a horizontal surface on ground level. Data source [8]..

and the slope's orientation, α can also be larger than 90°. Here, the slope does not receive any direct radiation since it is facing away from the Sun.

The Sun's path across the sky is very important for the daily or annual amount of solar radiation energy received by the ground. For horizontal surfaces, α will be 90° at sunrise. As the Sun ascends, α will decrease and reach its lowest value in the middle of the day at solar noon. Then in the afternoon, α will increase and reach 90° again at sunset. The Sun's daily path across the sky chiefly depends on the day of the year and the latitude. On average, solar radiation hits the ground more straight on in the tropical regions than in the mid and high latitudes (Figure 54), which makes tropical regions the warmest.

Figure 55 shows the Sun's path across the sky for Ljubljana on the winter and summer solstices. On the summer solstice, when the northern hemisphere is more exposed to the Sun than the southern hemisphere, the length of a day is about 15.5 h and the Sun is about 68° above the horizon at midday. It is different on the winter solstice when a day is only about 8.5 h long, and the Sun is only about 21° above the horizon in the middle of the day. It is completely different on the Svalbard archipelago, located at 78° north in the Arctic Circle. During the summer solstice, there is a polar day when the Sun is always between 12° and 36° above the horizon. It is different again in Kuala Lumpur, which lies almost on the equator (3° north). At the equinox, a day there is almost exactly 12 h long, with the Sun rising in the east, rising steadily almost to its zenith, and then descending steadily to the west.

Figure 55 also shows the average annual solar radiation energy a horizontal surface receives on ground level. It is noticeable that the regions at the equator do not receive

Problem 19: Determine the fraction of direct solar radiation that reaches the ground in clear weather if the Sun is at its zenith, 45° above the horizon, and 5° above the horizon. Assume that the atmosphere's optical thickness when the Sun is at its zenith is 0.22.

Solution: Equation 53 can be used. We are interested in the ratio between the radiation flux density at the ground and above the atmosphere, thus

$$\frac{j_\mathrm{s}}{j_0} = \mathrm{e}^{-\frac{\tau_\mathrm{zen}}{\cos\beta}}.$$

If the Sun is at its zenith, $\beta = 0°$ and the result is

$$\frac{j_\mathrm{s}}{j_0} = \mathrm{e}^{-\frac{0.22}{\cos 0°}} = 0.8.$$

In the same way, we obtain the results 0.73 and 0.08, for $\beta = 45°$ and $\beta = 85°$, respectively. This indicates that at $\beta = 45°$ the weakening of direct solar radiation is not yet significant, and only about 10% less radiation comes through the atmosphere compared to when the Sun is at its zenith. It is completely different when the Sun is only 5° above the horizon and ten times less radiation comes through the atmosphere than at its zenith. This also explains why the Sun can be looked at with the naked eye at sunset or sunrise, whereas would cause pain in the middle of the day due to the radiation being too strong.

the most energy (e.g., the Congo basin and Amazon lowlands). The reason is cloudiness, which often occurs there. The greatest energy is hence received by areas around a latitude of 30° where there are mostly deserts and clouds are less frequent, with the energy received mostly exceeding 2200 kWh/m^2 (e.g., the Sahara, Arabian desert, deserts in Australia, Atacama, Namib, Kalahari). Some regions located at higher altitudes also receive a relatively large amount of energy (e.g., the Tibetan Plateau) because the solar radiation's path through the atmosphere is shorter there. As one moves more towards the poles, the amount of received energy decreases, and at a latitude of about 50° it reaches a value of about 1200 kWh/m^2. A similar value also applies to Slovenia where the energy tends to range between 1050 and 1400 kWh/m^2. In Slovenia, cloud cover is relatively common; approximately half of the energy is provided by diffuse radiation and half by direct radiation. The Primorska, Kras, and Goriška regions, along with some other high-altitude locations in the Julian and Kamnik–Savinja Alps, receive the greatest energy, while the eastern foothills of the Julian Alps, most of the Kamnik–Savinja Alps, and southern Slovenia with Mount Snežnik, receive the least.

Problem 20: In Slovenia, approximately 16 500 GWh of electricity is consumed per year (which is equal to $5.94 \cdot 10^{16}$ J). We obtain it from hydroelectric power plants, thermal power plants, a nuclear power plant, a little bit from solar and wind-powered plants, and the rest from imports. If a solar power plant were constructed in the Sahara, how large would its area size need to be to produce the mentioned amount of electricity annually? Assume that the solar cells are placed horizontally, that every day is the same, with the Sun being above the horizon for 12 h and, instead of a realistic path of the Sun across the sky, assume that the Sun is at a fixed position at half-zenith height (45° above the horizon) and that the flux density of direct solar radiation at the ground is 1000 W/m^2. Further, assume that clear weather means that only direct solar radiation is falling on the ground and that the efficiency of the solar panels is 20% (the fraction of the incident radiation that is converted into electrical energy).

Solution: To calculate the power of radiation during the day, which is converted into electrical energy, we can use the equation 54 in which the term $(1 - a)$ is replaced by the solar panel efficiency $\eta = 0.2$.

$$P = \eta \cdot j_{\mathrm{s}} S \cdot \cos\alpha.$$

In order to calculate the energy produced during the whole year, the power must be multiplied by the time in which the electricity is produced (the time when the Sun is above the horizon), which is equal to $t = 12 \text{ h} \cdot 365 = 15.77 \cdot 10^6$ s. The amount of energy produced is thus

$$A = P \cdot t = \eta \cdot j_{\mathrm{dir}} S \cdot \cos\alpha \cdot t,$$

from where we can determine the required area size of the solar panels

$$S = \frac{A}{\eta \cdot j_{\mathrm{dir}} \cdot \cos\alpha \cdot t} = \frac{5.94 \cdot 10^{16}\ \mathrm{J}}{0.2 \cdot 1000\ \mathrm{W/m^2} \cdot \cos 45^\circ \cdot 15.77 \cdot 10^6\ \mathrm{s}} = 26.6\ \mathrm{km}^2.$$

We would therefore need an area approximately 5 km × 5 km large. If the efficiency of the solar panels were better, the area could also be smaller.

33 Equilibrium Temperature and the Greenhouse Effect

By good approximation, the Earth is in a ***radiative equilibrium***, with the total energy of longwave radiation it emits being the same as the total energy it receives from absorption of the Sun's shortwave radiation (Figure 56). Since the amount of emitted radiation depends strongly on the temperature while the energy received from the Sun is more or less constant, we can determine the so-called 'equilibrium temperature'. This is the temperature the Earth's surface would need to be at for the total energy it emits to be equal to the energy it receives.

A few simplifications can be used to calculate the equilibrium temperature. First, we shall ignore the atmosphere's influence and assume that the Earth's surface has a uniform temperature denoted by T. We also assume that the Earth is completely spherical and that the average albedo of the Earth for solar radiation is $a = 0.3$ (i.e., 30% of the solar radiation is reflected away from the Earth, while the rest is absorbed). We also assume that the Earth's surface radiates as a black body.

With these assumptions, we can write the power of radiation received by the Earth's surface from the Sun as $P_{\mathrm{received}} = (1-a)S_{\mathrm{p}}j_0$, where j_0 is the solar constant, and S_{p} is the size of the surface with which the Earth intercepts the solar radiation. From the direction of the Sun, the Earth looks like a circle with a radius equal to the radius of the Earth on which the solar radiation is falling perpendicularly, hence $S_{\mathrm{p}} = \pi R_{\mathrm{E}}^2$ where R_{E} is the Earth's radius. Accordingly, $P_{\mathrm{received}} = (1-a)\pi R_{\mathrm{E}}^2 j_0$. The power of the radiation emitted can be written as $P_{\mathrm{emitted}} = S_{\mathrm{E}} \cdot j_{\mathrm{emitted}}$, where $S_{\mathrm{E}} = 4\pi R_{\mathrm{z}}^2$ represents the entire surface of the Earth, and j_{emitted} the flux density of the emitted black-body radiation (Equation 51). Therefore, $P_{\mathrm{emitted}} = 4\pi R_{\mathrm{E}}^2 \sigma T^4$.

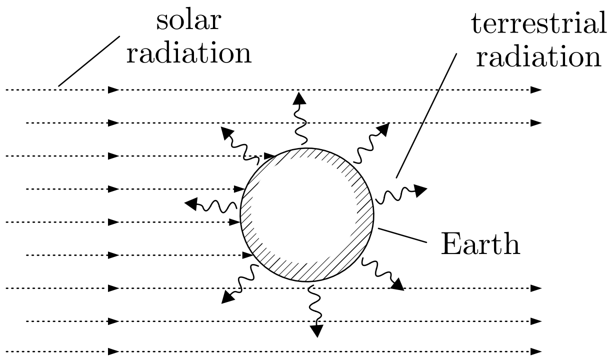


Figure 56: The Earth's radiative equilibrium with the total energy of longwave radiation it emits being the same as the total energy it receives from the Sun.

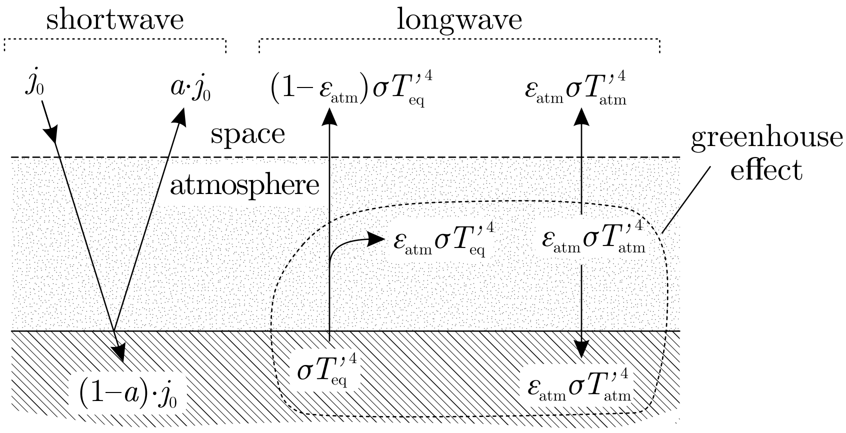


Figure 57: The global-mean energy balance for the Earth's surface-atmosphere system assuming that the surface and the atmosphere are two separate bodies with different temperatures.

In the case of the radiative equilibrium, the received power will be equal to the emitted power, and T will be equal to the equilibrium temperature, denoted by T_{eq}, thus

$$\begin{aligned} P_{\mathrm{received}} &= P_{\mathrm{emitted}}, \\ (1-a)\pi R_{\mathrm{E}}^2 j_0 &= 4\pi R_{\mathrm{E}}^2 \sigma T_{\mathrm{eq}}^4, \end{aligned} \tag{55}$$

from where T_{e} can be expressed as

$$T_{\mathrm{eq}} = \sqrt[4]{\frac{(1-a)j_0}{4\sigma}} = \sqrt[4]{\frac{(1-0.3)\cdot 1366\ \mathrm{W/m^2}}{4\cdot 5.67\cdot 10^{-8}\ \mathrm{W/(m^2\,K^4)}}} = 255\ \mathrm{K} = -18\ °\mathrm{C}. \tag{56}$$

The equilibrium temperature of the Earth's surface given by Equation 56 is unrealistically low and differs from the true average temperature near the surface (about 15 °C) by about 30 °C. This happens because we did not consider the influence of the atmosphere.

The calculation of the equilibrium temperature can be significantly improved if we consider that the atmosphere is a separate body with a different temperature, which is transparent for most of the solar radiation and yet also absorbs part of the longwave radiation coming from the surface. Here, we denote the equilibrium temperature of the Earth's surface as T'_{eq}, and the atmosphere's equilibrium temperature as T'_{atm}. We assume that the atmosphere is completely transparent for solar radiation (in fact, this is not completely true since part of the solar ultraviolet and infrared radiation is absorbed in the atmosphere) and that the emissivity of the atmosphere is $\varepsilon_{\mathrm{atm}} = 0.7$. According to Kirchhoff's law of thermal radiation, emissivity is equal to absorptivity (the share of incoming radiation absorbed by a medium), meaning that the absorptivity of the atmosphere is also equal to $\varepsilon_{\mathrm{atm}}$.

The radiation balance for this situation is visualized in Figure 57. The equilibrium temperature of the Earth's surface can be expressed as (the derivation is shown in

Appendix A.12)

$$T'_{\mathrm{eq}} = T_{\mathrm{eq}} \cdot \sqrt[4]{\frac{1}{1-\varepsilon_{\mathrm{atm}}/2}} = 255\ \mathrm{K} \cdot \sqrt[4]{\frac{1}{1-0.7/2}} = 284\ \mathrm{K} = 11\ °\mathrm{C}. \tag{57}$$

Compared to Equation 56, there is an additional term in Equation 57 that depends on the value of $\varepsilon_{\mathrm{atm}}$ – namely, on how well the atmosphere absorbs and emits longwave radiation. The surface equilibrium temperature given by Equation 57 is still slightly lower than the actual average air temperature near the ground (mainly because we did not take the influence of cloudiness on longwave radiation into account). However, at the same time, it is about 30 °C higher than T_{eq} given by Equation 56 and therefore much more realistic. The difference between the two equations is due to the influence of the atmosphere, which causes an effect called the ***greenhouse effect***. This is a process where part of the longwave radiation emitted by the Earth's surface is intercepted in the atmosphere and returned to the surface (Figure 57).

Gases that are mainly transparent for shortwave solar radiation and yet also absorb longwave terrestrial radiation are called ***greenhouse gases***. The most powerful greenhouse gas is water vapor, which has many strong and broad absorption bands in the IR part of the spectrum (Figure 51. Other important greenhouse gases are carbon dioxide, methane, nitrous oxide, and ozone. Greenhouse gases in the atmosphere have a very important role since they markedly increase the temperature near the Earth's surface through the greenhouse effect. Without these gases, the Earth's surface would be much colder, and the question is what kind of life would have developed arises. Recently, a major problem has emerged as the concentrations of certain greenhouse gases in the atmosphere have increased due to human activity. This is strengthening the greenhouse effect, thereby causing climate change and global warming (for more on this, see Section 34).

Figure 58 shows a more detailed and realistic global-mean energy balance on Earth, including, alongside radiation, some other forms of energy transfer. On average, about 31% of the energy of solar radiation falling on the Earth is reflected back into space. Of this, 6% is reflected through scattering by air molecules, 20% from clouds, and 5% from the ground. Simultaneously, 20% of solar radiation is absorbed in the atmosphere by some gases, aerosols, and clouds. The ground absorbs the remaining 49% of solar radiation. Most of the longwave radiation emitted by the Earth's surface is absorbed by greenhouse gases and aerosols, while a smaller part is also absorbed by clouds. The atmosphere returns a considerable part of the absorbed radiation back to the surface through the greenhouse effect. In addition to radiation, energy passes from the ground to the atmosphere via latent and sensible heat flows. The sensible heat flow, represented by the heat conduction between the Earth's surface and the atmosphere, is responsible for an energy flow equal to 5% of solar radiation. Latent heat transfer primarily occurs when water evaporates from the ground (primarily from the oceans), whereby a certain amount of energy from the ground is used up for evaporation. This energy is later released during condensation in the clouds and heats the surrounding air. The transfer of energy through latent heat is relatively large and, in terms of the energy flow, is equal to 26% of solar radiation.

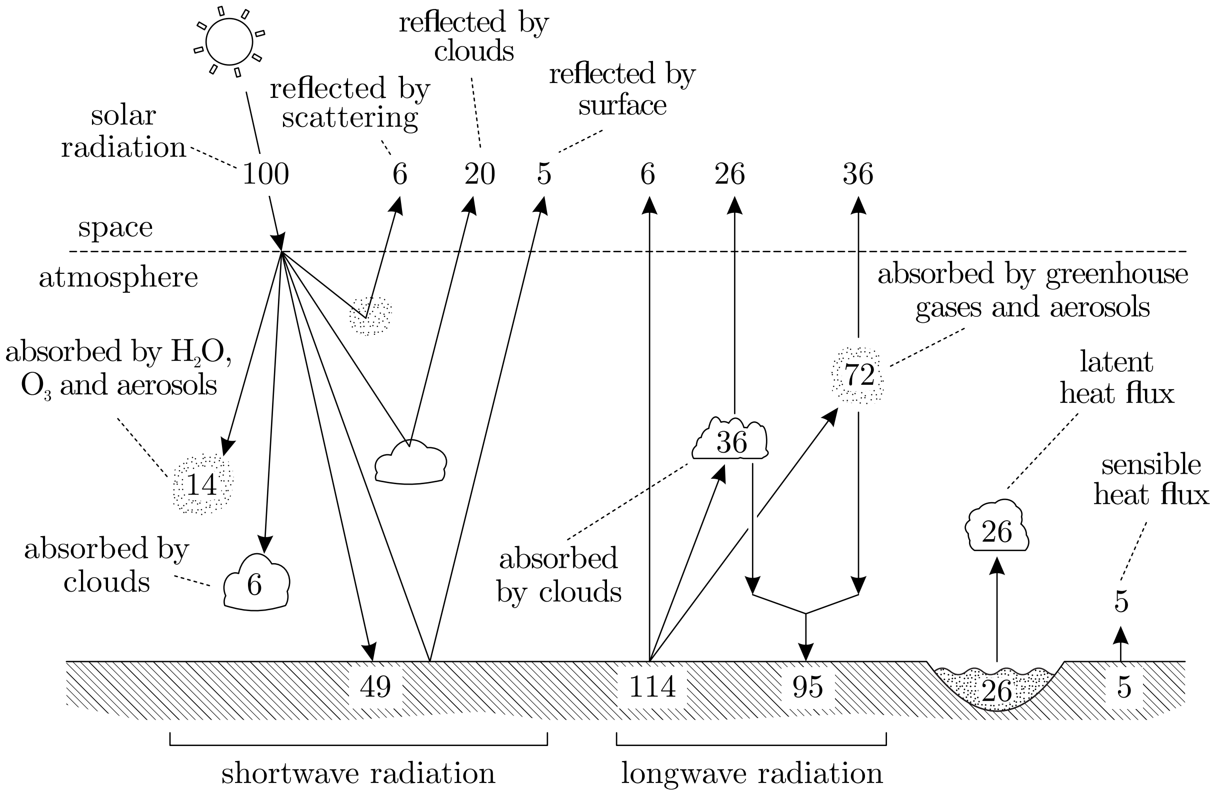


Figure 58: The global-mean energy balance on Earth. The numbers represent the fraction, expressed as a percentage, compared to the total power of solar radiation falling on the Earth (100% corresponds to a power of approximately $1.74 \cdot 10^{17}$ W). Adapted from [9].

In reality, the Earth is currently not entirely in a radiative equilibrium. The temperatures have not yet had time to adjust completely to the increased concentrations of greenhouse gases, and the concentrations of these gases are continuing to grow due to anthropogenic emissions. The Earth is presently receiving about 1 W/m^2 more energy than it is emitting and is thus warming. Most of this excess energy (about 90%) ends up heating the oceans.

34 Climate Change and Global Warming

Climate change and global warming are the consequences of an increase in the amount of certain greenhouse gases in the atmosphere, which in turn increases the strength of the greenhouse effect. The most problematic is carbon dioxide CO_2, the amount of which has increased by over 40% since the beginning of the industrial revolution. Its concentration is usually given in parts-per-million-volume (ppmv for short). The unit ppmv represents the ratio between the number of CO_2 molecules and the number of all air molecules in a certain volume, expressed in parts per million. The concentration of CO_2 250 years ago was about 280 ppmv, but by now has increased to about 410 ppmv (data valid for 2018).

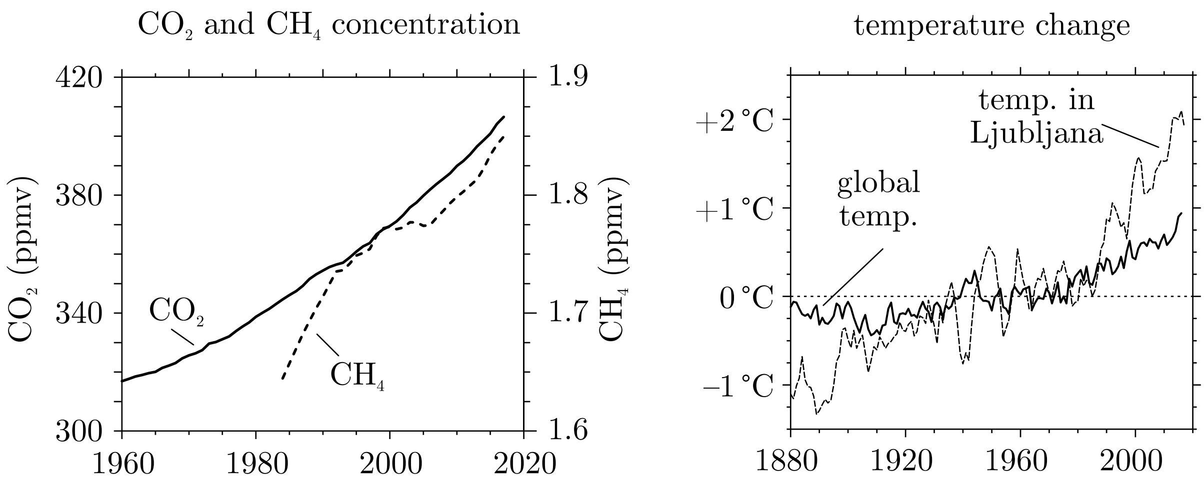


Figure 59: Changes in the concentration of CO_2 and CH_4 in the atmosphere in recent decades and the change in the average global near-surface temperature and the temperature in Ljubljana since 1880. The temperature deviation from the average temperature in the 20th century (1900–2000) is shown. Data sources: [10], [11], [12], and the Slovenian Environment Agency.

The rise in CO_2 is largely the result of the burning of fossil fuels, which is increasing due to humanity's ever-larger energy needs. The amount of CO_2 is increasing faster and faster (Figure 59), and current anthropogenic emissions of CO_2 are six times higher than they were in 1950. The concentration these days of CO_2 is also high compared to the last period in Earth's history. For example, chemical analyses of air bubbles trapped deep in polar ice reveal that in the last 800 000 years the concentration of CO_2 has not exceeded 300 ppmv.

Methane CH_4 is also problematic with amounts of it also rising due to anthropogenic causes, primarily farming and mining. The concentration of CH_4 250 years ago was about 0.7 ppmv, but by now has increased by more than 150% to about 1.8 ppmv (Figure 59). Although there is much less methane in the atmosphere than carbon dioxide, the influence of a single CH_4 molecule on the greenhouse effect is much greater than the influence of a single CO_2 molecule. The methane molecule has a much greater ability to absorb IR radiation than the CO_2 molecule. Moreover, the methane absorption bands are located at wavelengths at which other gases do not absorb significantly, which is not the case for CO_2, whose absorption bands overlap considerably with the bands of other gases, especially water vapor (Figure 51). A relatively small increase in methane concentration (about 1.1 ppmv) therefore has a noticeable impact on the greenhouse effect. The same also applies to N_2O, whose concentration has also grown following anthropogenic activities, yet its influence on the greenhouse effect is smaller. Currently, the combined effect of all other greenhouse gases (e.g., CH_4, N_2O) on the greenhouse effect changes is considered to be about half that of CO_2.

Increased concentrations of greenhouse gases also have some secondary effects that can further add to the greenhouse effect and raise the temperature (so-called ***climate***

change feedback loops). The temperature increase due to anthropogenic emissions of other greenhouse gases means that more water is evaporating from the ground (mainly the oceans) and there is more water vapor in the atmosphere than before. Since water vapor is a powerful greenhouse gas, the temperature is rising more than it would otherwise. Due to warming, the ice in the polar regions is also melting, reducing the size of the Earth's surface covered by ice. Ice-covered or snow-covered surfaces have a high albedo compared to land surfaces or oceans, which absorb most of the solar radiation. As the size of ice-covered areas is shrinking, more solar radiation is being absorbed by the ground, and temperatures are rising further. Another example is permafrost in which large amounts of CO_2 and CH_4 are trapped. Increased temperatures cause it to melt, whereby these gases escape into the atmosphere, further adding to the greenhouse effect.

The most obvious consequence of increased concentrations of greenhouse gases is the rise in the average temperature on Earth, or ***global warming***. Figure 59 shows the average global near-surface temperature change since 1880. Since then, the average global temperature has increased by about 1 °C. This temperature rise results from the strengthening of the greenhouse effect, in which a growing proportion of longwave radiation from the Earth's surface is intercepted in the atmosphere and returned to the surface. It is important to stress that an average global temperature rise of 1 °C does not mean that the temperature has risen by that much all around the planet. In fact, some areas of the Earth are warming much faster than others. One example is Ljubljana, where the temperature has increased by 2–3 °C in the last 140 years (Figure 59). It should be emphasized that the temperature increase in Ljubljana is not only due to climate change but is also an outcome of the city's expansion, with more and more natural land cover being replaced with buildings and roads (the 'urban heat island' effect). Measurements at other locations in Slovenia reveal that the average temperature has grown by around 2 °C since the 1960s.

Alongside warming, there are other changes as well such as ***changes in typical weather patterns*** which, for example, affect the spatial distribution of precipitation. Thus, there may be less precipitation than before in some areas and more in others. A ***reduction in precipitation*** can increase the frequency of droughts and make them last longer which, when combined with a higher temperature, can increase the water and heat stress on plants, in turn affecting food production. Measurements confirm a decrease in precipitation in many regions of the Earth. An example of two prominent areas with less precipitation are the Sahel region in Africa and southern Europe along the Mediterranean Sea. Even in Slovenia, especially in the western half of the country, a statistically significant decrease in precipitation compared to the situation a few decades ago has been observed at many stations. In connection with the increased temperatures, a significant decrease in snowfall has also been observed in Slovenia. At the same time, the average duration of solar radiation has increased (the time when the flux density of solar radiation at the ground exceeds a certain threshold value – 120 W/m^2 is typically used).

Climate change is also affecting the oceans. The temperature of water in the upper layer of the oceans (down to a depth of 700 m) has gone up on average by about 0.10 °C since 1961. The melting of ice on land, especially glaciers in the mountains and ice in polar regions, and the temperature expansion of water in the oceans is bringing a ***sea-level rise***. On average, sea levels have risen by about 25 cm over the last 150 years, with some low-lying coastal regions already experiencing serious problems. There are difficulties with more frequent flooding, stronger coastal erosion, and drinking water supply. In the long term, the most problematic are densely populated areas along large river deltas in Asia and Africa, and some island countries, notably in the Pacific. Due to the increased amount of CO_2 in the atmosphere, ever more of the gas is dissolving in the oceans, which is causing ***ocean acidification***. To date, the pH in the upper layer of the oceans has dropped by about 0.1, making it more difficult for organisms like oysters and corals to form their hard shells and skeletons.

One consequence of increased temperatures is changes in regions where certain ***diseases*** typically occur. One example are diseases transmitted by mosquitoes and ticks (e.g., malaria and Lyme disease) because the carriers of these diseases can today survive in regions where they could not do so in the past.

Computer-based climate simulations can be used to predict future climate changes. Simulations are made using ***climate models***, which are similar to numerical weather prediction models that are employed for weather forecasting (for more on this, see Section 36) which, in addition to the weather, simulate the oceans and certain other factors that can affect the climate (e.g., the change in vegetation). Forecasts are usually made for the period until the end of the 21st century. Long-term climate simulations are associated with a certain degree of uncertainty as current models cannot completely realistically simulate all the factors that influence the climate. Climate models are constantly being improved and hence give an increasingly accurate picture of what will happen in the future. Climate simulations also depend strongly on assumptions made about future anthropogenic greenhouse gas emissions. A few standard scenarios are typically used for these emissions. The most optimistic scenario assumes that anthropogenic emissions of greenhouse gases will reach a maximum in the near future and then begin to decrease rapidly. The most pessimistic scenario assumes that anthropogenic emissions will increase until the end of the 21st century.

Figure 60 shows the average global near-surface temperature and sea level forecasts for both scenarios. In the optimistic scenario, the average temperature would rise by about 1.5 °C by the end of the 21st century, while the pessimistic scenario predicts a much bigger temperature rise of about 4.5 °C. For the rise in the sea level, the difference between the two scenarios is smaller, with the first one predicting a further rise of about 0.4 m and the second one a rise of about 0.7 m. In the pessimistic scenario, the sea level would rise by about 1 m compared to the period before the industrial revolution. There is also a considerable difference in ocean acidity predictions between the scenarios. According to the optimistic scenario, the pH could further drop by about 0.1 while in the pessimistic scenario it would decrease by much more (by around 0.7).

Future climate changes depend highly on the amount of future anthropogenic greenhouse gas emissions. A significant emissions reduction could markedly reduce the magnitude of climate change and its associated negative consequences. The only way to effectively reduce emissions is through political agreements on the global level in which

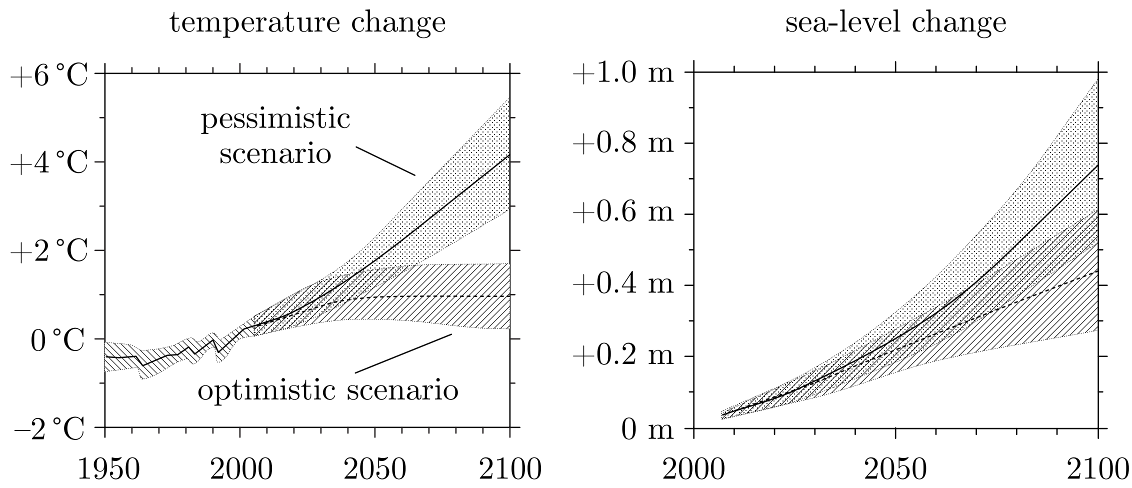


Figure 60: Forecasts of the average global near-surface temperature and sea level until the end of the 21st century. Simulations are made based on optimistic and pessimistic scenarios of anthropogenic greenhouse gas emissions. The changes are shown relative to the average temperature or sea level in the period 1986–2005. The shaded areas indicate the degree of the uncertainty of the forecasts. Adapted from [13].

most countries commit themselves to implementing certain measures. Thus far, attempts at such agreements have largely been unsuccessful and emissions have continued to grow, with multiple reasons being responsible for the failure. Differences between developed and developing countries play a big role. The developing countries note that the developed countries are responsible for most of the anthropogenic greenhouse gases currently found in the atmosphere and that it is unfair that every country should limit its emissions to the same extent. There is also the issue of responsibility for damage caused by past emissions. Since developed countries are mostly responsible for those past emissions whereas climate change is today affecting every country, developing countries expect financial compensation for the damage and help with adapting to climate change. Another big question is what will happen to climate migrants – those who live in regions that have become or will become unsuitable for living following climate change (e.g., due to increasingly difficult food production or the lack of drinking water), who will need to find a new place to live. A special group are countries that are large fossil fuel exporters – it is not in their interest to drastically reduce fossil fuel consumption as that would negatively impact their economy. The situation is also complicated because some countries might also benefit from climate change in the short term; for example, increased temperatures could increase crop yields in some countries that are currently experiencing cold climates.

One of the latest attempts to reach an agreement is the Paris Agreement signed by 195 countries in 2015. This agreement set a target to reduce greenhouse gas emissions to the extent that the rise in global temperature would be less than 2 °C (compared to the situation before the industrial revolution) while trying to limit the temperature rise to 1.5 °C. However, many details of the agreement have yet to be undecided and some countries have already withdrawn from it, making its success far from guaranteed (this was the situation in 2019).

35 The General Circulation of the Atmosphere

Although the Earth is in radiative balance to a good approximation (i.e., as much energy as it receives from the Sun, it also emits in the form of longwave radiation), this is not the case for individual regions on Earth. Particular regions receive significantly more energy than they emit, while the opposite is true for others. Figure 61 shows the approximate power of radiation received and emitted at different latitudes. In the tropics, up to a latitude of about 35°, the Earth receives more energy than it emits, while at higher latitudes, the opposite is true. This discrepancy results in a surplus of energy in tropical regions and a deficit at higher latitudes. With such an imbalance between the energy received and emitted, for example, the temperature in the tropics would be expected to rise continuously; however, this is not happening. This suggests that there is some mechanism that is continuously transferring the excess energy from the tropics to higher latitudes. This happens with the large-scale global flow of the atmosphere and oceans, also called ***general circulation***. As the air in the atmosphere and water in the oceans move, heat is also transferred between different regions of the Earth, causing the temperature differences to be reduced, with tropical regions being cooler and polar regions warmer than they would otherwise be. It would be different on some solid-surface planet or moon that has no atmosphere and oceans (e.g., the Earth's moon, which has almost no atmosphere). In this case, the two curves in Figure 61 would coincide, with each individual region of the planet's surface being in a radiative balance because there would be no effective mechanism that would allow the transfer of energy between different regions.

A detailed discussion of the global circulation of the atmosphere lies beyond the scope of this textbook, and we will not deal with circulation in the oceans. Figure 62 shows an idealized visualization of the global circulation of the atmosphere and the prevailing near-surface winds.

Regions near the equator receive the most solar radiation and are the warmest. There, the air near the surface converges from both hemispheres and rises in the so-called ***intertropical convergence zone*** (or ***ITCZ*** for short). The ITCZ is a region of

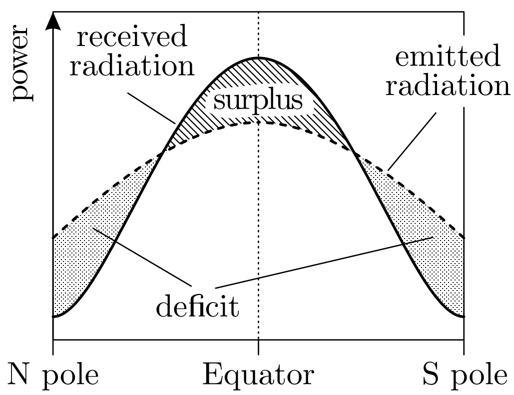


Figure 61: Approximate power of the received absorbed solar radiation (solid line) and emitted terrestrial radiation (dashed line) at different latitudes.

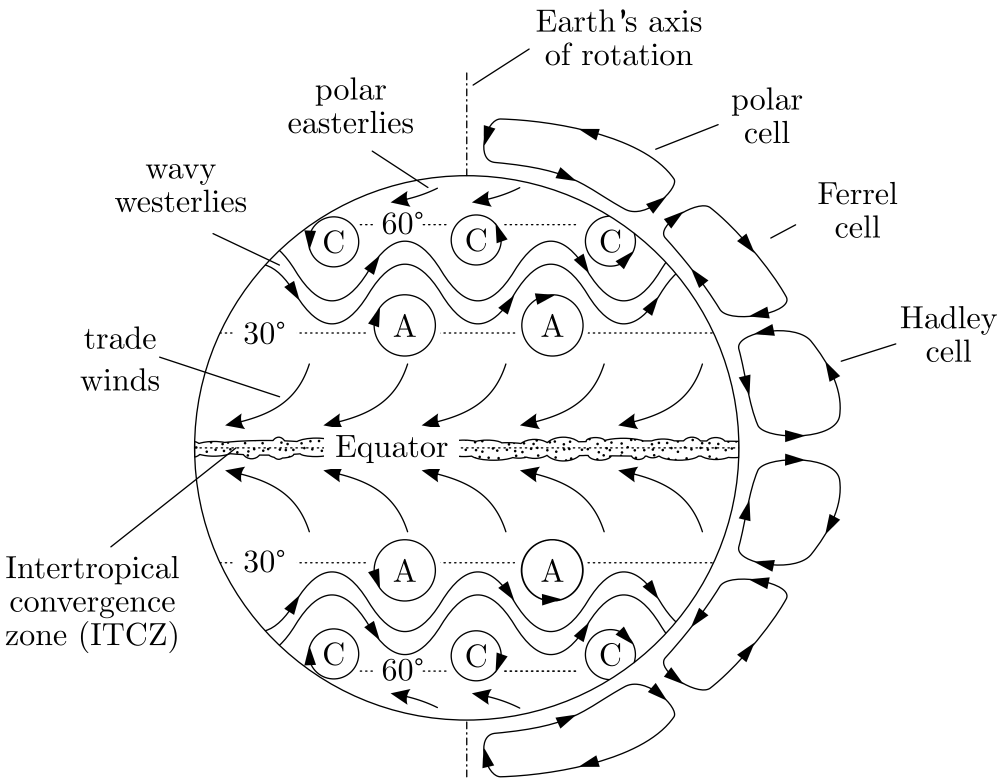


Figure 62: An idealized visualization of the global circulation of the atmosphere. Labels C and A denote cyclones and anticyclones.

frequent convection, cloudiness, and precipitation. A significant portion of the Earth's global precipitation falls in the ITCZ. The location of the ITCZ changes by the seasons (it is more to the north in the northern hemisphere's summer and more to the south in winter), and its shape is also significantly influenced by the land distribution. Ascending air in the ITCZ causes air to flow toward the pole at high altitudes. Due to the Coriolis force, which deflects the flow sideways, the poleward flow is interrupted at about 30° and the circulation splits into three characteristic cells (Figure 62).

The ***Hadley cell*** consists of rising air in the ITCZ, a poleward flow at high altitudes, descending air at about 30°, and airflow towards the ITCZ in the lower atmosphere. The outcome is the so-called ***trade winds***, the prevailing north-easterly and south-easterly winds in the tropics that blow near the surface from about 30° towards the equator or the ITCZ. Due to the existence of descending air (which causes good weather), there are large desert regions at 30°. For example, in the northern hemisphere, there are the Sahara, the deserts in Mexico and the United States, the Arabian Desert, and the deserts in southern Asia. In the southern hemisphere, there are the Atacama, Namib, Kalahari, and Australian deserts. The ***Polar cell*** is present in the polar regions where air descends above the pole and rises at about 60°, and in between ***polar easterlies*** blow near the surface.

Between the Hadley and polar cells is the ***Ferrel cell***, which is dominated by a westerly airflow represented by ***wavy westerlies***. These waves are called ***Rossby waves*** or ***planetary waves*** and have a typical wavelength of several thousand kilometers. Rossby waves generally move eastward but can also be stationary or move westward. In the case of a stationary wave, the same weather situation can endure for a long time over a geographical area, often referred to as blocking. Between the waves, large eddies in the airflow occur on both sides, with cyclones usually occurring on the polar side and anticyclones on the equatorial side.

Compared to the idealized situation shown in Figure 62, the actual general circulation of the atmosphere is much more complex and spatiotemporally variable. For example, the differences in surface temperatures of the land and oceans have a big influence and modify the circulation. Moreover, the orography can obstruct or block the airflow.

36 Numerical Weather Prediction

Nowadays, weather forecasting is done on the basis of computer simulations of the atmosphere. These simulations are made using computer programs called ***numerical weather prediction models***. All models work on the same underlying principle – they rely on a system of basic meteorological equations to determine the future state of the atmosphere. These are equations that describe physical processes in the atmosphere, and we already mentioned them in previous sections:

$$\begin{aligned}
\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} &= \vec{f}_{\mathrm{grad}} + \vec{f}_{\mathrm{Cor}} + \vec{f}_{\mathrm{g}} + \vec{f}_{\mathrm{fr}} && \text{the momentum equation,} \\
p &= \rho R T && \text{ideal gas law,} \\
\frac{\mathrm{d}\rho}{\mathrm{d}t} &= -\rho \nabla \cdot \vec{v} && \text{the continuity equation,} \\
\frac{\mathrm{d}T}{\mathrm{d}t} &= \frac{1}{\rho c_{\mathrm{p}}}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{1}{m c_{\mathrm{p}}}\frac{\mathrm{d}Q}{\mathrm{d}t} && \text{The thermodynamic energy equation.}
\end{aligned} \tag{58}$$

Equations 58 represent a system of partial differential equations for four main meteorological variables ($\vec{v}$, p, ρ, and T). Additional equations deal with other processes not described by Equations 58 such as humidity, clouds and precipitation, the interaction between the atmosphere and the ground, and radiation.

The simulations are done by integrating these equations in time, which allows for determining the value of meteorological variables in the future. The integration is performed numerically (by calculating on a computer) because the equations are too complex to be integrated in any other way. The most straightforward approach is to represent the atmosphere by a three-dimensional ***mesh of computational grid points***. During the simulation, the values of the meteorological variables are defined and forecasted only at the locations of these points. The grid points can be more or less evenly distributed across the Earth's surface, whereby the typical horizontal distance between adjacent points is called the ***spatial resolution of the model***. The resolution of the model should

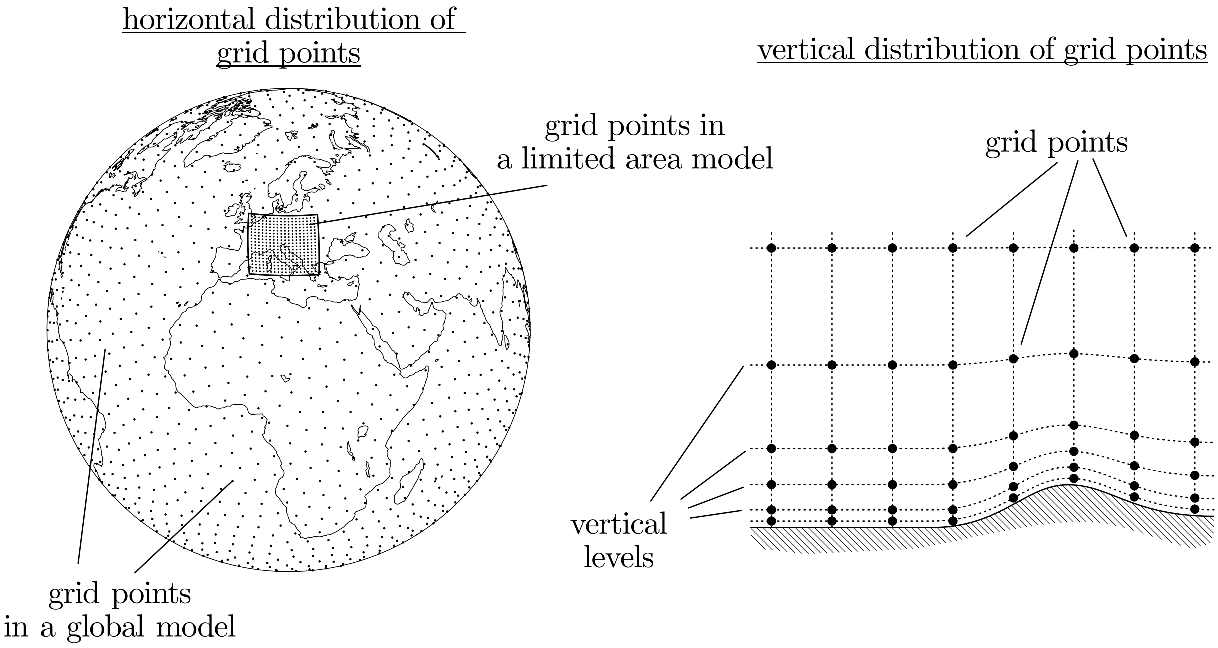


Figure 63: On the left is an example of computational grid points in a global model and a limited area model. The number of points nowadays is much larger than shown. The number in the figure corresponds to a resolution of approximately 500 km for the global model or 110 km for the limited area model. Global models these days have a resolution of about 10 km, and limited area ones of between 1 and 4 km. The limited area model area shown in the figure corresponds relatively well to the area currently used by the ALADIN model relied on at the Slovenian Environment Agency. On the right is an example of the vertical distribution of points into vertical levels. Closer to the ground, the levels generally follow the orography and are usually closer together, while higher up, they are further apart and more horizontal.

be as good as possible (the distance between neighboring points as small as possible) since the smallest spatial changes of meteorological variables can thereby be described, potentially resulting in a more realistic simulation. The problem is that better resolution requires more computational points and thus a more powerful computer is needed for the calculations. This is also the reason that very powerful computers are needed to efficiently calculate the weather forecast. In the vertical direction, the computational points are arranged on vertical levels. Closer to the ground, the levels usually follow the orography and tend to be closer together, whereas higher up they are further apart and more horizontal (Figure 63). Current models typically use between 60 and 140 vertical levels.

We distinguish between global and limited area models depending on the region in which the models simulate the atmosphere. ***Global models*** have computational points distributed across the entire Earth and thus simulate the whole atmosphere (Figure 63). Global high-resolution models require the most powerful computers that only the wealthiest countries can afford. Nowadays, the resolution of the best global models is about 10 km. Such resolution is still too poor for models to simulate small-scale phenomena like individual thunderstorms realistically, yet is sufficient for larger phenomena such as cyclones and weather fronts.

Limited area models simulate the atmosphere over a geographically limited region, generally much smaller than the Earth's entire surface. Since the computational region is considerably smaller than in global models, the computational points can be much closer together, and less powerful computers can be used. The majority of countries therefore run their own limited area models. Most of the limited area models that national meteorological services use operationally for weather forecasting these days have a resolution of between 1 and 4 km. The Slovenian Environment Agency (SEA) uses the ALADIN model, whose current resolution is 4.4 km. The resolution of the models is improving with the development of more powerful computers – for example, in 1997, when the SEA started to use ALADIN the resolution was 12.5 km, later it was improved to 9.5 km, while in 2019 it was 4.4 km, and preparations are underway for a further improvement. Figure 63 shows the approximate extent of the area used by the SEA in the ALADIN model. The area covers not only Slovenia but a large part of Central Europe as well. The reason is that weather phenomena can move quickly and it is important to simulate them with better resolution even before they enter the territory of Slovenia. While the shown area may seem large, it represents only about 0.9% of the size of the entire surface of the Earth.

Although better resolution enables limited area models to simulate some small-scale weather processes, it should be noted that limited area models cannot function entirely independently. Namely, they need the values of the variables at the edges of the computational area, which they can only obtain from global models (or other limited area models that cover a larger region). Therefore, the process of making a forecast usually starts by first calculating the forecast of global models, which are generally used to calculate a forecast up to 2 weeks into the future. It takes a few hours for the global model forecast to be calculated, and the results are transferred via the Internet to the meteorological services of individual countries where the calculation of limited area models immediately begins. These usually calculate forecasts for only a few days into the future and also finish the calculations in a few hours. Hence, in about 5 hours, all calculations are finished, which are then checked by an ***operational meteorologist*** who prepares and issues an official weather forecast. The job of an operational meteorologist is not simply to copy results from the model forecast into an official forecast. Numerical models often do not give completely correct predictions for reasons to be discussed later. Based on many years of experience with the model's behavior and its typical errors, an operational meteorologist critically interprets the model's results and adjusts the official forecast accordingly, which tends to be more correct.

A detailed description of numerical integration and the workings of the models is beyond the scope of this textbook. Still, it is noted that the integration is performed in successive time steps. The length of a time step is usually a few minutes. For example, if the time step is 5 minutes and the forecast calculation starts at midnight, the values of the variables are first calculated for the time that corresponds to 5 minutes past midnight (Figure 64). The new values of all variables at all computational points must be calculated. When this calculation is completed, new values are calculated for the next

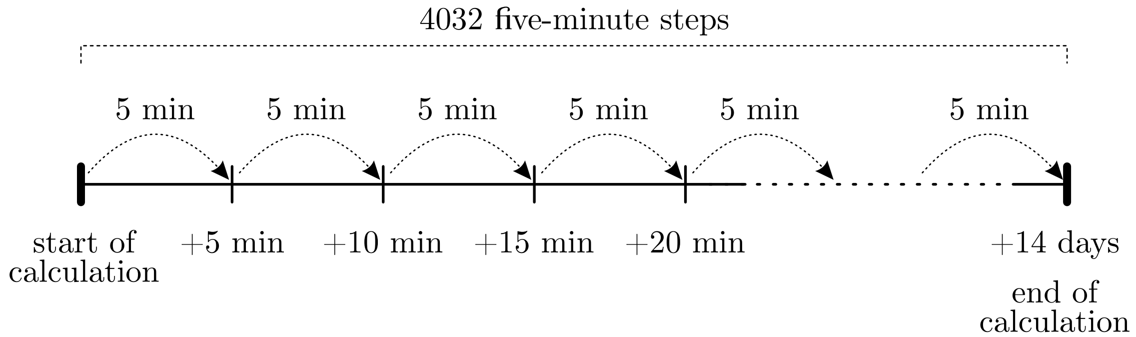


Figure 64: The sequence of 5-minute steps for calculating a 14-day weather forecast. A 14-day forecast requires 4,032 of such steps.

step, corresponding to the time 10 minutes past midnight. This process is repeated until the desired forecast-time duration is reached. For example, a 14-day forecast requires 4032 five-minute steps.

In order to perform the calculations for the first time step, it is necessary to know the values of all meteorological variables for all computational points at the initial time – the so-called ***initial conditions***. These values are determined based on values from the previous forecast and current meteorological measurements – the process of preparing the initial conditions is called ***data assimilation***. The initial conditions must be as correct as possible because otherwise the forecast can already contain a considerable error at the initial time, which only increases with each subsequent time step and makes the forecast less accurate. Practically all types of meteorological measurements are used to determine the initial conditions, for example, near the ground at weather stations, on ships and buoys, and in the sky with sounding balloons, aircraft, and radars. Measurements from meteorological satellites are also essential as they are almost the only source of information on the state of the atmosphere high above the oceans. Even when all the measurements are available, there is still not nearly enough of them or they are not accurate enough to reliably determine the initial values of all meteorological variables at all computational points around the Earth – the initial conditions thus always have an error, which makes the forecasts less accurate.

The result of the numerical forecast is the calculated values of all meteorological variables at all computational points for the entire forecast period – the so-called ***deterministic weather forecast***. Experience shows that the longer the forecast period, the less correct the forecast. Namely, a forecast for a day or two into the future is mostly accurate but, by the fourteenth day, the errors tend to be so large that the forecast is no longer useful. This often causes a dilemma as to what extent the results of the deterministic weather forecast can be trusted – that is, to what extent can we trust that the calculated values will match the actual values.

An estimate of forecast uncertainty can be made using a ***probabilistic weather forecast***. Unlike a deterministic forecast, which forecasts a single value for each meteorological variable at some location and time, a probabilistic forecast forecasts a range of values (i.e., a probability distribution). This is generally done by using ***ensemble***

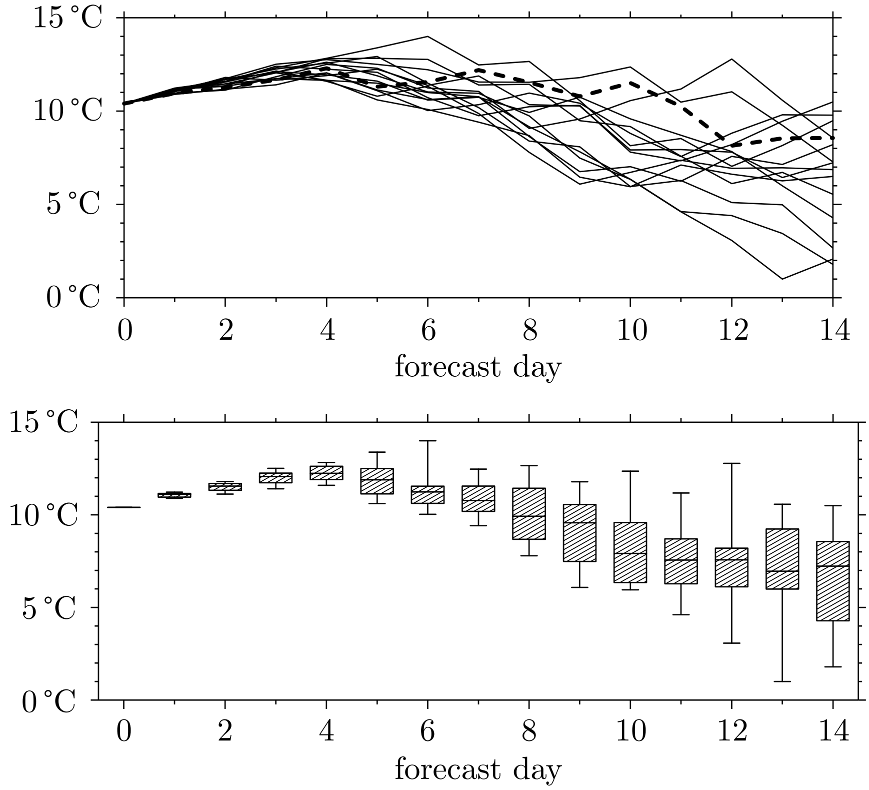


Figure 65: An example of a 14-day probabilistic forecast of the minimum daily temperature in Ljubljana where the forecast ensemble consists of 15 members. Above: forecast results are shown separately for all 15 members. The bold dashed line represents the primary operational forecast made at a higher resolution. Below: forecast results are shown with box plots. The shaded rectangles represent the range of predicted values between the 1st and 3rd quartiles, with the horizontal line in between representing the median and the handles the maximum and minimum values.

forecasting. Here, several similar forecasts are made for the same period instead of a single forecast. Such a group of forecasts is called an ensemble while individual forecasts in the ensemble are called ensemble members. There are usually a few dozen members in an ensemble. The primary operational forecast is made in the same way described in the above paragraphs. The other forecasts that constitute the ensemble are usually made at a lower resolution (typically, the distance between the computational points is about twice that of the primary forecast), with the initial conditions for each member being slightly modified. Hence, each member forecast starts from a slightly different initial state, giving more or less different results. At the beginning, the results of all the forecasts tend to agree fairly well with each other, although the differences between the ensemble members increase over time.

Figure 65 shows an example of a 14-day probabilistic forecast of the minimum daily temperature in Ljubljana made based on an ensemble containing 15 members. For the first few days, all members forecast very similar minimum daily temperature values – this means that the forecast for this period is very trustworthy as the calculated values of all members are in close agreement. Over time, the results for the different members diverge more and more, and after 5 days the range of possible temperatures is several degrees Celsius. Later, the uncertainty further increases, and after 12 days the range of possible temperatures is already around 10 °C. In this case, the model's results can no longer be trusted and the forecast can be deemed useless in the sense that it cannot predict the temperature with an accuracy that may be considered useful.

Using the results of an ensemble forecast, one can also estimate the probability of the occurrence of an event. For example, one can determine the probability of the minimum temperature on the eighth day being higher than 10 °C. Figure 65 shows that for the eighth day the median forecast value is at approximately 10 °C, meaning that half of the members forecasted a higher and half a lower value, and thus the probability is 50%. Similarly, probabilities for events defined by other variables, e.g., precipitation, cloudiness, and wind, could be determined.

The forecast error usually increases with the length of a forecast, with several reasons explaining this:

- ***Limited resolution*** – Due to limited computing power, the distance between computational points in the model cannot be arbitrarily small. This means that models cannot realistically simulate phenomena smaller in size than the distance between points.
- ***Imperfect measurements*** – Limited and partially inaccurate measurements make it impossible to accurately determine the initial values of meteorological variables in all computational points around the Earth. Therefore, an error is always present in the initial conditions, which increases with the length of a forecast.
- ***Nonlinearity*** – The underlying equations used by the models (Equations 58) contain some non-linear terms that cause the errors (e.g., those that are always present in the initial conditions) to increase very quickly. Since the error increases quite fast and also quickly spreads over the whole atmosphere, it is very difficult to produce useful meteorological forecasts for more than 2 weeks into the future. This is quite different from, for example, the astronomical equations that describe the movement of the planets in our solar system and allow the planets' positions to be relatively accurately calculated and predicted even more than 1 million years into the future.
- ***Approximate treatment of processes*** – Some processes in the atmosphere, for example, droplet formation, precipitation, the absorption and emission of radiation, and turbulence in the ground layer, are treated in a very simplified and approximate manner in the models due to their complexity, which leads to forecast errors.

Despite the problems listed above, the accuracy of weather forecasting has steadily improved over the years. This has been enabled by the ever-increasing amount of meteorological measurements (mainly measurements from satellites) that allow the initial conditions to be ever more accurately determined, the development of more and more powerful computers, and progress in the understanding of processes in the atmosphere and the improvement of their description in models.

Example Questions

The list of questions below can help with preparation for the theoretical part of the exam. The list is not final, and the exam questions may differ from those listed below.

1. List and describe the gases that make up the atmosphere.
2. List and describe the characteristic layers of the atmosphere.
3. Describe the international standard atmosphere.
4. List the most important meteorological variables and describe their connection with what happens on a microscopic scale. Describe the difference between weather and climate.
5. Describe the meteorological form of the gas equation and explain all the physical quantities that appear in it.
6. Explain why water vapor is a special gas in the atmosphere, and list which quantities can be used to describe the amount of water vapor in the air.
7. Describe relative humidity and dewpoint temperature.
8. List and describe instruments for measuring temperature, atmospheric pressure, wind, humidity, and precipitation.
9. Describe meteorological measurements by sounding balloons, radars, and satellites.
10. Explain how atmospheric pressure changes with altitude, assuming the atmosphere is isothermal.
11. Describe the thermodynamic energy equation.
12. Explain why air cools as it rises and state how fast it cools.
13. Explain the lifting condensation level and how it can be determined.
14. Describe convection.
15. Describe how static stability is defined and what it depends on. List some examples of very stable layers that are often observed in the atmosphere.
16. Describe the convective stability of the atmosphere and the measures by which it can be expressed.
17. Describe the formation of clouds and the typical sizes and shapes of the hydrometeors that constitute them.
18. List and describe the main types of clouds.

19. Explain how condensation trails can form behind airplanes.
20. Explain what fog is. Describe the different types of fog and how they form.
21. Describe how precipitation forms and list its types according to the type of hydrometeors that fall. Describe convective and stratiform precipitation.
22. Explain how thunderstorms form. Further, describe their main characteristics and list the different types of thunderstorms.
23. Explain what advection is and describe the advection equation.
24. Describe the momentum equation.
25. Describe the Coriolis and the pressure gradient forces. What do the direction and the magnitude of these forces depend on?
26. Describe the force of gravity and the centrifugal force caused by the curved path of the air parcel. What do the direction and magnitude of these forces depend on?
27. Describe balanced flow winds. What do their direction and speed rely on?
28. Explain how frictional force affects balanced flow winds. Where in the atmosphere is the effect of friction greater and where is it smaller?
29. Explain what a cyclone and an anticyclone are. Explain how the wind blows in them at high altitudes and near the surface and what kind of weather prevails in them.
30. Describe turbulence and the surface layer. Describe how the wind changes with height in the surface layer.
31. Describe and list the main types of weather fronts. What kind of weather can be expected with the passage of different types of fronts?
32. Describe local coastal, slope, Foehn, and Bora winds.
33. Describe the continuity equation.
34. Describe the difference between solar and terrestrial radiation and explain why the difference occurs.
35. Describe the solar constant, state its value, and what the value depends on.
36. Explain how radiation passes through the atmosphere and why the sky is blue.
37. Explain what determines how much solar radiation is received and absorbed by the ground, and list the types of this radiation. Describe which regions of the Earth receive the most and the least energy from solar radiation.

38. Describe the greenhouse effect and list the most important greenhouse gases.
39. Explain why climate change and global warming are occurring. List the changes that have already happened and explain what the consequences will be in the future.
40. Describe the general circulation of the atmosphere and explain why it occurs.
41. Explain how weather forecasts are made these days.
42. Describe the difference between global and limited area models.
43. Explain why current meteorological measurements are vital to weather forecasting and list the types of measurements used for this purpose.
44. Describe ensemble weather forecasting and explain why it is useful.
45. Explain why a weather forecast is never entirely accurate and why the error increases with the length of the forecast, and list and describe the reasons for this.

List of Constants and Symbols

A list and the values of some of the constants and symbols used in the textbook:

$\vec{a}$	acceleration (m/s^2)
a	albedo – the fraction of incoming radiation that is reflected
α	the angle between the direction of the incident radiation and a normal to the ground (°)
β	the angle by which the wind deviates from the isobars and the zenith angle (°)
c_p	specific heat of air at constant pressure ($c_\mathrm{p} = 1004$ J/(kg K))
c_v	specific heat of air at constant volume ($c_\mathrm{v} = 717$ J/(kg K))
$CAPE$	Convective Available Potential Energy (J/kg)
δ	zero plane displacement – usually approximated by multiplying the height of obstacles on the ground, e.g., vegetation or buildings, by about 0.7 (m)
$e\ (= p_\mathrm{v})$	partial pressure of water vapor or vapor pressure (Pa)
e_s	saturation vapor pressure (Pa)
e_s0	vapor pressure at the triple point of water ($e_\mathrm{s0} = 6.1$ hPa)
ε	emissivity
ε_atm	emissivity of the atmosphere
f	relative humidity (%) and the Coriolis parameter (s^{-1})
$\vec{f}_\mathrm{cent}$	specific centrifugal force due to the curved path of an air parcel (m/s^2)
$\vec{f}_\mathrm{cent.rot.Earth}$	specific centrifugal force of rotation of the Earth (m/s^2)
$\vec{f}_\mathrm{Cor}$	specific Coriolis force (m/s^2)
$\vec{f}_\mathrm{fr}$	specific frictional force (m/s^2)
$\vec{f}_\mathrm{grad}$	specific pressure gradient force (m/s^2)

$\vec{f}_{\mathrm{grav}}$	specific gravitational force (m/s^2)
$\vec{f}_{\mathrm{g}}$	specific force of gravity (m/s^2)
$\vec{F}_{\mathrm{g}}$	gravity force (N)
$\vec{F}_{\mathrm{v}}$	buoyancy force (N)
φ	latitude and angle between the horizontal wind and gradient vectors (°)
g	the gravity acceleration of the Earth (the standard value of the gravity acceleration is $g = 9.81$ m/s^2)
γ	lapse rate of the temperature in the environment, or negative vertical temperature gradient in the atmosphere $\gamma \equiv -\frac{\partial T_{\mathrm{e}}}{\partial z}$ (K/m)
$\mathit{\Gamma}_{\mathrm{a}}$	the adiabatic lapse rate of the temperature of unsaturated moist air that is rising ($\mathit{\Gamma}_{\mathrm{a}} \approx 10$ K/km)
$\mathit{\Gamma}_{\mathrm{s}}$	the adiabatic lapse rate of the temperature of saturated moist air that is rising ($\mathit{\Gamma}_{\mathrm{s}} =$ from SI4.5 to 10 K/km)
h_{s}	latent heat of sublimation of water ($h_{\mathrm{s}} = 2.83 \cdot 10^6$ J/kg at 0 °C)
h_{v}	latent heat of vaporization of water ($h_{\mathrm{v}} = 2.50 \cdot 10^6$ J/kg at 0 °C)
j	radiative flux density (W/m^2)
j_0	solar constant ($j_0 = 1366$ W/m^2)
j_{s}	radiative flux density of direct solar radiation at the ground (W/m^2)
k_{fr}	friction coefficient (s^{-1})
k	von Kármán constant ($k \approx 0.4$)
m	mass of air (kg)
m_{e}	mass of displaced air from the surrounding environment (kg)
M_{z}	molar mass of air ($M_{\mathrm{z}} \approx 29$ kg/kmol)
∇	vector differential operator $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
ω	Earth's angular velocity ($\omega = 7.27 \cdot 10^{-5}$ s^{-1})
p	atmospheric pressure (Pa)
p_{v} $(= e)$	partial pressure of water vapor or vapor pressure (Pa)
∇p	atmospheric pressure gradient (Pa/m)
P	power (W)
P_{emitted}	emitted power (W)
P_{received}	received power (W)
q	specific humidity (kg/kg)
r	mixing ratio (kg/kg)
R	specific gas constant for dry air ($R = 287$ J/(kg K))
R^*	universal gas constant ($R^* = 8314$ J/(kmol K))
R_{u}	radius of curvature (m)
R_{v}	specific gas constant for water vapor ($R_{\mathrm{v}} = 461$ J/(kg K))
R_{z}	radius of the Earth (m)
ρ	air density (kg/m^3)
ρ_{v}	water vapor density or absolute humidity (kg/m^3)
S	area size (m^2)
S_{ind}	Showalter index (K)
σ	Stefan-Boltzmann constant ($\sigma = 5.67 \cdot 10^{-8}$ W/(m^2 K^4))

t	time (s)
T	temperature (K)
T'_{atm}	equilibrium temperature of the atmosphere (K)
T_{d}	dewpoint temperature (K)
T_{e}	ambient air temperature (K)
T_{eq}	equilibrium temperature of the Earth's surface (K)
T'_{eq}	equilibrium temperature of the Earth's surface (K)
T_{s0}	temperature of the triple point of water ($T_{\mathrm{s0}} = 273$ K)
T_{500}	air temperature at the altitude where atmospheric pressure is equal to 500 hPa (K)
T_{L500}	the temperature that the air would have if it rose adiabatically from an altitude where the atmospheric pressure is 850 hPa to an altitude where it is 500 hPa. (K)
∇T	temperature gradient (K/m)
τ_{zen}	optical thickness of the atmosphere if the Sun is at its zenith
V	volume (m^3)
$\vec{v}$	wind or horizontal wind (m/s)
v	wind or horizontal wind speed (m/s)
v_{g}	geostrophic wind speed (m/s)
v_x	wind component along coordinate x (m/s)
v_y	wind component along coordinate y (m/s)
v_z	wind component along coordinate z (m/s)
$\nabla \cdot \vec{v}$	wind divergence (s^{-1})
v_*	friction velocity (m/s)
x	one of the horizontal local coordinates (m)
y	one of the horizontal local coordinates (m)
z	altitude and the vertical local coordinate (m)
z_0	among other things also the surface roughness parameter (m)
z_{LCL}	lifting condensation level height (m)

List of Acronyms

List of acronyms used in the text:

ALADIN	name of the limited area model operationally used by the SEA
CAPE	Convective Available Potential Energy
CAT	Clear Air Turbulence
CFC	Chlorofluorocarbons
IR	Infrared
ISA	International Standard Atmosphere
ITCZ	Intertropical Convergence Zone
LCL	Lifting Condensation Level
SEA	The Slovenian Environment Agency
UV	Ultraviolet

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Index

A Appendices

A.1 Derivation of the Ideal Gas Law Equation for Air

Assuming that air is a mixture of ideal gases, we can write the ideal gas law equation for each gas

$$p_{N_2}V = \frac{m_{N_2}}{M_{N_2}}R^*T,$$
$$p_{O_2}V = \frac{m_{O_2}}{M_{O_2}}R^*T, \tag{59}$$
$$....$$

where p_X is the partial pressure of each gas, and m_X and M_X are its mass and molar mass. An ideal gas law equation can also be written for a mixture of ideal gases

$$pV = \frac{m}{M_z}R^*T, \tag{60}$$

where p is the total atmospheric pressure, m is the mass of air, and M_z is the molar mass of air, for which we do not yet know the value. At the same time, the total atmospheric pressure is the sum of the partial pressures of all gases in the air

$$p = p_{N_2} + p_{O_2} + ... \tag{61}$$

If we express the pressures from equations 59 and 60 and insert them into Equation 61, we can cancel out V, R^*, and T and obtain

$$\frac{m}{M_z} = \frac{m_{N_2}}{M_{N_2}} + \frac{m_{O_2}}{M_{O_2}} + ... \tag{62}$$

If we divide Equation 62 by m, we obtain

$$\frac{1}{M_z} = \frac{\left(\frac{m_{N_2}}{m}\right)}{M_{N_2}} + \frac{\left(\frac{m_{O_2}}{m}\right)}{M_{O_2}} + ... \tag{63}$$

and

$$M_z = \left[\frac{\left(\frac{m_{N_2}}{m}\right)}{M_{N_2}} + \frac{\left(\frac{m_{O_2}}{m}\right)}{M_{O_2}} + ...\right]^{-1}. \tag{64}$$

The terms of the form $\left(\frac{m_X}{m}\right)$ represent the mass fractions of individual gases in the air as listed in Table 1. If we consider only N_2 and O_2, which have the mass fractions 0.753 and 0.23, we obtain

$$M_z = \left[\frac{0.753}{28\ \mathrm{kg/kmol}} + \frac{0.23}{32\ \mathrm{kg/kmol}}\right]^{-1} = \tag{65}$$
$$=29.34\ \mathrm{kg/kmol}.$$

If we also took the other gases into account, we would obtain a more accurate value, which is slightly smaller ($M_z = 28.96$ kg/kmol).

A.2 Derivation of the equation for a change in atmospheric pressure with altitude for an isothermal atmosphere

If the air is still or moving only horizontally, a hydrostatic equilibrium is quickly established in the atmosphere in which there is no vertical movement, and the specific force of gravity and the specific pressure gradient force are in balance in the vertical direction

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = g, \tag{66}$$

where $g = 9.81\ \mathrm{m/s^2}$ is the gravity acceleration of the Earth. Equation 66 is a differential equation that can be solved under certain assumptions. It is easiest to solve assuming that the air temperature does not change with the altitude (the atmosphere is isothermal). From the ideal gas law (Equation 2), the density is determined and inserted into Equation 66. The equation is then multiplied by ∂z and divided by p, R and T, which yields

$$\frac{\partial p}{p} = -\frac{g}{RT}\cdot \partial z. \tag{67}$$

The differential equation is solved by integrating both sides separately, with the limits of the integration for altitude being z_0 and z and for atmospheric pressure $p(z_0)$ and $p(z)$.

$$\int_{p(z_0)}^{p(z)} \frac{\partial p}{p} = -\int_{z_0}^{z} \frac{g}{RT}\cdot \partial z, \tag{68}$$

from where it follows

$$\ln\left[\frac{p(z)}{p(z_0)}\right] = -\frac{g}{RT}\cdot (z - z_0). \tag{69}$$

From here $p(z)$ can be expressed

$$p(z) = p(z_0)\cdot \mathrm{e}^{-\frac{g}{RT}(z-z_0)}. \tag{70}$$

A.3 Derivation of the equation for a change in atmospheric pressure with altitude for a linear temperature change

Equation 67 can also be solved by relying on the assumption that the temperature changes linearly with the altitude. We assume that the temperature change can be expressed as

$$T(z) = T(z_0) - \gamma\cdot (z - z_0). \tag{71}$$

where $T(z)$ and $T(z_0)$ are the temperatures at altitudes z and z_0 and γ is the temperature lapse rate in the environment. If γ is positive, this means that the temperature decreases with the altitude, and vice versa, if γ is negative (e.g., $\gamma = 8$ K/km means that the air located 1 km lower down will be colder by 8 °C, while $\gamma = -8$ K/km means the air will be 8 °C warmer there). Equation 71 can be inserted into Equation 67

$$\frac{\partial p}{p} = -\frac{g}{R \cdot T(z)} \cdot \partial z \tag{72}$$

and integrated similarly as for the case of an isothermal atmosphere

$$\int_{p(z_0)}^{p(z)} \frac{\partial p}{p} = -\int_{z_0}^{z} \frac{g}{R \cdot \left(T(z_0) - \gamma \cdot (z - z_0)\right)} \cdot \partial z. \tag{73}$$

The integral on the left side of the equation is the same as in Equation 68, and its value is $\ln\left[p(z)/p(z_0)\right]$. For the integral on the right side of the equation, we can introduce a new variable $u = T(z_0) - \gamma \cdot (z - z_0)$ and rearrange Equation 73 as

$$\ln\left[\frac{p(z)}{p(z_0)}\right] = \frac{g}{R \cdot \gamma} \cdot \int_{T(z_0)}^{T(z_0)-\gamma\cdot(z-z_0)} \frac{\partial u}{u}, \tag{74}$$

and

$$\ln\left[\frac{p(z)}{p(z_0)}\right] = \frac{g}{R \cdot \gamma} \ln\left[\frac{T(z_0) - \gamma \cdot (z - z_0)}{T(z_0)}\right], \tag{75}$$

and then

$$\ln\left[\frac{p(z)}{p(z_0)}\right] = \ln\left[\left(\frac{T(z_0) - \gamma \cdot (z - z_0)}{T(z_0)}\right)^{\frac{g}{R\cdot\gamma}}\right], \tag{76}$$

from where $p(z)$ can be expressed

$$p(z) = p(z_0) \cdot \left(1 - \gamma \cdot \frac{z - z_0}{T(z_0)}\right)^{\frac{g}{R\cdot\gamma}}. \tag{77}$$

where, similarly to Equation 9, $p(z_0)$ and $p(z)$ are the atmospheric pressures at altitudes z_0 and z, while γ is the temperature lapse rate, that is assumed to be constant in the layer between z_0 and z.

A.4 Derivation of the thermodynamic energy equation for unsaturated air

The change in the internal energy of an ideal gas depends only on the temperature change and can be written as $\mathrm{d}W_\mathrm{n} = mc_\mathrm{v}\mathrm{d}T$, where m is the mass of air, $\mathrm{d}T$ is a small temperature change and $c_\mathrm{v} =$ 717 J/(kg K) the specific heat of air at a constant volume. Air can compress or expand, pushing the surrounding air away, thereby doing work $\mathrm{d}A = p\mathrm{d}V$ on it, where $\mathrm{d}V$ is a small change in the volume of an air parcel. Equation 11 can thus be written as

$$mc_\mathrm{v}\mathrm{d}T = \mathrm{d}Q - p\mathrm{d}V. \tag{78}$$

The ideal gas law (Equation 2) can be written in the form $pV = mRT$, where the terms on the left and right sides of the equation can be written in terms of the total derivative, and we obtain

$$\begin{aligned} \mathrm{d}(pV) &= \mathrm{d}(mRT), \\ p\mathrm{d}V + V\mathrm{d}p &= mR\mathrm{d}T. \end{aligned} \tag{79}$$

We express the term $p\mathrm{d}V$ from the equation and insert it into Equation 78, taking into account that for ideal gases the identity $c_\mathrm{v} + R = c_\mathrm{p}$ holds true, and we obtain

$$mc_\mathrm{p}\mathrm{d}T = \mathrm{d}Q + V\mathrm{d}p. \tag{80}$$

If we divide Equation 80 by $mc_\mathrm{p}\mathrm{d}t$, we obtain Equation 12.

A.5 Derivation of the temperature change of unsaturated air when it is ascending or descending

The temperature change during the ascent or descent of unsaturated air can most easily be derived using Equation 80. We assume that the process happens fast enough so that the air parcel does not exchange any heat with the surrounding environment (the process is adiabatic). In this case, $\mathrm{d}Q = 0$ and Equation 80 takes the form

$$0 = mc_\mathrm{p}\mathrm{d}T - V\mathrm{d}p. \tag{81}$$

If we assume that the atmosphere is in a hydrostatic equilibrium (Equation 66) and that the air parcel adjusts to the atmospheric pressure in the surrounding environment ($\partial p = \mathrm{d}p$ and $\partial z = \mathrm{d}z$), we can express the pressure change as

$$\mathrm{d}p = -\rho g\mathrm{d}z. \tag{82}$$

If we insert the expression for $\mathrm{d}p$ into Equation 81, we obtain

$$0 = mc_\mathrm{p}\mathrm{d}T + V\rho g\mathrm{d}z, \tag{83}$$

where m is cancelled out by ρV, yielding

$$0 = c_\mathrm{p}\mathrm{d}T + g\mathrm{d}z. \tag{84}$$

If we divide Equation 84 by $c_\mathrm{p}\mathrm{d}z$ and express $\frac{\mathrm{d}T}{\mathrm{d}z}$, we obtain

$$\frac{\mathrm{d}T}{\mathrm{d}z} = -\frac{g}{c_\mathrm{p}}, \tag{85}$$

which is equal to Equation 13.

A.6 Derivation of the dewpoint temperature change of unsaturated air when it is ascending or descending

The dew point temperature depends only on the partial pressure of water vapor (e) and is given by Equation 8. Due to the expansion or compression of air, e changes during vertical movement and thus the dewpoint temperature also changes. At the same time, the mass of the individual air parcels is conserved, and the mass of water vapor in the same air parcel is also conserved, which means that the ratio of these two masses is preserved as well, which can be expressed by

$$\mathrm{d}\left(\frac{m_\mathrm{v}}{m}\right) = 0. \tag{86}$$

The mass ratio can be expressed through the ideal gas law equations for air and water vapor, from which one obtains

$$\frac{m_\mathrm{v}}{m} = \frac{R}{R_\mathrm{v}} \cdot \frac{e}{p}. \tag{87}$$

If we insert the expression from Equation 87 into Equation 86 and differentiate, we obtain

$$\mathrm{d}\left(\frac{R}{R_\mathrm{v}} \cdot \frac{e}{p}\right) = \frac{R}{R_\mathrm{v}}\left(\frac{\mathrm{d}e}{p} - e\frac{\mathrm{d}p}{p^2}\right) = 0 \tag{88}$$

and hence the identity

$$\frac{\mathrm{d}e}{e} = \frac{\mathrm{d}p}{p}. \tag{89}$$

Expressing total derivative for Equation 8 gives

$$\mathrm{d}T_\mathrm{d} = \frac{R_\mathrm{v}T_\mathrm{d}^2}{h_\mathrm{v}} \cdot \frac{\mathrm{d}e}{e}, \tag{90}$$

where $\frac{\mathrm{d}e}{e}$ is replaced by $\frac{\mathrm{d}p}{p}$, and for $\mathrm{d}p$ we use the expression from Equation 82. This yields

$$\mathrm{d}T_\mathrm{d} = -\frac{R_\mathrm{v}T_\mathrm{d}^2}{h_\mathrm{v}} \cdot \frac{\rho g \cdot \mathrm{d}z}{p}. \tag{91}$$

We divide Equation 91 by $\mathrm{d}z$ and take into account that $\rho/p = 1/RT$ is valid due to the ideal gas law, which yields

$$\frac{\mathrm{d}T_\mathrm{d}}{\mathrm{d}z} = -\frac{gR_\mathrm{v}}{h_\mathrm{v}R}\frac{T_\mathrm{d}^2}{T}. \tag{92}$$

Equation 92 shows that the rate of change in the dewpoint temperature will not always be the same because it depends on the current values of the temperature and dewpoint temperature.

For example, if we assume that the temperature at the ground is the same as in ISA (15 °C) and that the dewpoint temperature is 10 °C, we obtain the value −1.75 K/km for $\frac{\mathrm{d}T_\mathrm{d}}{\mathrm{d}z}$. This value is approximately equal to $-\frac{1}{6}\Gamma_\mathrm{a}$, thus Equation 14 is frequently used instead of Equation 92 to determine an approximate value for $\frac{\mathrm{d}T_\mathrm{d}}{\mathrm{d}z}$ near the surface.

A.7 Derivation of the temperature change of saturated air when it is ascending or descending

The temperature change of a saturated air parcel during its ascent or descent can most easily be derived using Equation 81, where an additional term related to the energy that is released/consumed during the phase changes of water needs to be added

$$0 = mc_\mathrm{p}\mathrm{d}T - V\mathrm{d}p + h_\mathrm{v}\mathrm{d}m_\mathrm{vs}. \tag{93}$$

m_vs represents the saturated mass of water vapor in the air parcel since we assume that when rising all excess water vapor immediately condenses ($\mathrm{d}m_\mathrm{v}$ represents the change in this mass as a result of condensation or evaporation). To derive the expression for $\mathrm{d}m_\mathrm{v}$, we use Equation 87, noting that the partial pressure of water vapor in saturated air is the same as the saturated vapor pressure

$$\frac{m_\mathrm{vs}}{m} = \frac{R}{R_\mathrm{v}} \cdot \frac{e_\mathrm{s}}{p}, \tag{94}$$

where we express m_vs as

$$m_\mathrm{vs} = \frac{mR}{R_\mathrm{v}} \cdot \frac{e_\mathrm{s}}{p}. \tag{95}$$

The total derivative of Equation 95, assuming that the total mass of the air parcel m does not change, is

$$\mathrm{d}m_\mathrm{vs} = \frac{mR}{R_\mathrm{v}p}\mathrm{d}e_\mathrm{s} - \frac{mRe_\mathrm{s}}{R_\mathrm{v}p^2}\mathrm{d}p. \tag{96}$$

The total derivative of Equation 6 can be expressed as

$$\mathrm{d}e_\mathrm{s} = \frac{h_\mathrm{v}e_\mathrm{s}}{R_\mathrm{v}T^2}\mathrm{d}T. \tag{97}$$

If we insert Equation 97 into Equation 96, we obtain

$$\mathrm{d}m_{\mathrm{vs}} = \frac{m h_{\mathrm{v}} e_{\mathrm{s}} R}{R_{\mathrm{v}}^2 p T^2}\mathrm{d}T - \frac{m R e_{\mathrm{s}}}{R_{\mathrm{v}} p^2}\mathrm{d}p. \tag{98}$$

We insert the expression from Equation 98 into Equation 93

$$0 = m c_{\mathrm{p}}\mathrm{d}T - V\mathrm{d}p + \frac{m h_{\mathrm{v}}^2 e_{\mathrm{s}} R}{R_{\mathrm{v}}^2 p T^2}\mathrm{d}T - \frac{m h_{\mathrm{v}} e_{\mathrm{s}} R}{R_{\mathrm{v}} p^2}\mathrm{d}p. \tag{99}$$

If we divide Equation 99 by m and $\mathrm{d}p$ and express the term $\frac{\mathrm{d}T}{\mathrm{d}p}$, we obtain

$$\frac{\mathrm{d}T}{\mathrm{d}p} = \frac{\frac{1}{\rho} + \frac{h_{\mathrm{v}} e_{\mathrm{s}} R}{R_{\mathrm{v}} p^2}}{c_{\mathrm{p}} + \frac{h_{\mathrm{v}}^2 e_{\mathrm{s}} R}{R_{\mathrm{v}}^2 p T^2}}. \tag{100}$$

If in Equation 100 we express $\frac{1}{\rho}$ in the numerator of the fraction and c_{p} in the denominator, and consider $\mathrm{d}p = -\rho g\, ddz$, we obtain

$$\frac{\mathrm{d}T}{\rho g \mathrm{d}z} = -\frac{1}{\rho c_{\mathrm{p}}} \cdot \left[\frac{1 + \frac{h_{\mathrm{v}} e_{\mathrm{s}}}{R_{\mathrm{v}} p} \cdot \frac{\rho R}{p}}{1 + \frac{h_{\mathrm{v}}^2 e_{\mathrm{s}} R}{c_{\mathrm{p}} R_{\mathrm{v}}^2 p T^2}}\right], \tag{101}$$

where by multiplying by g and taking into account that $\frac{\rho R}{p} = \frac{1}{T}$ is true due to the ideal gas law, and that $\frac{g}{c_{\mathrm{p}}} = \mathit{\Gamma}_{\mathrm{a}}$, we obtain

$$\frac{\mathrm{d}T}{\mathrm{d}z} = -\mathit{\Gamma}_{\mathrm{a}} \cdot \left[\frac{1 + \frac{h_{\mathrm{v}} e_{\mathrm{s}}}{R_{\mathrm{v}} p T}}{1 + \frac{h_{\mathrm{v}}^2 e_{\mathrm{s}} R}{c_{\mathrm{p}} R_{\mathrm{v}}^2 p T^2}}\right], \tag{102}$$

and taking into account Equation 17, we obtain

$$\mathit{\Gamma}_{\mathrm{s}} = \mathit{\Gamma}_{\mathrm{a}} \cdot \left[\frac{1 + \frac{h_{\mathrm{v}} e_{\mathrm{s}}}{R_{\mathrm{v}} p T}}{1 + \frac{h_{\mathrm{v}}^2 e_{\mathrm{s}} R}{c_{\mathrm{p}} R_{\mathrm{v}}^2 p T^2}}\right]. \tag{103}$$

At the top of the troposphere, temperatures are very low and hence the saturated vapor pressure is almost zero. From Equation 103 it can be seen that for $e_{\mathrm{s}} \approx 0$ Pa, $\mathit{\Gamma}_{\mathrm{s}}$ is the same as $\mathit{\Gamma}_{\mathrm{a}}$. This means it is approximately true that in the upper troposphere saturated and unsaturated air cools very similarly (in both cases by about 10 K/km). Near to the surface, the situation is different, and the value of $\mathit{\Gamma}_{\mathrm{s}}$ is smaller

For example, for ISA at sea level, the temperature is $T = 15$ °C, atmospheric pressure $p =$ 1013.25 hPa, and saturated vapor pressure $e_{\mathrm{s}}(15\ °\mathrm{C}) =$ 17.19 hPa, thus

$$\mathit{\Gamma}_{\mathrm{s}} = \mathit{\Gamma}_{\mathrm{a}} \cdot \left[\frac{1 + \frac{2.50 \cdot 10^6\ \mathrm{J/kg} \cdot 1719\ \mathrm{Pa}}{101\,300\ \mathrm{Pa} \cdot 461\ \mathrm{J/(kg\,K)} \cdot 288\ \mathrm{K}}}{1 + \frac{(2.50 \cdot 10^6\ \mathrm{J/kg})^2 \cdot 1719\ \mathrm{Pa} \cdot 287\ \mathrm{J/(kg\,K)}}{101\,300\ \mathrm{Pa} \cdot (461\ \mathrm{J/(kg\,K)})^2 \cdot 1004\ \mathrm{J/(kg\,K)} \cdot (288\ \mathrm{K})^2}}\right] = 4.9\ \mathrm{K/km}.$$

A similar calculation can also be made for an altitude of 11 km, yet the temperature and pressure at this height must first be determined. In the standard atmosphere, the stratosphere begins at an altitude of 11 km where the temperature is −56.5 °C. The saturated vapor pressure at this temperature is 3.42 Pa. The atmospheric pressure at 11 km is obtained using Equation 77, where the parameters of the ISA are assumed ($\gamma = 6.5$ K/km, $T(0\ \mathrm{m}) = 288$ K, $p(0\ \mathrm{m}) = 1013$ hPa), which yields 226 hPa. $\mathit{\Gamma}_{\mathrm{s}}$ is calculated similarly as before

$$\mathit{\Gamma}_{\mathrm{s}} = \mathit{\Gamma}_{\mathrm{a}} \cdot \left[\frac{1 + \frac{2.50 \cdot 10^6\ \mathrm{J/kg} \cdot 3.42\ \mathrm{Pa}}{22\,600\ \mathrm{Pa} \cdot 461\ \mathrm{J/(kg\,K)} \cdot 216.5\ \mathrm{K}}}{1 + \frac{(2.50 \cdot 10^6\ \mathrm{J/kg})^2 \cdot 3.42\ \mathrm{Pa} \cdot 287\ \mathrm{J/(kg\,K)}}{22\,600\ \mathrm{Pa} \cdot (461\ \mathrm{J/(kg\,K)})^2 \cdot 1004\ \mathrm{J/(kg\,K)} \cdot (216.5\ \mathrm{K})^2}}\right] = 9.8\ \mathrm{K/km}.$$

In general, $\mathit{\Gamma}_{\mathrm{s}}$ will be the smallest near the surface where its value is approximately 5 K/km, which means that the cooling speed of saturated air during ascent is almost half of that of unsaturated air. The value of $\mathit{\Gamma}_{\mathrm{s}}$ increases with height and almost reaches the value of $\mathit{\Gamma}_{\mathrm{a}}$ at the top of the troposphere.

A.8 Calculation of convective available potential energy (CAPE)

CAPE is defined as the integral of the vertical acceleration a_{vert} along the altitude assuming that the air rises from the ground, integrating only from the level of free convection z_{LFC} to the equilibrium level z_{EQ} (this reflects the altitude interval when the rising air is warmer than the surroundings and the vertical acceleration points upwards). Assuming that only the buoyant force $\vec{F}_{\mathrm{v}}$ and the gravity force $\vec{F}_{\mathrm{g}}$ exist in the vertical direction, their sum is given by Equation 21, thus

$$g \cdot m\left(\frac{T - T_{\mathrm{e}}}{T_{\mathrm{e}}}\right) = m \cdot a_{\mathrm{vert}}, \tag{104}$$

which yields

$$a_{\mathrm{vert}} = g\left(\frac{T - T_{\mathrm{e}}}{T_{\mathrm{e}}}\right). \tag{105}$$

The CAPE value is calculated as the integral of the vertical acceleration

$$CAPE = g\int_{z_{\mathrm{LFC}}}^{z_{\mathrm{EQ}}}\left(\frac{T - T_{\mathrm{e}}}{T_{\mathrm{e}}}\right)\mathrm{d}z. \tag{106}$$

To calculate the CAPE value, it is necessary to numerically integrate the integral into the equation

106. Typically, temperature profile data are obtained from a sounding balloon measurement or the results of a numerical weather prediction model. Usually, in Equation 106, the virtual temperatures of the rising and ambient air are considered instead of temperatures.

The virtual temperature is the temperature that dry air of the same density as moist air would have at a given pressure. By using virtual temperature, the influence of humidity on air density is taken into account more realistically, which affects the balance of forces in the vertical direction. The virtual temperature value can be calculated as

$$T_\mathrm{v} = (1 + q(R_\mathrm{v}/R - 1))T' \approx (1 + 0.61q)T', \quad (107)$$

where T' is the temperature and T_v is the virtual temperature. A more detailed explanation and derivation of Equation 107 is available in [14].

A.9 Derivation of the advection equation

If we have a scalar variable f that depends on the coordinates x, y, z, and time t, a small change in the value of the variable $\mathrm{d}f$ can be written as a total derivative

$$\mathrm{d}f = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + \frac{\partial f}{\partial z}\mathrm{d}z + \frac{\partial f}{\partial t}\mathrm{d}t. \quad (108)$$

If we divide Equation 108 by $\mathrm{d}t$, we obtain

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial f}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}t} + \frac{\partial f}{\partial t}. \quad (109)$$

The terms of the form $\frac{\mathrm{d}x}{\mathrm{d}t}$ represent the components of the velocity vector in different directions, which can be written as $\frac{\mathrm{d}x}{\mathrm{d}t} = v_x$, $\frac{\mathrm{d}y}{\mathrm{d}t} = v_y$ and $\frac{\mathrm{d}z}{\mathrm{d}t} = v_z$, and $\vec{v} = (v_x, v_y, v_z)$. Equation 109 can thus be written as

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial x}v_x + \frac{\partial f}{\partial y}v_y + \frac{\partial f}{\partial z}v_z + \frac{\partial f}{\partial t}, \quad (110)$$

which can be expressed in shorter form as

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \vec{v} \cdot \nabla f + \frac{\partial f}{\partial t}, \quad (111)$$

where ∇f is the gradient of the variable f, defined as $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$ (for more on the gradient, see Section 21), and $\vec{v} \cdot \nabla f$ is the scalar product of the velocity and gradient vectors. If we express the term $\frac{\partial f}{\partial t}$, we obtain

$$\frac{\partial f}{\partial t} = -\vec{v} \cdot \nabla f + \frac{\mathrm{d}f}{\mathrm{d}t}. \quad (112)$$

Noting that the scalar product of two vectors is identical to the product of their lengths and the cosine of the intermediate angle φ, we also obtain

$$\frac{\partial f}{\partial t} = -|\vec{v}| \cdot |\nabla f| \cdot \cos\varphi + \frac{\mathrm{d}f}{\mathrm{d}t}. \quad (113)$$

A.10 Derivation of the continuity equation

The mass of air in volume V can be expressed by the integral of the density over the volume

$$m = \iiint\limits_V \rho \cdot \mathrm{d}V. \quad (114)$$

If Equation 114 is derived in terms of time, and we assume that the mass in the volume can change only as a result of the mass flow across the boundaries of the volume, we can write

$$\frac{\partial m}{\partial t} = \iiint\limits_V \frac{\partial \rho}{\partial t} \cdot \mathrm{d}V = -\oiint\limits_S \vec{j}_m \cdot \mathrm{d}\vec{S}. \quad (115)$$

S represents the surface area of volume V, while $\vec{j}_m$ is the mass flux vector. Using Gauss's theorem, instead of the surface integral of the vector field over a closed surface, we can write the volume integral of the vector divergence over the enclosed volume, thus

$$\oiint\limits_S \vec{j}_m \cdot \mathrm{d}\vec{S} = \iiint\limits_V \left(\nabla \cdot \vec{j}_m\right) \mathrm{d}V. \quad (116)$$

If we insert Equation 116 into Equation 115, we can write

$$\iiint\limits_V \frac{\partial \rho}{\partial t} \cdot \mathrm{d}V = -\iiint\limits_V \left(\nabla \cdot \vec{j}_m\right) \mathrm{d}V. \quad (117)$$

The expression in Equation 117 is valid for an arbitrarily small volume V, which means that the following must also be true

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{j}_m. \quad (118)$$

If we assume that the airmass flux is solely due to the movement of air (wind), we can write $\vec{j}_m = \rho\vec{v}$, where $\vec{v}$ is the wind vector. We then obtain

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho\vec{v}) = -\rho\nabla \cdot \vec{v} - \vec{v} \cdot \nabla\rho. \quad (119)$$

At the same time, we can write the advection equation for the air density (Equation 112)

$$\frac{\partial\rho}{\partial t} = -\vec{v}\cdot\nabla\rho + \frac{\mathrm{d}\rho}{\mathrm{d}t}. \tag{120}$$

If we insert Equation 120 into Equation 119, we obtain

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\vec{v}, \tag{121}$$

which is equal to Equation 50.

A.11 Derivation of the attenuation of direct solar radiation in the atmosphere

The decrease in the flux density of direct solar radiation while passing through matter (e.g., through the atmosphere) can be described by

$$\mathrm{d}j' = -j'k\rho\mathrm{d}s', \tag{122}$$

where j' is the flux density of the radiation, k is the attenuation coefficient, ρ is the density of matter, and $\mathrm{d}j'$ represents the flux density reduction on a short path of length $\mathrm{d}s'$. The equation is only valid when the radiation is coming from only a single direction (e.g., for direct solar radiation). Moreover, the possible increase in the flux density due to emission and scattering is neglected. Equation 122 is a differential equation that can be solved by dividing the equation by j' and integrating both sides

$$\int_{j_0}^{j}\frac{\mathrm{d}j'}{j'} = -\int_0^s k\rho\mathrm{d}s'. \tag{123}$$

The integral on the right side of the equation represents a quantity called the optical thickness $\tau = \int_0^s k\rho\mathrm{d}s'$. By integrating the left side of the equation, we obtain

$$\ln\left(\frac{j}{j_0}\right) = -\tau, \tag{124}$$

and

$$j = j_0\mathrm{e}^{-\tau}, \tag{125}$$

which is called the Lambert-Beer law. The greater the optical thickness of the substance, the more the radiation will be weakened on its way through the matter. If we are interested in the attenuation of radiation through the entire atmosphere when the Sun is at its zenith, instead of $\mathrm{d}s'$ we write $\mathrm{d}z$ and define the atmosphere's optical thickness at the zenith passage $\tau_{\mathrm{zen}} = \int_0^\infty k\rho\mathrm{d}z$, after which, with a very similar derivation as above, we obtain Equation 52.

If the Sun is not at its zenith, the path the radiation takes through the atmosphere is longer (Figure 53). This can be taken into account by writing $\frac{\mathrm{d}z}{\cos\beta}$ instead of $\mathrm{d}z$, where β is the deviation angle from the zenith direction. Following the same procedure as before, we obtain the expression by integration

$$j = j_0\mathrm{e}^{-\frac{\tau_{\mathrm{zen}}}{\cos\beta}}, \tag{126}$$

which is the same as the expression in Equation 53.

A.12 Derivation of the equilibrium temperature of the Earth's surface taking the influence of the atmosphere into account

A simplified version of the radiation balance for Earth is shown in Figure 57. In a radiative balance, the surface emits longwave radiation with a power of $4\pi R_E^2\sigma {T'_{\mathrm{eq}}}^4$. At the same time, it receives shortwave solar radiation with a power of $(1-a)\pi R_E^2 j_0$ and longwave radiation from the atmosphere with a power of $4\pi R_E^2\sigma {T'_{\mathrm{atm}}}^4$. The atmosphere receives some of the longwave radiation emanating from the surface with a power of $4\pi R_E^2\varepsilon_{\mathrm{atm}}\sigma {T'_{\mathrm{eq}}}^4$, and at the same time emits radiation to the ground and space with a total power of $8\pi R_E^2\varepsilon_{\mathrm{atm}}\sigma {T'_{\mathrm{atm}}}^4$. Thus

$$\begin{aligned}&\text{surface :}\\ &(1-a)\pi R_{\mathrm{E}}^2 j_0 + 4\pi R_{\mathrm{E}}^2\varepsilon_{\mathrm{atm}}\sigma {T'_{\mathrm{atm}}}^4 = 4\pi R_{\mathrm{E}}^2\sigma {T'_{\mathrm{eq}}}^4,\\ &\text{atmosphere :}\\ &4\pi R_{\mathrm{E}}^2\varepsilon_{\mathrm{atm}}\sigma {T'_{\mathrm{eq}}}^4 = 8\pi R_{\mathrm{E}}^2\varepsilon_{\mathrm{atm}}\sigma {T'_{\mathrm{atm}}}^4,\end{aligned}$$

which can be simplified to

$$(1-a)j_0 + 4\sigma {T'_{\mathrm{atm}}}^4 = 4\sigma {T'_{\mathrm{eq}}}^4, \tag{127}$$

$${T'_{\mathrm{eq}}}^4 = 2{T'_{\mathrm{atm}}}^4. \tag{128}$$

This is a system of two equations for two unknown temperatures (T'_{atm} and T'_{eq}). The easiest solution is to express ${T'_{\mathrm{atm}}}^4 = {T'_{\mathrm{eq}}}^4/2$ from the second equation and insert it into the first equation, from which T'_{eq} is expressed as

$$T_{\mathrm{tal}} = \sqrt[4]{\frac{(1-a)j_0}{4\sigma}}\cdot\sqrt[4]{\frac{1}{1-\varepsilon_{\mathrm{atm}}/2}}. \tag{129}$$

The expression under the first root is identical to the expression for T_{eq} from Equation 56, thus

$$T_{\mathrm{tal}} = T_{\mathrm{z}} \cdot \sqrt[4]{\frac{1}{1-\varepsilon_{\mathrm{atm}}/2}}. \tag{130}$$