COMPUTATION OF ELECTRIC CHARGE ON POWER TRANSMISSION LINES

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Abstract: A system of parallel lines above a conducting or insulating plane serves as a model of a transmission line system. We present a few computational steps and results that address the question of the synchronicity of electric potential and charge on a given wire of the power line system. The differences in phase angles of the oscillating charge and the associated potential depend on the geometry of the system. For a benchmark and three additional cases the charges on the wires were computed using the described procedure. They are presented in the results section.

Izračun električnega naboja na močnostnih prenosnih linijah

Kjučne besede: kvazistatični izračuni električnega naboja, daljnovodni sistemi

Izvleček: Dvodimenzionalni sistem vzporednih vrvi končnih polmerov, ki se razpenjajo nad prevodno ali včasih neprevodno ravnino, služi kot osnovni model pri obravnavi sistemov daljnovodnih napetostnih vodov.

V članku pokažemo, da naboj danega vodnika in njegov potencial v splošnem ne nihata sofazno. Razlika med faznima kotoma potenciala in njemu pripadajočega naboja je odvisna od geometrije sistema. Predlagamo ustrezno pot do iskanih nabojev. Le-ti so osnova za izračun ostalih električnih količin, predvsem električne poljske jakosti v okolici sistema, in prikažemo nekaj rezultatov za izbrane postavitve daljnovodnih vrvi.

1 Introduction

The most basic among the models of power transmission line systems is two-dimensional. It consists of conducting parallel straight lines with known phase angles and r.m.s. values of electric potentials. The diameters of the lines are small compared to the distances between wires. The task is to determine linear charge densities on the lines from the given electric potentials of the lines.

There are more realistic models of power transmission lines, for instance, the diameter of the conductors may not be small compared to the distances between the lines, or the gravity and string forces may be included, which distort the straight lines into the chain curves. Further more, some computer programs consider the electrical properties and geometry of the pylons and even terrain.

For all the cases mentioned the computational algorithm is basically the same, although the matrix coefficients may be a way more difficult to compute and the size of the matrix tends to grow considerably /1/.

2 Methods

The notation used in this article for a-priori known quantities is:

r_i: position of the i-th wire, (x_i, y_i),

V_i: electrical potential of the i-th wire,

ϑ_i: phase angle of the electrical potential V_i, and for a-priori unknowns:

q_k: linear charge density of the k-th wire,

φ_k: phase angle of the linear charge density q_k.

The potential of the i-th wire has a form $V_i \cdot \cos(\omega t+\vartheta_i)$, where $\omega = 2\pi\nu$ and ν is the frequency.

The frequency ν = 50 Hz justifies a quasi-static approach for power transmission lines, so at any given time the potential of i-th wire may be written as a superposition of the charge on all wires /5/:

$$V_i \cdot \cos(\omega t+\vartheta_i) = \frac{q_i \cdot \cos(\omega t+\varphi_i)}{2\pi\varepsilon_0} \ln\frac{1}{r_{i0}} + \sum_{k\neq i} \frac{q_k \cdot \cos(\omega t+\varphi_k)}{2\pi\varepsilon_0} \ln\frac{1}{|\mathbf{r}_i - \mathbf{r}_k|}. \tag{1}$$

The introduction of parameter P_{ik} defined as

$$P_{ik} = \begin{cases} \frac{1}{2\pi\varepsilon_0} \ln\frac{1}{r_{i0}} & ; \quad i = k \\ \frac{1}{2\pi\varepsilon_0} \ln\frac{1}{|\mathbf{r}_i - \mathbf{r}_k|} & ; \quad i \neq k \end{cases}$$

gives a shorter form of equation (1):

$$V_i \cdot \cos(\omega t+\vartheta_i) = \sum_k P_{ik} \cdot q_k \cdot \cos(\omega t+\varphi_k). \tag{2}$$

The identity cos ($\alpha + \beta$) = cos α cos β - sin α sin β and equation (2) lead to two separate parts of the system of equations, the first oscillating as $\cos(\omega t)$,the second as $\sin(\omega t)$:

$$V_i \cdot \cos\vartheta_i = \sum_k P_{ik} \cdot q_k \cdot \cos\varphi_k \tag{3}$$

$$V_i \cdot \sin\vartheta_i = \sum_k P_{ik} \cdot q_k \cdot \sin\varphi_k \tag{4}$$

Any attempt to solve equations (3) and (4) directly for unknown q_k and ρ_k is bound to fail for almost any set of input parameters $\mathbf{r}_i$, V_i, and ϑ_i. However, with the introduction of new variables, as shown in the following paragraph, the system of equations can be linearized, its matrix becomes diagonally dominant, and therefore suitable for further numerical manipulation.

The system of equations (3) and (4) can be linearized by introducing variables

$$a_k = q_k \cdot \cos\varphi_k$$
$$b_k = q_k \cdot \sin\varphi_k$$
$$V_{c;i} = V_i \cdot \cos\vartheta_i$$
$$V_{s;i} = V_i \cdot \sin\vartheta_i,$$

and takes the form of two separate sets of linear equations:

$$\mathrm{V_c} = \mathrm{P} \cdot \mathrm{a} \quad (5)$$
$$\mathrm{V_s} = \mathrm{P} \cdot \mathrm{b}, \quad (6)$$

where a = $(a_1, a_2, \ldots)$, b = $(b_1, b_2, \ldots)$, and

$$\mathrm{P} = \begin{pmatrix} P_{11} & P_{12} & \cdots \\ P_{21} & P_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}.$$

Since $r_{i0} \ll |\mathrm{r}_i - \mathrm{r}_k|$ for each *i* and *k* it follows that **P** is diagonally dominant.

After solving (5) and (6) one can obtain the unknown linear charge densities q_k and phase angles φ_k as:

$$q_k = \sqrt{a_k^2 + b_k^2}$$
$$\varphi_k = \arctan \frac{b_k}{a_k}.$$

3 Results

This section presents computational outcomes - the linear charge densities and their phase angles - for three different systems of power transmission lines. All cases deal with 400 kV systems, with wires of 1 centimeter in diameter, but differ in some other aspects. The zeroth example serves as a benchmark. It is followed by the first case, which is a realistic example of the 400 kV system. The second example and the third example are a bit exotic: the second only because of the geometry chosen, while the third deals also with the number of the wires and their potentials that can hardly be found in practice.

In a view of the conductivity of the ground both extreme possibilities were taken into account. When the ground is considered to be a perfect insulator the computations are performed as explained in the previous section, and their results in the examples section may be found under $q_{min}/(2\pi\varepsilon_0)$ and φ_{min}. With the ground as a perfect conductor the solution is obtained by applying the method of images, and the results for each wire in these cases may be found in $q_{max}/(2\pi\varepsilon_0)$ and φ_{max} columns. No additional unknowns are introduced in the case of a perfectly conducting ground, since the image charge of q_k-th linear charge density at time *t* has a value $q_k \cdot \cos(\omega t + \varphi_k + \pi)$.

Each of the following examples has three parts: input data, two-dimensional (*x, y*) sketch of the wires with the ground, and the resulting q_k and φ_k for all the wires in cases of insulating and conducting grounds.

3.1 Example 0

Table 1

input data				
i	x [m]	y [m]	V_{max} [V]	θ [°]
0	0.0	10.0	100000.0	0.0

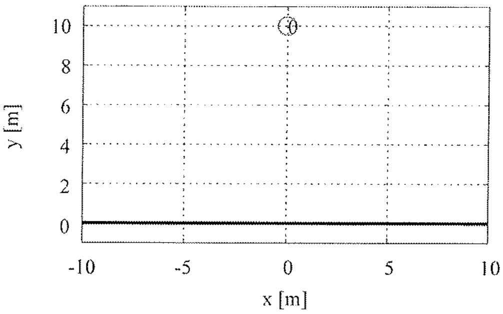

Table 2

output data				
i	$\frac{q_{min}}{2\pi\varepsilon_0}$ [V]	φ_{min} [°]	$\frac{q_{max}}{2\pi\varepsilon_0}$ [V]	φ_{max} [°]
0	18873.9	0.0	12056.8	0.0

Consider a wire with potential $V = V_0 \cdot \cos(\omega t)$, where V_0 = 100 kV. We can obtain an analytical result for the charge on the wire if it is suspended above a conducting plane by the method of images

$$V_0 \cdot \cos(\omega t) = \frac{q_0 \cdot \cos(\omega t + \varphi_0)}{2\pi\varepsilon_0} \ln \frac{2h}{r_0}, \quad (7)$$

where r_0 is the radius of the wire, and *h* is the ele-vation of the wire above the ground. The solution of (7) gives φ_0 = 0, where for given r_0 = 5 mm and *h* = 10 m we get: $\frac{q_0}{2\pi\varepsilon_0} = V_0/\ln\frac{2h}{r_0} \approx 12.057$kV.

The results of the computational algorithm below give the same result for $q_{max}/(2\pi\varepsilon_0)$. The charge of the wire above a conducting ground matches the analytical result, but the result for an insulating ground should be ignored, since a single infinite conducting line does not have a uniquely defined electric potential. The numerical value q_{min} in the output data equals $\frac{1}{2\pi\varepsilon_0}\ln(1/r_0)$ and cannot be connected to the electric potential of the wire.

3.2 Example 1

Six parallel wires serve as a first model for a double 400 kV power transmission line system.

Table 3

input data				
i	x [m]	y [m]	V_{max} [V]	θ [°]
0	-1.0	6.0	230940.0	0.0
1	0.0	6.0	230940.0	-120.0
2	1.0	6.0	230940.0	120.0
3	-1.0	10.0	230940.0	120.0
4	0.0	10.0	230940.0	-120.0
5	1.0	10.0	230940.0	0.0

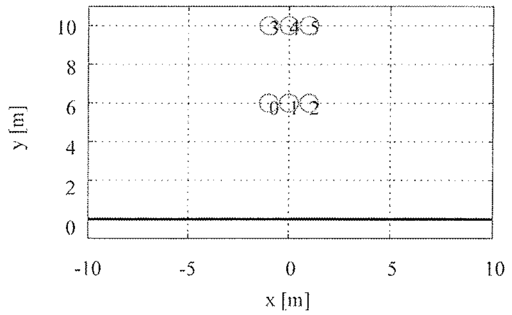

Table 4

	output data			
i	$\frac{q_{min}}{2\pi\varepsilon^{o}}$ *[V]*	*φ min [°]*	$\frac{q_{max}}{2\pi\varepsilon^{o}}$ *[V]*	*φ max [°]*
0	*57059.9*	*23.4*	*41393.0*	*4.7*
1	*25853.9*	*-120.0*	*44887.5*	*-120.0*
2	*57059.9*	*96.6*	*41393.0*	*115.3*
3	*57059.9*	*96.6*	*41291.5*	*115.4*
4	*25853.9*	*-120.0*	*44983.3*	*-120.0*
5	*57059.9*	*23.4*	*41291.5*	*4.6*

The maximum value of the electric potential of the wires is $V_{max} = 400000 = \sqrt{3} \approx 230.94$kV, conside-ring the electric potential of the conducting ground is zero.

The results show that the phase angles of the middle wires i = 1 and i = 4 are the same for the input potentials and for the resulting charges, but differ for other wires, as one would expect considering the geometrical symmetries of the system. For any taken wire the difference between the phase angle of the electric potential and the phase angle of the line charge does not exceed 5°.

3.3 Example 2

In the next example we consider six scattered parallel wires of the 400 kV power line system. The geometry we choose introduces no symmetries, and the results show none. Nevertheless, the differences between the phase angles of the potential and charge again do not exceed 5.

Table 5

	input data			
i	x [m]	y [m]	V_{max} [V]	θ [°]
0	-3.0	5.0	230940.0	0.0
1	0.0	4.0	230940.0	-120.0
2	4.0	6.0	230940.0	120.0
3	-1.0	8.0	230940.0	120.0
4	0.0	11.0	230940.0	-120.0
5	2.0	10.0	230940.0	0.0

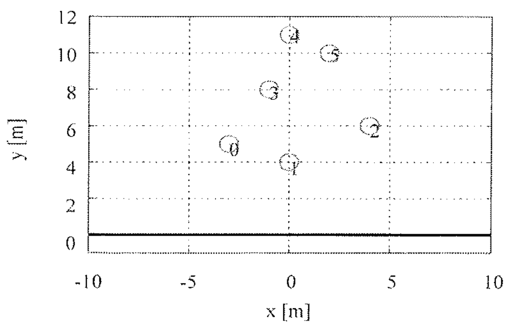

Table 6

	output data			
i	$\frac{q_{min}}{2\pi\varepsilon^{o}}$ *[V]*	φ min [°]	$\frac{q_{max}}{2\pi\varepsilon^{o}}$ *[V]*	φ max [°]
0	34911.5	2.6	35897.4	0.7
1	36301.6	-121.2	37000.7	-119.7
2	34183.5	119.9	33535.9	120.1
3	38125.3	120.4	37194.1	120.1
4	37119.6	-123.9	37348.7	-122.4
5	37314.3	2.2	37808.8	1.3

3.4 Example 3

As the last one, we present a highly exotic example in the theory and practice of 400 kV power transmission line systems. It consists of nine wires with some unusual phase angles of electric potential, and therefore represents an electrically non-symmetric case. There is no apparent geometrical symmetry, either.

Table 7

	input data			
i	x [m]	y [m]	V_{max} [V]	θ [°]
0	-3.0	5.0	230940.0	0.0
1	0.0	4.0	230940.0	-120.0
2	4.0	6.0	230940.0	120.0
3	-1.0	8.0	230940.0	120.0
4	0.0	11.0	230940.0	-120.0
5	2.0	10.0	230940.0	0.0
6	6.0	2.0	230940.0	50.0
7	7.0	6.0	230940.0	170.0
8	8.0	9.0	230940.0	-80.0

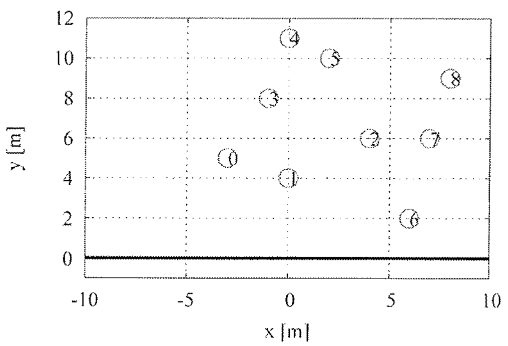

Table 8

	output data			
i	$\frac{q_{min}}{2\pi\varepsilon^{o}}$ *[V]*	φ min [°]	$\frac{q_{max}}{2\pi\varepsilon^{o}}$ *[V]*	φ max [°]
0	34207.6	3.7	36038.4	1.1
1	36981.1	-122.2	36893.5	-119.5
2	34012.1	117.3	33301.3	115.8
3	39140.8	119.9	37643.3	119.1
4	35552.5	-125.8	35909.1	-123.0
5	37776.4	6.0	38716.0	4.5
6	33940.1	46.8	36733.5	45.8
7	34302.0	169.8	33058.4	170.0
8	32441.4	-75.5	33802.6	-73.9

As shown before, for a given wire the phase angles of the potential and the charge do not differ significantly. In this particular case the differences are larger than in the previous examples, but still below 10°. Given a set of values of potential phase angles and a set of phase angles of line charges, the matching pairs can be found only by checking input and output tables even for this geometry. However, one can easily imagine an example where this is not so, no matter how more exotic it might be.

4 Discussion

The importance of the electric charge computation for quasi-static low-frequency sources, like power transmission lines, is vast. Since the charge is the source of the electric field, its distribution only enables us to find the appropriate values of the surrounding field. Traditionally, the electric field values were used for the estimation of the loses due to corona discharges, while today with the increasing interest in possible health issues associated with the non-thermal effects of electromagnetic radiation they play a role in the design of power line grids /3, 2, 4/.

As shown in the article, the charge is not oscillating synchronously with the electric potential in general, so its computation is a task on its own. The system of equations is non-linear, but can be linearized, or it better should be, in order to avoid computational problems. Even the simplest of models, the model of infinite straight lines presented here, gives us the elliptically polarized results for the electric field strength vector. This we intend to discuss in our future work.

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