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Theoretical and Experimental Study of Fatigue Strength of Plain Woven Glass/Epoxy Composite

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In this paper an energy-based model for predicting fatigue life and evaluation of progressive damage in composite materials is proposed. The damage model is based on the concepts of continuum damage mechanics. The applicability of the proposed energy model was studied in fatigue experiments on 10-layer composite laminates made of glass fabric impregnated with epoxy-phenolic resin. Experimental results were processed by the method of least squares to determine the unknown parameters of the model. Theoretical fatigue strength curves are in good agreement with experimental data.
Keywords: composite materials, fatigue, damage mechanics, fracture, finite element modelling, mechanical properties

0 INTRODUCTION

With the increasing use of composite materials comes an increasing need to understand their fatigue behavior. They exhibit very complex failure mechanisms under fatigue loading because of anisotropic characteristics in their strength and stiffness. Cyclic loading causes extensive damage throughout the composite volume, leading to failure from general degradation of the material instead of a predominant single crack. A predominant single crack is the most common failure mechanism in static loading of isotropic, brittle materials such as metals. There are four basic failure mechanisms in composite materials as a result of fatigue: matrix cracking, delamination, fiber breakage and interfacial debonding [1]. The different failure modes combined with the inherent anisotropies, complex stress fields, and overall non-linear behavior of composites severely limit our ability to understand the true nature of fatigue.

Different fatigue models have been established during the last decades, which are based on the well-known S-N curves. They usually require extensive experimental work and do not take into account the actual damage mechanisms, such as matrix cracking and fiber breakage. These models make up the first class of the so-called ‘fatigue life models’ [2]. The second class comprises the phenomenological models for residual stiffness and strength. The reliability of a composite component can change over time because of the strength and stiffness loss the material exhibit. These models propose an evolution law which can describe the gradual deterioration of the stiffness or strength of the composite specimens in terms of macroscopically observable properties. The third class of models introduces one or more properly chosen damage variables which describe the deterioration of the composite component. These models are based on a physically sound modeling of the underlying damage mechanisms, which lead to the macroscopically observable degradation of the mechanical properties. They can predict the damage growth in composite component, such as the number of transverse matrix cracks per unit length and the size of the delamination area. One of the important outcomes of all established fatigue models is the prediction of fatigue life and each of these three categories uses its own criterion for determination of fatigue life [2].

In this paper a model for predicting fatigue life and evaluation of progressive damage is proposed. The unknkown parameters of this model were estimated using experimental results of fatigue tests on glass fiber/epoxy composite mark STEF-1.

1 EFFECTIVE STRESS CONCEPT

In contrast to fracture mechanics which considers the process of an equilibrium condition or initiation and growth of microcracks as a discontinuous phenomenon, continuum damage mechanics uses a continuous internal variable, which is related to the density of these microdefects. The damage variable, based on the effective stress concept, represents average material degradation, which reflects the various types of damage at the micro-scale level like nucleation and growth of voids, cavities, micro-cracks, and other microscopic defects. In the literature various forms of damage have been proposed in recent years, for example, scalars, vectors, second and forth order tensors. The complexity and variety of mechanisms of accumulation of fatigue damage and degradation of strength properties of the composite component make reasonable use of an internal scalar variable for the quantitative description of damage. For the case of isotropic damage, the damage variable is scalar and associated with a decrease in the effective area of any

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cross-section in the vicinity of this point of the body and is defined using the concept of effective stress in the following manner [3]:

$$D = \frac{A - \tilde{A}}{A}, \tag{1}$$

where $\tilde{A}$ is the effective resisting area corresponding to the damaged area A. The effective area $\tilde{A}$ is obtained from A by removing the surface intersections of the micro-cracks and cavities and correcting for the micro-stress concentrations in the vicinity of discontinuities and for the interactions between closed defects. By definition, theoretical value of D should be within $0 \leq D \leq 1$. In the multiaxial case of isotropic damage, all the stress components act on the same effective area, so the effective stress tensor is:

$$\tilde{\sigma}_{ij} = \frac{\sigma_{ij}}{1 - D}. \tag{2}$$

In the case of anisotropic damage, the damage variable has been shown to be tensorial in nature. The case of anisotropic damage is much more complicated to ensure a good representation of the physics as well as compatibility with thermodynamics. In a general state of deformation and damage, the effective stress tensor $\tilde{\sigma}$ is related to the stress tensor σ by the following linear equation:

$$\tilde{\sigma}_{ij} = M_{ijkl} \cdot \sigma_{kl}, \tag{3}$$

but this lead to a nonsymmetric tensor and a complicated theory. As only the symmetric part accounts for the constitutive equations of elasticity, a symmetrical form for $\tilde{\sigma}_{ij}$ can be obtained through the transformation [4]:

$$\tilde{\sigma}_{ij} = \frac{1}{2}\left[\sigma_{ik}(\delta_{kj} - D_{kj})^{-1} + (\delta_{il} - D_{il})^{-1}\sigma_{lj}\right], \tag{4}$$

where δ_{ij} is the Kronecker delta and D_{ij} is a second order damage tensor. Firs must use the second principle of thermodynamics to derive the proposed energy model mut be used, because accumulation of damage is a dissipative process that is governed by the laws of thermodynamics. The second principle of thermodynamics, also referred to as the Clausius-Duhem inequality, can be written as [5]:

$$\sigma_{ij}\dot{\varepsilon}_{ij} - \rho(\dot{\psi} + s\dot{T}) - q_i \frac{T_{,i}}{T} \geq 0, \tag{5}$$

where $\vec{q}$ is the heat flux vector associated with the temperature gradient for the non isothermal processes, ψ Helmholtz free energy and s entropy density. The Helmholtz free energy is a function of all the state variables. If it is assumed that no plastic deformation occurs in ccan write its rate can be written as:

$$\dot{\psi} = \frac{\partial\psi}{\partial\varepsilon_{ij}^{e}}\dot{\varepsilon}_{ij}^{e} + \frac{\partial\psi}{\partial T}\dot{T} + \frac{\partial\psi}{\partial D}\dot{D}, \tag{6}$$

together with the definition of the associated variables, Eq. (5) becomes:

$$\left(\sigma_{ij} - \rho\frac{\partial\psi}{\partial\varepsilon_{ij}^{e}}\right)\dot{\varepsilon}_{ij}^{e} - \rho\left(s + \frac{\partial\psi}{\partial T}\right)\dot{T} - \rho\frac{\partial\psi}{\partial D}\dot{D} - q_i\frac{T_{,i}}{T} \geq 0. \tag{7}$$

On the other hand, the analytical expression for the Helmholtz free energy together with the principle of strain equivalence and the concept of effective stress for isothermal processes can be written as [4]:

$$\psi = \frac{1}{2\rho}a_{ijkl}\varepsilon_{ij}^{e}\varepsilon_{kl}^{e}\left(1 - D\right), \tag{8}$$

where a_{ijkl} is the elastic stiffness tensor. The state laws are derived from the state potential so the law of elasticity coupled with damage will have the form:

$$\sigma_{ij} = \rho\frac{\partial\psi}{\partial\varepsilon_{ij}^{e}} = a_{ijkl}\varepsilon_{kl}^{e}\left(1 - D\right). \tag{9}$$

The thermodynamics of irreversible processes defines its associated variable Y, (a positive quadratic function) called the "energy density release rate" for scala damage D, that can be defined as [5]:

$$Y = -\rho\frac{\partial\psi}{\partial D} = \frac{1}{2}a_{ijkl}\varepsilon_{ij}^{e}\varepsilon_{kl}^{e}, \tag{10}$$

this means that to always satisfy inequality of positive dissipation Eq. (7), the damage rate $\dot{D}$ must be a non-negative function, so:

$$Y \cdot \dot{D} \geq 0. \tag{11}$$

2 DEFINING RELATIONS FOR THE DEVELOPMENT OF DAMAGE IN COMPOSITE

The direct measurement of damage as the surface density of microdefects (cracks, pores or inclusions) is difficult to perform and is used only in laboratories well equiped for micrography. It is easier to take advantage of the coupling between damage and elasticity to evaluate the damage by inverse methods [6]. Throughout the composite's life, growth of damage can be monitored nondestructively by measuring one of the properties of the material: the moduli, for instance, or the electrical conductivity, or light scattering, or the x-ray absorption, or ultrasonic attenuation, or the damping coefficient, or by acoustic emission detection [7]. In general, the damage variable D is described as a function of cyclic stress range $\Delta\sigma$, number of loading cycles N, stress ratio R, environmental conditions such as temperature T, and material properties such as stiffness E, as given below:

$$D = D\left(\Delta\sigma, N, E_{ijkl}, R, T, ...\right). \tag{12}$$

In the make the further assumption is made that the damage accumulation rate depends on the maximum value of specific elastic strain energy W_e per cycle, the load ratio R and on the current level of scalar isotropic damage D, then:

$$\frac{dD}{dN} = f\left(R, D, W_e\right). \tag{13}$$

If it is assumed that a power relation exists between the maximum value of specific elastic strain energy per cycle and damage growth rate, kinetic equation for the damage parameter can be written in the form [8]:

$$\frac{dD}{dN} = m \cdot k\left(R\right) \cdot \left(W_e\right)_{max}^{n}, \tag{14}$$

where W_e is the elastic strain energy per unit volume of the body which is computed from the individual stress components and elastic strains, $k(R)$ function which depends on the stress ratio, m, n constants which define the rate of damage accumulation. According to the principle of strain equivalence, with the definition of the effective stress expressecan write Eq. (14) can be written in the form:

$$\frac{dD}{dN} = m \cdot k\left(R\right) \cdot \left[\frac{1}{2}\tilde{\sigma}_{ij}\varepsilon_{ij}\right]_{max}^{n}, \tag{15}$$

or:

$$\frac{dD}{dN} = m \cdot k\left(R\right) \cdot \left[\frac{1}{2\left(1-D\right)} \cdot C_{ijkl}\sigma_{ij}\sigma_{kl}\right]_{max}^{n}, \tag{16}$$

where C_{ijkl} is the tensor of elastic constants of undamaged composite. It is also possible to replace stresses by strains:

$$\frac{dD}{dN} = m \cdot k\left(R\right) \cdot \left[\frac{1}{2}a_{ijkl}\varepsilon_{ij}^{e}\varepsilon_{kl}^{e}(1-D)\right]_{max}^{n}. \tag{17}$$

The fatigue life-time, meaning the number of cycles to increase damage parameter from D_1 to D_2 is found by integrating Eq. (13) to give N:

$$N = \int_{D_1}^{D_2} \frac{dD}{f\left(R, D, W_e\right)}. \tag{18}$$

In the case of uniaxial loading, the dependence of scalar damage parameter on the number of cycles will have the following form calculated by means of this model:

$$D = 1 - \left(-\frac{\left(n+1\right) \cdot m \cdot k\left(R\right) \cdot \sigma^{2n}}{2^n \cdot E^n} \cdot N + 1\right)^{\frac{1}{n+1}}, \tag{19}$$

where E is the modulus of elasticity of the corresponding direction. Because of considerable nonlinearity dependence of the damage parameter on the number of cycles, at the stage preceding failure, the growth rate increases and tends to infinity, material becomes unstable and ruptures. Therefore, integration of Eq. (13) over the interval 0 to 1 gives the number of cycles to rupture corresponding to the critical value of the damage:

$$\sigma = \left(\frac{2^n \cdot E^n}{\left(n+1\right) \cdot m \cdot k\left(R\right)} \cdot \frac{1}{N_f}\right)^{\frac{1}{2n}}, \tag{20}$$

where N_f is the number of cycles to failure and E is the modulus of elasticity of the corresponding direction. For investigating the validity of this presented energy model assumptions series of fatigue experiments were performed on plain woven glass/epoxy composite laminates.

3 EXPERIMENTAL PROCEDURES

3.1 Description of Material and Experimental Determination of the Elastic Moduli

The material used in this study is a plain woven glass/epoxy composite. The plain woven glass fabric was stacked in 10 layers impregnated with epoxy-phenolic resin. Material manufactured in sheets with the dimensions of 890×1020 mm and nominal thickness of 2 mm. It is macroscopically orthotropic, having two orthogonal axes of symmetry of mechanical properties, coinciding with the directions of warp and weft threads (Fig. 1).

For fatigue experiments 30 identical specimens were cut in the warp and 30 specimens in weft directions of the fabric. Specimens had rectangular cross-section 2×15 mm with length of 175 mm. A resonance frequency technique was first applied to determine the Young's modulus of specimens.

The test method is nondestructive in nature and can be used for specimens prepared for other tests. One end of the composite test specimen was fixed to an electrodynamic shaker (vibration exciter) and the frequency was slowly increased. At a particular instance of time, the input frequency becomes equal to the first natural frequency of the system and the amplitude level increases significantly. This is the resonance peak and can be clearly distinguished in the response curve.

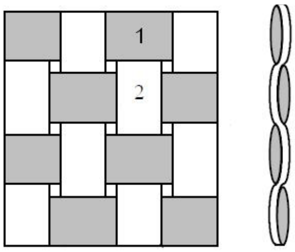

Fig. 1. *Structure of material (plain-woven fabric, 1-warp; 2-weft threads)*

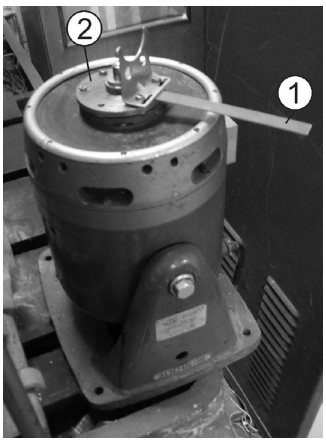

Fig. 2. *The fixing scheme of test specimen on the shaker platform: 1-test specimen; 2-vibration platform*

For a uniform clamped-free beam the differential equation of motion can be written as [9]:

$$EI\frac{\partial^4 Y(x,t)}{\partial x^4}+\rho F\frac{\partial^2 Y(x,t)}{\partial t^2}=0, \qquad (21)$$

where E is Young's modulus of the corresponding direction, I the moment of interia, F the cross-section area, ρ density. Using boundary conditions and general solution of Eq. (21) we can determine the first natural resonance frequency of the composite beam:

$$T=\frac{1}{f}=\frac{2\pi}{3.515}\sqrt{\frac{\rho FL^4}{EI}}, \qquad (22)$$

where L = 147 mm is the working length of specimen in vibration tests. A series of vibration tets were performed on 30 specimens that were cut in warp direction and the mean value of their first natural resonance frequencies was determined:

$$\langle f_1\rangle=\sum_{i=1}^{30}\frac{f_i}{30}=26\text{ Hz}. \qquad (23)$$

Once resonance frequency is determined we can define Young's modulus in the warp direction using Eq. (22):

$$E_{11}=\frac{4\pi^2\langle f_1\rangle^2\rho FL^4}{(3.515)^2 I_z}=5.62\cdot 10^9\text{ Pa}. \qquad (24)$$

The same tests were carried out on specimens that were cut in weft direction and the mean value of their first natural frequencies was determined:

$$\langle f_2\rangle=\sum_{i=1}^{30}\frac{f_i}{30}=23.5\text{ Hz}. \qquad (25)$$

The value of Young's modulus of corresponding specimens can be defined as:

$$E_{22}=\frac{4\pi^2\langle f_2\rangle^2\rho Fl^4}{(3.515)^2 I_z}=4.59\cdot 10^9\text{ Pa}. \qquad (26)$$

Other physical and mechanical properties like Poisson's ratio and density of this composite material were taken from literature [10]. They are given in Table 1.

Table 1. *Physical and mechanical properties of the composite laminates*

Density, [kg/m^3]		1860
Young's modulus, [MPa]	Warp direction	5620
	Weft direction	4590
Poisson's ratio	Warp direction	0.22
	Weft direction	0.18

3.2 Fatigue Tests

Tests were performed on a special machine type DP-5/3 which is used to determine the bending fatigue resistance of sheet fibrous woven specimens. Machine imposes an alternating bending angle on the upper clamp (point B). At the down end the specimen is clamped (point A). Machine allows simultaneous testing of three specimens at the same angle of bending with or without the preliminary tension of specimens with removable weights. Testing machine DP5/3 is shown in Fig. 3.

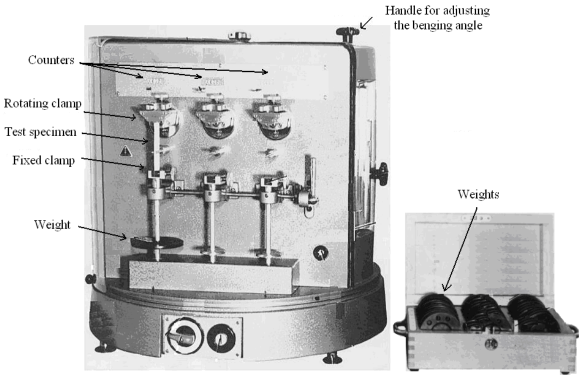

Fig. 3. *Machine type DP-5/3 used for fatigue tests*

The value of the imposed bending angle is a controllable parameter over a wide range of 20 to 180°. Working length of specimens in fatigue tests was L = 92 mm (Fig. 4).

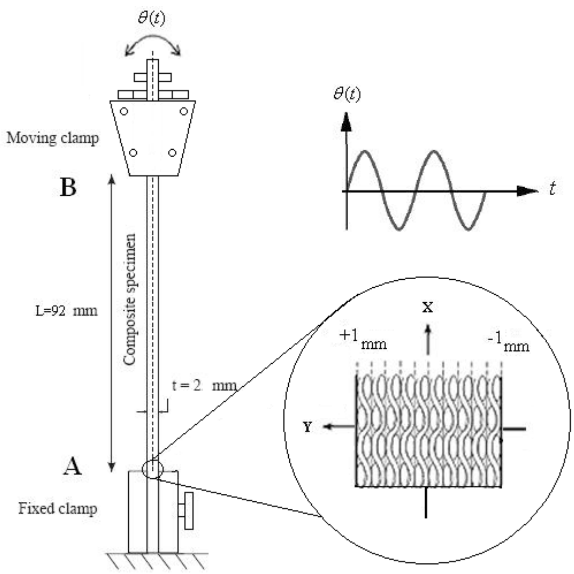

Fig. 4. *Schematic drawing of the bending fatigue setup*

The desired frequency of fully-reversed bending can be installed on 100 or 300 bending cycles in a minute. Two sets of fatigue experiments were carried out with no preliminary tension. In the first set, experiments were performed to examine the fatigue bending resistance of 30 specimens that were cut in the warp direction with different values of the imposed bending angle of 60, 50, 45, 40, 35 and 30°. A total of five specimens were tested at each bending angle. In the second set, the same tests were carried out on 30 specimens that were cut in the weft direction. Specimens were all tested at the same frequency of 100 fully-reversed bending cycles in a minute. Tests were continued until a complete failure in room temperature. After the automatic shutdown of the machine due to the destruction of specimens the number of cycles at failure established by the counters was recorded.

The constitutive equations for evaluation of stresses and bending moments are based on the classical beam theory. A composite test specimen cannot experience any deflection at points A and B. A derivative of the deflection function is zero at the point A. Because the other end is free to rotate about the Z axis, a derivative of deflection function is equal to θ at point B. The point of maximal deflection C occurs between points A and B. This deflection is a function of x and bending angle θ:

$$Y = Y(x,\theta). \tag{27}$$

In the linear formulation we can determine the value of the deflections, using differential equation:

$$EI\frac{d^4Y}{dx^4} = 0, \tag{28}$$

where E is the Young's modulus of the corresponding direction. Eq. (28) has a solution of the form:

$$Y(x) = c_1 + c_2x + c_3x^2 + c_4x^3, \tag{29}$$

where c_1, c_2, c_3 and c_4 constants that can be defined from boundary conditions. The boundary conditions at points A and B are:

$$Y = 0, \quad \mathrm{d}Y/\mathrm{d}x = 0, \quad x = 0, \tag{30}$$

$$Y = 0, \quad \mathrm{d}Y/\mathrm{d}x = \theta, \quad x = L. \tag{31}$$

Once deflections are determined, we can define the values of bending moment and normal stress:

$$\sigma_x = -y \cdot E\frac{d^2Y}{dx^2} = -y \cdot E\left[-\frac{2\theta}{l} + \frac{6\theta}{l^2}x\right]. \tag{32}$$

In the tests when deflection is greater than the thickness of specimen a non-linear analysis and formulation is required to determine the deflection [11]:

$$EI\frac{d^4Y}{dx^4} - \left[\frac{3}{2}EF\left(\frac{dY}{dx}\right)^2 \cdot \left(\frac{d^2Y}{dx^2}\right)\right] = 0. \tag{33}$$

A finite element analysis (FEA) was carried out using Ansys 11 software to assess the validity of the calculated values of stresses. The element used for the analysis was 8-node structural Shell93 which has six degrees of freedom at each node. This element is particularly well suited to model curved orthotropic shells in case of large deflections [12]. A boundary condition of fixing all displacements and rotations on the upper line (point A), fixing all displacements with a given rotation about the Z axis $Rot\ z = \theta$ on the lower line (point B) was applied (Fig. 5).

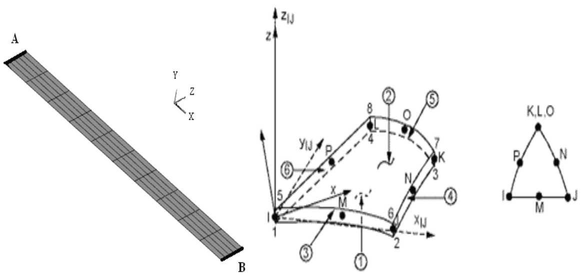

Fig. 5. *Shell element for modeling the composite specimen*

It should be noted that the difference between the stress values obtained from Eqs. (32) and (33) and values calculated by Ansys software do not exceed 5%.

4 PROCESSING THE RESULTS OF FATIGUE EXPERIMENTS

For practical application of the proposed model it is necessary to identify the unknown functional dependency parameters Eq. (14) on the basis of experimental results. Therefore, the results of fatigue tests were processed by the method of least squares to determine constants m and n in such way that the sum of the squared errors over all the observations is minimized. i.e., the quantity Q we are interested in minimizing is:

$$Q(m,n,E_{11},E_{22}) = \sum_{i=1}^{j}\left[\sigma_{i1}(N_{i1}) - F(N_i,E_{11},m,n)\right]^2 + \sum_{i=1}^{j}\left[\sigma_{i2}(N_{i2}) - F(N_i,E_{22},m,n)\right]^2. \quad (34)$$

Function F has the form of a power law Eq. (20), if it is assumed that the function of cycle parameter has the constant value of $k(R) = 1$ under stationary loading conditions, then function Q will take the form:

$$Q = \sum_{i=1}^{30}\left(\sigma_{i1} - \left(\frac{2^n E_{11}{}^n}{(n+1)\cdot m}\right)^{\frac{1}{2n}} \cdot N_{i1}^{\frac{-1}{2n}}\right)^2 + \sum_{i=1}^{30}\left(\sigma_{i2} - \left(\frac{2^n E_{22}{}^n}{(n+1)\cdot m}\right)^{\frac{1}{2n}} \cdot N_{i2}^{\frac{-1}{2n}}\right)^2. \quad (35)$$

By taking logarithms of both sides of the Eq. (35) and introducing new variable $t = \ln N$ there is a new function Q':

$$Q' = \sum_{i=1}^{30}\left(ln\,\sigma_{i1} - \left(\frac{1}{2}ln\,E_{11} + \frac{1}{2n}ln\frac{2^n}{m\cdot(n+1)} - \frac{1}{2n}t_{i1}\right)\right)^2 + \sum_{i=1}^{30}\left(ln\,\sigma_{i2} - \left(\frac{1}{2}ln\,E_{22} + \frac{1}{2n}ln\frac{2^n}{m\cdot(n+1)} - \frac{1}{2n}t_{i2}\right)\right)^2. \quad (36)$$

If $a = \frac{1}{2n}ln\left(\frac{2^n}{m\cdot(n+1)}\right)$ and $b = -\frac{1}{2n}$, leads to the following:

$$Q' = \sum_{i=1}^{30}\left(ln\,\sigma_{i1} - \left(\frac{1}{2}ln\,E_{11} + a + bt_{i1}\right)\right)^2 + \sum_{i=1}^{30}\left(ln\,\sigma_{i2} - \left(\frac{1}{2}ln\,E_{22} + a + bt_{i2}\right)\right)^2. \quad (37)$$

Taking the derivative of Q' with respect to a and b, setting them to zero gives the following set of equations:

$$\frac{\partial Q'(a,b,E_{11},E_{22})}{\partial a} = 0,\ \frac{\partial Q'(a,b,E_{11},E_{22})}{\partial b} = 0. \quad (38)$$

Solving these equations gives the following least square estimates of m and n as:

$$m = 1.034 \cdot 10^{-27}\ [\text{Pa}],\ n = 3.521\ . \quad (39)$$

These values are substituted into Eq. (20) to obtain two theoretical S-N curves for fatigue study of specimens cut in different directions:

$$\sigma_1(N_1) = \left(\frac{2^n \cdot E_{11}{}^n}{(n+1)\cdot m}\cdot\frac{1}{N_{f1}}\right)^{\frac{1}{2n}}, \quad \sigma_2(N_2) = \left(\frac{2^n \cdot E_{22}{}^n}{(n+1)\cdot m}\cdot\frac{1}{N_{f_2}}\right)^{\frac{1}{2n}}. \quad (40)$$

The theoretical S-N curve and fatigue test results in warp direction obtained at different stress levels are plotted in Fig. 6, as the number of cycles to failure, N_f, against the applied stress, σ.

In the same way for test specimens that were cut in weft direction in Fig. 7 shown theoretical S-N curve and fatigue tests results.

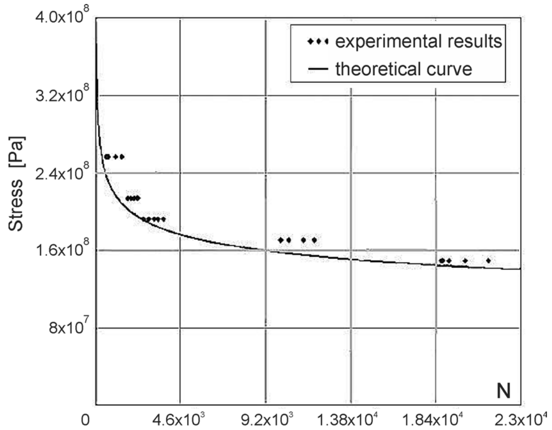

Fig. 6. *Theoretical fatigue curve and experimental results from tests in warp direction*

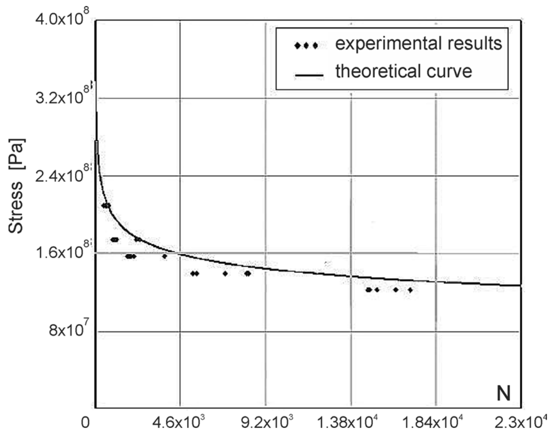

Fig. 7. *Theoretical fatigue curve and experimental results from tests in weft direction*

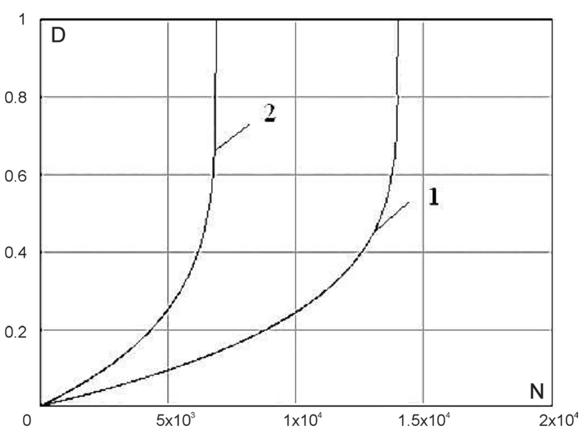

Fig. 8. *Evolution of damage parameter in specimens cut; 1-in warp 2- in weft directions.*

After identifying the damage model parameters, it can be applied to predict quantitatively the damage evolution of structural elements. It is possible to draw the graphs of the evolution of scalar damage variable as a function of number of fatigue cycles for two types of specimens cut in different directions using Eq. (19), as it is shown in Fig. 8.

Note that graphs were constructed at the same given value of stress amplitude, for example $\sigma = 1.5 \cdot 10^8$ Pa and $k(R) = 1$. It is possible to observe that the damage parameter consistently increases until the end of the test when it increases very fast until rupture.

5 CONCLUSIONS

In the present work an energy-based model for predicting fatigue life and quantitative evaluation of progressive damage using Lemaitre's concept of equivalent stress hypothesis was proposed. The model allows the prediction of fatigue durability by taking into account the principal directions of the stress tensor relative to the planes of elastic symmetry of material. The unknkown parameters of this model defining the fatigue damage accumulation rate, were identified using experimental results from fatigue tests of glass fiber/epoxy composite specimens that were cut in both the warp and weft directions. Then, the model has been applied to study the evolution of damage in specimens cut in different directions under fatigue tests. It has been shown that the theoretical fatigue strength curves obtained by means of this model were in good agreement with experimental data.

6 REFERENCES

[1] Harris, B. (2003). *Fatigue in composites*. CRC Press, Woodhead Publishing Ltd., Cambridge.

[2] Van Paepegem, W., Degrieck, J. (2000). Fatigue Damage Modelling of Fiber-reinforced Composite Materials: Review. *Applied Mechanics Reviews*, vol. 54, no. 4, p. 279-300.

[3] Kachanov, L.M. (1986). *Introduction to Continuum Damage Mechanics*. M. Nijhoff Publ., Dordrecht.

[4] Voyiadjis, G.Z., Kattan, P.I. (2005). *Damage mechanics*. CRC Press, A Taylor & Francis Company, Florida, DOI:10.1201/9781420027839.

[5] Lemaitre, J. (1996). *A course on damage mechanics* (2nd Edition). Springer-Verlag Berlin, DOI:10.1007/978-3-642-18255-6.

[6] Lemaitre, J., Desmorat, R. (2005). *Engineering damage mechanics: ductile, creep, fatigue and brittle failure*. Springer, New York.

[7] Seruga, D., Fajdiga, M., Nagode, M. (2011). Creep damage calculation for thermo mechanical fatigue. *Strojniški vestnik - Journal of Mechanical Engineering*,

vol. 57, no. 5, p. 371-378, DOI:10.5545/sv-jme.2010.108.
[8] Movaghghar, A., Lvov, G.I. (2010). An energy model for fatigue life prediction of composite materials using continuum damage mechanics. *ICMAE 2010 proceedings*, p. 275-279.
[9] Meirovitch, L. (2001). *Fundamentals of vibration*. MacGraw-Hill Companies, Inc.
[10] *Handbook of composites* (1982), (ed. Lubin, G.), Van Nostrand Reinhold, New York.
[11] Reddy, J.N. (2007). *Nonlinear finite element analysis*. Oxford University Press.
[12] *Ansys V.11 Help Reference* (2011).