UPORABA METODE VIKOR PRI ISKANJU OPTIMALNE REŠITVE ZA ZASNOVO GEODETSKE MREŽE ZA POSEBNE NAMENE

THE USE OF VIKOR METHOD IN FINDING THE OPTIMAL SOLUTION FOR A SPECIAL-PURPOSE GEODETIC NETWORK DESIGN

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IZVLEČEK

Študija obravnava uporabo metode VIKOR pri iskanju optimalne rešitve za načrtovanje namenske geodetske mreže. Metoda ponuja večkriterijski kompromisni način rangiranja, ki je bil razvit leta 1986. Na podlagi predhodno opredeljenih kriterijskih funkcij, povezanih z natančnostjo in zanesljivostjo, so bile najprej postavljene štiri sprejemljive alternativne rešitve, ki smo jih nato v iskanju optimalne podvrgli metodi VIKOR. Geodetska mikromreža, uporabljena v študiji, je bila simulirana kot trilateracijska in je sestavljena iz šestih kontrolnih točk, ki diskretizirajo strukturno geometrijo, ter štirih, petih ali šestih referenčnih točk (stebrov), odvisno od tega, za katero alternativno rešitev je šlo. Optimalna zasnova omrežja bi morala zagotoviti zaznavanje odstopanj položajev kontrolnih točk v skladu z vnaprej uvedenimi tolerancami in omejitvami. Izkazalo se je, da je pristop, ki temelji na metodi VIKOR, zelo učinkovito orodje pri nalogah, ki vključujejo več nasprotujočih si zahtev, naloženih geodetu, in ko je treba odločitev sprejeti v kratkem času.

ABSTRACT

This study deals with the use of VIKOR method in finding the optimal solution for a special-purpose geodetic network design. It is about a multi-criteria compromise ranking method developed in 1986. Based on previously defined criteria functions related to precision and reliability, four acceptable alternative solutions were firstly established. Then, with the aim of finding the optimal one, those solutions were subjected to VIKOR method. The geodetic micro-network used in this study was simulated as a trilateration one and consisted of six control points discretizing a structural geometry as well as four, five or six reference points (pillars), depending on what alternative solution it was about. The optimal network design should have ensured the detection of deviations of the control points' positions in accordance with a priori introduced tolerances and constraints. It turned out the approach based on VIKOR method is a very efficient tool in tasks involving multiple conflicting requirements referred to a geodetic engineer and in the cases when the decision must be made in a short term.

KLJUČNE BESEDE

namenska geodetska mreža, natančnost, zanesljivost, tolerance, alternativne rešitve zasnove mreže, večkriterijska kompromisna lestvica, metoda VIKOR, optimalna zasnova mreže

KEY WORDS

special-purpose geodetic network, precision, reliability, tolerances, alternative network design solutions, multi-criteria compromise ranking, VIKOR method, optimal network design

1 INTRODUCTION

Many authors have dealt with the geodetic network optimization issue in their papers so far. It is common knowledge that, when it comes to optimization approaches in that sense, we have four well-known geodetic network design orders, and those are (Grafarend, 1974): the Zero-Order Design (ZOD), the First-Order Design (FOD), the Second-Order Design (SOD) and the Third-Order Design (THOD). This classification of design orders is conditional, because it also has its own shortcomings, which are mainly related to the interpenetration and connection of individual designs when solving problems. Namely, each task of the optimal designing of geodetic networks is solved individually, or with a partial solving of another task (e.g. in the SOD, the design matrix A is varied, which partially solves the FOD, etc). However, optimization tasks according to the above division are not discussed here.

An extensive and detailed review of all geodetic network optimization procedures can be found in Тамутис (1979) and Grafarend and Sansò (1985). Besides, there are many other recent studies dealing with the optimization of geodetic networks, and some of them are: Bagherbandi et al (2009), Doma and El Shoney (2011), Dwivedi and Dikshit (2013), Pachelski and Postek (2016), Postek (2021).

In this study, the authors present a multi-criteria optimization method that is used with the aim of presenting the decision-making process with the special-purpose geodetic networks in a completely new way. It is about VIKOR (in serbian: **VI**šekriterijumsko **KO**mpromisno **R**angiranje/**R**ešenje; in english: Multi-Criteria Optimization and Compromise Ranking /Solution) method that was developed by Opricović (1986). It has been widely used in non-geodetic problems (see, for example, Opricović and Tzeng (2004), Nikolić et al (2010), Kuo and Liang (2011), Nisel (2014), Chatterjee and Chakraborty (2016), Jokić et al (2019), etc). Consequently, a curiosity regarding its application in geodetic network designing has arisen. Such an approach significantly differs from the ZOD, FOD, SOD and THOD, beacause it involves a greater number of conflicting criteria. In other words, the main characteristic of this new approach is, actually, the presence of more opposing requirements. Some criteria functions that represent drawbacks or losses should be minimzed, while the remaing ones that present benefits or gains should be maximized. That is the main issue that makes the entire optimization process more complex.

The use of VIKOR method in designing of a special-purpose geodetic network is representatively shown through this study.

2 METHODS AND MATERIALS

This section presents the theoretical basis of VIKOR method as well as characteristics of the system to be optimized herein.

2.1 Theoretical background of VIKOR method

Basically, this method was developed on the basis of compromise programming elements and it starts from the ''boundary'' forms of L_p-metric. The metric is written as follows (Opricović, 1998):

$$L_p(F^*, F(x)) = \left\{\sum_{i=1}^{n}[f_i^* - f_i(x)]^p\right\}^{\frac{1}{p}},\ 1 \le p < \infty \quad (1)$$

and it represents the distance between the "ideal" point $F^*(f_1^*, \ldots, f_n^*)$ and the point $F(x) = (f_1(x), \ldots, f_n(x))$ in the space of criteria functions. The "ideal" point is determined from n values of criteria functions by means of the following equation:

$$f_i^* = ext_j f_{ij}, \quad (i, j) \in \{1, 2, \ldots, n_c\} \times \{1, 2, \ldots, n_a\}, \tag{2}$$

where ext_j (j is the ordinal number of alternative) stands for the maximum if i-th criterion function represents a benefit or gain, or minimum if the same function represents a drawback or loss.

Since in practice the values f_{ij} are often given in different units, a transformation is introduced to avoid the unit-variety problem. This transformation gives the following corresponding dimensionless values:

$$d_{ij} = \frac{f_i^* - f_{ij}}{f_i^* - f_i^-}, \quad (i, j) \in \{1, 2, \ldots, n_c\} \times \{1, 2, \ldots, n_a\}. \tag{3}$$

Now, for the alternative a_j, we can write the "boundary" forms of the L_p–metric as follows:

$$S_j = \sum_{i=1}^{n} w_i d_{ij} = \sum_{i=1}^{n} w_i \frac{f_i^* - f_{ij}}{f_i^* - f_i^-}, \quad p = 1 \tag{4}$$

$$R_j = \max_i (w_i d_{ij}) = \max_i \left(w_i \frac{f_i^* - f_{ij}}{f_i^* - f_i^-} \right), \quad p = \infty, \tag{5}$$

with f_i^* and f_i^- which correspond to the best and worst alternative of the system, respectively, and w_i representing the weight of the criterion function f_{ij} (i.e. the decision-maker preference regarding the i-th crtiterion), whereby $\Sigma_{i=1}^{n} w_i = 1$.

If $R_j = R^-$ for two or more j-indices, the following modification is introduced:

$$R_{j,\text{mod}} = R_j + \frac{S_j - R^-}{100} = R_j + \frac{S_j - \max_j R_j}{100}. \tag{6}$$

The essence of VIKOR method is to calculate the Q_j value for all alternatives and then separate the one which the minimal value (the minimal distance from the "ideal" point) corresponds to. This measure for multi-criteria ranking of the j-th alternative is calculated as follows:

$$Q_j = v \cdot QS_j + (1 - v) \cdot QR_j, \tag{7}$$

where v is the weight of the strategy of fulfilling most of the criteria (it is chosen by a decision maker, but VIKOR method assumes the value of 0.50).

In Eq. (7) we have

$$QS_j = \frac{S_j - S^*}{S^- - S^*} = \frac{S_j - \min_j S_j}{\max_j S_j - \min_j S_j} \quad \text{and} \tag{8}$$

$$QR_j = \frac{R_j - R^*}{R^- - R^*} = \frac{R_j - \min_j R_j}{\max_j R_j - \min_j R_j} \tag{9}$$

that are, respectively, the measure of deviation expressing the requirement for maximum group benefit (serves to form the first ranking list) and measure of deviation expressing the requirement for minimizing the maximal distance of an alternative from the ''ideal'' alternative (serves to form the second ranking list). These two measures, combined as given in Eq. (7), provide the third, compromise ranking list.

In multi-criteria ranking using VIKOR method, alternative a_j is better in total (according to all criteria) than alternative a_k if $Q_j(v = 0.50) < Q_k(v = 0.50)$. However, it is still not enough to consider a_j the best alternative. Namely, to consider it the best one, it has to be first-ranked on the compromise ranking list and fulfill two conditions given below (Opricović, 1998).

Condition C1: The first-ranked alternative (a') on the compromise ranking list for $v = 0.50$ has to have a ''sufficient advantage'' over the alternative from the next position (a''), what means that

$$Q(a'') - Q(a') \geq DQ = \min(0.25; 1/(n_a - 1)), \tag{10}$$

where DQ is the ''sufficient advantage'' threshold which is equal to 0.25 when the number of alternatives is less than six.

Condition C2: The first-ranked alternative (a') on the compromise ranking list for $v = 0.50$ has to have a ''sufficiently stable'' first position meaning that it also fulfills at least one of the following requirements: (1) it is first-ranked on the first ranking list (according to QS); (2) it is first-ranked on the first ranking list (according to QR); (3) it is first-ranked on the third ranking list (according to Q, for $v = 0.25$ and $v = 0.75$).

Conclusions are made as follows:

- **If the first-ranked alternative on the compromise list fulfills both conditions (C1 and C2), it is considered the only and best solution**;
- **If the first-ranked alternative on the compromise list does not fulfill only the condition C2**, it is considered not ''sufficiently'' better than the second-ranked alternative, and then **a set of compromise solutions is formed so that it includes only the first- and second-ranked alternative**;
- **If the first-ranked alternative on the compromise list does not fulfill only the condition C1 or both conditions (C1 and C2)**, it is considered not ''sufficiently'' better than the second-ranked alternative and any other alternative (a^k) on the list that fulfills

  $$Q(a^k) - Q(a') < DQ = \min(0.25; 1/(n_a - 1)), \tag{11}$$

  and then a set of compromise solutions is formed so that it includes the first-, second-ranked and any other alternative for which the above inequality is valid.

The results of VIKOR method are the three ranking lists (formed according to QS, QR and Q values) and the alternative with ''sufficient advantage'' as well as, at the same time, ''sufficiently stable'' first position (if both conditions, C1 and C2, are fulfilled) or a set of compromise solutions (when one or both conditions, C1 and C2, are not fulfilled). On the basis of these results, the final solution is adopted.

2.2 Description of the system to be optimized in the study

For the purpose of the study herein, a four-variant special-purpose geodetic network was simulated. Each of the four variants represents an individual acceptable solution that fulfills all pre-set constrains (see subsection 2.2.2).

Namely, it is about a trilateration control network that should serve as a base for detecting deviations of the projected from the marked structure pillars' positions. The nework consists of four, five or six reference points (pillars), and six points representing the structure pillars' centers (i.e. the points whose positions are controlled). The change in the number of reference points and lengths to be measured basically alternates the network design (solution). In this way, four alternative solutions for the geodetic control network (hereinafter: alternatives) were established. They are denoted as A1, A2, A3 and A4.

2.2.1 Mathematics of the Least Squares Method used in this study

In this study, we use the well-known principles of the LS (Least Squares) method (for a detailed insight into the method, see e.g. Perović (2005)), by means of the following matrices: **A** – design matrix, **P** – weight matrix, N – normal equation coefficient matrix.

Design *n*-by-*u* matrix (**A**) is formed based on the following:

$$\mathbf{A}_{n\times u}=\begin{matrix} y_{R1} & x_{R1} & \cdots & y_{Rm} & x_{Rm} & y_{S1} & x_{S1} & \cdots & y_{S6} & x_{S6} \end{matrix}\\ \begin{pmatrix} B_{R1;1} & A_{R1;1} & \cdots & B_{Rm;1} & A_{Rm;1} & B_{S1;1} & A_{S1;1} & \cdots & B_{S6;1} & A_{S6;1} \\ B_{R1;2} & A_{R1;2} & \cdots & B_{Rm;2} & A_{Rm;2} & B_{S1;2} & A_{S1;2} & \cdots & B_{S6;2} & A_{S6;2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{R1;n} & A_{R1;n} & \cdots & B_{Rm;n} & A_{Rm;n} & B_{S1;n} & A_{S1;n} & \cdots & B_{S6;n} & A_{S6;n} \end{pmatrix}, \quad (12)$$

with $r(\mathbf{A}_{n\times u}) = u - d = u - 3$, where we have the following denotations:

R1, ..., R*m* – reference points (pillars), with m equal to 4, 5 or 6, depending on the alternative;

S1, ..., S6 – six points representing the centers of the structure pillars;

n – the number of the lengths to be measured in the alternative;

u – the number of the unknown parameters (i.e. the coordinates of the geodetic network points) in the alternative;

The coefficients which figure in matrix **A** are taken from the corresponding correction equations for measured lengths. The correction equation for the measured length between points j and k is:

$$v_{D_j^k} = B_{jk}y_j + A_{jk}x_j + B_{kj}y_k + A_{kj}x_k + f_{D_j^k}\ , \quad f_{D_j^k} = D_{j,0}^k - d_j^k, \quad (13)$$

and, in addition to the mentioned coefficients, it contains unknown parameters in the form of differential increments of the coordinates Y_j, X_j, Y_k and X_k, as well as the absolute term representing the difference between approximate and measured length (not used herein). The coefficients are calculated as follows:

$$B_{jk} = \left(\frac{\partial D_j^k}{\partial y_j}\right)_0 = -\sin v_{j,0}^k = -B_{kj}\ , \quad (14)$$

$$A_{jk} = \left(\frac{\partial D_j^k}{\partial x_j}\right)_0 = -\cos v_{j,0}^k = -A_{kj}, \quad (15)$$

whereby the index ''0'' is introduced to indicate the approximate value obtained from the approximate coordinates $Y_{j,0}$, $X_{j,0}$, $Y_{k,0}$ and $X_{k,0}$ (see subsection 2.2.4). Index j in Eqs. (13), (14) and (15) represents a reference point (pillar), and index *k* can be represent of a reference point or a structure pillar's center. The symbol ν is introduced to denote the bearing.

Weight matrix (**P**) is a diagonal *n*-by-*n* matrix, with the general term calculated from the adopted dispersion coefficient *a priori* (with index "0") and the length mensurement variance obtained in accordance with a manufacturer's declaration, and it is formed as follows:

$$\mathbf{P}_{n\times n} = diag\{P_i\} = diag\{P_1, P_2, ..., P_n\} = diag\left\{\frac{\sigma_0^2}{\sigma_{D_1}^2}, \frac{\sigma_0^2}{\sigma_{D_2}^2}, ..., \frac{\sigma_0^2}{\sigma_{D_n}^2}\right\}. \quad (16)$$

Normal equation coefficient *u*-by-*u* matrix (**N**) is calculated using the following equation:

$$\mathbf{N}_{u\times u} = (\mathbf{A}_{n\times u})^{\mathrm{T}}\mathbf{P}_{n\times n}\mathbf{A}_{n\times u}. \quad (17)$$

On the basis of the metrices given by Eqs. (12), (16) and (17), we can calculate the remaining matrices which are used in the study. Those matrices are: the generalized inverse of the singular matrix **N** (i.e. cofactor matrix for the unknown parameters' estimates), the cofactor matrix for the measured length values' estimates, the cofactor matrix for the measured length corrections' estimates and the redundancy matrix.

Pseudoinverse of the matrix **N** is extracted from the inverse of the singular matrix **N**, that is previously extended by a chosen datum constraint matrix in the following way:

$$\begin{pmatrix} \mathbf{N}_{u\times u} & \mathbf{B}_{u\times 3} \\ (\mathbf{B}_{u\times 3})^{\mathrm{T}} & \mathbf{0}_{3\times 3} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{N}^{+}_{u\times u} & (\mathbf{B}^{+}_{3\times u})^{\mathrm{T}} \\ \mathbf{B}^{+}_{3\times u} & \mathbf{0}_{3\times 3} \end{pmatrix}, \quad (18)$$

or calculated as:

$$\mathbf{N}^{+}_{u\times u} = (\mathbf{N}_{u\times u} + \mathbf{B}_{u\times 3}(\mathbf{B}_{u\times 3})^{\mathrm{T}})^{-1}\mathbf{N}(\mathbf{N}_{u\times u} + \mathbf{B}_{u\times 3}(\mathbf{B}_{u\times 3})^{\mathrm{T}})^{-1}, \quad (19)$$

where the datum constraint matrix has the following form (provides the minimal trace of the cofactor matrix for the unknown parameters' estimates, but only for the reference points):

$$\mathbf{B}^{\mathrm{T}} = \begin{matrix} y_{\mathrm{R1}} & x_{\mathrm{R1}} & \cdots & y_{\mathrm{R}m} & x_{\mathrm{R}m} & y_{\mathrm{S1}} & x_{\mathrm{S1}} & \cdots & y_{\mathrm{S6}} & x_{\mathrm{S6}} \end{matrix} \\ \begin{pmatrix} 1/\sqrt{m} & 0 & \cdots & 1/\sqrt{m} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1/\sqrt{m} & \cdots & 0 & 1/\sqrt{m} & 0 & 0 & \cdots & 0 & 0 \\ -\xi_{\mathrm{R1}} & \eta_{\mathrm{R1}} & \cdots & -\xi_{\mathrm{R}m} & \eta_{\mathrm{R}m} & 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \quad (20)$$

In Eq. (20), for the denotations in the third row of the matrix, and taking into account that $j \in \{\mathrm{R1}, ..., \mathrm{R}m\}$, we write:

$$\xi_j = \frac{X_{j,0} - \bar{X}_0}{\sqrt{\sum_{j=1}^{m}(Y_{j,0} - \bar{Y}_0)^2 + \sum_{j=1}^{m}(X_{j,0} - \bar{X}_0)^2}}, \quad (21)$$

$$\eta_j = \frac{Y_{j,0} - \bar{Y}_0}{\sqrt{\sum_{j=1}^{m}(Y_{j,0} - \bar{Y}_0)^2 + \sum_{j=1}^{m}(X_{j,0} - \bar{X}_0)^2}}, \quad (22)$$

whereby $Y_{j,0}$, and $X_{j,0}$ represent the approximate coordinates, while $\overline{Y}_0$ and $\overline{X}_0$ are the figure centroid coordinates.

The cofactor matrix for the measured length values' estimates, the cofactor matrix for the measured length corrections' estimates and the redundancy matrix are, respectively, calculated as follows:

$$\mathbf{Q}_{\hat{\mathbf{l}}} = \mathbf{AN}^{+}\mathbf{A}^{\mathrm{T}} = \mathbf{AQ}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathrm{T}} (\hat{\mathbf{x}} \text{ is the vector of unknown parameters' estimates}), \quad (23)$$

$$\mathbf{Q}_{\hat{\mathbf{v}}} = \mathbf{P}^{-1} - \mathbf{Q}_{\hat{\mathbf{l}}}, \quad (24)$$

$$\mathbf{R} = \mathbf{Q}_{\hat{\mathbf{v}}}\mathbf{P}. \quad (25)$$

2.2.2 Precision and reliability constraints which were pre-set

As the constraints which should have been fulfilled in the case of each design being an alternative for the subject geodetic control network, the following nine were chosen:

$$\frac{A_j}{B_j} \leq 1.5, \quad \forall j \in \{\text{R1}, ..., \text{R}m\},\ m \in \{4,5,6\}, \quad (26)$$

$$\frac{A_s}{B_s} \leq 2.2, \quad \forall s \in \{\text{S1}, ..., \text{S6}\}, \quad (27)$$

$$\sigma_{p,j} \leq 1.0\,\text{mm}, \quad \forall j \in \{\text{R1}, ..., \text{R}m\},\ m \in \{4,5,6\}, \quad (28)$$

$$\sigma_{p,s} \leq 1.5\,\text{mm}, \quad \forall s \in \{\text{S1}, ..., \text{S6}\}, \quad (29)$$

$$r_{ii} = (\mathbf{Q}_{\hat{\mathbf{v}}}\mathbf{P})_{ii} = r_{D_i} \geq 0.20, \quad \forall i \in \{1,2,...,n\}, \quad (30)$$

$$G_i^{*} \leq 7.64\sigma_{D_i}, \quad \forall i \in \{1,2,...,n\}, \quad (31)$$

$$(1-\beta_0)_i \geq 0.80, \quad \forall i \in \{1,2,...,n\}, \quad (32)$$

$$(1-\beta_{cs})_r \geq 0.80, \quad r \in \{cs_{\text{S1,S2}}, cs_{\text{S2,S3}}, cs_{\text{S4,S5}}, cs_{\text{S5,S6}}, cs_{\text{S1,S4}}, cs_{\text{S2,S5}}, cs_{\text{S3,S6}}\}, \quad (33)$$

$$(1-\beta_{ccp})_s \geq 0.80, \quad s \in \{\text{S1}, ..., \text{S6}\}, \quad (34)$$

where the following, previously unmentioned, denotations are present:

- $A_{j(s)} = \sigma_0\sqrt{\frac{1}{2}(Q_{\hat{y}_{j(s)}\hat{y}_{j(s)}} + Q_{\hat{x}_{j(s)}\hat{x}_{j(s)}} + \sqrt{(Q_{\hat{x}_{j(s)}\hat{x}_{j(s)}} - Q_{\hat{y}_{j(s)}\hat{y}_{j(s)}})^2 + 4Q^2_{\hat{x}_{j(s)}\hat{y}_{j(s)}}})}$ is the semi-major axis of the standard error ellipse for the point *j*, i.e. *s*; s, devided by square root of the Chi-square distribution quantile for two degrees of freedom and significance level α.
- $B_{j(s)} = \sigma_0\sqrt{\frac{1}{2}(Q_{\hat{y}_{j(s)}\hat{y}_{j(s)}} + Q_{\hat{x}_{j(s)}\hat{x}_{j(s)}} - \sqrt{(Q_{\hat{x}_{j(s)}\hat{x}_{j(s)}} - Q_{\hat{y}_{j(s)}\hat{y}_{j(s)}})^2 + 4Q^2_{\hat{x}_{j(s)}\hat{y}_{j(s)}}})}$ is the semi-minor axis of the standard error ellipse for the point *j*, i.e. *s*, devided by square root of the Chi-square distribution quantile for two degrees of freedom and significance level α.
- $\sigma_{p,j(s)} = \sigma_0\sqrt{Q_{\hat{y}_{j(s)}\hat{y}_{j(s)}} + Q_{\hat{x}_{j(s)}\hat{x}_{j(s)}}}$ is the standard positional error of the point j, i.e. s;
- r_{ii} is the redundancy coefficient for the measurement *i*;

- $G_i^* = \frac{t_{1-\beta_0} + t_{1-\alpha_0/2}}{\sqrt{r_{ii}}}\sigma_{D_i}$ is the detectable marginal gross-error value in the measurement *i* (probabilities $1-\beta_0$ and α_0 are, respectively, the test power and significance level for one-dimensional hypotheses in *data snooping test* – for $1-\beta_0 = 0.80$, $\alpha_0 = 0.01$ and $r_{ii} = 0.20$, $\forall i = \{1, 2, ..., n\}$ we have $G_i^* \approx 7.64\sigma_{D_i}$; for details about the test, see Baarda (1968));
- $\sigma_{D_i\,[\mathrm{mm}]} = a_{[\mathrm{mm}]}\ \ b_{[\mathrm{mm/km}]}\ D_{i\,[\mathrm{km}]}$ is the measured length standard (*a* and *b* are empirical coefficients provided by manufacturer)
- $(1-\beta_0)_i$ = normsdist $(7.64\sqrt{r_{ii}} - t_{1-\alpha_0/2})$ is the test power in detecting gross error limit value in the measurement *i*;
- $(1-\beta_{cs})_r = \text{normsdist}\left(\frac{d_{cs}}{\sigma_0\sqrt{\mathbf{h}_{pq}^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{x}},pq}\mathbf{h}_{pq}}} - t_{1-\alpha/2}\right)$ is the test power in detecting minimal deviation of the projected from the marked structure pillar span of 5mm(= d_{cs}), with the vector $\mathbf{h}_{pq}^{\mathrm{T}} = (B_{pq}\ \ A_{pq}\ \ -B_{pq}\ \ -A_{pq})$, $pq \in \{S1S2, S2S3, S4S5, S5S6, S1S4, S2S5, S3S6\}$, the matrix $\mathbf{Q}_{\hat{\mathbf{x}},pq}$ that represents the submatrix (of the cofactor matrix $\mathbf{Q}_{\hat{\mathbf{x}}}$) related to the coordinates of points *p* and *q*; and the significance level $\alpha = 0.05$;
- $(1-\beta_{ccp})_s = \text{normsdist}\left(\frac{d_{ccp}\sqrt{(\sin\theta_s\quad\cos\theta_s)(\mathbf{H}_s\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{H}_s^{\mathrm{T}})(\sin\theta_s\quad\cos\theta_s)^{\mathrm{T}}}}{\sigma_0} - t_{1-\alpha_{tab}/2}\right)$ is the test power in detecting minimal deviation of the projected from the marked structure pillar's center position of 4mm(= d_{ccp}), where α_{tab} = 0.02367 (derived from the tabular value for non-centrality paremeter used in the test for congruence of 2D point position, for adopted test power of 0.80 and figure rank equal to 2), θ_s is the standard error ellipse azimuth angle obtained from $\mathrm{tg}2\theta_s = 2Q_{\hat{x}_s\hat{y}_s} / (Q_{\hat{x}_s\hat{x}_s} - Q_{\hat{y}_s\hat{y}_s})$, and $\mathbf{H}_s$ is a 2-by-*u* matrix that separates the cofactor submatrix $\mathbf{Q}_{\hat{\mathbf{x}},s}$, related to the coordinates of the point *s*, from the cofactor matrix $\mathbf{Q}_{\hat{\mathbf{x}}}$.

Remark: The thresholds in inequalities (26) and (27) are introduced on the basis of the fact that, in real conditions, i.e. in praxis, it is very difficult, almost impossibly, to establish such a geodetic network's configuration that will provide ratio values that are significantly smaller than the corresponding introduced ones.

2.2.3 Criteria functions used in the application of VIKOR method in the study

The authors introduced eight criteria functions here. Those functions are the following:

1) Deviation of the average redundancy coefficient from the optimal value:

$$f_1 = \overline{r} - r_{opt} = \frac{n - r(\mathbf{A}_{n\times u})}{n} - 0.40 \tag{35}$$

with $f = n - r(\mathbf{A}_{n\times u}) = n - u + d = n - u + 3$ d.f;

2) Sum of cofactors for the unknown coordinates' estimates of the structure pillars centers:

$$f_2 = \sum_{s=\mathrm{S1}}^{\mathrm{S6}} (Q_{\hat{y}_s\hat{y}_s} + Q_{\hat{x}_s\hat{x}_s}), \quad s \in \{\mathrm{S1}, \ldots, \mathrm{S6}\}; \tag{36}$$

3) Deviation of the average ratio of semi-major axis to semi-minor axis from the optimal value for the structure pillar's center:

$$f_3 = \overline{\left(\frac{A}{B}\right)} - \left(\frac{A}{B}\right)_{opt} = \left(\frac{1}{6}\sum_{s=\mathrm{S1}}^{\mathrm{S6}} \frac{A_s}{B_s}\right) - 1, \quad s \in \{\mathrm{S1}, \ldots, \mathrm{S6}\}; \tag{37}$$

4) Ratio of the maximal to minimal semi-major axis in the part related to the structure pillars centers:

$$f_4 = \frac{\max_s A_s}{\min_s A_s}, \quad s \in \{\mathrm{S1}, \ldots, \mathrm{S6}\}; \tag{38}$$

5) Average standard positional error of the structure pillar's center:

$$f_5 = \overline{\sigma_p} = \frac{1}{6}\sum_{s=\mathrm{S1}}^{\mathrm{S6}} \sigma_{p,s}, \quad s \in \{\mathrm{S1}, \ldots, \mathrm{S6}\}; \tag{39}$$

6) Difference between the minimal value obtained for the test power in detecting minimal deviation (in this study $d_{sc} =$ 5mm) of the projected from the marked structure pillar span and the corresponding threshold value:

$$f_6 = \min_r (1-\beta_{cs})_r - (1-\beta_{cs})_{threshold} = \min_r (1-\beta_{cs})_r - 0.80, \tag{40}$$

where $r \in \{cs_{\mathrm{S1,S2}}, cs_{\mathrm{S2,S3}}, cs_{\mathrm{S4,S5}}, cs_{\mathrm{S5,S6}}, cs_{\mathrm{S1,S4}}, cs_{\mathrm{S2,S5}}, cs_{\mathrm{S3,S6}}\}$;

7) Difference between the minimal value obtained for the test power in detecting minimal deviation (in this study $d_{ccp} =$ 4mm) of the projected from the marked structure pillar's center position and the corresponding threshold value:

$$f_7 = \min_s (1-\beta_{ccp})_s - (1-\beta_{ccp})_{threshold} = \min_s (1-\beta_{ccp})_s - 0.80, \tag{41}$$

with $s \in \{\mathrm{S1}, \ldots, \mathrm{S6}\}$;

8) Difference between the average value obtained for the test power in detecting gross error limit value in the measurement and the corresponding threshold value:

$$f_8 = \overline{1-\beta_0} - (1-\beta_0)_{threshold} = \left(\frac{1}{n}\sum_{i=1}^{n} (1-\beta_0)_i\right) - 0.80, \quad i \in \{1, 2, \ldots, n\}. \tag{42}$$

The first five functions represent losses so they should be minimized. On the contrary, the last three functions represent gains to be maximized.

Because the main emphasis was placed on the precision and reliability of the considered geodetic control network, as well as the negligible differences between the costs of geodetic and other works related to the four alternatives are present, no criteria related to price were directly introduced. Even though, a price criterion is indirectly involved through the first criterion function (see Eq. (35)).

2.2.4 Main characteristics of the simulated multivariant geodetic network used in the study

In this subsection, the main characteristics of each established alternative (previously denoted as A1, A2, A3 and A4) are shown.

Approximate coordinates (Y and X) of the reference points R1, R2, R3 and R4 are the same for all the four alternatives, but those of the reference points R5 and R6 differ (see Table 1). Of course, approximate coordinates of the structure pillars centers, S1, S2, ..., S6, are common for all alternatives (see Table 2).

Table 1: Approximate values of the reference points' coordinates for all the four alternatives.

Reference point	Coordinate	Approximate values of coordinates for each alternative			
		A1	A2	A3	A4
R1*	*Y* [m]	990.000	990.000	990.000	990.000
	X [m]	1030.000	1030.000	1030.000	1030.000
R2*	*Y* [m]	1220.000	1220.000	1220.000	1220.000
	X [m]	1041.000	1041.000	1041.000	1041.000
R3*	*Y* [m]	1172.000	1172.000	1172.000	1172.000
	X [m]	1229.000	1229.000	1229.000	1229.000
R4*	*Y* [m]	1015.000	1015.000	1015.000	1015.000
	X [m]	1220.000	1220.000	1220.000	1220.000
R5	*Y* [m]	1097.000	1091.000	927.000	*no value*
	X [m]	1157.000	1127.000	1125.000	*no value*
R6	*Y* [m]	1095.000	*no value*	1300.000	*no value*
	X [m]	1099.000	*no value*	1117.000	*no value*

** Reference points with coordinates common for all the four alternatives*

Table 2: Approximate values of the structure pillar centers' coordinates common for all the four alternatives.

Coordinate	Approximate values of coordinates for each structure pillar's center					
	S1	S2	S3	S4	S5	S6
Y [m]	1070.000	1070.000	1070.000	1110.000	1110.000	1110.000
X [m]	1070.000	1130.000	1190.000	1070.000	1130.000	1190.000

Relevant data for this study related to the established alternatives are shown in Table 3.

Table 3: Values for main design characteristics and criteria functions for all the four alternatives (with denotations introduced previously).

Main design characteristics and criteria functions	Values for main design characteristic and criteria functions for each alternative			
	A1	A2	A3	A4
$m + 6$	12	11	12	10
n	42	38	47	32
u	24	22	24	20
d	3	3	3	3
$f = n - u + d$	21	19	26	15
f_1	0.10	0.10	0.15**	0.07*
f_2	9.9360 mm^2*	10.5944 mm^2	10.8042 mm^2	11.5296 mm^2 **
f_3	0.24	0.15*	0.61**	0.33
f_4	1.18*	1.40**	1.32	1.39
f_5	1.29 mm*	1.32 mm	1.34 mm	1.38 mm**
f_6	0.927099**	0.927104	0.932376	0.941131*
f_7	0.922113*	0.838633	0.829618	0.810279**
f_8	0.985734	0.984275**	0.991219*	0.984607

** Best criterion value*
*** Worst criterion value*

It should be mentioned that all criteria function values given in Table 3 were obtained using the length measurement standard of σ_i = 1mm + 1.5mm/km$\cdot D_{i[\mathrm{km}]}$, which, for the four readings in a series gives $\sigma_{D_i} = \sigma_i / \sqrt{4}$ = 0.5mm + 0.75mm/km$\cdot D_{i[\mathrm{km}]}$, $i \in \{1, 2, ..., n\}$, what was used in all calculations.

All four alternative designs with the associated standard error ellipses are shown in figures 1, 2, 3 and 4.

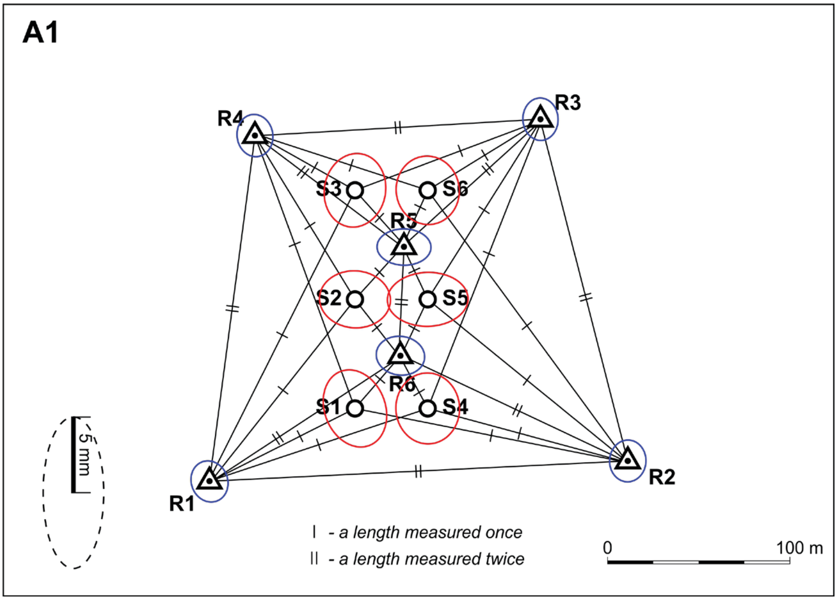

Figure 1: Alternative A1 geodetic control network with standard error ellipses.

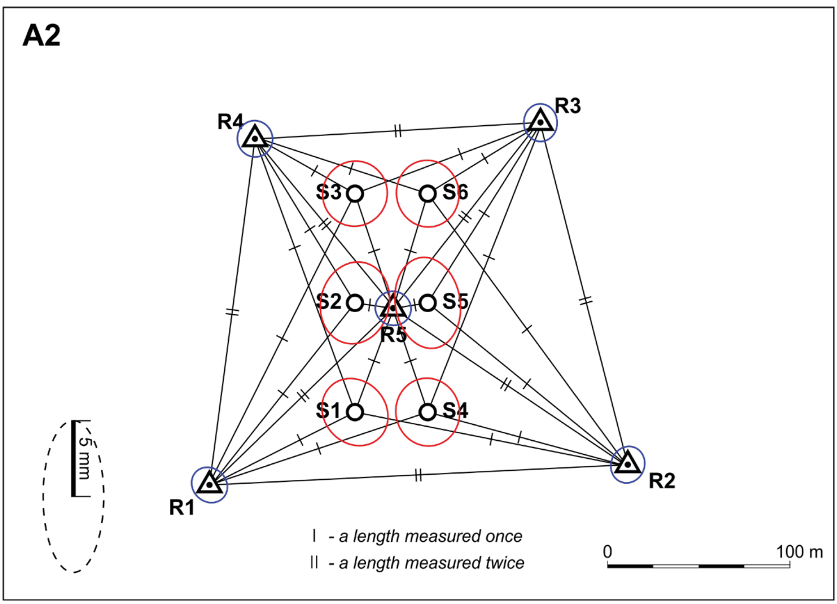

Figure 2: Alternative A2 geodetic control network with standard error ellipses.

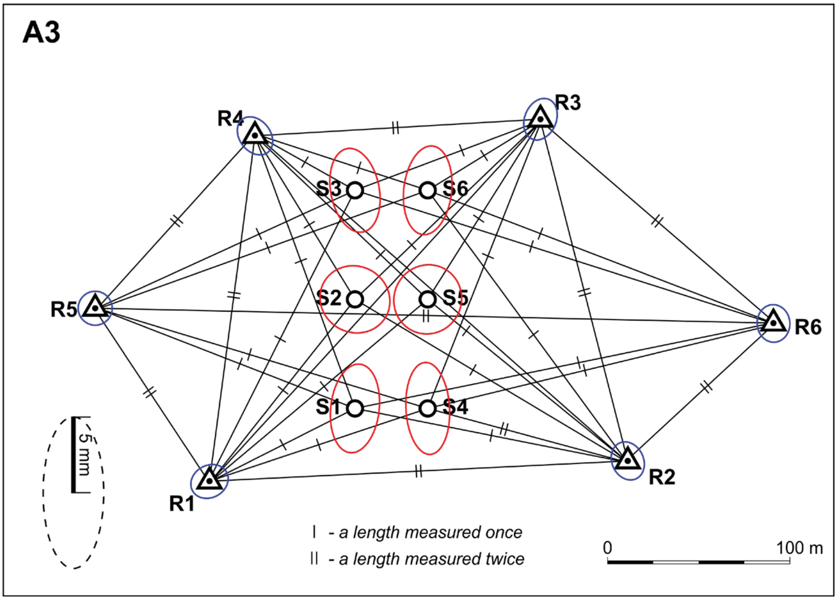

Figure 3: Alternative A3 geodetic control network with standard error ellipses.

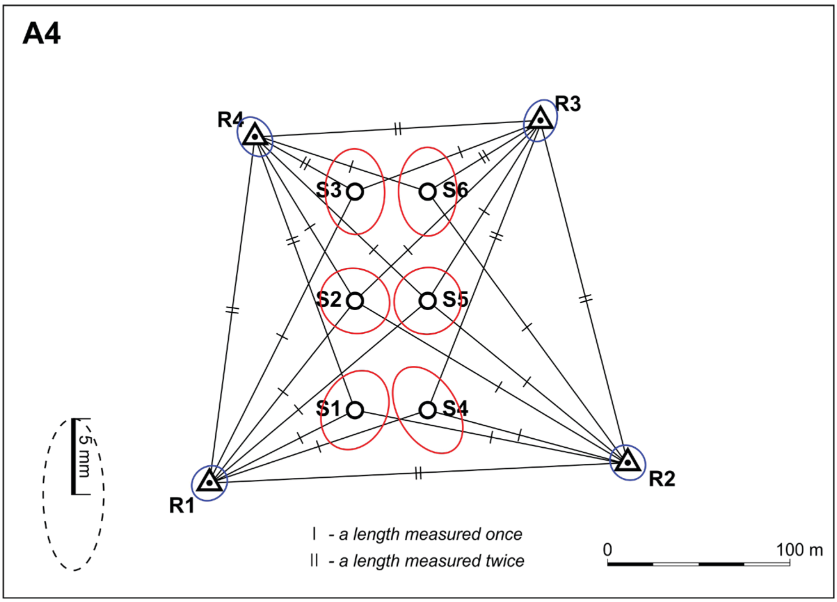

Figure 4: Alternative A4 geodetic control network with standard error ellipses.

3 RESULTS

Based on the values for the eight criteria functions given in Table 3, we can establish initial ranking lists of alternatives. Those lists are presented in Table 4.

Table 4: Ranking lists of alternatives based on the criteria functions' values from Table 3.

Position on the ranking list	Ranking lists of alternatives for each criterion function							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
First	A4	A1	A2	A1	A1	A4	A1	A3
Second	A2	A2	A1	A3	A2	A3	A2	A1
Third	A1	A3	A4	A4	A3	A2	A3	A4
Fourth	A3	A4	A3	A2	A4	A1	A4	A2

In the continuation of the presentation, all relavant results regarding the use of VIKOR method in the study are shown. Those results were obtained after an four-approach ranking. Each individual ranking approach involved a set of eight weights, one for each criterion function.

As can be spotted by looking at Table 5, the ratio of weights by approaches are as follows: 3:4:4:3:10:15:15:12 (for the Ranking approach I); 4:7:7:5:7:5:5:5 (for the Ranking approach II); 3:10:5:3:10:10:10:7 (for the Ranking approach III); and 4:10:9:5:10:6:6:5 (for the Ranking approach IV).

Table 5: Sets of criteria weights for all the four ranking approaches.

Criteria weight	Sets of criteria weights for each ranking approach			
	Ranking approach I	Ranking approach II	Ranking approach III	Ranking approach IV
w_1	0.045	0.089	0.052	0.073
w_2	0.061	0.156	0.172	0.182
w_3	0.061	0.156	0.086	0.164
w_4	0.045	0.111	0.052	0.091
w_5	0.152	0.156	0.172	0.182
w_6	0.227	0.111	0.172	0.109
w_7	0.227	0.111	0.172	0.109
w_8	0.182	0.111	0.121	0.091

As previously said, the criteria functions f_1, f_2, f_3, f_4 and f_5 are minimized, while f_6, f_7 and f_8 are maximized.

Dimensionless criterion function values d_{ij}, $i \in \{1, 2, ..., 8\}, j \in \{1, 2, 3, 4\}$, are shown in Table 6. It is easy to conclude that, for the alternative A1, the second, fourth, fifth and seventh criterion functions have the minimal values (0), and the sixth criterion function has the maximal value (1). Analogously, the corresponding conclusions can be drawn regarding the remaining three alternatives (A2, A3 and A4).

Table 6: Dimensionless criterion function's values d_{ij} (i-th criterion function and j-th alternative).

Dimensionless criterion function	Values for the dimensionless criterion function for each alternative			
	A1	A2	A3	A4
d_{1j}	0.3701	0.3701	1	0
d_{2j}	0	0.4131	0.5448	1
d_{3j}	0.2078	0	1	0.3981
d_{4j}	0	1	0.6622	0.9519
d_{5j}	0	0.3876	0.5660	1
d_{6j}	1	0.9996	0.6240	0
d_{7j}	0	0.7465	0.8271	1
d_{8j}	0.7899	1	0	0.9521

In tables 7, 8, 9 and 10 the main results of the use of VIKOR method in the ranking approaches I, II, III and IV, respectively, are presented.

Table 7: Main results after using VIKOR method in the Ranking approach I.

Measure	Main results after using VIKOR method – Ranking approach I					
	A1	A2	A3	A4		
S_j	0.4003	0.7247	0.5847	0.6799	S^*	0. 4003
					S^-	0.7247
R_j	**0.2273**	0.2272	0.1880	**0.2273**	R^*	0.1880
					R^-	**0.2273**
$R_{j,\mathrm{mod}}$	0.2290	0.2272	0.1880	0.2318	R^*_{mod}	0.1880
					R^-_{mod}	0.2318
QS_j	0	1	0.5685	0.8619		
$QR_{j,\mathrm{mod}}$	0.9362	0.8948	0	1		
$Q_{j,\mathrm{mod}}(v = 0.50)$	0.4681	0.9474	0.2842	0.9309		
$Q_{j,\mathrm{mod}}(v = 0.25)$	0.7022	0.9211	0.1421	0.9655		
$Q_{j,\mathrm{mod}}(v = 0.75)$	0.2341	0.9737	0.4263	0.8964		

Table 8: Main results after using VIKOR method in the Ranking approach II.

Measure	Main results after using VIKOR method – Ranking approach II					
	A1	A2	A3	A4		
S_j	0.2641	0.5737	0.6520	0.6957	S^*	0.2641
					S^-	0.6957
R_j	0.1111	0.1111	**0.1556**	**0.1556**	R^*	0.1111
					R^-	**0.1556**
$R_{j,\mathrm{mod}}$	0.1111	0.1111	0.1605	0.1610	R^*_{mod}	0.1111
					R^-_{mod}	0.1610
QS_j	0	0.7173	0.8988	1		
$QR_{j,\mathrm{mod}}$	0	0	0.9912	1		
$Q_{j,\mathrm{mod}}(v = 0.50)$	0	0.3587	0.9450	1		
$Q_{j,\mathrm{mod}}(v = 0.25)$	0	0.1793	0.9681	1		
$Q_{j,\mathrm{mod}}(v = 0.75)$	0	0.5380	0.9219	1		

Table 9: Main results after using VIKOR method in the Ranking approach III.

Measure	Main results after using VIKOR method – Ranking approach III					
	A1	A2	A3	A4		
S_j	0.3048	0.6307	0.6139	0.7157	S^*	0.3048
					S^-	0.7157
R_j	**0.1724**	0.1723	0.1426	**0.1724**	R^*	0.1426
					R^-	**0.1724**
$R_{j,\mathrm{mod}}$	0.1737	0.1723	0.1426	0.1778	R^*_{mod}	0.1426
					R^-_{mod}	0.1778
QS_j	0	0.7931	0.7522	1		
$QR_{j,\mathrm{mod}}$	0.8834	0.8440	0	1		
$Q_{j,\mathrm{mod}}(v = 0.50)$	0.4417	0.8185	0.3761	1		
$Q_{j,\mathrm{mod}}(v = 0.25)$	0.6626	0.8313	0.1880	1		
$Q_{j,\mathrm{mod}}(v = 0.75)$	0.2209	0.8058	0.5641	1		

Table 10: Main results after using VIKOR method in the Ranking approach IV.

Measure	Main results after using VIKOR method – Ranking approach IV					
	A1	A2	A3	A4		
S_j	0.2418	0.5448	0.6568	0.7110	S^*	0.2418
					S^-	0.7110
R_j	0.1091	0.1091	0.1636	**0.1818**	R^*	0.1091
					R^-	**0.1818**
QS_j	0	0.6459	0.8846	1		
QR_j	0.0006	0	0.7501	1		
$Q_j(v = 0.50)$	0.0003	0.3229	0.8174	1		
$Q_j(v = 0.25)$	0.0004	0.1615	0.7838	1		
$Q_j(v = 0.75)$	0.0001	0.4844	0.8510	1		

Based on the data from tables 7, 8, 9 and 10, the following ranking lists are established: **(a) Ranking approach I** (the first ranking list: A1, A3, A4, A2; the second ranking list: A3, A2, A1, A4; the compromise ranking list for v = 0.50: A3, A1, A4, A2); **(b) Ranking approach II** (the first ranking list: A1, A2, A3, A4; the second ranking list: A1, A2, A3, A4; the compromise ranking list for v = 0.50: A1, A2, A3, A4); **(c) Ranking approach III** (the first ranking list: A1, A3, A2, A4; the second ranking list: A3, A2, A1, A4; the compromise ranking list for v = 0.50: A3, A1, A2, A4); and **(d) Ranking approach IV** (the first ranking list: A1, A2, A3, A4; the second ranking list: A2, A1, A3, A4; the compromise ranking list for v = 0.50: A1, A2, A3, A4).

Considering the results given in Table 7 as well as the established ranking lists for the Ranking approach I, it can be said that the first alternative on the compromise ranking list, A3, does not have a ''sufficient advantage'' over the alternative A1, which is the second ranked on the same list (condition C1 not fulfilled). On the other hand, alternative A3 has a ''sufficient advantage'' over the alternative A4, which is the third on the compromise ranking list. Besides, alternative A3 is the first ranked on the *QR* ranking list, so condition C2 is fulfilled. Taking into account this facts, one can conclude that, *in the case of the Ranking approach I, we have the set of compromise solutions that contains alternatives A3 and A1*.

When it comes to the results obtained using Ranking approach II (Table 8), we have alternative A1 as the first-ranked one on the compromise ranking list and it has a ''sufficient advantage'' over the second-ranked alternative, A2 (condition C1 fulfilled). Alternative A1 has a ''sufficiently stable'' first position, because it is also the first-ranked one on the compromise lists for $v = 0.25$ and $v = 0.75$. In addition, it is also first-ranked on *QS* and *QR* ranking list, so condition C2 is fully met. Thus, *when applying the Ranking approach II, we get alternative A1 as the best solution*.

After applying the Ranking approach III (the corresponding results are shown in Table 9), it turned out that *we have again the set of compromise solutions that involved alternatives A3 and A1*. Here, the conclusions were analogous to those obtained using the Ranking approach I.

Looking at Table 10, we can conclude that *if apply the Ranking approach IV, we again get alternative A1 as the best solution*.

On the basis of the previously stated, the following conclusion can be made: ***the optimal solution for the local geodetic control network design is the one related to alternative A1***. Although, it would not be wrong to declare the set of compromise solutions that contains alternatives A1 and A3 as the solution.

4 DISCUSSION

This study showed that the use of VIKOR method in designing a special-purpose geodetic network is very desirable when an engineering surveying professional has to make a decision in a sea of conflicting requirements, especially those related to precision and reliability. What is the most important, such a multi-criteria optimization approach is an efficient tool that gives the best solution for complex geodetic systems.

On the basis of the results obtained, the best alternative was adopted as the solution for the design of the considered geodetic network in the study. What is very interesting is the fact that, before using the VIKOR algorithm, this alternative was the best according to only four criteria. It was ranked third according to internal reliability (the first criterion in the study) and, even, fourth (the last one on the list) according to the test power in detecting minimal deviation (adopted herein) of the projected from the marked structure pillar span.

All of the above leads to the conclusion that the best solution, i.e. compromise solution, does not always have to be the best one according to all or most of introduced criteria. On the contrary, it may be the worst one according to some criteria, but it will be the best in the compromise. Thus, that's where VIKOR method is, we dare say, the best ally in solving problems.

At the very end, the authors propose the use of VIKOR method whenever there is a need to make a decision in a complex engineering system involving a number of conflicting requirements that are difficult to achieve all at the same time.

Literature and references:

Baarda, W. (1968). A Testing Procedure for Use in Geodetic Networks. Delft: Netherlands Geodetic Commission, Publications on Geodesy, New Series, 2 (5).

Bagherbandi, M., Eshagh, M., Sjöberg, L.E. (2009). Multi-Objective versus Single-Objective Models in Geodetic Network Optimization. Nordic Journal of Surveying and Real Estate Research, 6 (1), 7–20.

Chatterjee, P., Chakraborty, S. (2016). A Comparative Analysis of VIKOR Method and Its Variants. Decision Science Letters, 5 (4), 469–486.

Doma, M.I., El Shoney, A.F. (2011). A New Method for Designing the Optimum Geodetic Networks Using Genetic Algorithms. Journal of Engineering and Applied Science, 58 (2), 109–125.

Dwivedi, R., Dikshit, O. (2013). A Comparison of Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) in Second Order Design (SOD) of GPS Networks. Journal of Applied Geodesy, 7 (2), 135–146.

Grafarend, E.W. (1974). Optimization of Geodetic Networks. Bolletino di Geodesia a Science Affini, 33 (4), 351–406.

Grafarend, E.W., Sansò, F. (1985). Optimization and Design of Geodetic Networks. Berlin • Heidelberg • New York • Tokyo: Springer-Verlag.

Jokić, Ž., Delibašić, B., Komljenović, S. (2019). Application of the VIKOR Method When Selecting Caliber for Automatic Rifles for the Purpose of Implementation in Operational Use in Units of the Serbian Army. Vojno delo, 71 (6), 200–221.

Kuo, M.S., Liang, G.S. (2011). Combining VIKOR with GRA Techniques to Evaluate Service Quality of Airports Under Fuzzy Environment. Expert Systems with Applications, 38 (3), 1304–1312.

Nikolić, M., Radovanović, Lj., Desnica, E., Pekez, J. (2010). Application of the VIKOR Method for the Selection of the maintenance Strategy. Tehnička dijagnostika, 9 (4), 25–32.

Nisel, S. (2014). An Extended VIKOR Method for Ranking Online Graduate Business Programs. International Journal of Information and Education Technology, 4 (1), 103–107.

Opricović, S. (1986). Višekriterijumska optimizacija. Beograd: Naučna knjiga.

Opricović, S. (1998). Višekriterijumska optimizacija sistema u građevinarstvu. Beograd: Građevinski fakultet Univerziteta u Beogradu.

Opricović, S., Tzeng, G.H. (2004). Compromise Solution by MCDM Methods: A Comparative Analysis of VIKOR and TOPSIS. European Journal of Operational Research, 156 (2), 445–455.

Pachelski, W., Postek, P. (2016). Optimization of observation plan based on the stochastic characteristics of the geodetic network. Reports on Geodesy and Geoinformatics, 101 (1), 16–26.

Perović, G. (2005). Least Squares. Monograph. Belgrade: Faculty of Civil Engineering.

Postek, P. (2021). New Computer Simulation Method for First-Order Design of Geodetic Network: Aiming for Low Cost. Journal of Surveying Engineering, 147 (1), 04020020.

Тамутис, З.П. (1979). Оптимальные методы проектирования геодезических сетей [Optimal Methods for Designing Geodetic Networks]. Москва: Недра.

Anđić D., Đurović R. (2023). The use of VIKOR Method in Finding the Optimal Solution for a Special-Purpose Geodetic Network Design. Geodestski vestnik, 67 (2), 196-212.
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