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Prispevek k parameterski identifikaciji dinamičnih sistemov z eno prostostno stopnjo

Parameter Identification for Single-Degree-of-Freedom Dynamic Systems

Nikola Jakšić - Miha Boltežar

V prispevku predstavljamo metodo identifikacije parametrov sistemov z eno prostostno stopnjo. Uvrščamo jo v skupino metod parametrične identifikacije sistemov, ki potrebujejo strukturiran matematični model. Metoda omogoča izračun parametrov gibalne enačbe modela na podlagi merjene časovne vrste pospeška obravnavanega sistema. Metodo smo preskusili na eksperimentalni napravi z lastnostmi Duffingovega nihala. Rezultati so pokazali, da metoda omogoča kakovostno identifikacijo parametrov na kratkih časovnih vrstah, pri razmeroma majhnem številu točk časovne vrste in za raznovrstne sisteme z eno prostostno stopnjo.

(Ključne besede: sistemi dinamski, ugotavljanje parametrov, stopnje prostosti, modeli matematični)

An approach to parameter identification for a single-degree-of-freedom system is presented. It fits into the group of parametric system identification methods that use a structured mathematical model. It uses the free acceleration response of the system in order to estimate the parameters of the equation of motion for the model under consideration. The approach has been tested on an experimental device with the features of a Duffing oscillator. The results show that our approach offers parameter identification with good quality for short time series using only a modest number of data points for a wide range of single-degree-of-freedom systems.

(Keywords: dynamical systems, parametric identification, degrees of freedom, mathematical model)

0 UVOD

V inženirski praksi se velikokrat zgodi, da je dober model sistema znan, ali pa ga je mogoče izpeljati iz osnovnih zakonitostih mehanike. Določitev vrednosti parametrov gibalne enačbe izbranega modela iz dinamičnega odziva sistema je naloga, ki jo rešuje pričujoče delo. To nalogo je mogoče rešiti na več različnih načinov.

Prispevek [1] obravnava lastna nihanja sistema z eno prostostno stopnjo z viskoznim dušenjem in s Coulombovim modelom suhega drsnega trenja. Postopek uporabi zmanjševanje amplitud pomika sistema za identifikacijo parametrov disipacije energije.

V članku [2] avtorja razvijeta metodo na podlagi aproksimacijske teorije polinomov Čebišova za sisteme z eno prostostno stopnjo, pri katerih predpostavita polinomsko togostno in dušilno karakteristiko. Identifikacija stabilnega linearnega sistema z uporabo polinomskih funkcij je predstavljena v [3]. Primer identifikacije

0 INTRODUCTION

In engineering practice a good model of the real system, or a few likely candidates for a good model of the real system, are usually known or can be deduced from basic mechanical principles. The task is to determine the parameters of the model's equation of motion based on information contained in the system's dynamical response. There are several ways of achieving this.

One study [1] considered the free vibrations of a single-degree-of-freedom (s.d.o.f.) system with combined viscous damping and Coulomb dry friction. This approach used only the amplitude decay of the displacement response of the system.

An approach to parameter identification of assumed polynomials for the description of non-linearities in restoring and damping forces within a forced dynamical system was used in [2]. This approach uses approximation theory with Tchebishev polynomials. The identification of a stable linear system using polynomial kernels was presented in [3].

ekvivalentnega viskoznega dušenja lahko najdemo v [4].

Parameterske identifikacije nelinearnih sistemov se lahko lotimo na nekaj načinov. Identifikacijo parametrov nelinearnega sistema omogoča model PHP (pospešek - hitrost - pomik) [5] prek poznavanja modela in kinematičnih spremenljivk pomika, hitrosti in pospeška sistema. Delo [6] predstavlja metodo za parametrično identifikacijo modelov večih vstopov in izstopov. Rekurzivno metodo identifikacije za nekatere nelinearne sisteme na podlagi šumnih meritev predstavlja [7]. Identifikacija z ekvivalentno linearizacijo le malo nelinearnih sistemov je predstavljena v [8]. Metoda, ki oceni parametre nelinearnega sistema na podlagi frekvenčnih odzivih funkcij višjih redov, je opisana v [9]. V delih [10] in [11] avtor uporabi Hilbertovo transformacijo za identifikacijo parametrov nelinearnega sistema z eno prostostno stopnjo. Identifikacija parametrov nelinearnih sistemov z uporabo valovne transformacije je opisana v [12] in z uporabo nevronskih mrež v [13] do [15]. Ocene parametrov sistemov s histereznim učinkom obravnavajo v [16] do [19].

V tem prispevku predstavljamo metodo identifikacije parametrov poljubnih sistemov z eno prostostno stopnjo, ki je preprosta in uspešna tudi na kratkih merjenih časovnih vrstah. Preliminarne raziskave [20] so pokazale, da daje metoda zelo dobre rezultate, kadar za identifikacijo parametrov uporabimo spremenljivke faznega prostora in kadar je pospešek glavni vir informacij o sistemu [21]. Nadaljnje raziskave ([22] do [24]) so potrdile uspešnost metode pri identifikaciji parametrov nelinearnih modelov z eno prostostno stopnjo na podlagi kratkih meritev pospeška sistemov.

Predstavljeno metodo smo preskusili na podlagi lastnega nihanja eksperimentalne naprave, ki ima lastnosti Duffingovega nihala. Merjeni pospešek smo uporabili za parametrsko identifikacijo sistema.

An approach to the identification of equivalent viscous damping parameters is discussed in [4].

Non-linear systems are approached in several different ways. The AVD (Acceleration-Velocity-Displacement) model [5] offers a way of achieving parameter identification for a non-linear system by knowing the model and time series of displacement, velocity and acceleration. A method of parameter identification for a multi-input multi-output model was also presented in [6]. A recursive approach for a class of non-linear systems from noisy measurements was introduced in [7]. An identification of weakly non-linear systems using equivalent linearization was presented in [8]. A method used for estimations of the non-linear systems based on high-order frequency-response functions was described in [9]. In [10] and [11] the Hilbert transform was used in order to identify the parameters of the s.d.o.f. non-linear system. The use of the wavelet transform entered the field of the non-linear system's parameter identification in [12]. Parameter identification via neural networks was presented in [13] to [15]. The identification of a hysteretic system was studied in [16] to [19].

In this paper an approach to parameter identification is proposed that is simple, convenient for short measured time series and can be used on different classes of s.d.o.f. systems. Preliminary studies [20] have shown that the method gives very good results when phase-space variables are used for the identification and when the acceleration is the main source of the system's information [21]. Further research ([22] to [24]) confirmed the success of the parameter identification method applied to the short measured acceleration response of the non-linear s.d.o.f. system.

The parameter identification method is tested against a real experimental device that resembles a Duffing's system by using the device's free acceleration response.

1 METODA

1 METHOD

V inženirski praksi velikokrat modeliramo dejanski sistem z modelom z eno prostostno stopnjo, katerega lastna nihanja popišemo z enačbo (1). To je lahko že končni model ali pa le prvi korak k modeliranju sistema.

It is not unusual in engineering practice to model a real dynamical system with a s.d.o.f. model in which the free vibrations are governed by equation (1). This can be either the final model or just the first approach to the problem.

$$\ddot{x} - F(x, \dot{x}; a_1, \ldots, a_n) = 0 \qquad (1),$$

kjer $a_1, \ldots, a_n$ pomenijo n neznanih parametrov, ki jih moramo identificirati. Metoda temelji na geometrijski predstavitvi rešitve gibalne enačbe sistema kot dvoparametrične družine krivulj. Glede na začetne pogoje je le ena možna krivulja, tir gibanja. Vsaka točka tira gibanja in njeni časovni odvodi zadovoljijo gibalno enačbo. Diferencialne enačbe sistema lahko

where $a_1, \ldots, a_n$ represent n unknown parameters, which need to be determined. The approach is based on a geometrical representation of the solutions of the differential equation of motion. The solutions consist of a family of curves governed by two parameters. Only one trajectory is realized with the initial conditions. The differential equation of motion can be represented by predstavimo kot sistem algebrskih enačb, če jih rešujemo na njihove parametre. Za izračun n parametrov potrebujemo teoretično le n točk tirnice in ustrezajočih *n* točk na časovnih odvodih kinematičnih spremenljivk, ki jo sestavljajo. Tako prevedemo problem v reševanje sistema algebrskih enačb oziroma v reševanje predefiniranega sistema algebrskih enačb. Slednje opravimo z metodo najmanjših kvadratov odstopanj.

Cilj je karakterizacija mehanskega sistema z izbranim modelom. Predpostavimo, da je tip diferencialne enačbe gibanja znan in da je časovna vrsta pospeška sistema izmerjena. Pod temi predpostavkami je mogoče metodo razdeliti na dva dela:

a) **Rekonstrukcija** prostora stanj; to je rekonstrukcija manjkajočih časovnih vrst hitrosti in pomika iz merjenega pospeška z numeričnim integriranjem. Če merjena časovna vrsta vsebuje šum ravni SNR ≤ 40 dB, je potrebno njeno glajenje. V ta namen smo uporabili kubične približne zlepke. Če pa je raven šuma v merjeni časovni vrsti manjša, oziroma če smo merjeno časovno vrsto že zgladili, uporabimo interpolacijske zlepke 3. ali 5. reda za numerično integracijo časovne vrste pospeška. V primeru daljših merjenih časovnih vrst (več ko 2 nihaja) priporočamo uporabo časovnih oken. Dobljeno časovno vrsto hitrosti interpoliramo in ponovno integriramo. Faza rekonstruiranja časovnih vrst je popolnoma neodvisna od izbranega modela.

b) **Oceno** vrednosti parametrov izvedemo z metodo najmanjših kvadratnih odstopanj ciljne funkcije (2). Slednjo izpeljemo iz diferencialne enačbe gibanja (1). Gibalna enačba (1) velja pri kateremkoli času, zato lahko seštejemo vrednosti leve strani enačbe pri vseh diskretnih časih in tako ustvarimo ciljno funkcijo:

an algebraic equation where the parameters are considered to be unknowns. Hence, to estimate the n parameters of the model's equation of motion, theoretically only n points on the trajectory and on the time derivatives of its kinematics variables are needed. The problem is transformed to one of solving a system of algebraic equations or a predefined system of algebraic equations by means of a least-squares approximation, if there are more points than parameters.

The aim is to characterize a mechanical system with a chosen model. Let us consider that the type of differential equation of motion is known and the acceleration time history of the system under consideration is measured. Then the approach to parameter identification can be divided into two parts:

a) **Reconstruction** of the state space, in other words, the reconstruction of the missing velocity and displacement time histories from the measured acceleration time history by numerical integration. If the noise level in the measured time history is SNR ≤ 40 dB then smoothing of the latter has to be performed. In this paper, the approximating cubic splines were used for this purpose. For the case of low-level noise in the acceleration time history and for the case of an already-smoothed acceleration time history the interpolation with splines of the 3rd or 5th degree was used in order to numerically integrate the acceleration time history. For the case of a long measured time history (more than 2 cycles) the time-window approach is strongly recommended. The obtained velocity time history has to be interpolated and integrated again. The reconstruction stage of the approach is completely model independent.

b) **Estimation** of the parameters is achieved by a least-squares fit of the least-squared merit function, equation (2), deduced from the equation of motion, equation (1). Since the equation of motion (1) is valid for any given time, we can sum up the values of the equation for all the discrete times and thus we can create the merit function equation:

$$\chi^2 = \sum_{i=1}^{m} \left[\ddot{x}_i - F(x_i, \dot{x}_i; a_1, \ldots, a_n)\right]^2 \qquad (2),$$

kjer *m* pomeni število točk merjene časovne vrste pospeška, $m > n$ in $x_i, \dot{x}_i, \ddot{x}_i$ pomenijo pomik, hitrost in pospešek *i*-te točke. $a_1, \ldots, a_n$ označuje *n* parameterov, ki jih želimo identificirati. Ker smo časovni vrsti hitrosti in pomika dobili z numerično integracijo iz časovne vrste pospeška, moramo dodati dve novi neznanki in tudi novo spremenljivko. Novi neznanki sta prosti integracijski konstanti - neznani začetna pogoja x_0 in $\dot{x}_0$. Nova spremenljivka pa je diskretni čas t_i pri *i*-ti točki časovne vrste pospeška. Ciljno funkcijo moramo zatorej napisati na novo:

where m denotes the number of points of the measured acceleration time history, $m > n$ and $x_i, \dot{x}_i, \ddot{x}_i$ denote displacement, velocity and acceleration at the *i*-th sampling point, respectively. $a_1, \ldots, a_n$ denote the *n* parameters to be identified. Because the velocity and displacement time histories have been numerically integrated from the acceleration time history, two new unknowns and a new variable are introduced. These two new unknowns are the free integration constants, i.e. the unknown initial conditions x_0 and $\dot{x}_0$. The new variable is the discrete time t_i at the *i*-th sampling point. Hence, the merit function must be rewritten as:

$$\chi^2 = \sum_{i=1}^{m} \left[\ddot{x}_i - F(x_i, \dot{x}_i, t_i; x_0, \dot{x}_0, a_1, \ldots, a_n)\right]^2 \qquad (3).$$

Reševanje s časovnimi okni zahteva razdelitev merjene časovne vrste na podkorake, ki se lahko prekrivajo. Vsak podkorak obravnavamo kot posebno časovno vrsto in jo izpostavimo identifikacijskemu postopku. Rezultate - identificirane vrednosti parametrov - povprečimo prek vseh podkorakov. Uporaba časovnih oken je nujna zaradi numeričnih napak, ki se pojavijo pri glajenju šumne časovne vrste pospeška in njene dvakratne numerične integracije.

The time-window approach requires segmentation of the original time history into sub-intervals, which may overlap. Each sub-interval is treated as a separate time history. A complete identification procedure is applied to each sub-interval and the results – identified parameters – are finally averaged over all sub-intervals. The time-window approach is necessary because of the numerical errors introduced by smoothing of the noisy acceleration time history and its double numerical integration.

2 DUFFINGOV SISTEM

2 DUFFING'S SYSTEM

Duffingov sistem lahko uporabimo pri modeliranju dinamičnega sistema z nelinearno togostjo v primeru nosilca v uklonjenem stanju ali velike deformacije nosilca [25]. Gibalno enačbo lastnega nihanja Duffingovega sistema lahko zapišemo kot:

The Duffing's system can be used for modelling dynamical systems with non-linear stiffness such as the post buckling or the large deflection of beams [25]. The equation of motion of free vibrations of Duffing's system with dry friction can be written as:

$$\ddot{x} + a\dot{x} + bx + cx^3 = 0 \qquad (4),$$

kjer je *a* parameter, ki opisuje viskozno dušenje, *b* je parameter, ki ponazarja linearni del togosti sistema, *c* pa opiše nelinearni del togosti sistema.

Edina mogoča atraktorja lastnega nihanja sistema z eno prostostno stopnjo sta točka in mejna zanka. Dinamično obnašanje obravnavanega sistema bo zategadelj preprosto.

Ciljno funkcijo predstavljene metode za identifikacijo parametrov Duffingovega sistema (4) lahko zapišemo kot:

where a is a parameter that describes viscous damping, *b* is a parameter representing the linear part of stiffness in the system and *c* denotes the non-linear part of stiffness.

In the case of free vibrations of a s.d.o.f. system the only attractor shapes possible are the point attractor and the limit cycle. The dynamical behavior of the system under consideration is expected to be simple.

Applying the approach of parameter identification to Duffing's system, equation (4), the least-squared merit function can be rewritten as:

$$\chi^2 = \sum_{i=1}^{m} \left[\ddot{x}_i + a(\dot{x}_i + \dot{x}_0) + b(x_i + x_0 + t_i\dot{x}_0) + c(x_i + x_0 + t_i\dot{x}_0)^3\right]^2 \qquad (5),$$

kjer t_i pomeni čas *i*-te točke.

Enačba (5) predstavlja nelinearni problem najmanjših kvadratov odstopanj. Rešujemo ga z iterativnim reševanjem:

1) Najprej moramo uganiti začetne vrednosti začetnih pogojev. Nelinearni optimizacijski problem tako prevedemo na linearno reševanje po metodi najmanjših kvadratov odstopanj - regresija. V vseh primerih smo izbrali ničné začetne pogoje, s katerimi je bila konvergenca metode vedno hitra.
2) Linearni optimizacijski problem rešimo po metodi najmanjših kvadratov odstopanj (regresija).
3) Novo vrednost začetnih pogojev izračunamo iz ocenjenih regresijskih koeficientov.
4) Ponavljamo drugi korak iteracijske zanke, dokler niso izpolnjeni pogoji konvergence.

Vrednosti parametrov konvergirajo v že nekaj iteracijskih korakih. V primeru uporabe časovnih oken identificirani začetni pogoji v določenem časovnem oknu predstavljajo začetne vrednosti le-teh za naslednje časovno okno. Pogoj je le, da korak časovnih oken ne sme biti prevelik (kar se ne zgodi pogosto).

where t_i denotes time at the *i*-th sampling point.

Equation (5) represents a non-linear least-squares-fit problem. It was solved by using the following iterative procedure:

1) The initial conditions must be guessed first. Thus the non-linear least-squares-fit problem is transformed into a linear one. The choice of zero initial conditions worked well in all cases.
2) The linear least-squares-fit problem is solved.
3) The new value for the initial conditions is computed from the estimated regression parameters.
4) The second step is repeated until the convergence criterion is met.

The values of the parameters converge after a few steps of the iteration. In the case of the time-window approach the estimated initial conditions from a certain time window are used as a good guess for the next time window if the time-window shift is not too big (which is rare).

3 PRESKUS

Preskusno delo smo opravili na namensko zgrajeni napravi, ki ima lastnosti Duffingovega nihala, ker omogoča nihanje z velikimi amplitudami. Najprej smo opravili preproste ločene teste: ocenili smo vzmetno karakteristiko sistema s statičnim testom in količino razsipane energije v sistemu z logaritemskim dekrementom. Nato smo primerjali rezultate, dobljene z našo metodo identifikacije parametrov, za katero verjamemo, da je preprosta za uporabo z rezultati, dobljenimi z ločenimi testi. Nadalje smo primerjali obnašanje metode in njene rezultate v primeru nihanja vztrajnostne mase, ko so grabljice v zraku in v vodi.

3.1 Preskusna naprava

Preskusno napravo sestavljata dve ločeni vzporedni listnati vzmeti, konzolno vpeti v stojalo na eni strani in pritrjeni na vzrajnostno maso na drugi strani. Velikost prečnega prereza posamezne vzmeti znaša $a \times h$ = 1 mm × 30 mm. Dolžina vzmeti je l = 512 mm. Velikost vztrajnostne mase znaša m_i = 1,892 kg. Celotno vztrajnostno maso ocenimo na $m_c = m_i + 2 \times m_s/3$ = 1,971 kg. Slika 1 prikazuje skico preskusne naprave.

3.2 Ločeni testi

Najprej smo določili vzmetno karakteristiko vzmeti in nato še razmernik dušenja iz merjene časovne vrste pospeška pri lastnem nihanju naprave.

Statično merjeno vzmetno karakteristiko smo ponazorili z linearno (6) in kubično (7) funkcijo. Merjeno ter tudi linearno in kubično približno vzmetno karakteristiko prikazuje slika 2. Na tej sliki prikazujemo le pozitivne vrednosti vzmetne karakteristike, ki je liha funkcija. Vrednost koeficienta k_1 linearne karakteristike (6) je ocenjena na k_1 = 71,172 N/m. Vrednosti koeficientov k_1 in k_3 kubične karakteristike (7) pa na k_1 = 78,072 N/m in k_3 =- 2470,504 N/m^3.

3 EXPERIMENT

The experimental work was undertaken on a purpose-made experimental device which resembles the features of a Duffing's oscillator by allowing high-amplitude oscillations. Since we belive that our method is relativly simple to apply we also considered simple, separate tests of the system by estimating the system's spring characteristic by static testing and by estimating the amount of dissipated energy by the logarithmic decrement. The comparison of both approaches is presented. After that we applied the method to measured responses of the inertial mass while rake oscilating in water and compared the results to those obtained in the air experiment.

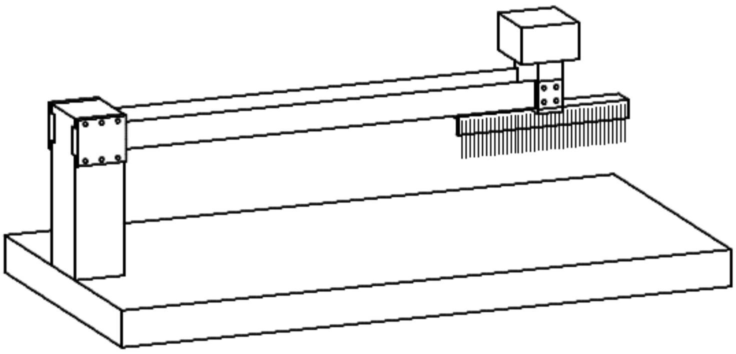
3.1 Experimental device

The experimental device is composed of two parallel but separated leaf springs clamped at one end and attached to an inertial mass at the other end. The dimensions of the spring's cross-section are $a \times h$ = 1 mm × 30 mm and the spring's length is l = 512 mm. The inertial mass is m_i = 1.892 kg. The complete inertial mass is estimated to be $m_c = m_i + 2 \times m_s/3$ = 1.971 kg. The experimental device is schematically shown in Fig. 1.

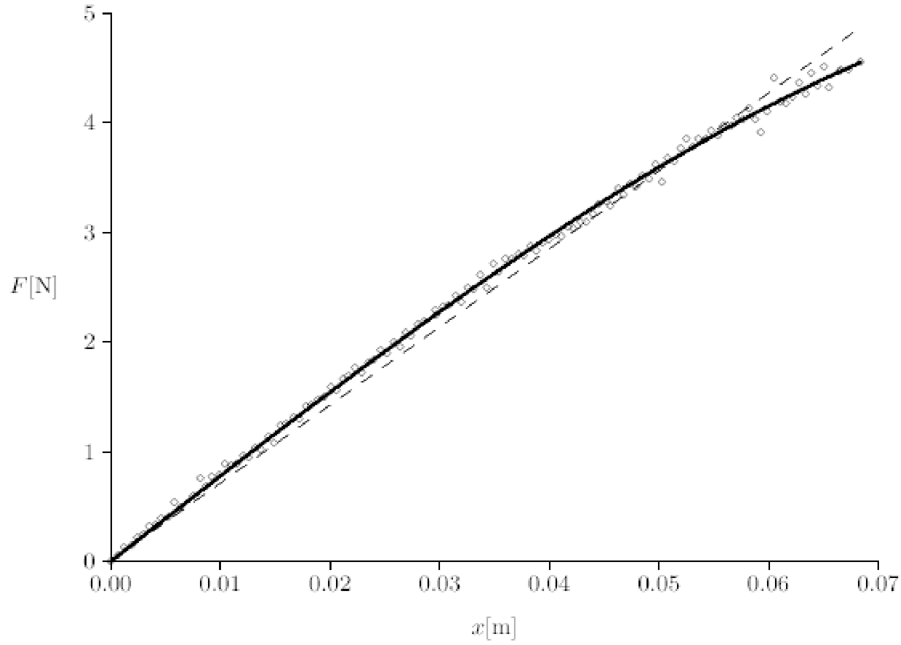
3.2 Separate tests

The spring characteristic was determined with a static test and then the damping ratio was determined from the measured acceleration free response of the device.

The statically measured spring characteristic was approximated with the linear, eq. (6), and the cubic, eq. (7), functions. The linear and cubic approximations of the measured spring characteristic are shown in Fig. 2. The characteristic of the spring is an odd function, but only positive values are shown in Figure 2. The value of the coefficient k_1 of the characteristic eq. (6) is k_1 = 71.172 N/m. The values of the coefficients k_1 and k_1 of the characteristic eq. (7) are k_1 = 78.072 N/m and k_1 = - 2470.504 N/m^3.



Sl. 1. *Preskusna naprava*
Fig. 1. *Experimental device*



Sl. 2. *Statično določena vzmetna karakteristika: merjene točke (◇◇◇), približna linearna karakteristika (– – –) in približna nelinearna karakteristika (——)*
Fig. 2. *Statically determined spring characteristic: measurement points (◇◇◇), approximate linear characteristic (– – –) and approximate non-linear characteristic (——)*

$$F(x) = k_1 \cdot x \tag{6}$$

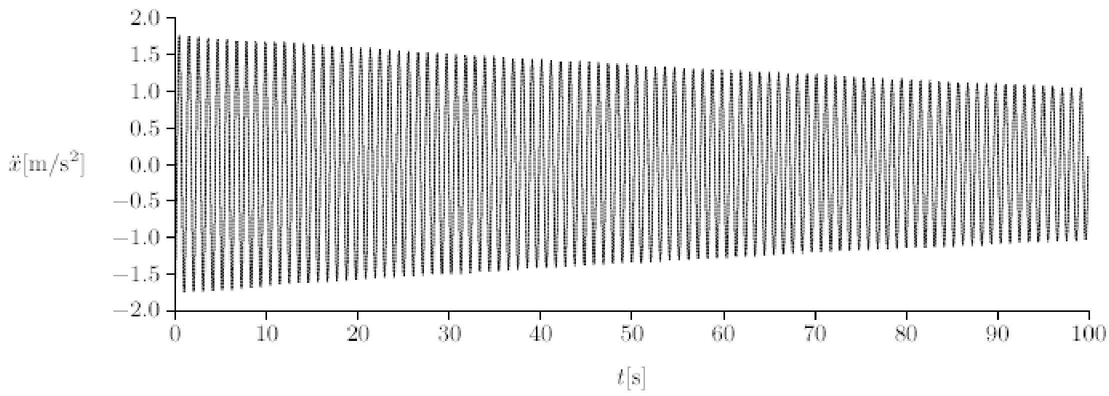
$$F(x) = k_1 \cdot x + k_3 \cdot x^3 \tag{7}$$

Če delimo k_1 in k_3 s celotno vztrajnostno maso, dobimo parametre Duffingovega modela b in c, enačba (4). Vrednosti parametra b izračunamo kot $b = k_1/m_c = 39{,}610$ in vrednost parametra c kot $c = k_3/m_c = -1253{,}427$.

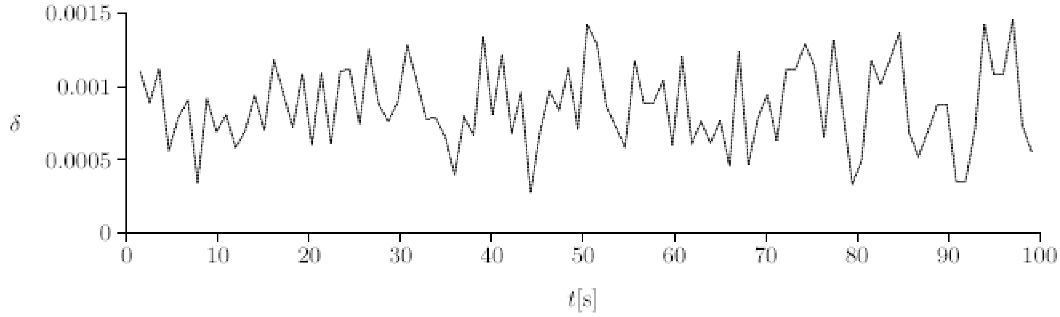
Merjeni odziv sistema (pospešek) pri lastnem nihanju prikazuje slika 3. Vrednosti razmernika dušenja v odvisnosti od časa so prikazane na sliki 4. Ekvivalentni razmernik viskoznega dušenja ocenimo z uporabo logaritemskega dekrementa linearnega modela s povprečenjem grafa na sliki 4. Vrednost razmernika dušenja δ smo ocenili na $\delta = 8{,}694 \cdot 10^{-4}$ oziroma pri upoštevanju parametrov Duffingovega modela (4): $a = 1{,}0943 \cdot 10^{-2}$.

If k_1 and k_3 are divided by the total inertial mass they fit to the parameters of Duffing's model b and c, respectively, equation (4). The value of b is computed as $b = k_1/m_c = 39.610$ and $c = k_3/m_c = -1253.427$.

The measured free acceleration response is presented in Fig. 3. Values of the damping ratio as a function of time are shown in Fig. 4. The equivalent viscous damping ratio was estimated by using the logarithmic decrement of the linear model approach by averaging the plot in Figure 4. The damping ratio δ was estimated to have a value of $\delta = 8.694 \cdot 10^{-4}$ or in terms of Duffing's model, eq. (4): $a = 1.0943 \cdot 10^{-2}$.



Sl. 3. *Merjena časovna vrsta pospeška, nihanje grabljic v zraku*
Fig. 3. *Measured acceleration time series, rake oscillating in the air*



Sl. 4. *Časovni potek razmernika dušenja* δ
Fig. 4. *The damping ratio* δ *as function of time*

3.3 Identifikacija parametrov - grabljice v zraku

Pospešek smo merili z merilnikom pospeška, pritrjenim na vztrajnostno maso. Merjeno časovno vrsto smo zajeli z 12-bitno A/D konverzijo in jo shranili na trdi disk računalnika. Frekvenco vzorčenja smo nastavili na 1 kHz. Merjena časovna vrsta pospeška je prikazana na sliki 3.

Spremenljivke prostora stanj smo rekonstruirali z uporabo interpolacijskih kubičnih zlepkov zaradi nizke ravni šuma v merjeni časovni vrsti. Parametre smo identificirali na prvih desetih nihajih časovne vrste. Uporabili smo tudi postopek reševanja s časovnimi okni zaradi dolžine identifikacijskega koraka.

Preverili smo vpliv sprememb dolžine časovnega okna, frekvence vzorčenja in tudi koraka časovnega okna na veljavnost ocene vrednosti parametrov. Merjeno časovno vrsto smo prevzorčili na 100 Hz in to vrednost označili kot privzeto vrednost. Privzeta vrednost dolžine časovnega okna je dva nihaja in privzeta vrednost koraka časovnega okna je 1/10 nihaja.

Vpliv sprememb dolžine časovnega okna

Rezultati identificiranih parametrov pri različnih dolžinah časovnega okna so prikazani v preglednici 1. V prvem stolpcu so navedene različne dolžine časovnega okna. V drugem stolpcu so oznake krivulj na sliki 5. V zadnjih treh stolpcih so zbrane identificirane vrednosti parametrov Duffingovega modela.

Na sliki 5 je prikazan detajl desete pozitivne amplitude. Merjeni pospešek je narisan z debelo črto. Odzivi Duffingovega modela pa so narisani s tankimi črtami. Oznake grafov odziva modela so opisane v preglednici 1. Vidimo lahko, da leži primerna izbira dolžine časovnega okna med enim nihajem in dvema nihajema.

Vpliv sprememb frekvence vzorčenja

Rezultati identificiranih parametrov pri različnih frekvencah vzorčenja so prikazani v preglednici 2. V prvem stolpcu so navedene različne frekvence vzorčenja. V drugem stolpcu so oznake krivulj na sliki 6. V zadnjih treh stolpcih so zbrane

3.3 Parameter identification: the rake in the air

The acceleration time history was measured by an accelerometer fixed to the inertial mass. The time history was aquired with a 12 bit A/D converter and stored on a PC's HDD. The sampling frequency was 1 kHz. The measured acceleration is presented in Fig. 3.

The state space was reconstructed by the cubic spline interpolation because of the low noise contamination of the measured time history. The parameters were identified on the first ten cycles of the response. The time-window approach to identification was adopted because of the length of the identification interval.

The impacts of variations of the length of the time window, the sampling frequency and the step of the time-window shift on the validity of the estimated parameters were studied. The time history was re-sampled at 100 Hz and this is set to be the default sampling frequency. The default time-window length was set to two cycles and the default time-window shift was set to 1/10 of the cycle.

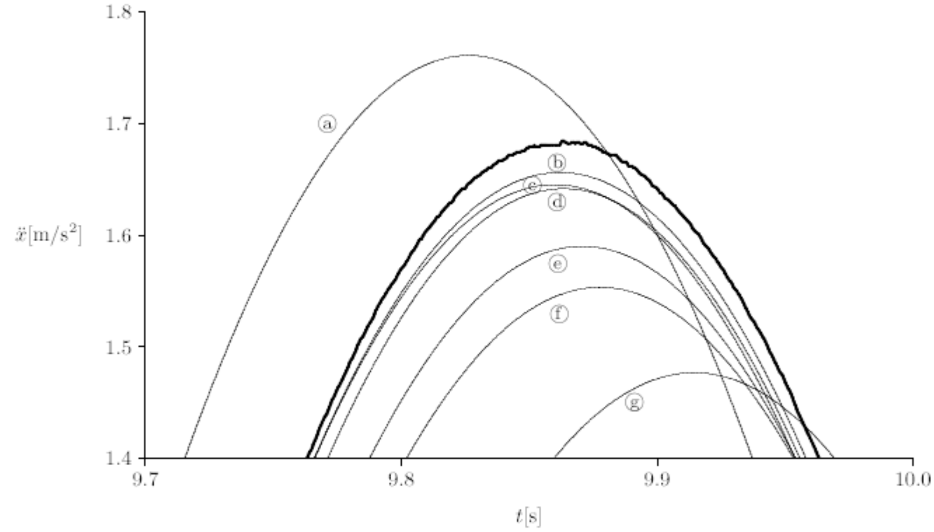
Influence of time-window length variation

The results of the identified parameters for varying time-window length are shown in Table 1. The first column has the chosen time-window lengths, the second column denotes the labels of the curve in Fig. 5 and the last three columns contain values of the identified parameters of Duffing's model.

In Fig. 5 a detail of the tenth positive amplitude is shown. The measured acceleration is drawn with a thick line and the acceleration responses of Duffing's model are drawn with thin lines. The labels of the model responses correspond to the labels in Table 1. We can see that the best choices for the length of the time window lie between one and two cycles.

Influence of sampling-rate variation

The results for parameters at various sampling rates are shown in Table 2. The first column lists the sampling rates, the second column indicates the label of the curve in Fig. 6 and the last three columns contain values of the identified



Sl. 5. *Merjeni pospešek (debela črta) in simulirani odziv Duffingovega modela pri parametrih, identificiranih pri različnih dolžinah časovnega okna, za oznake glej preglednico 1. Detajl desete pozitivne amplitude.*
Fig. 5. *Measured acceleration (thick line) and simulated responses of Duffing's model for the parameters identified at different time-window lengths, for labels see Table 1. Details of the tenth positive amplitude.*

Preglednica 1. *Ocenjene vrednosti parametrov Duffingovega modela pri različnih dolžinah časovnega okna*
Table 1. *Estimated values of the parameters of Duffing's model at different time-window lengths*

Dolžina časovnega okna Time-window length	Krivulja na sliki 5 Curve in figure 5	a	b	c
0.5 nihaja/cycle	ⓐ	$-8.409 \cdot 10^{-3}$	37.078	-451.256
1 nihaj/cycle	ⓑ	$1.232 \cdot 10^{-2}$	36.778	-442.403
1.5 nihaja/cycle	ⓒ	$1.153 \cdot 10^{-2}$	36.686	-387.403
2 nihaja/cycles	ⓓ	$1.209 \cdot 10^{-2}$	36.283	-162.403
3 nihaji/cycles	ⓔ	$1.427 \cdot 10^{-2}$	36.287	-212.612
4 nihaji/cycles	ⓕ	$1.537 \cdot 10^{-2}$	36.250	-234.609
5 nihajev/cycles	ⓖ	$2.069 \cdot 10^{-2}$	35.932	-221.224

identificirane vrednosti parametrov Duffingovega modela. Merjeno časovno vrsto smo prevzorčili tako, da ustreza vrednostim spreminjanih frekvenc vzorčenja.

Na sliki 6 je prikazan detajl desete pozitivne amplitude. Merjeni pospešek je narisan z debelo črto. Odzivi Duffingovega modela so narisani s tankimi črtami. Oznake grafov odziva modela so opisane v preglednici 2. Na sliki 6 lahko vidimo, da ni bistvene razlike med različnimi frekvencami vzorčenja.

Vpliv sprememb koraka časovnega okna

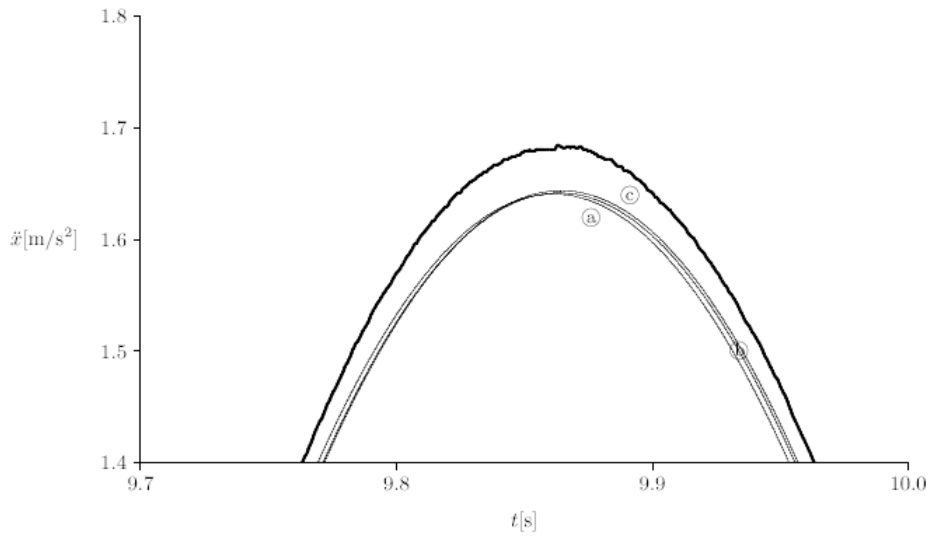
Rezultati identificiranih parametrov pri različnih korakih časovnega okna so prikazani v preglednici 3. V prvem stolpcu so navedeni različni koraki časovnega okna. V drugem stolpcu so oznake krivulj na sliki 7. V zadnjih treh stolpcih so zbrane

parameters of Duffing's model. The measured time history of the acceleration was resampled to match the desired sampling rate.

In Fig. 6 a detail of the tenth positive amplitude is shown. The measured acceleration is drawn with a thick line and the acceleration responses of Duffing's model are drawn with a thin line. The labels of the model responses correspond to the labels in Table 2. In Fig. 6 we can see that there are no major differences between the different sampling rates.

Influence of time-window shift variation

The results for parameters at various time-window shifts are shown in Table 3. The first column lists the time-window shifts, the second column indicates the label of the curve in Fig. 7 and the last three columns contain



Sl. 6. *Merjeni pospešek (debela črta) in simulirani odziv Duffingovega modela pri parametrih, identificiranih pri različnih frekvencah vzorčenja, za oznake glej preglednico 2. Detajl desete pozitivne amplitude.*
Fig. 6. *Measured acceleration (thick line) and simulated responses of Duffing's model for the parameters identified at different sampling rates, for labels see Table 2. Details of the tenth positive amplitude.*

Preglednica 2. *Ocenjene vrednosti parametrov Duffingovega modela pri različnih frekvencah vzorčenja*
Table 2. *Estimated values of the parameters of Duffing's model at different sampling rates*

Frekvenca vzorčenja Sampling rate	Krivulja na sliki 6 Curve in figure 6	a	b	c
10 Hz	ⓐ	$1.227 \cdot 10^{-2}$	36.291	-159.511
100 Hz	ⓑ	$1.209 \cdot 10^{-2}$	36.283	-162.403
1000 Hz	ⓒ	$1.188 \cdot 10^{-2}$	36.169	-157.292

identificirane vrednosti parametrov Duffingovega modela.

Na sliki 7 je prikazan detajl desete pozitivne amplitude. Merjeni pospešek je narisan z debelo črto. Odzivi Duffingovega modela so narisani s tankimi črtami. Oznake grafov odziva modela so opisane v preglednici 3. Na sliki 7 lahko vidimo, da ni bistvene razlike med različnimi koraki časovnega okna.

Ponovljivost preskusa

Parametre Duffingovega modela smo identificirali z najboljšo mogočo kombinacijo frekvence vzorčenja (1000 Hz), dolžine časovnega okna (1 nihaj) in koraka časovnega okna (1/1000 nihaja) na merjenem pospešku in dobili naslednje vrednosti parametrov: $a = 1{,}067 \cdot 10^{-2}$, $b = 36{,}780$ in $c = -444{,}702$.

Ponovljivost preskusa in metode smo preverili na enajstih različnih merjenih časovnih vrstah pospeška z enako frekvenco vzorčenja, dolžino časovnega okna in njegovim korakom kakor pri prvi časovni vrsti. Rezultati identifikacije so zbrani v preglednici 4. Vidimo lahko, da je raztros najmanjši pri

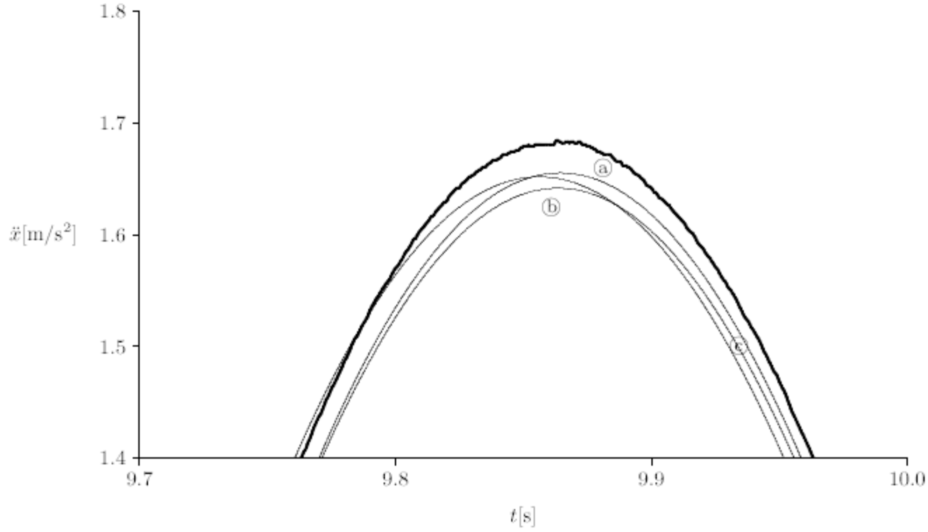
values of the identified parameters of Duffing's model.

In Fig. 7 a detail of the tenth positive amplitude is shown. The measured acceleration is drawn with a thick line and the acceleration responses of Duffing's model are drawn with a thin line. The labels of the model responses correspond to the labels in Table 3. In Fig. 7 we can see that there are no major differences between the different time-window shifts.

The experimental repeatability

The best possible combination of the sampling rate (1000 Hz), the time-window length (1 cycle) and the time-window shift (1/1000 cycle) applied to the identification procedure on the measured acceleration yield results for Duffing's model parameters of $a = 1.067 \cdot 10^{-2}$, $b = 36.780$ and $c = -444.702$.

The repeatability of the experiment and the method were tested on eleven different measured acceleration time histories with the same combination of the sampling rate, the time-window length and the time-window shift as for the first time history. The results of the identification are presented in Table 4. We can



Sl. 7. *Merjeni pospešek (debela črta) in simulirani odziv Duffingovega modela pri parametrih, identificiranih pri različnih korakih časovnega okna, za oznake glej preglednico 3. Detajl desete pozitivne amplitude.*
Fig. 7. *Measured acceleration (thick line) and simulated responses of Duffing's model for the parameters identified at different time-window shifts, for labels see Table 3. Details of the tenth positive amplitude.*

Preglednica 3. *Ocenjene vrednosti parametrov Duffingovega modela pri različnih korakih časovnega okna*
Table 3. *Estimated values of the parameters of Duffing's model at different time-window shifts*

Korak časovnega okna Time-window shift	Krivulja na sliki 7 Curve in figure 7	a	b	c
1/100 nihaja/cycle	ⓐ	$1.029 \cdot 10^{-2}$	36.690	-496.632
1/10 nihaja/cycle	ⓑ	$1.209 \cdot 10^{-2}$	36.283	-162.403
1 nihaj/cycle	ⓒ	$1.055 \cdot 10^{-2}$	36.169	-95.593

parametru *b*, manj ko 1 odstotek srednje vrednosti. Raztros parametra *c* je približno 4,4% srednje vrednosti. Parameter *a* ima največji raztros glede na svojo srednjo vrednost, približno 18%, kar je posledica modeliranja strukturnega dušenja vzmeti in zračnega upora z ekvivalentnim viskoznim dušenjem in tudi zaradi majhne količine raztrosa energije na nihaj.

Duffingov ali linearni model?

Primerjajmo obnašanje Duffingovega modela z linearnim. Identificirane parametre obeh modelov najdemo v preglednici 5. Na sliki 8 je prikazan detajl 32. pozitivne amplitude. Merjeni pospešek je narisan z debelo črto. Odziva Duffingovega in linearnega modela sta narisana s tankima črtama. Oznaki grafov odzivov modelov sta opisani v preglednici 5. Na sliki 8 lahko vidimo, da Duffingov model bolje popiše amplitudo in predvsem frekvenco merjene časovne vrste v primerjavi z linearnim modelom. Vendar tudi odziv Duffingovega modela zaostaja za merjenim odzivom. Proti koncu merjene časovne vrste se razlike še povečujejo. Na tem mestu poudarimo, da se te

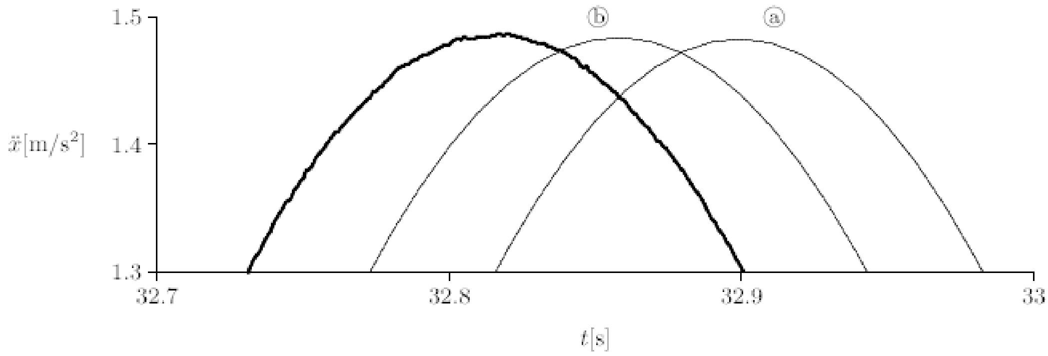
see that the deviations relative to the average value are the smallest with parameter *b*, less than 1%. Parameter *c* experiences greater deviations, approximately 4%. The greatest deviations relative to the average value are experienced by parameter *a*, due to the modelling of the structural damping and the air resistance with the equivalent viscous damping and also due to the small amount of the dissipated energy over a cycle.

Duffing's or linear model?

Let us compare the dynamical behavior of the Duffing's and the linear models. The identified values of the parameters can be found in Table 5. A detail of the 32nd positive amplitude is shown in Fig. 8. The measured acceleration is drawn with a thick line and the acceleration responses of Duffing's and the linear models are drawn with a thin line. The labels of the responses correspond to the labels in Table 5. In Fig. 8 we can see that the Duffing's model is better at describing the amplitude as well as the frequency of the measured time series in comparison to the linear model. We can also see that the Duffing's model response is also lagging the measured response. The differences are increasing

Preglednica 4. *Primerjava ocen parametrov Duffingovega modela enajstih neodvisnih meritev*
Table 4. *Comparison of the estimated values of the Duffing's model parameters for eleven independent measurements*

Meritev Measurement	a	b	c
01	$1.067 \cdot 10^{-2}$	36.780	-444.702
02	$1.099 \cdot 10^{-2}$	36.787	-441.901
03	$0.964 \cdot 10^{-2}$	36.870	-403.251
04	$1.197 \cdot 10^{-2}$	36.540	-468.860
05	$0.931 \cdot 10^{-2}$	36.763	-451.457
06	$1.036 \cdot 10^{-2}$	36.298	-415.578
07	$0.947 \cdot 10^{-2}$	36.196	-424.219
08	$1.486 \cdot 10^{-2}$	36.162	-424.179
09	$0.997 \cdot 10^{-2}$	36.057	-427.190
10	$0.857 \cdot 10^{-2}$	35.980	-423.418
11	$1.170 \cdot 10^{-2}$	36.136	-415.392
povprečje average	$1.068 \cdot 10^{-2}$	36.415	-430.923
std. deviacija std. deviation	$0.172 \cdot 10^{-2}$	0.337	18.906



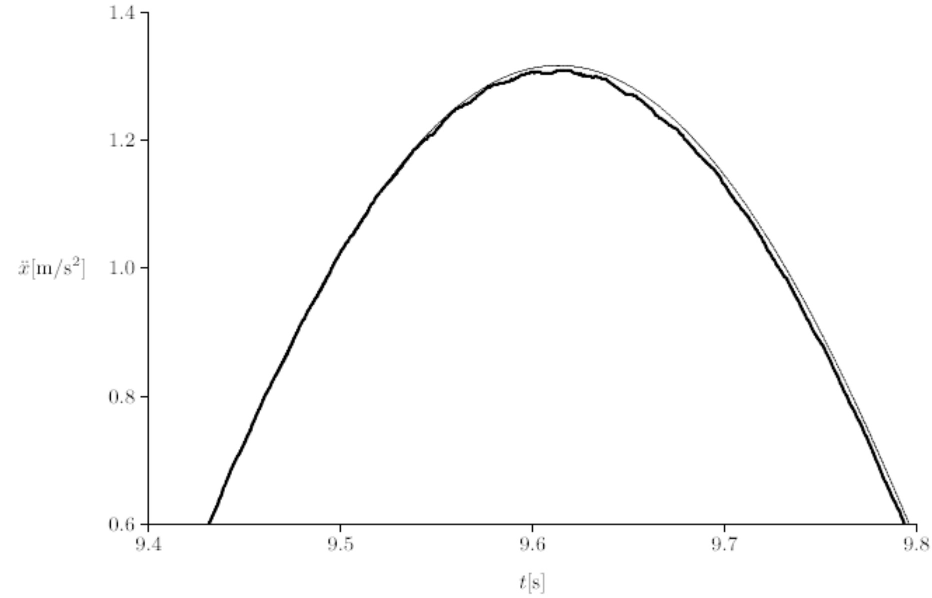
Sl. 8. *Merjeni pospešek (debela črta) in simulirana odziva Duffingovega in linearnega modela, za oznake glej preglednico 5. Detajl 32. pozitivne amplitude.*
Fig. 8. *Measured acceleration (thick line) and simulated responses of the Duffing's and the linear model, for labels see Table 5. Details of the 32nd positive amplitude.*

razlike pokažejo zunaj območja identifikacije in zato spadajo v področje napovedi obnašanja dinamičnih sistemov. Naš namen pa ni poiskati najprimernejši model eksperimentalne naprave, ampak predstaviti metodo parametrične identifikacije.

with the time. Let us stress here that these differences appeared outside the identification interval. Hence, this is a subject of the prediction rather then the identification. Our aim is to present the method of parametric identification and not to derive the most adequate model of the experimental device.

Preglednica 5. *Primerjava ocen parametrov Duffingovega in linearnega modela*
Table 5. *Comparison of the estimated values of the Duffing's and linear models*

Model Model	Krivulja na sliki 8 Curve in figure 8	a	b	c
Linearni Linear	ⓐ	$1.116 \cdot 10^{-2}$	36.012	
Duffingov Duffing's	ⓑ	$1.067 \cdot 10^{-2}$	36.780	-444.702



Sl. 9. *Merjeni pospešek (debela črta) in simulirana odziva Duffingovega modela (tanka črta). Detajl 10. pozitivne amplitude.*
Fig. 9. *Measured acceleration (thick line) and simulated responses of the Duffing's model (thin line). Details of the 10th positive amplitude.*

3.4 Identifikacija parametrov - grabljice v vodi

V vodo smo pomočili le grabljice, ki so pritrjene na vztrajnostno maso. S tem smo povečali raztros energije sistema.

Meritve v vodi smo izvedli enako kakor pri identifikaciji parametrov v zraku. Tudi spreminjanja parametrov identifikacije (frekvenca vzorčenja, dolžina in korak časovnega okna) postrežejo s podobnimi ugotovitvami kakor pri identifikaciji parametrov v zraku.

Parametre Duffingovega modela smo identificirali z najboljšo mogočo kombinacijo frekvence vzorčenja (1000 Hz), dolžine časovnega okna (1 nihaj) in koraka časovnega okna (1/1000 nihaja) na pospešku preskusne naprave in dobili naslednje vrednosti parametrov: $a = 3{,}599 \cdot 10^{-2}$, $b = 36{,}911$ in $c = -355{,}038$.

Na sliki 9 je prikazan detajl desete pozitivne amplitude. Merjeni pospešek je narisan z debelo črto, odziv Duffingovega modela pa s tanko črto. Na tej

3.4 Parameter identification: the rake in the water

Only the rake was partially submerged in the case of the identification of the parameters in the water. In this way the energy dissipation was increased.

The experiment in water was conducted in the same way as the experiment in the air. The variations of the identification parameters (the sampling rate, the length and the shift of the time window) gives similar results to the experiment in air.

The best possible combination of sampling rate (1000 Hz), time-window length (1 cycle) and time-window shift (1/1000 cycle) applied to the identification procedure on the measured acceleration of the experimental set-up yield results for Duffing's model parameters of $a = 3.599 \cdot 10^{-2}$, $b = 36.911$ and $c = -355.038$.

In Fig. 9 a detail of the tenth positive amplitude is shown. The measured acceleration is drawn with a thick line and the acceleration response of Duffing's

Preglednica 6. *Primerjava ocen parametrov Duffingovega modela pri različnih postopkih identifikacije*
Table 6. *Comparison of the estimated values of the Duffing's model for different approaches of the identification*

Analiza Analysis	a	b	c
Statični test Static test		39.610	-1253.427
Logaritmski dekrement Logarithmic decrement	$1.094 \cdot 10^{-2}$		
Identifikacija parametrov v zraku Parameter identification in the air	$1.067 \cdot 10^{-2}$	36.780	-444.702
Identifikacija parametrov v vodi Parameter identification in the water	$3.599 \cdot 10^{-2}$	36.911	-355.038

sliki lahko vidimo odlično ujemanje merjenega in simuliranega odziva, kar govori v prid predpostavljeni linearni upornosti tekočine.

4 ANALIZA REZULTATOV

Vrednosti identificiranih parametrov Duffingovega modela, dobljene z različnimi postopki, so zbrane v preglednici 6. Posebej se bomo osredotočili na primerjavi med ločenimi testi in identifikacijo parametrov v zraku ter med identifikacijo parametrov v zraku in v vodi.

4.1 Primerjava med ločenimi testi in identifikacijo parametrov v zraku

Primerjavo med parametri Duffingovega modela najdemo v preglednici 6. Vidimo lahko, da se najmanj razlikuje vrednost parametra *a* (-2,5 %). Obe vrednosti parametra *a* smo določili iz merjenega odziva sistema, torej dinamično. Nekaj večjo razliko med vrednostima parametrov najdemo pri parametru b (-7,7 %). Ocenili smo negativno vrednost parametra *c*, kar je konsistentno in popisuje degresivno karakteristiko vzmeti. Tako velika razlika, kakršno opazimo pri parametru *c*, je posledica majhne občutljivost Duffingovega modela na ta parameter in tudi s frekvenco povezanih vplivov pri dinamičnem testiranju, ki jih pri statičnem testu ni.

Primerjavo med merjenim pospeškom, simuliranim pospeškom, dobljenim na podlagi identificiranih parametrov Duffingovega modela, in simuliranim pospeškom, dobljenim na temelju parametrov Duffingovega modela, ocenjenih z ločenimi testi, prikazuje slika 10. Povsem jasno lahko vidimo, da ni bistvene razlike med merjenim odzivom

model is drawn with a thin line. In Fig. 9 we can see that there is very good agreement between the measured and the simulated responses, which is in favour of the asumption of the linearity of the fluid resistance.

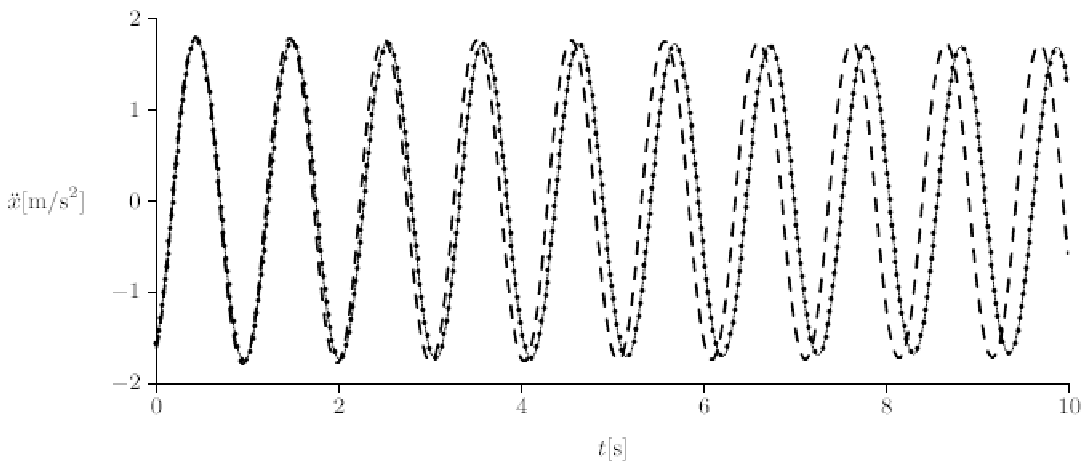
4 ANALYSES OF THE RESULTS

The identified values of the parameters of the Duffing's model, obtained by different approaches, are presented in Table 6. The focus was given to a comparison between the separate tests and the parameter identification in the air as well as between the parameter identification in the air and in the water.

4.1 Comparison between the separate tests and the parameter identification in the air

The comparison of the parameters of Duffing's model is shown in Table 6. We can see that the values of parameter *a* differ the least (-2.5 %). Both values of parameter *a* were estimated from a measured system response, i.e. dynamically. A somewhat larger difference in values can be found with the parameter *b* (-7.7 %). The parameter c is identified to be negative, which is consistent and describes the degressive spring characteristic. Such a big difference as seen with parameter *c* is due to the lower sensitivity of Duffing's model to that particular parameter and due to the frequency-dependent effects during dynamical testing, which are not present during static testing.

The comparison of the measured acceleration, the simulated acceleration based on identified parameters of Duffing's model and the simulated acceleration based on the separate tests of the parameters of Duffing's model are shown in Fig. 10. We can see clearly that there is no major difference between the measured response and the response



Sl. 10. *Primerjava med merjenim pospeškom (——) z odzivom Duffingovega modela, dobljenega na podlagi ocenjenih parametrov, dobljenih z identifikacijo v zraku (·· ····), in parametrov, dobljenih z ločenimi testi (– – –).*
Fig. 10. *The comparison of the measured acceleration (——) with the Duffing's model response based on identified parameters from the approach in this paper (·· ····) and based on the static test and logarithmic decrement (– – –).*

in z odzivom Duffingovega modela z identificiranimi parametri. Razlika pa je očitna med merjenim odzivom in odzivom Duffingovega modela, dobljenim s parametri in ocenjenimi z ločenimi testi.

4.2 Primerjava med identifikacijo parametrov v zraku in v vodi

Primerjavo med parametri Duffingovega modela najdemo v preglednici 6. Vidimo lahko, da se najmanj razlikuje vrednost parametra *b* (0,4 %). Večjo razliko najdemo pri parametru *c* (-25,3 %). Razliko lahko pripišemo majhni občutljivosti Duffingovega modela na ta parameter. Opazimo lahko, da je vrednost parametra *a* več ko trikrat večja v vodi kakor v zraku.

5 SKLEPI

V prispevku predstavljamo postopek parametrske identifikacije mehanskih sistemov z eno prostostno stopnjo na podlagi merjenega pospeška. Za prikaz metode smo uporabili Duffingov sistem. Postopek sledi zamisli, da je mogoče izračunati parametre diferencialne enačbe gibanja, ki jo lahko predstavimo kot algebrsko enačbo, če štejemo parametre za neznanke.

Eksperimentalno delo smo razdelili na tri sklope. V prvem delu smo statično določili vzmetno karakteristiko, v drugem pa smo določili vzmetno karakteristiko dinamično z uporabo predstavljene metode. Ekvivalentno viskozno dušenje, ki ga popisuje parameter *a*, smo prav tako določili na dva načina: najprej z uporabo logaritemskega dekrementa in v drugem delu z našim postopkom. Pokazali smo, da je težko razlikovati med merjenim odzivom in odzivom, dobljenim z uporabo našega postopka. Na drugi strani pa se odziv, dobljen na podlagi ločenih testov, vidno razlikuje od

based on the identification procedure. The difference between the measured response and the response based on parameters that have been determined by separate tests is clearly visible in Fig. 10.

4.2 Comparison between the parameter identification in the air and in the water

The comparison of the identified parameters of Duffing's model are shown in Table 6. We can see that the values of parameter *b* differ the least (0.4 %). A larger difference in values can be found for parameter *c* (-25.3 %). Such a difference, as seen with parameter *c*, is due to the lower sensitivity of Duffing's model to that particular parameter. We can also see that the value of parameter *a* increases three times in the water in comparison to the air.

5 CONCLUSIONS

In this paper an approach to parameter identification for the s.d.o.f mechanical system based on measured acceleration is presented. Duffing's system was taken into consideration. The approach follows the idea of computing the parameters of the differential equation of motion, which can be represented as an algebraic equation if the parameters are considered to be unknowns.

The experimental work was divided into three parts. In the first part the spring characteristic was determined statically and in the second part the spring characteristic was determined dynamically using our approach. The equivalent viscous damping described by parameter *a* was also estimated by two different methods, firstly by logarithmic decrement and secondly by our approach. It was shown that it is difficult to draw a distinction between the measured response and the response gained by our approach and that the response simulated on the basis of the static test and the logarithmic decrement significantly differs from the measured time history. In the third part the merjenega. V tretjem delu preskusa smo dodali vodo in tako povečali raztros energije. Pokazali smo, da dobimo dobro ujemanje parametrov b in c pri identifikaciji v zraku in v vodi.

Rezultati kažejo, da predstavljena metoda identifikacije parametrov omogoča kakovostno oceno parametrov na kratkih časovnih vrstah (nekaj nihajev). Potrebuje tudi razmeroma skromno število točk in je uspešna na različnih sistemih z eno prostostno stopnjo. Omogoča preprosto določevanje parametrov in začetnih pogojev iz merjenega pospeška pri lastnem nihanju sistema. Metoda je neobčutljiva na začetne pogoje in poznati moramo le tip gibalne enačbe.

water is added and thus the energy dissipation of the system increased. It was shown that there is a good agreement with the identification in the air concerning the parameters b and c.

The results show that our approach offers parameter identification with good quality for short time series (a few cycles) using only a modest number of data points for a wide range of s.d.o.f. systems. If offers easy ways for computing the parameters and initial conditions from the free-acceleration-response data of the s.d.o.f. system. The approach is insensitive to initial conditions and only the type of equation of motion needs to be known.

6 LITERATURA
6 REFERENCES

[1] Badrakhan, F. (1985) Separation and determination of combined dampings from free vibrations. *Journal of Sound and Vibration*, 100(2):243–255.

[2] Natke, H.G. and M. Zamirowski (1990) On methods of structure identification for the class of polynomials within mechanical systems. *ZAMM - Z. angew. Math. Mech*., 70(10):415–420.

[3] Ying, C.-M. and B. Joseph (1999) Identification of stable linear systems using polynomial kernels. *Industrial and Engineering Chemistry Research,* 38:4712–4728.

[4] Sekavčnik, M. (2000) Analysis of turbocharger impeller vibrations. *Journal of Mechanical Engineering - Strojniški vestnik,* 46(11/12):750–761.

[5] Chen, Q. and G.R. Tomlinson (1996) Parametric identification of systems with dry friction and nonlinear stiffness using a time series model. *Transactions of the ASME - Journal of Vibration and Acoustic,* 118:252–263.

[6] Boukhrist, A., G. Mourot, and J. Ragot (1999) Non-linear dynamic system identification: a multi-model approach. *International Journal of Control*, 72(7/8):591–604.

[7] Moustafa, K.A.F., and H.E. Emara-Shabaik (2000) Recursive parameter identification of a class of nonlinear systems from noisy measurements. *Journal of Acoustic and Control*, 6:49–60.

[8] Rice, H.J. (1995) Identification of weakly non-linear systems using equivalent linearization. *Journal of Sound and Vibration*, 185(3):473–481.

[9] Lee, G.-M. (1997) Estimation of non-linear system parameters using higher-order frequency response function. *Mechanical Systems and Signal Processing*, 11(2):219–228.

[10] Feldman, M. (1994) Non-linear system vibration analysis using Hilbert transform - i. free vibration analysis method 'freevib'. *Mechanical Systems and Signal Processing*, 8(2):119–127.

[11] Feldman, M. (1994) Non-linear system vibration analysis using Hilbert transform - ii. forced vibration analysis method 'forcevib'. *Mechanical Systems and Signal Processing*, 8(3):309–318.

[12] Staszewski, W.J. (1998) Identification of non-linear systems using multi-scale ridges and skeletons of the wavelet transform. *Journal of Sound and Vibration*, 214(4):639–658.

[13] Masri, S.F., A.G. Chassiakos, and T.K. Caughey (1993) Identification of nonlinear dynamic system using neural networks. *Transactions of the ASME - Journal of Applied Mechanics*, 60:123–133.

[14] Chassiakos, A.G. and S.F. Masri (1996) Identification of structural systems by neural network. *Mathematic and Computers in Simulation*, 40:637–656.

[15] Yadaiah, N., N. Sivakumar, and B.L. Deekshatulu (2000) Parameter identification via neural networks with fast convergence. *Mathematics and Computer in Simulation*, 51:157–167.

[16] Yar, M.and J.K. Hammond (1987) Parameter estimation for hysteretic systems. *Journal of Sound and Vibration,* 117(1):161–172.

[17] Ni, Y.Q., J.M. Ko, and C.W. Wong (1998) Identification of non-linear hysteretic isolators from periodic vibration tests. *Journal of Sound and Vibration*, 217(4):737–756.

[18] Chassiakos, A.G., S.F. Masri, A.W. Smyth, and T.K. Caughey (1998) On-line identification of hysteretic systems. *Transactions of the ASME - Journal of Applied Mechanics*, 65:194–2038.

[19] Smyth, A.W., S.F. Masri, A.G. Chassiakos, and T.K. Caughey (1999) On-line parametric identification of mdof nonlinear hysteretic systems. *Journal of Engineering Mechanics - ASCE*, 125(2):133–142.

[20] Jakšić, N., M. Boltežar, and A. Kuhelj (2000) Parameter identification of a single degree of freedom dynamical system based on phase space variables. In N.S. Ferguson, H.F. Wolfe, M.A. Ferman, and S.A. Rizzi, editors, Seventh International Conference on Recent Advances in Structural Dynamics, volume 2, pages 951–964, Southampton: University of Southampton, 24-27 July 2000*., Institute of Sound and Vibration Research.*

[21] Jakšić, N., M. Boltežar, and A. Kuhelj (2000) Identifikacija parametrov lastnega nihanja mehanskih sistemov z eno prostostno stopnjo. In L. Škerget, editor, *Slovensko društvo za mehaniko*, Kuhljevi dnevi 2000, Zbornik del, strani 199–206, Maribor, 21.-22. September 2000. Slovensko društvo za mehaniko. (Parameter identification of natural vibrations of s.d.o.f. systems).

[22] Jakšić, N., M. Boltežar, and A. Kuhelj (2001) Parameter identification of a single-degree-of-freedom system with spline interpolation and approximation. *Z. angew. Math. Mech*., 81(suppl. 4):S895-S896.

[23] Jakšić, N. and M. Boltežar (2001) An approach to parameter identification for the s.d.o.f. dynamical system based on short free acceleration response. *Journal of Sound and Vibration*, 250(3):465-483.

[24] Jakšić, N. (2002) Identifikacija parametrov avtonomnih dinamičnih sistemov drugega reda. Doktorska teza (Parameter identification of the second-order autonomous dynamical system. PhD thesis) *Univerza v Ljubljani, Fakulteta za strojništvo*.

[25] Thompson, J.M.T. and H.B. Stewart (1986) Nonlinear dynamics nad chaos. *John Wiley and Sons Ltd*., 6th reprint edition.

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