Original scientific paper

Received: June 16, 2014
Accepted: January 30, 2015

Fragility curves of RC columns estimated by the CAE method

Ocena krivulj ranljivosti AB-stebrov s CAE-metodo

Klemen Sinkovic, Iztok Perus*
University of Ljubljana, Faculty of Civil and Geodetic Engineering, Jamova cesta 2, 1000 Ljubljana, Slovenia
*Corresponding author. E-mail: iperus@siol.net

Abstract

In the paper an alternative empirical approach for the estimation of fragility curves for RC columns is proposed. The CAE (Conditional Average Estimator) method was used. The procedure includes the LHS method, which was applied in order to take into account different uncertainties. The result of the study are estimated fragility curves for typical reinforced concrete (RC) columns (1) designed without seismic detailing, (2) designed according to first seismic codes used in the former Yugoslavia and (3) designed according to Eurocode 8. The obtained results clearly reveal the higher deformation capacity of RC columns, designed according to Eurocode 8. Additionally, the fragility curves related to various damage states, i.e. concrete crushing, longitudinal bar buckling, and longitudinal bar fracture were estimated. It is concluded that the proposed procedure offers a viable alternative to existing approaches.

Key words: CAE method, drift, fragility, RC columns, performance-based earthquake engineering (PBEE)

Izvleček

V prispevku je predlagan alternativni empirični način za oceno krivulj ranljivosti za armiranobetonske (AB) stebre. Uporabljena je bila CAE-metoda s cenilko pogojnega povprečja. Postopek vključuje tudi LHS-metodo, ki je bila uporabljena z namenom upoštevanja različnih negotovosti. Rezultati študije so ocene krivulj ranljivosti tipičnih AB-stebrov treh različnih obdobij, in sicer za (1) AB-stebre, projektirane brez potresnih predpisov, (2) AB-stebre, projektirane v skladu s prvimi potresnimi predpisi v nekdanji Jugoslaviji, in (3) AB-stebre, projektirane v skladu s predpisi Evrokod 8. Dobljeni rezultati jasno kažejo na večjo deformacijsko kapaciteto stebrov, projektiranih po predpisih Evrokod 8. Dodatno so bile krivulje ranljivosti ocenjene tudi za druga stanja poškodovanosti, kot je drobljenje betona, lokalni uklon vzdolžne armature in zlom vzdolžne armature. Mogoče je skleniti, da predlagani postopek ponuja dobro alternativo sedanjim načinom.

Ključne besede: CAE-metoda, zamik, ranljivost, AB-stebri, potresno inženirstvo

Introduction

In contrast to many of the existing models for the prediction of the deformation capacity of RC columns, which provide only deterministic (point) estimations, in the performance-based design of seismic resistant buildings predictive capacity models that are unbiased and explicitly account for all the uncertainties are needed. Only in probabilistic models of capacity is variability explicitly taken into account[1]. The aim of the paper is thus to propose an alternative method for the estimation of fragility curves for RC columns, which is based on the CAE probabilistic drift capacity model.

As well as in performance-based seismic design, fragility curves are very important for the estimation of the overall risk to the civil infrastructure in earthquake-prone areas. They can be used for emergency response and disaster planning by local and national authorities. Insurance companies can use them to make assessments of potential losses due to a particular scenario earthquake. Due to their numerous possible applications, a number of different research projects have been carried out worldwide. For example, Singhal and Kiremidjian[2] presented vulnerability curves and damage probability matrices for low-, mid- and high-rise RC frame structures, using Monte Carlo simulation techniques and non-linear dynamic analyses. Dumova-Jovanoska[3] presented damage probability matrices and damage indices as functions of intensity for different damage states for selected RC frame and RC wall-frame structures. Kappos et al.[4] applied a hybrid approach for the development of vulnerability curves for reinforced concrete and unreinforced masonry structures in terms of peak ground acceleration and spectral displacement. Panagiotakos and Fardis[5] evaluated the performance of generic archetypal RC buildings according to Eurocode 8, using non-linear analyses. This study was later upgraded by the development of corresponding fragility functions[6]. Akkar et al[7] estimated vulnerability curves for low- and mid-rise infilled frame RC buildings. Push-over analyses of a number of existing buildings in Duzce were performed for buildings with a low-level of seismic design, having between two and five storeys. Good agreement of the estimated vulnerability curves with observed damage after the 1999 Duzce earthquake was observed. Rossetto & Elnashai[8] produced vulnerability curves for low-rise infilled RC frames, designed on the basis of the old Italian seismic code. The proposed analytical curves are in reasonable agreement with the empirical curves. There are several other applications. A more detailed survey of works related to fragility can be found in[6], and on the probabilistic approach to capacity models in[9].

Due to the complexity of the problem, fragility studies generally focus on generic types of structures[6]. Consequently, simplified structural models, having properties that account for the uncertainties and randomness in the structural parameters, are used to mathematically represent real buildings. The fragility curves are then presented as a function of different intensity measures, *IM*, (e.g. *PGA*, *PGV*, S_a, S_d) for different types of structures, different numbers of storeys, etc., taking into account different damage states (e.g. yielding, collapse). Fragility curves can be presented for buildings as a whole or for any of their structural elements. In the latter case, demand can also be expressed in terms of deformation, not only in terms of *IM*. This is because the fragility of a structural member is defined as the conditional probability of failure for given values of the selected demand parameter. Hence, fragility curves can be obtained by simulations of the seismic response of structures at varying demand thresholds. Moreover, by ignoring the uncertainty in the demand, each fragility curve can display the probability that the capacity is less than or equal to a particular displacement/drift demand[1].

Empirical, expert opinion based, analytical and hybrid methods can be used for the assessment of fragility curves. In this paper an empirical approach to the estimation of the fragility curves of RC columns using the CAE method is presented. For example, the fragility of a RC column can be defined as the conditional probability of failure (at an ultimate drift defined at a 20 % drop in maximum strength) for given values of drift demand. Additionally, fragility can be derived in terms of other damage states, e.g. concrete crushing, longitudinal bar buckling, longitudinal bar fracture and axial loss

failure. In this paper the CAE method is briefly presented. The different databases used in the study are described. The obtained results are verified by making comparisons with existing solutions from the literature. Fragility curves for typical RC columns, designed without seismic detailing, designed according to the first seismic codes used in the former Yugoslavia, and designed according to EC8, are provided as the results of the study.

Procedure for the estimation of fragility curves

The CAE method

The fragility curves presented in this study are estimated by the CAE method. A detailed description of this method from the engineering point of view is given in Peruš et al.[10]. Here only a brief description is given.

The phenomenon of the capacity of RC columns and then the estimation of the corresponding fragility curves can be described by observing *N* RC column specimens during the experiments. The mathematical description of the observation of one specimen during the experiment is called a model vector. As a result, the whole phenomenon can be described by a finite set of model vectors.

$$\{\boldsymbol{X_1}, ..., \boldsymbol{X_n}, ..., \boldsymbol{X_N}\} \tag{1}$$

It is assumed that the observation of one particular specimen can be described by a number of variables, which are treated as components of a model vector

$$\boldsymbol{X_n} = \{b_{n1}, ..., b_{nl}, ...,b_{nD}, c_{n1}, ..., c_{nk}, ...,c_{nM}\} \tag{2}$$

The vector $\boldsymbol{X_n}$ can be further composed of two truncated vectors $\boldsymbol{B}$ and $\boldsymbol{C}$

$$\begin{aligned}&\boldsymbol{B_n} = \{b_{n1}, ..., b_{nl}, ...,b_{nD}\} \text{ and}\\ &\boldsymbol{C_n} = \{c_{nl}, ..., c_{nk}, ...,c_{nM}\}\end{aligned} \tag{3a}$$

Vector $\boldsymbol{B_n}$ is complementary to vector $\boldsymbol{C_n}$ and therefore their concatenation yields the complete data model, vector $\boldsymbol{X_n}$. The prediction vector, too, is composed of two truncated vectors, i.e., the given truncated vector $\boldsymbol{B}$ and the unknown complementary vector $\hat{\boldsymbol{C}}$

$$\begin{aligned}&\boldsymbol{B} = \{b_1, ..., b_l, ...,b_D\} \text{ and}\\ &\hat{\boldsymbol{C}} = \{\hat{c}_1, ..., \hat{c}_k, ...,\hat{c}_M\}\end{aligned} \tag{3b}$$

The problem now is how an unknown complementary vector $\hat{\boldsymbol{C}}$ can be estimated from a given truncated vector $\boldsymbol{B}$ and the model vectors $\{\boldsymbol{X_1}, ..., \boldsymbol{X_n}, ..., \boldsymbol{X_N}\}$, i.e., how the drift capacity δ can be estimated from known input parameters and the available data in the database. By using the conditional probability density function, the optimal estimator for the given problem can be expressed as

$$\hat{\delta} = \sum_{n=1}^{N} \delta_n A_n \tag{4}$$

where

$$A_n = \frac{a_n}{\sum_{i=1}^{N} a_i} \tag{5}$$

and

$$a_n = \frac{1}{(2\pi)^{D/2} w_n^D} \exp\left[-\sum_{l=1}^{D} \frac{(b_l - b_{nl})^2}{2w_n^2}\right] \tag{6}$$

Note that in the above equations *M* is assumed to be 1 and consequently $\hat{c}_1 = \hat{\delta}$.

In equations (4–6) δ is an estimate of the displacement/drift at failure or any other damage state (e.g. flexural and shear failure, defined at a 20 % reduction in lateral strength, axial failure, bar buckling, etc.), δ_n is the same output variable corresponding to the *n*-th model vector in the database, *N* is the number of model vectors in the database, b_{nl} is the *l*-th input variable (e.g. axial load index, P^*, shear span index, L^*) of the *n*-th model vector in the database, and b_l is the *l*-th input variable corresponding to the prediction vector. Note that each model vector corresponds to the results of one experiment from the database. *D* is the number of input variables, and defines the dimension of the sample space. A Gaussian function is used in order to achieve a smooth interpolation between the points of the model vectors. In this context the width w_n is called the "smoothing" parameter that corresponds to *n*-th model vector from the

database. In our case the same width w_n of the Gaussian function is used for all the input variables. It is therefore important that the input parameters in the equation for a_n are normalized, i.e. generally in the range from 0 to 1.

An intermediate result in the computational process is parameter $\hat{\rho}$, which is defined as:

$$\hat{\rho} = \frac{1}{N}\sum_{n=1}^{N} a_n \tag{7}$$

It provides a measure of how the influence of all the model vectors in the database is spread over the sample space, and strongly depends on the smoothing parameter w. It helps to detect possible less accurate predictions (indicated by low values of $\hat{\rho}$) due to the manner of data distribution in the database, and due to local extrapolation outside the data range.

When the expression for the displacement/ drift $\hat{\delta}$ at failure (Equation 4) is compared with the expression for the first order moment of the random variable X, which corresponds to the mean value m_x

$$E[X] = \sum_{i=1}^{N} x_i p_x(x_i) = m_x \tag{8}$$

the similarity between the two expressions becomes evident[11]. $p_x(x_i)$ is the probability of the random variable $X = x_i$ and corresponds to the weights A_n (Equation 5), which depend on the similarity between the input variables of the prediction vector, and on the corresponding input variables pertinent to the model vectors stored in the database. Also, there is clear similarity when the central second order moment of the probability distribution of the random variable X, called its variance, given by the expression

$$\mu_x = Var[X] = \sum_{i=1}^{N} (x_i - m_x)^2 p_x(x_i) \tag{9}$$

is compared with the prediction of so-called "local standard deviation" in the CAE method:

$$\hat{E}_\sigma^2 = \sum_{n=1}^{N} (\delta_n - \hat{\delta})^2 A_n \tag{10}$$

The above interpretation of the CAE equations enables the estimation of the corresponding probability distribution, including the median value, as well as different percentile values. The proposed procedure is demonstrated by the simple example presented in Appendix A.

The LHS method

The LHS (Latin hypercube sampling) method was first described by Mackay et al.[12] and then further elaborated by Iman et al.[13]. The method, which has become increasingly popular, was used in this study in order to take into account different uncertainties (e.g. aleatory and epistemic). Aleatory uncertainties, which are inherent in the phenomenon itself, are irreducible, and cannot be influenced by the observer or the manner of observation. On the other hand, epistemic uncertainties arise from errors in measurement, from the finite size of the available observation data, and from the adopted mathematical model. Note that in the presented study no distinction was made between these two types of uncertainties.

LHS allows the creation of experimental samples with as many points as needed or desired[14]. (Note, however, that the number of samples needed should not be too low[15].) It permits the use of very different statistical assumptions, and is able to treat both small and large design spaces (there are no constraints in terms of the data density and location). Additionally, LHS is flexible. If, for example, a few dimensions have to be dropped out, the resulting design is still a Latin hypercube. Moreover, the existing data can be reused without having to make any reduction in the number of sampled points.

The use of LHS in its basic form does not account for any possible correlations between the variables. However, in earthquake engineering, some of the parameters of the RC structural elements are correlated (e.g. the yield strength of the reinforcement, and the concrete compressive strength), so some authors have made us of more advanced variants of LHS (e.g.[15–17]). In the presented study, however, derived input parameters were used, and since it was assumed that they are independent, a basic variant of LHS was used. This is well-justified since parametric studies (not presented here) have shown that the influence of derived input parameters has only a minor effect on the results presented in this study.

The databases used in the study

The application of the CAE method requires a representative database, many different databases of RC structural elements being presently available (e.g.[1, 10, 18–29]). The deformation capacity expressed in terms of ultimate drift, representing a "near collapse" limit state, which is used in this study, is defined as a drift at a predefined drop below maximum strength. A 20 % drop in maximum strength (i.e. when the restoring force reaches 80 % of its maximum value) is commonly used, although this definition may significantly underestimate the true axial load-carrying capacity of the columns. In other cases, when lateral force resistance in columns is not reached because of premature load reversal, and also in cases, when deformation capacity is not reached because of limitation in the applied maximum displacement, the measured maximum drifts provided a lower bound of the deformation capacity.

For validation and comparison purposes, the first database used in the study was the database on RC columns at axial failure[1]. In this paper it is called the DB1 database, and contains data on 28 RC column specimens. Only two input parameters are considered, namely the normalized axial load and the parameter *s/d*, which is related to the confinement (*s* is the hoop spacing, and *d* is the depth to the centre-line of the outermost tension reinforcement).

For the estimation of fragility curves for RC columns which failed in flexure, the PEER database was used[18]. The same input parameters as proposed by Perus et al.[10] were taken into account (i.e. an axial load index, a shear span index, the concrete compressive strength, the confinement effectiveness factor multiplied by the confinement index and the longitudinal reinforcement index). The effective database was called DB2, and contains data on 156 RC column specimens. In order to develop fragility curves for other damage states, different from flexural or axial failure, an effective database, derived from the PEER database, and called the DB3 database, was used. It contains data on 80, 20 and 38 RC column specimens for which the drift at concrete crushing, longitudinal bar buckling, and longitudinal bar fracture, respectively, was measured.

Fragility curves of RC columns

Comparison with the existing solution

In this section the results obtained by using the proposed procedure are compared with the results of the existing solution. It is assumed that all the random variables which account for different uncertainties in the input parameters (e.g. the material properties and the applied loads) are log-normally distributed.

Fragility curves for axial failure were estimated, using the database DB1. The following two parameters and corresponding coefficients of variation were taken into account:

— the axial load index ($P^* = P/P_o$; [0.07–0.22]): $CoV = 0.11$;
— the parameter *s/d*, related to the confinement [0.6–1.23]: $CoV = 0.02$

Note that the bounds which are used in the normalization process of the CAE method (see[10]) take into account the distribution of the two input parameters in the database DB1.

By means of the LHS method, a database with 500 samples for each of the columns (3CLH18 and 2CLD12) from Zhu et al.[1], taking into account different uncertainties, was prepared. The CAE method was then applied. Due to the small number of input parameters, the smoothing parameter was given values of $w_{min} = 0.05$ and $w_{max} = 0.1$.The drift point estimates (i.e. the mean and the local standard deviation) at axial failure were determined, using Eqs. 4–6 and 10, as a function of the axial load index and the parameter *s/d*. Considering Eqs. 4 and 8–10, the CAE empirical cumulative distribution function (CDF) was determined for each sample from the LHS generated database. The corresponding smoothed CDF for the median drift values, as well as for the 15 % and 85 % bounds, were then calculated. The estimate is based on the counted values obtained from the CAE empirical CDF.

The results presented in Figure 1 indicate relatively good agreement between the CAE approach and the approach proposed by Zhou et al.[1]. The discrepancy between the results can be attributed to the functional form, which is not *a priori* assumed in case of the CAE method. The CAE functional form follows the data more closely, which can, however, in some cases

(e.g. extrapolation) lead locally to illogical results. In the presented case, the ρ value (Equation 7) amounts to 0.6 and 1.2 for both of the RC sample columns, respectively. The relatively high values indicate accurate CAE predictions. The observed discrepancy between the results suggests that the uncertainties may be different (i.e. higher) to those usually estimated.

Fragility curves of flexural failure for different designs of RC columns

Fragility curves corresponding to flexural failure for different designs of RC columns (designed without seismic detailing - NO, designed according to first seismic codes used in the former Yugoslavia – YU, and designed according to Eurocode 8 – EC8) were estimated by CAE, using database DB2. Five input parameters were taken into account, with the following statistical parameters:

- axial load index ($P^* = P/P_o$): $CoV = 0.11$,
- shear span index ($L^* = L_V/h$): $CoV = 0.05$,
- concrete compressive strength ($f_c^{'}$): $CoV = 0.05$,
- confinement index ($\rho_s^* = \rho_s f_{ys}/f_c^{'}$): $CoV = 0.08$
- longitudinal reinforcement index ($\rho_l^* = \rho_l f_{yl}/f_c^{'}$): $CoV = 0.08$.

Fragility curves were estimated for $L^* = 3.5$ and three different values of P^*, which amounted to 0.05, 0.15 and 0.25, respectively. Additionally, in the case of the RC columns designed without seismic detailing, a fragility curve for $P^* = 0.35$ was estimated. (Here it should be noted RC columns in such older buildings are, in many cases, heavily loaded due to the applied vertical loads.) Mean values of other three input parameters were assessed from the collected data on past designs. Note that only data on columns in the first storey, which is usually the critical one, were taken into account. The mean values and corresponding coefficients of variations, which account for different uncertainties, are presented in Table 1.

The results which correspond to fragility curves with an 85 % confidence level are presented in Figure 2. A very small difference in the drift capacity between the RC columns designed without seismic detailing and RC columns designed according to YU codes can be observed. Table 1

Table 1: *Mean values and corresponding coefficients of variation which accounts for different uncertainties*

	NO	YU	EC8
ρ_s^*	0.002 / 0.51	0.002 / 0.51	0.05 / 0.22
ρ_l^*	0.15 / 0.41	0.15 / 0.22	0.2 / 0.22
$f_c^{'}$	20 / 0.21	25 / 0.21	30 / 0.21

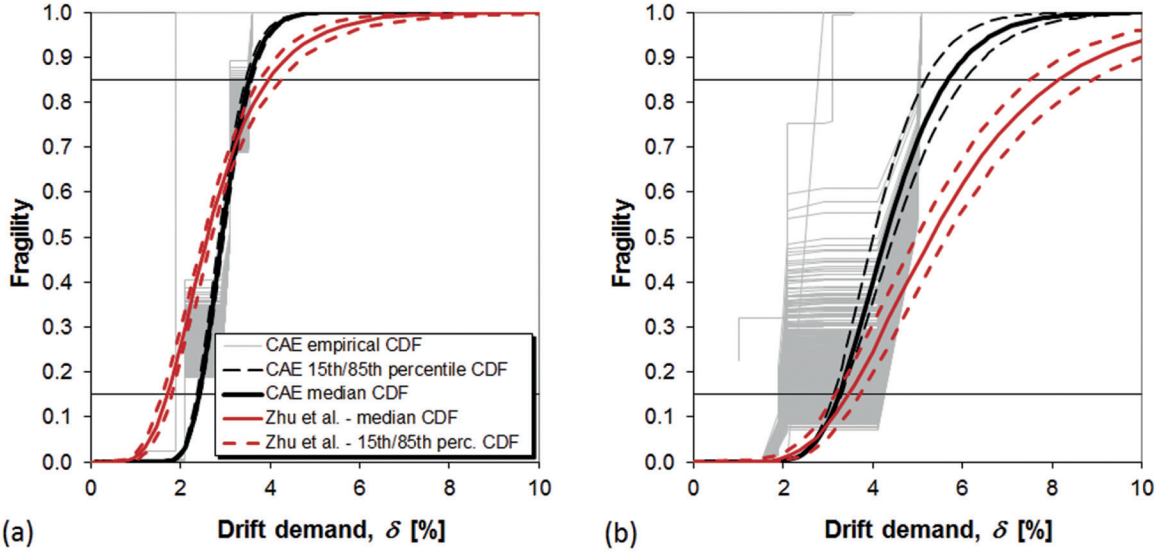
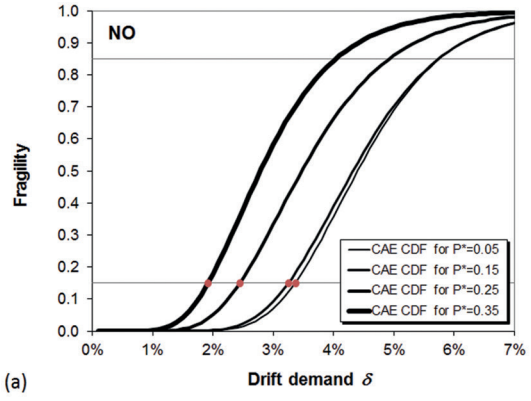
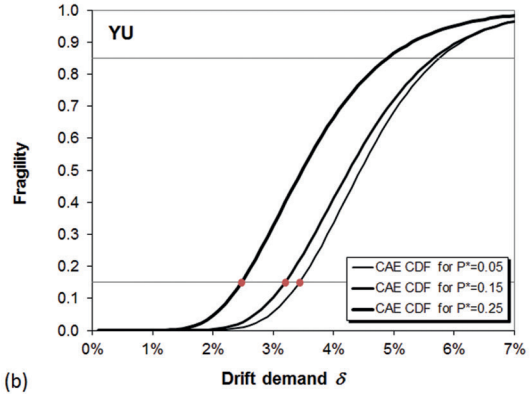


Figure 1: *Fragility estimates for RC columns at axial failure for (a) column 3CLH18 and (b) column 2CLD12[1]. Shown are discrete empirical CDF, obtained by the CAE method and by the corresponding smoothed (log-normal) CDF, and the same estimates, as obtained by Zhu[1].*

reveals, that the most notable change in the input parameters is the increase in concrete compressive strength, which only slightly improves the drift capacity and decreases dispersion. It should be noted, that despite of validation of YU codes for that particular time of being, the requirements for shear reinforcement were insufficient, which can be also observed from the data, taken from actual designs of buildings.





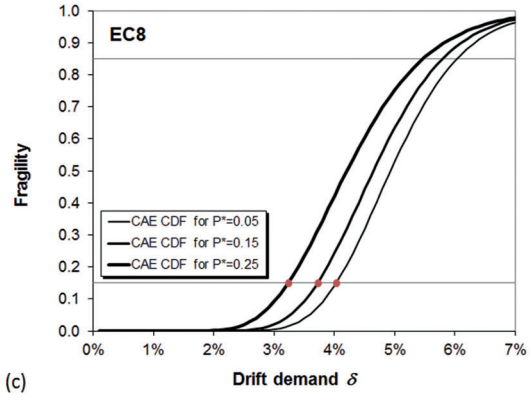


Figure 2: *Fragility curves with an 85 % confidence level for a typical column, designed (a) without seismic codes, (b) according to seismic codes used in the former Yugoslavia, and (c) according to the EC8 code. In all cases L^* amounts to* 3.5.

An important increase in drift capacity can be observed in the case of Eurocode 8, where all three parameters, especially the shear reinforcement which can significantly contribute to the drift capacity, are also increased. RC columns designed without seismic detailing exhibit a surprisingly high drift capacity. In the case of Eurocode 8 the ultimate drift capacity is reduced by a corresponding factor due to poor seismic detailing and the use of smoothed bars. The presented results suggest that a better CAE model should include additional parameters which would take into account proper seismic detailing.

Fragility curves of RC columns for different damage states

Besides the fragility curves corresponding to flexural and axial failure, fragility curves can also be estimated for other damage states, e.g. concrete crushing, longitudinal bar buckling and longitudinal bar fracture. The procedure is the same, only a proper database is required. In order to demonstrate the estimation of fragility curves of RC columns for other damage states, the database DB3 was used. Note that in this case uncertainties were not taken into account. The results are compared with the measured drift obtained in the case of the pseudo-dynamic testing of the 3-storey SPEAR building[30], which is representative building of old constructions in southern European Countries without specific provisions for earthquake resistance. It was designed for gravity loads alone, using the concrete design code applying in Greece between 1954 and 1995, with the construction practice and materials typical of the early 70s: for a concrete a nominal strength f_c' = 25 MPa was assumed while based on the scarcity of the current production, it was only possible to find reinforcement with a characteristics yield strength larger than initially requested f_y ≈ 450 MPa. The structure is regular in elevation since typical storey height is 3 m. The plan configuration is doubly non symmetric (Figure 3), with 2-bay frames spanning from 3 m to 6 m. Eight out of the nine columns have a square 25 cm × 25 cm cross-section (also the interior column C3), whereas the ninth column has a 25 cm × 75 cm cross section.

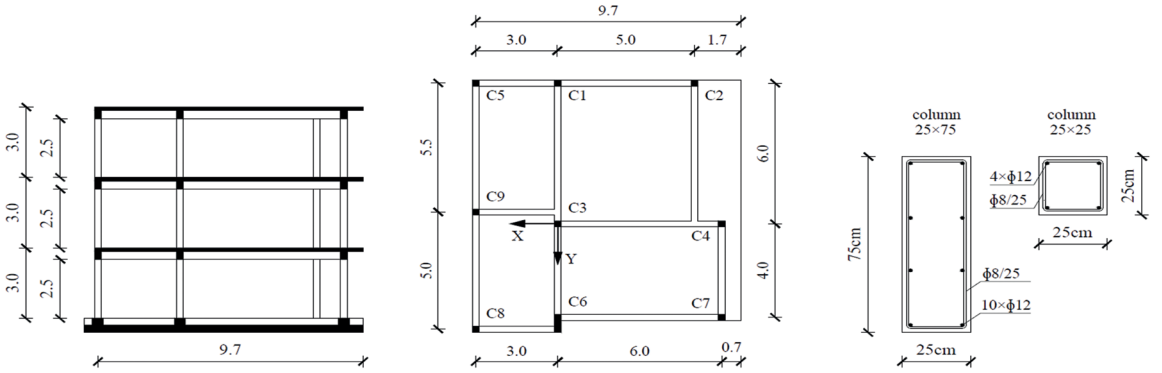


Figure 3: *The elevation view, the plan view and the typical reinforcement in RC columns of the SPEAR building.*

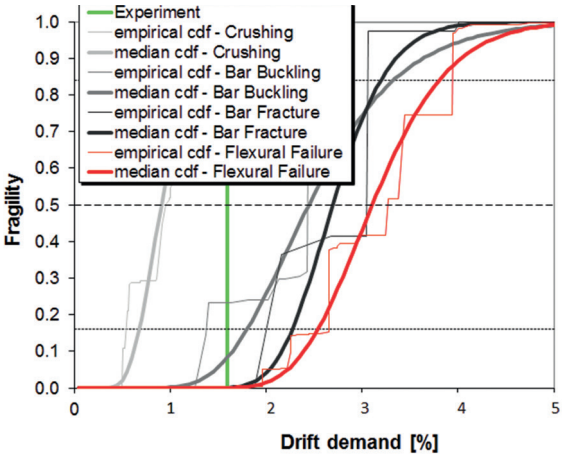


Figure 4: *Fragility curves at an* 85 % *confidence level for column C3 in the first storey of the SPEAR building. Considered are concrete crushing, longitudinal bar buckling, longitudinal bar fracture, and flexural failure of the column, respectively.*

Figure 4 reveals that concrete crushing should occur at early stages of the experiment. Figure 5 confirms the obtained result. Namely, extensive spalling and cracking was observed at the top of the column. The estimated smoothed CDF indicates that neither longitudinal bar buckling, nor longitudinal bar fracture or ultimate drift was achieved. Indeed, the ultimate drift was not achieved, and other heavy damage was not observed. At an 85 % confidence level, longitudinal bar buckling is expected at the increased drift (approx. 1.8 %). This may be followed by longitudinal bar fracture at 2.3 % drift. Flexural failure may be expected at a drift of 2.6 %.

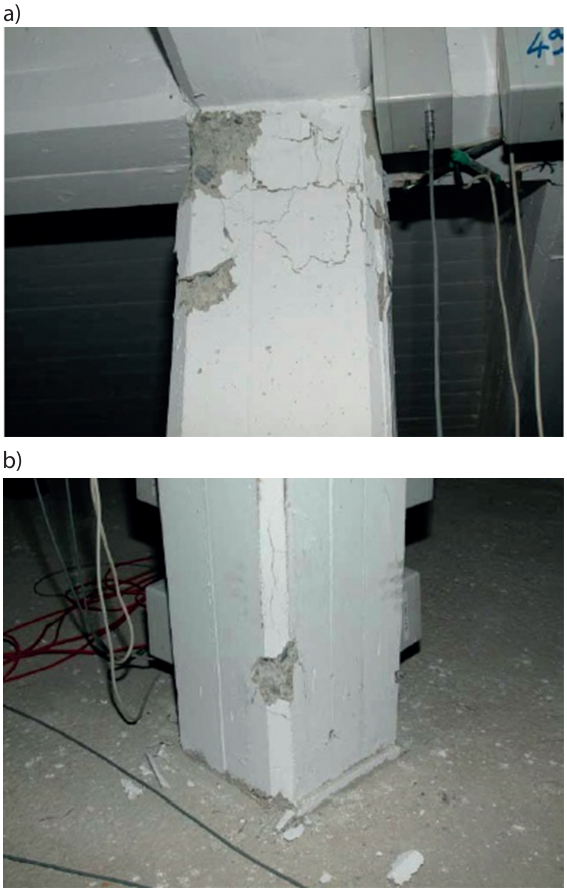


Figure 5: *Damage after the PGA* = 0.2 g *pseudo-dynamic testing of the SPEAR building*[30]*: (a) extensive spalling at the top of the column, and (b) minor damage at the base of the column.*

Conclusions

In the paper an empirical approach to the estimation of fragility curves of RC columns using the CAE method is presented. The LHS method was applied in order to take into account different uncertainties.

As a result of the study fragility curves at an 85 % confidence level for typical RC columns, designed without seismic detailing, according to first seismic codes in the former Yugoslavia, and according to Eurocode 8 were proposed. The resulting increasing trend can be observed, while a small difference in the drift capacity was observed between the RC columns designed without seismic detailing and those designed according to the YU codes. Both types of columns were designed with relatively bad detailing for shear reinforcement and usually with smooth longitudinal reinforcement, which have the biggest role in case of EC8 codes and provide favourable flexural behaviour with adequate deformation capacity of RC columns. One of the possible reasons could be also the fact that the discussed existing buildings are not fully representative for buildings built before and after YU regulations. The other reason can be imperfect CAE model. Those differences were not fully discussed since the primary purpose of the article was to propose a relatively simple method for the determination of the fragility curves. On the other hand an important increase in drift capacity was observed in the case of EC8 codes, especially due to the increase and better detailing of shear reinforcement and also due to better detailing of longitudinal reinforcement with better material characteristics, which typically significantly contributes to the higher deformation capacity of RC columns.

Further research is still needed in order to increase the sample size of existing buildings designed according to YU codes, and to improve the presented CAE model. Obtained conclusions may be very general, but other specific statements could be highly speculative at the moment.

Fragility curves related to other damage states, i.e. concrete crushing, longitudinal bar buckling, and longitudinal bar fracture, were also estimated. It can be concluded that the proposed procedure offers a viable alternative to other procedures, and that there is no need to use closed-form equations for the prediction of point estimates. However, the presented results suggest that a better CAE model is needed to accurately describe the older designs of RC columns. It should include an additional parameter which would account for poor seismic detailing.

Appendix A

In order to better illustrate the use of the CAE method for the estimation the fragility curves, the calculations for the input data P^* = 0.25 and L^* = 3 are shown in Table A1. Note that, for illustration purposes, only two input parameters are considered. The database consists of 7 test samples of RC columns. Equations 1–7 are used, taking into account w = 0.15. It can be clearly seen that the influence of the 7 different input drift values (based on the results of measurements) on the predicted drift value depends on the similarity of the input parameters P^* and L^* between the measured and predicted deformation. The highest weight A_n is assigned to the sample #6 because its values of P^* = 0.27 and L^* = 2.4 are the nearest to the target values P^* = 0.25 and L^* = 3.

In the next step the drifts δ are sorted from the lowest to the highest value, together with the corresponding coefficients A_n (see Table A2). The results are presented in Figure A1, where

Table A1: *An example of the prediction of ultimate drift δ for P^* = 0.5 and L^* = 3 by the CAE method (w = 0.15)*

Sample	P^*	L^*	Norm. P^*	Norm. L^*	δ	a_n	A_n	$\delta_n A_n$	$(\delta_n-\delta)^2 A_n$
#1	0.13	1.50	0.74	0.70	0.034	0.042	0.015	0.000	0.000 006 20
#2	0.35	3.50	0.30	0.30	0.042	0.371	0.129	0.005	0.000 020 22
#3	0.05	2.10	0.90	0.58	0.049	0.016	0.005	0.000	0.000 000 17
#4	0.00	3.30	1.00	0.34	0.078	0.004	0.001	0.000	0.000 000 77
#5	0.18	2.80	0.64	0.44	0.063	0.703	0.244	0.015	0.000 017 46
#6	0.27	2.40	0.46	0.52	0.051	0.789	0.274	0.014	0.000 003 43
#7	0.21	3.15	0.58	0.37	0.057	0.957	0.332	0.019	0.000 002 01
					$\sum \rightarrow$	2.882	1.000	0.0545	0.000 050 26

δ_{mean} = 0.045 45;

$\sigma = \sqrt{0.00005026} = 0.0071$

the dotted line represents the values from the last column in Table A2 (CAE empirical CDF). The corresponding smoothed log-normal cumulative distribution function (smoothed CDF) is estimated from the mean and local standard deviation (δ_{mean} = 0.045 45, σ = 0.007 1), calculated above.

Table A2: *Determination of the CAE empirical and corresponding smoothed lognormal cumulative distribution function (CDF)*

Sample	δ	A_n	$\sum A_n$
#1	0.034	0.014 70	0.014 70
#2	0.042	0.128 61	0.143 31
#3	0.049	0.005 43	0.148 74
#6	0.057	0.273 78	0.422 52
#7	0.063	0.332 18	0.754 70
#5	0.068	0.243 90	0.998 61
#4	0.081	0.001 39	1.000 00

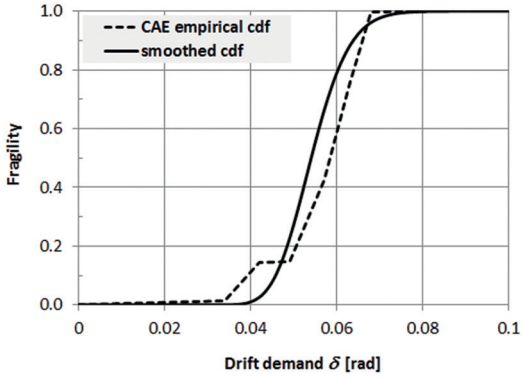


Figure A1: *Fragility function determined from the CAE empirical CDF and the corresponding smoothed lognormal CDF.*

Acknowledgements

The results presented in this paper are based on work supported by the Slovenian Research Agency. The constructive comments provided by Prosessors P. Fajfar and M. Dolšek are greatly appreciated.

References

[1] Zhu, L., Elwood, K. J., Haukaas, T. (2007): Classification and seismic safety evaluation of existing reinforced concrete columns. *Journal of Structural Engineering*, 133 (9), pp. 1316–1330.

[2] Singhal, A, Kiremidjian, A. (1995): *Method for developing motion damage relationships for reinforced concrete frames*, Report #115, Blume Earthquake Engineering Centre, Stanford University, Stanford.

[3] Dumova-Jovanoska, E. (2000): Fragility curves for reinforced concrete structures in Skopje (Macedonia) region. *Soil Dynamics and Earthquake Engineering,* 19 (6), pp. 455–466.

[4] Kappos, A., Panagopoulos, G., Panagiotopoulos, C., Penelis, G. (2006): A hybrid method for the vulnerability assessment of R/C and URM buildings. *Bull. Earthquake Eng.,* 4, pp. 391–413.

[5] Panagiotakos, T. B., Fardis, M. N. (2004): Seismic performance of RC frames designed to Eurocode 8 or to the Greek Codes 2000. *Bulletin of Earthquake Engineering,* 2, pp. 221–259.

[6] Papailia, A. (2011): *Seismic fragility curves for reinforced concrete buildings*. MSc Thesis (Supervisor M. N. Fardis), University of Patras.

[7] Akkar, S., Sucuoglu, H., Yakut, A. (2005): Displacement-based fragility functions for low- and mid-rise ordinary concrete buildings. *Earthquake Spectra*, 21 (4), pp. 901–927.

[8] Rossetto, T., Elnashai, A. (2005): A new analytical procedure for the derivation of displacement-based vulnerability curves for populations of RC structures. *Engineering Structures,* 27 (3), pp. 397–409.

[9] Zhu, L. (2005): *Probabilistic drift capacity models for reinforced concrete columns*. MSc Thesis (Supervisor K. J. Elwood, Co-supervisor T. Haukaas), The University of British Columbia.

[10] Perus, I., Poljansek, K., Fajfar, P. (2006): Flexural deformation capacity of rectangular RC columns determined by the CAE method. *Earthquake Eng. Struct. Dyn.*, 35 (12), pp. 1453–1470.

[11] Perus, I., Dolsek, M. (2009): The error estimation in the prediction of ultimate drift of RC columns for performance-based earthquake engineering. *RMZ-Mater. Geoenviron.*, 56 (3), pp. 322–336.

[12] McKay, M. D., Beckman, R. J., Conover, W. J. (1979): A Comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics,* 21, pp. 239–245.

[13] Iman, R. L., Helton, J. C., Campbell, J. E. (1981): An approach to sensitivity analysis of computer models, Part 1. Introduction, input variable selection and preliminary variable assessment. *Journal of Quality Technology,* 13, pp. 174–183.

[14] Viana, F. A. C. (2013): *Things you wanted to know about the Latin hypercube design and were afraid to ask*. 10th World Congress on Structural and Multidisciplinary Optimization, May 19–24, 2013, Orlando, Florida, USA.

[15] Dolsek, M. (2009): Incremental dynamic analysis with consideration of modeling uncertainties. *Earthquake Eng. Struct. Dyn.*, 38, pp. 805–825.

[16] Vorechovský, M., Novak, D. (2009): Correlation control in small-sample Monte Carlo type simulations I: A simulated annealing approach. *Probabilistic Engineering Mechanics,* 24, pp. 452–462.

[17] Dolsek, M. (2012): Simplified method for seismic risk assessment of buildings with consideration of aleatory and epistemic uncertainty. *Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance,* 8 (10), pp. 939–953.

[18] Berry, M., Parrish, M., Eberhard, M. (2004): PEER structural performance database, User's Manual, Version 1.0, 2004, Pacific Earthquake Engineering Research Center; Available on: < http://www.ce.washington.edu/~peera1/.>

[19] Panagiotakos, T. B., Fardis, M. N. (2001): "Deformations of reinforced concrete members at yielding and ultimate." *ACI Structural Journal*, 98 (2), pp. 135–148.

[20] Biskinis, D. E., Roupakias, G. K., Fardis, M. N. (2004): Degradation of Shear Strength of RC Members with Inelastic Cyclic Displacements. *ACI Structural Journal,* 101 (6), pp. 773–783.

[21] Biskinis, D. E. (2007): *Resistance and Deformation Capacity of Concrete Members with or without Retrofitting*. PhD Thesis (Supervisor M. N. Fardis), Civil Engineering Department, University of Patras, Greece. (in Greek).

[22] Biskinis, D. E., Fardis, M. N. (2010): Deformations at Flexural Yielding of Members with Continuous or Lap-Spliced Bars. *Structural Concrete,* 11 (3), pp. 127–138.

[23] Biskinis, D. E., Fardis, M. N. (2010): Flexure-Controlled Ultimate Deformations of Members with Continuous or Lap-Spliced Bars, *Structural Concrete,* 11 (2), pp. 93–108.

[24] Grammatikou, S. (2013): *Strength, deformation capacity and failure mode of RC walls in seismic loading*. MSc Thesis (Supervisor M. N. Fardis), Civil Engineering Department, University of Patras, Greece. (in Greek).

[25] Lignos, D. G., Krawinkler, H. (2012): *Sidesway collapse of deteriorating structural systems under seismic excitations*. Report No. 177, 2012, The John A. Blume Earthquake Engineering Center, Stanford, CA, USA.

[26] Lignos, D. G., Krawinkler, H. (2013): Development and utilization of structural component databases for performance-based earthquake engineering. *Journal of Structural Engineering*, Special issue 139, pp. 1382–1394.

[27] Poljansek, K., Perus, I., Fajfar, P. (2009): Hysteretic energy dissipation capacity and the cyclic to monotonic drift ratio for rectangular RC columns in flexure. *Earthquake Eng. Struct. Dyn.*, 38 (7), pp. 907–928.

[28] SERIES database (2013): RC column, beam and wall database. Available on: < http://www.dap.series.upatras.gr/>.

[29] Perus, I., Biskinis, D., Fajfar, P., Fardis, M. N., Grammatikou, S., Krawinkler, H., Lignos, D. (2014): *The SERIES database of RC elements*. 2nd European conference on Earthquake Engineering and Seismology, Istanbul, August 2014, Paper #145.

[30] Negro, P., Mola, E., Molina, F. J., Magonette, G. E. (2004): *Full-scale PSD testing of a torsionally unbalanced three-storey non-seismic RC frame*. 13th WCEE, Paper # 968, August 2004, Vancouver, B. C., Canada.