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Matematični modeli dinamike helikopterskega letenja

Mathematical Models of Helicopter Flight Dynamics

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Helikopter se razlikuje od drugih prometno transportnih sredstev, ne le po svoji sestavi temveč tudi po možnostih svojega gibanja. Helikopter se lahko premika navpično, lahko lebdi v zraku, lahko se vrti na istem mestu, lahko se premika naprej in v stran, a lahko tudi kombinira vse te premike. Zaradi tega sta modeliranje in testiranje dinamike helikopterja zelo zapleten problem. Trenutno se problemi dinamike helikopterskega letenja v glavnem rešujejo z uporabo sodobnih računalnikov. Čeprav so za mnoge zahtevne probleme računalniki neizogibni, ne omogočajo razumevanja fizikalne plati problema. Na srečo je veliko problemov v zvezi s helikopterji mogoče analizirati brez preveč zapletenih izračunov in navadno je mogoče priti do preproste formule. Čeprav niso ustrezne za preračune, te formule, pri konstrukciji helikopterja, omogočajo zadovoljivo interpretacijo potrebnih aerodinamičnih in dinamičnih pojavov. Helikopter spada v skupino zračnih sistemov in njegovo tradicionalno modeliranje se deli na: a) prostorsko geometrijo in kinematiko in na b) dinamiko togega telesa in dinamiko zraka, skozi katerega se giblje. V zadnjem času, so se razvili naslednji modeli: c) elastični model, odvisen od aerodinamičnih sil, d) model pogonskega sistema, e) model hidravličnih in drugih izvršilnikov za aerodinamično krmiljenje, f) model obnašanja pilota, g) model sistema navigacije in h) model problema vodenja. Matematični model, opisan v tem prispevku, se nanaša na a) in b).

(Ključne besede: helikopterji, dinamika letenja, matematični modeli)

The helicopter is a specific form of traffic-transportation, not just in terms of its structure but also in terms of its possibilities for motion. The helicopter can move vertically, hover in the air, turn around, move forward and move laterally, and it can also perform combinations of these movements. As a result, modeling and testing helicopter dynamics is a very complex problem. The problems in helicopter flight dynamics are mostly solved with the aid of modern computers. Though inevitably, with many complex problems, computers do not make it possible to understand the physical nature of the problem. Fortunately, many problems related to helicopters can be analyzed without overly complex calculus, and usually it is possible to obtain simple formulae. Though not suitable for calculus, these formulae, when designing the helicopter, enable a satisfactory interpretation of the required aerodynamic and dynamic phenomena. The helicopter belongs to the group of aerospace systems and its traditional modeling may be divided into: a) three-dimensional (space) geometry and kinematics, and b) rigid-body dynamics and the fluid dynamics through which it moves. Recently, the following models were developed: c) the elasticity model in intersubordinance with a fluid, d) the propulsion system model, e) the hydraulic model and other actuators that achieve aerodynamic control, f) the pilot-behavior model, g) the navigation-system model, and h) the beacon-problem model. The mathematical model described in this paper is related to a) and b).

(Keywords: helicopters, flight dynamics, mathematical models)

1 GIBANJE ROTORSKIH KRAKOV

Da bi razumeli dinamiko letenja helikopterja in določili dinamične momente in sile, ki delujejo na helikopter, je nujno potrebno najprej raziskati

1 MOTION OF ROTOR BLADES

To understand the flight dynamics of helicopters and determine the dynamic moments and forces that act upon the helicopter, it is a necessity to pre-investigate gibanje nosilnih krakov rotorja. Iz velikega števila različnih helikopterjev smo izbrali helikopter z enim rotorjem, katerega kraki so povezani na glavo rotorja s členkom, okoli katerega se prosto gibljejo. Treba je povedati, da so kraki trdno povezani z glavo rotorja.

the motion of the supporting rotor blades. From a vast number of different types of helicopters, we chose the single-rotor helicopter that has its blades coupled with the main rotor by a hinge about which they can move freely. It should be noted that there are also rotors that have the blade connected in a fixed manner to the hub.

1.1 Enačbe mahanja kraka

Krake rotorja vzamemo kot togo telo. Vodoravni členek je na razdalji eR od osi vrtenja. Kotna hitrost gredi je Ω=konst, a krak maha s kotno hitrostjo dβ/dt. Vzdolžna os kraka je vzporedna vztrajnostni osi kraka in gre skozi členek(sl. 1).

Na sliki 1 je R dolžina kraka, kot β je položaj kraka. Po zapletenih preračunih, dobimo enačbe gibanja krakov:

1.1 Equations of blade flapping

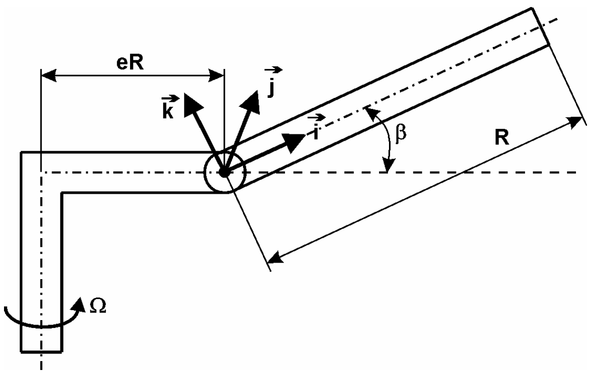
Rotor blades are regarded as a rigid body. The horizontal hinge is placed at a length eR from the rotation axis. The shaft rotates at an angular speed Ω=const, and the blade flappers at an angular speed of dβ/dt. The axis that passes through the blade is parallel to the axis of inertia of the blade and passes through the hinge (Fig. 1).

In Figure 1, R represents the length of the blade, β represents the flapping angle of the blade. Following some complex calculus, the equations for blade flapping are obtained:

$$\ddot{\beta} + \Omega^2 (1+\varepsilon)\ \beta = M_{Ay}/\mathrm{J_y} \tag{1}$$

$$\mathrm{J_x}\ \dot{\beta}\ \Omega\ \cos\beta + J_x\left(-\dot{\beta}\right)\Omega\ \sin\beta = 0 \tag{2}$$

$$-2\ J_y\ \Omega\ \dot{\beta}\ \sin\beta = M_z \tag{3}$$



Sl. 1. *Prikaz mahanja kraka*
Fig. 1. *Explanatory drawing for blade flapping*

1.2 Enačbe zaostajanja kraka

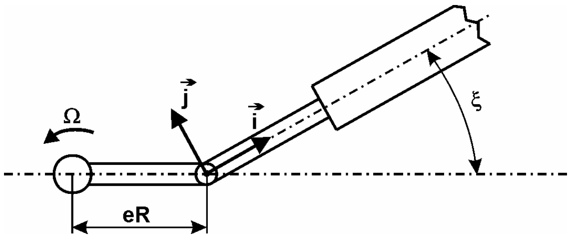
Predpostavimo, da je β=0 in da se krak giblje naprej glede na navpični členek za kot zaostajanja ξ. Navpični členek je na razdalji eR od osi vretena. Koordinatni sistem postavimo enako kot v prejšnjem primeru. Slika 2 prikazuje poenostavljeno skico za določanje zaostajanja kraka.

Iz tega izhaja enačba zaostajanja kraka:

1.2 Equations of blade throwback

It is assumed that β=0 and that the blade is moving forward in relation to the vertical hinge by the throwback angle amount ξ. The vertical hinge is placed at a distance eR from the shaft axis. The coordinate system is positioned as in the previous case. Figure 2 presents a simplified scheme for determining the blade throwback.

From this the equation for blade throwback follows:



Sl. 2. *Prikaz zaostajanja kraka*
Fig. 2. *Explanatory scheme for blade throwback*

$$\ddot{\xi} + \Omega^2 \varepsilon\ \xi - 2\ \Omega\ \beta\ \dot{\beta} = M_z / J_z \tag{4}$$

Če je kot med položajem kraka in smerjo letenja $\psi=\Omega t$, sledi:

If the azimuth angle is described as $\psi=\Omega t$, then it follows that:

$$\frac{d^2\xi}{d\psi^2} + \varepsilon\ \xi - 2\ \beta \frac{d\beta}{d\psi} = \frac{M_z}{J_z \Omega^2} \tag{5}$$

1.3 Enačba vzpenjanja kraka

Vzemimo, da sta kota mahanja in zaostajanja enaka nič. Korak kraka je kot med tetivo profila kraka in ravnino glave rotorja, označen kot θ_k. Na sliki 3 vidimo koordinatni sistem, povezan s krakom.

Enačbe gibanja kraka okoli vzdolžne osi so:

1.3 Equation of blade climb

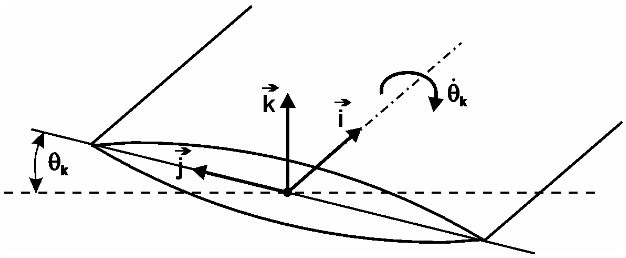
It is assumed that the flapping and the throwback angles are equal to zero. The blade step is the angle between the blade cross-section chord and the plane of the hub, designated as θ_k. Figure 3 shows the coordinate system attached to the blade.

The equations of blade motion about the longitudinal axis are:

$$\ddot{\theta}_k + \Omega^2\ \theta_k = M_x / J_x \tag{6}$$

$$-2\ J_z\ \Omega\ \dot{\theta}_k \sin\theta_k = M_z \tag{7}$$

$$J_y\ \Omega\ \dot{\theta}_k \cos\theta_k - J_y\ \dot{\theta}_k \cos\theta_k = 0 \tag{8}$$



Sl. 3. *Koordinatni sistem na profilu kraka*
Fig. 3. *Coordinate system at the blade cross-section*

2 ROTORSKE SILE

Za projekcijo sil lahko uporabimo naslednje osi: os v smeri vlečne sile rotorja, os rotorskega diska, ki je pravokotna na ravnino rotorja, to je na ravnino, kjer ležijo konci krakov in os gredi.

Ko izberemo eno izmed teh osi, bosta preostali dve osi v koordinatnem sistemu pravokotni nanjo, usmerjeni bočno, oziroma proti repu helikopterja. Običajno se komponenta sile v smeri izbrane osi

2 ROTOR FORCES

To project the forces the following axes may be used: the control axis, the rotor disc axis (which is normal to the rotor plane, i.e., to the plane on which the blade tips reside), and the shaft axis.

Once the axis is chosen, the remaining axes of the coordinate system will be normal to it and pointed laterally, i.e., to the tail of the helicopter. The force component along the chosen axis is normally referred imenuje **vlečna sila**, komponenta sile proti repu se imenuje **sila *H***, a komponenta sile, usmerjena bočno, se imenuje **sila *Y***. Če komponente sile označimo brez indeksiranja, menimo da se nanašajo na os v smeri včene sile. Indekse "*D*" in "*S*" uporabljamo, če se komponente nanašajo na os rotorja oziroma na os pogonske gredi.

Ker sta kota mahanja in zaostajanja ponavadi majhna (do 10°), lahko napišemo:

to as the **tow force**, the force component pointed towards the tail is called the ***H* force**, and the force component pointed laterally is said to be the ***Y* force**. If the force components are designated without subscripts, it is assumed that they are determined relative to the control axis, whereas the subscripts "*D*" and "S" are used when they relate to the rotor axis, i.e., the shaft axis.

Since the flutter and mount angles are usually small (to 10°), a relation between these components can be obtained:

$$T \approx T_D \approx T_S$$
$$H \approx H_D + T_D\ a_1 \cong H_S + T_S\ B_1$$

2.1 Vzdolžno ravnotežje sil

Kot B_1 je vzdolžna amplituda ciklične spremembe koraka kraka; kot a_{1s} je kot med gredjo in osjo rotorskega diska. Po obsežnih izračunih dobimo izraz za vzdolžno amplitudo ciklične spremembe koraka kraka:

2.1 Longitudinal equilibrium of forces

Angle B_1 is the longitudinal amplitude of a cyclic change in the blade step; angle a_{1s} is the angle between the shaft and the axis of the rotor disc. After extensive calculus the expression for the longitudinal amplitude of cyclic change in the blade step is obtained:

$$B_1 = \frac{M_f - G \cdot fR + H \cdot hR + M_S \cdot a_1}{T \cdot hR + M_S} \tag{9}$$

Za e=0, lahko vzamemo da je M_s=0 in M_f=0. Ker je T=G, sledi:

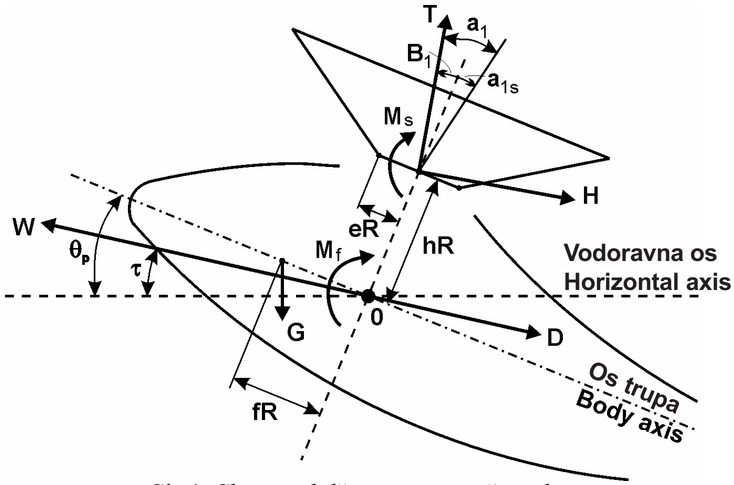
For e=0, we can say that M_s=0 and M_f=0, and since T=G, it follows that:

$$B_1 = -\frac{f}{h} + \frac{H}{G} \tag{10}$$

$$\theta_p = -\frac{D}{G}\ \cos\tau - \frac{f}{h} + \frac{M_f}{G \cdot hR} \tag{11}$$

Za enačbo (10) imamo preprosto fizikalno pojasnilo: amplituda vzdolžnega cikličnega krmiljenja krakov mora imeti tako vrednost, da postavi smer rezultirajoče sile rotorja skozi masno središče.

Equation (10) has a simple physical interpretation: the amplitude of the longitudinal cyclic control must have such a value in order to position the direction of the resultant rotor force through the center of mass.



Sl. 4. *Skica vzdolžnega ravnotežja sil*
Fig. 4. *Drawing for determining the longitudinal equilibrium of forces*

2.2 Prečno ravnotežje sil

Kot A_1 pomeni amplitudo prečne ciklične spremembe koraka kraka rotorja:

2.2 Lateral equilibrium of forces

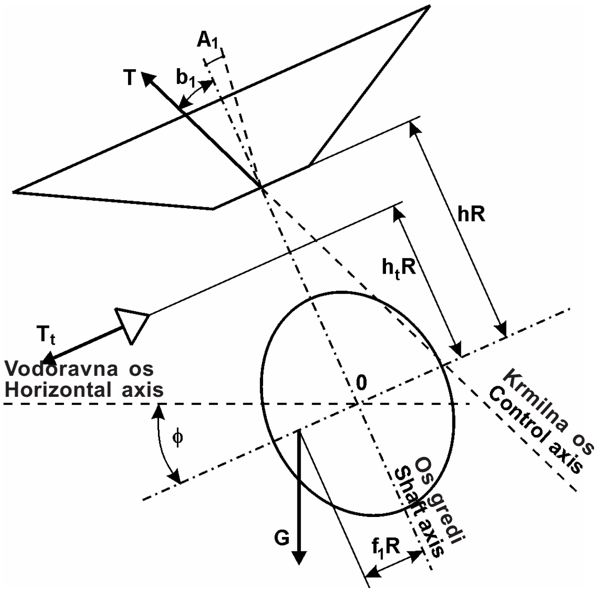
Angle A_1 represents the amplitude of lateral cyclic change in the blade step of the supporting helicopter rotor:

$$A_1 = -\frac{G \cdot f_1 R + M_S \cdot b_1 + T_t \cdot h_t R}{G \cdot hR + M_S} \tag{12}$$

Če vrednost A_1 uvrstimo v ustrezne enačbe, dobimo vrednost kota ϕ, ki določa lego trupa.

By replacing the value A_1 in the corresponding equations, we obtain the value of angle ϕ, which determines the position of the fuselage.

$$\phi = -\frac{T_t}{G} + \frac{G \cdot f_1 R + M_S \cdot b_1 + T_t \cdot h_t R}{G \cdot hR + M_S} \tag{13}$$



Sl. 5. *Skica prečnega ravnotežja sil*
Fig. 5. *Drawing for determining the lateral equilibrium of forces*

Če je M_S=0 in h_t=h, kar pogosto lahko vzamemo, sledi:

If M_S=0 and h_t=h, which can often be assumed, it follows that:

$$\phi \approx \frac{f_1}{h},$$

kar pomeni, da je glava rotorja navpično nad masnim središčem. Vse vrednosti izračunanih kotov so tako imenovane *uravnovešene vrednosti*.

which means the rotor hub is positioned vertically above the center of mass. All the values of these determined angles are the so-called *trimmed values*.

3 NELINEARNI MATEMATIČNI MODEL DINAMIKE LETENJA

Matematično modeliranje helikopterskega gibanja je izredno zahtevna naloga in zato je nujno privzeti številne predpostavke in približke. Za analizo dinamičnih značilnosti helikopterja ni nujno treba, razen v izjemnih primerih, poznati gibanja

3 NON-LINEAR MATHEMATICAL MODEL OF THE FLIGHT DYNAMICS

Mathematical modeling of a helicopter's motion is a very complex task and, therefore, it is necessary to introduce a series of assumptions and approximations. Knowledge of the motion of the individual helicopter blades is not necessary for investigating the dynamic posameznih krakov. Za definiranje sil in momentov pri motenem letu je dovolj opazovati rotor kot celoto. Zaradi velikega števila različnih helikopterjev, v tem prispevku analiziramo helikopter z enim samim rotorjem, katerega kraki so s členki pritrjeni na glavo rotorja. Kakor je že bilo rečeno, je helikopter zmožen različnih gibanj in bi zato bilo zelo težko narediti matematični model za kombinacijo vseh gibanj. Vzeli bomo, da je helikopter vzletel in da leti premočrtno. Komponente hitrosti helikopterja pri nominalni vrednosti in premočrtnem letenju so: W_x, W_y in W_z. Koti spremembe smeri ψ, nagiba ϕ in vzpenjanja θ, veljajo dokler so velikosti motenj v dovoljenih mejah. Na sliki 6 je predstavljena shema helikopterja s premičnim koordinatnim sistemom, vezanim na njegovo masno središče, a na sliki 7 je blokovni diagram helikopterja.

Po uvedbi nekoliko predpostavk, na primer da:

- je masa helikopterja konstantna,
- je helikopter togo telo,
- ravnina *xz* simetrijska ravnina,
- so kotni prirastki $\Delta\Psi$, $\Delta\theta$, $\Delta\phi$ majhni in tako dalje;

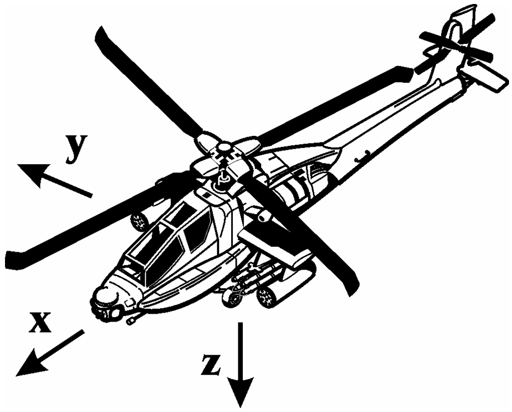
pridemo do nelinearnega matematičnega modela z odkloni v obliki:

characteristics of the helicopter, except in a special case, but rather for defining the forces and moments in a disturbed flight it is sufficient to view the rotor as a whole. Because of the large number of different helicopters, in this paper a single-rotor helicopter that has its blades connected to the hub by hinges was studied. As mentioned before, the helicopter can perform different movements and it would be very difficult to make a mathematical model that would combine all those movements. It is assumed that the helicopter is airborne and in straightforward flight. It is necessary that the helicopter, during its straightforward flight, has the following velocity components, W_x, W_y, and W_z, at nominal values, and the angle of turn ψ, the angle of roll ϕ, and the angle of climb θ, as long as the intensity of disturbance is within permitted limits. Figure 6 presents a schematic of the helicopter with a floating coordinate system tied to its center of mass, and Figure 7 presents a helicopter block diagram.

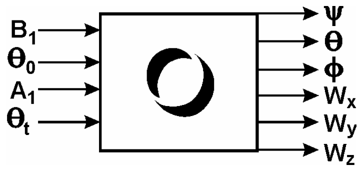
After introducing a series of assumptions, such as:

- the helicopter mass is a constant value,
- the helicopter is a rigid body,
- the 0*xz* is a plane of symmetry,
- the angle increments $\Delta\Psi$, $\Delta\theta$, $\Delta\phi$ are too small, and so on,

we come to a non-linear mathematical model with deviations in the form:



Sl. 6. *Shema helikopterja*
Fig. 6. *Schematic of helicopter*



Sl. 7. *Blokovni diagram helikopterja*
Fig. 7. *Helicopter block diagram*

$$\frac{d(\Delta W_x)}{dt}=\frac{1}{m}\left[f_1\left(\Delta W_x,\Delta W_z,\Delta\dot{\theta},u_1,u_2\right)-\left(mg\cos\tau\right)\Delta\theta\right] \tag{14}$$

$$\frac{d(\Delta W_z)}{dt}=\frac{1}{m}\left[f_2\left(\Delta W_x,\Delta W_z,\Delta\dot{\theta},u_1,u_2\right)+W_{zN}m\Delta\dot{\theta}-\left(mg\sin\tau\right)\Delta\theta\right] \tag{15}$$

$$\frac{\mathrm{d}(\Delta\theta)}{\mathrm{dt}}=\Delta\dot{\theta} \tag{16}$$

$$\frac{\mathrm{d}(\Delta\dot{\theta})}{dt}=\frac{1}{J_y}f_3\left(\Delta W_x,\Delta W_z,\Delta\dot{W}_z,\Delta\dot{\theta},u_1,u_2\right) \tag{17}$$

$$\frac{d(\Delta W_y)}{dt}=\frac{1}{m}\left[f_4\left(\Delta W_y,\Delta\dot{\phi},\Delta\dot{\psi},u_3,u_4\right)+W_{zN}m\Delta\dot{\psi}+mg\cos\tau\Delta\theta+mg\sin\tau\Delta\psi\right] \tag{18}$$

$$\frac{\mathrm{d}(\Delta\phi)}{\mathrm{dt}}=\Delta\dot{\phi} \tag{19}$$

$$\frac{\mathrm{d}(\Delta\dot{\phi})}{dt}=\frac{1}{J_x}\left[f_5\left(\Delta W_y,\Delta\dot{\phi},\Delta\dot{\psi},u_3,u_4\right)+J_{xz}\Delta\dot{\psi}\right] \tag{20}$$

$$\frac{\mathrm{d}(\Delta\psi)}{\mathrm{dt}}=\Delta\dot{\psi} \tag{21}$$

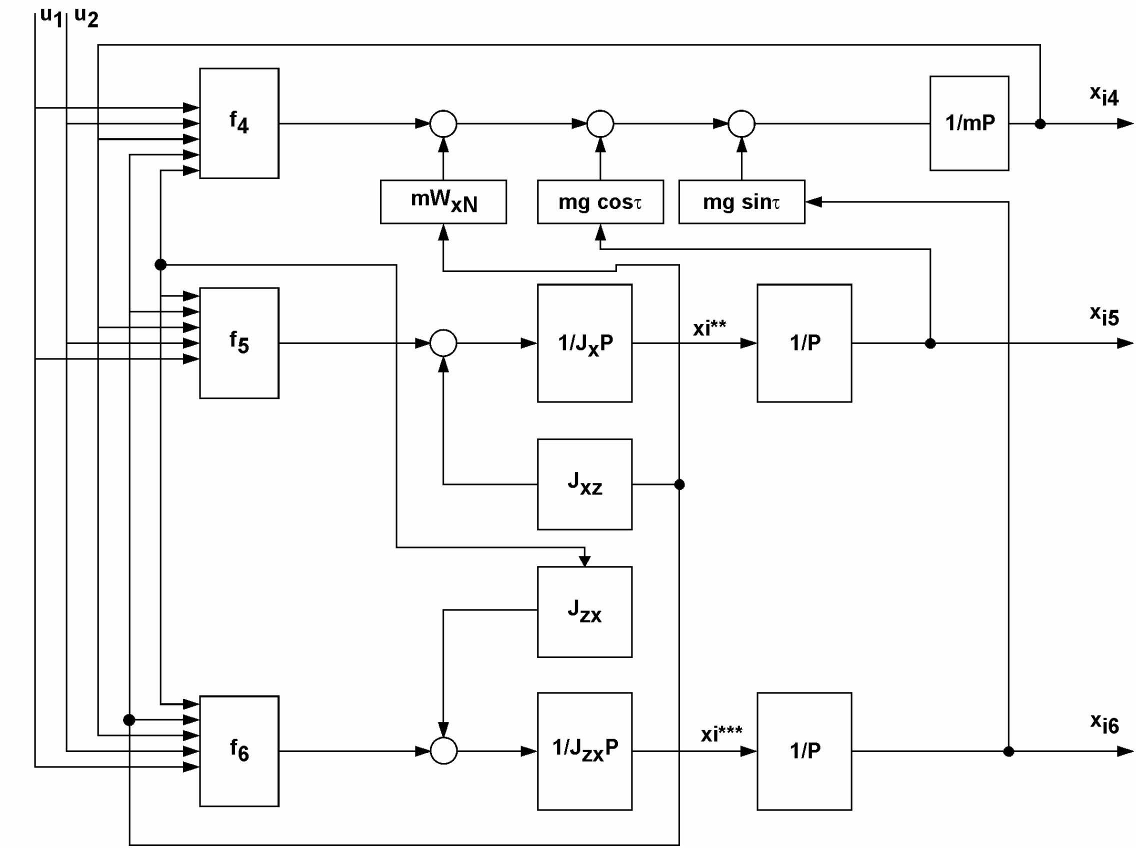
$$\frac{\mathrm{d}(\Delta\dot{\psi})}{dt}=\frac{1}{J_z}\left[f_6\left(\Delta W_y,\Delta\dot{\phi},\Delta\dot{\psi},u_3,u_4\right)+J_{xz}\Delta\dot{\phi}\right] \tag{22}$$

Kjer so:

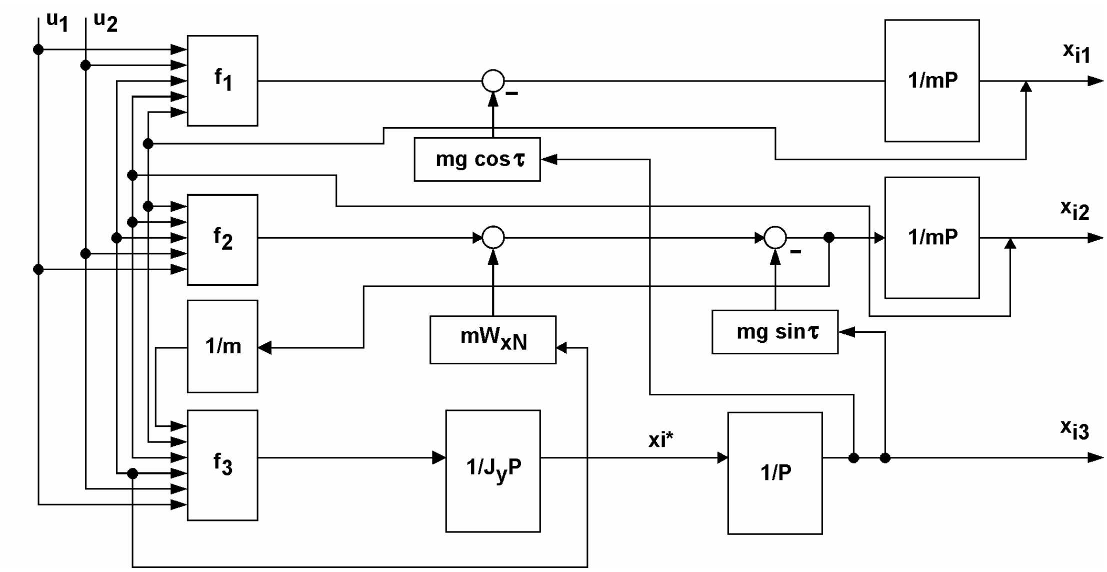
- $u_1=\Delta B_1$ – amplituda ponovitvene spremembe koraka v vzdolžni smeri (za vzdolžno gibanje),
- $u_2=\Delta\theta_0$ – sprememba skupnega koraka kraka rotorja helikopterja (za vzdolžno gibanje),

Where:

- $u_1=\Delta B_1$ – the amplitude of the cyclic change in step in the longitudinal direction (in terms of longitudinal motion),
- $u_2=\Delta\theta_0$ – the change of the collective step of the helicopter rotor blade (in terms of longitudinal motion),



Sl. 8. *Shema prečnega gibanja*
Fig. 8. *Schematic for lateral motion*



Sl. 9. *Shema vzdolžnega gibanja*

Fig. 9. *Schematic for longitudinal motion*

- u_3=ΔA_1 – amplituda ponovitvene spremembe koraka v prečni smeri (za prečno gibanje) in
- u_4=$\Delta\theta_t$ – sprememba skupnega koraka repnega rotorja (za prečno gibanje).

Shematska diagrama predstavljamo na slikah 8 in 9.

4 LINEARIZIRANI MATEMATIČNI MODEL DINAMIKE LETENJA

Dokazano je, da v tehniki lahko uporabimo s sprejemljivo natančnostjo linearizirane matematične modele pod pogojem, da imajo fizikalne veličine majhna odstopanja od nominalnih vrednosti. Nelinearni matematični model dinamike letenja helikopterja ni primeren za določanje splošnih rešitev v analitični obliki, čeprav se problem rešuje s sodobno računalniško tehnologijo.

Zaradi sprejetih predpostavk bodo izstopne vrednosti, vstopne vrednosti in vektor stanja tako za vzdolžno kakor za prečno gibanje:

- u_3=ΔA_1 – the amplitude of the cyclic change in step in the lateral direction (in terms of lateral motion),
- u_4=$\Delta\theta_t$ – the change of the collective step of the tail rotor (in terms of lateral motion).

Schematic diagrams are presented in Figures 8 and 9.

4 LINEARIZED MATHEMATICAL MODEL OF THE FLIGHT DYNAMICS

In technical applications it has been shown that, with an acceptable accuracy, linearized mathematical models may be used under the condition that the deviations of the physical quantities from their nominal values are small. A non-linear mathematical model of the helicopter's flight dynamics is inadequate for finding general solutions in an analytical form, even though the problem is solved with the aid of modern computer technology.

The outcome of the adopted presumptions is that the output values, input values, and the vector of state for both the longitudinal and lateral motion will be:

$$\underline{X} = \left(X_1 \ldots X_9\right)^T \tag{23}$$

$$\underline{X_i} = \left(X_{i1} \ldots X_{i6}\right)^T \tag{24}$$

$$\underline{u} = \left(u_1 \ldots u_4\right)^T \tag{25}$$

Vektorska enačba stanja za linearizirani matematični model z brezrazsežnimi veličinami, odstopanji, to je veličinami stanja, je enačba (26). Enačba izstopnih veličin je (27).

The vector equation of state for the linearized mathematical model with non-dimensional quantities, deviations, i.e., the quantities of state, is shown in Equation (26). Equation (27) presents the output values.

$$\underline{\dot{X}} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} & 0 & a_{58} & a_{59} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{75} & 0 & a_{77} & 0 & a_{79} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & a_{95} & 0 & a_{97} & 0 & a_{99} \end{bmatrix} \underline{X} + \begin{bmatrix} b_{11} & b_{12} & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_{41} & b_{42} & 0 & 0 \\ 0 & 0 & b_{53} & b_{54} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_{73} & b_{74} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_{93} & b_{94} \end{bmatrix} \underline{u} \tag{26}$$

$$\underline{X_i} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \underline{X} \tag{27}$$

Čeprav so tukaj prikazana v skupni matrici, je treba poudariti, da so vzdolžna in prečna gibanja ločena, ker je to bil eden izmed pogojev za izpeljavo tega matematičnega modela. V enačbah (26) in (27), so enačbe za vzdolžno gibanje predstavljene v prvih štirih vrsticah matrike, medtem ko preostalih pet vrstic pomeni enačbo stanja in enačbo prečnega gibanja. V enačbi (26) uporabljamo naslednje označbe:

In addition to the way this is presented, in the form of a common matrix, it should also be noted that the longitudinal and lateral motions are separated, because this was the condition for deriving this mathematical model. In Equations 26 and 27, the equations for the longitudinal motion are presented within the first four rows of the matrice, while the remaining five rows present the equation of state and the equation of lateral motion. The designations used in Equation 26 are:

$$C^* = \frac{1}{1 - i_{xz}^2 / i_x i_z} \qquad a_{11} = x_u \qquad a_{12} = x_w \qquad a_{13} = -m_c \cos\tau \qquad a_{14} = x_q \qquad a_{21} = z_u$$

$$a_{22} = z_w \qquad a_{23} = -m_c \sin\tau \qquad a_{24} = \hat{W}_{xN} + z_q \qquad a_{55} = y_v \qquad a_{56} = m_c \cos\tau \qquad a_{58} = m_c \sin\tau$$

$$a_{59} = \hat{W}_{xN} \qquad a_{41} = m_u + m_{\dot{w}} z_u \qquad a_{42} = m_w + m_{\dot{w}} z_w \qquad a_{43} = -m_{\dot{w}} m_c \sin\tau \quad a_{44} = m_q + m_{\dot{w}} \left(\hat{W}_{xN} + z_q \right)$$

$$b_{11} = x_{B_1} \qquad b_{12} = z_{B_1} \qquad b_{21} = x_{\theta_0} \qquad b_{22} = z_{\theta_0} \qquad b_{41} = m_{\dot{w}} z_{B_1} + m_{B_1} \qquad b_{42} = m_{\dot{w}} z_{\theta_0} + m_{\theta_0}$$

$$b_{53} = y_{A_1} \qquad b_{54} = y_{\theta_t} \qquad b_{73} = \left(l_{A_1} + n_{A_1} i_{xz} / i_x \right) C^* \quad b_{74} = \left(l_{\theta_t} + n_{\theta_t} i_{xz} / i_x \right) C^* \quad b_{94} = \left(n_{\theta_t} + l_{\theta_t} i_{xz} / i_z \right) C^*$$

$$a_{75} = \left(n_v i_{xz} / i_x + l_v \right) C^* \quad a_{77} = \left(l_p + n_p i_{xz} / i_x \right) C^* \qquad a_{95} = \left(n_v + l_v i_{xz} / i_z \right) C^* \quad a_{97} = \left(n_p + l_p i_{xz} / i_z \right) C^*$$

$$a_{99} = \left(n_r + l_r i_{xz} / i_z \right) C^* \qquad b_{93} = \left(n_{A_1} + l_{A_1} i_{xz} / i_z \right) C^* \quad a_{79} = \left(l_r + n_r i_{xz} / i_x \right) C^*$$

Na slikah 10 in 11 so predstavljene sheme lineariziranega matematičnega modela v vzdolžnem in prečnem gibanju.

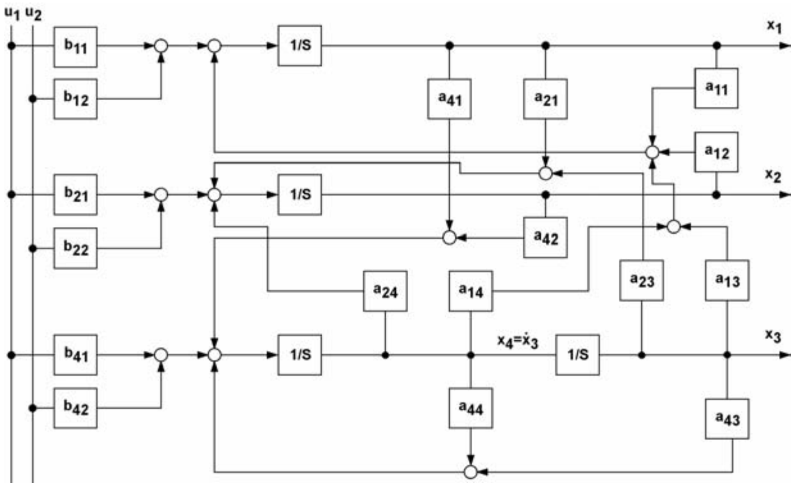
Figures 10 and 11 present schematics of the linearized mathematical model in a longitudinal and lateral motion.

5 REZULTATI PROGRAMA

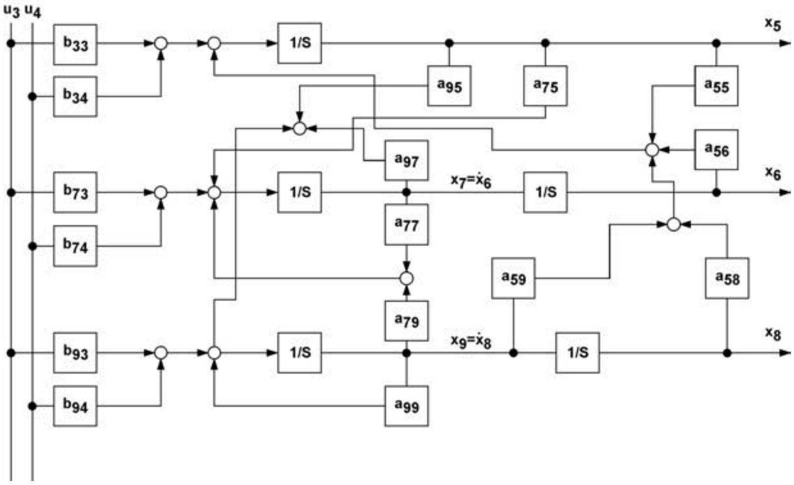
5 PROGRAM RESULTS

Program je testiran na primeru helikopterja z enim samim rotorjem, ki ima krake členkasto vpete na glavo rotorja. Helikopter je opisan z naslednjimi vstopnimi podatki: teža helikopterja *G*=45042N, količnik pokritja rotorja *s*=0,058, polmer rotorja *R*=8,1m, koeficient višine glave rotorja *h*=0,25, količnik upora *δ*=0,013, število krakov glavnega rotorja *b*=4, masa kraka *m*=79,6kg, količnik napredovanja rotorja

The program was tested on the example of a single-rotor helicopter for which the main rotor blades are tied to the hub over hinges. The helicopter is described by the following input data: helicopter weight, *G*=45042N; rotor abundance degree, *s*=0.058; rotor radius, *R*=8.1m; hub height coefficient, *h*=0.25; drag coefficient, *δ*=0.013; number of blades of the main rotor, *b*=4; blade mass, *m*=79.6kg; rotor



Sl. 10. *Shema lineariziranega matematičnega modela pri vzdolžnem gibanju*
Fig. 10. *Schematic of a linearized mathematical model when in longitudinal motion*



Sl. 11. *Shema lineariziranega matematičnega modela pri prečnem gibanju*
Fig. 11. *Schematic of linearized mathematical model when in lateral motion*

μ=0,3, vzgonski gradient profila a=5,65 1/rad, hitrost konca kraka ΩR=208m/s, masno središče kraka x_g je na 45% radija kraka R, oddaljenost členka kraka od gredi pa je 0,04R, gostota zraka na višini letenja (100m) ρ=1,215 kg/m^3. Za vzdolžno gibanje je matematični model v vektorski obliki:

operating mode coefficient, μ=0.3; gradient of lift, a=5.65; velocity of blade top, ΩR=208m/s; distance of blade mass center coefficient, x_g=0.45; distance of hinge from shaft, eR=0.04R; and air density at flight altitude (100m), ρ=1.215 kg/m^3. For longitudinal motion the mathematical model in vector form is:

$$\underline{\dot{X}} = A\ \underline{X} + b\ \underline{u}$$

$$\underline{\dot{X}} = \begin{bmatrix} -0,0509 & 0,1323 & -0,0734 & 0,00263 \\ 0,1216 & -1,2525 & 0 & 0,3 \\ 0 & 0 & 0 & 1 \\ 6,512 & 12,1 & 0 & -0,844 \end{bmatrix} \underline{X} + \begin{bmatrix} 0,1344 & 0,066 \\ 0,3578 & -0,9477 \\ 0 & 0 \\ -28,329 & 17,88 \end{bmatrix} \underline{u}$$

$$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Enačba izstopnih veličin je:

The equation at the exit is:

$$\underline{X_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underline{X} \quad \text{kjer je/where is} \quad \underline{X_i} = \begin{bmatrix} X_{i1} \\ X_{i2} \\ X_{i3} \end{bmatrix}$$

Matrika A lineariziranega modela helikopterja za prečno gibanje je:

Matrix A of the linearized model of the helicopter for lateral motion is:

$$A = \begin{bmatrix} -0,15125 & 0,0734 & 0 & 0 & 0,3 \\ 0 & 0 & 1 & 0 & 0 \\ -64,42 & 0 & -2,948 & 0 & 1,378 \\ 0 & 0 & 0 & 0 & 1 \\ 55,297 & 0 & 0,413 & 0 & -1,64 \end{bmatrix}$$

$$\underline{\dot{X}} = A\ \underline{X} + B\ \underline{u}$$

$$\underline{X} = \begin{bmatrix} X_5 & X_6 & X_7 & X_8 & X_9 \end{bmatrix}^T$$

$$\underline{u} = \begin{bmatrix} u_3 & u_4 \end{bmatrix}$$

6 SKLEPI

Da bi bil matematični model dinamike helikopterskega letenja, strogo določen, bi moral biti sestavljen iz sistema nelinearnih, neustaljenih, parcialnih diferencialnih enačb. Da bi te enačbe poenostavili, smo vzeli nekaj predpostavk. Zanemarili smo elastične lastnosti helikopterja, da bi helikopter analizirali kot togo telo in tako eliminirali razpršitev parameterov. Poleg tega smo zanemarili porabo goriva in s tem neustaljenost, ki bi se pojavila zaradi časovne spremembe mase helikopterja.

Glede na to, da ima helikopter šest prostostnih stopenj, smo zaradi poenostavitve vzeli, da se gibanje da ločiti na vzdolžno in prečno gibanje in da se ta gibanja analizirajo posamično. Povdarimo, da se matematični model helikopterja nanaša na helikoptersko premočrtno gibanje s hitrostjo W. Matematični model, ki bi obsegal vsa gibanja helikopterja, vključno z vzletom in pristankom, bi bil veliko bolj zapleten. Vpliv resonance in vibracije se prav tako zanemari. Zaostajanje kraka se tukaj tudi ne upošteva, ker drugače kotna hitrost kraka v ravnini rotacije ne bi bila več nespremenljiva. Ločene analize posamičnih gibanj krakov so velika poenostavitev, zato ker obstaja velika medsebojna odvisnost med gibanji kraka. Če bi ta gibanja ne bili ločili, bi bilo nujno analizirati stabilnost vseh gibanj kraka.

Postavljanje koordinatnega začetka v vztrajnostno središče omogoča odstranitev nekaterih vztrajnostnih momentov, tako da se

6 CONCLUSIONS

The flight dynamics mathematical model of a helicopter that would be strictly determined would comprise a system of non-linear, non-stationary, partial differential equations. To simplify these equations we introduce a number of assumptions. Ignored are the elastic characteristics of the helicopter so the helicopter can be thought as a rigid body and, in this way, the dispersal of parameters is eliminated. Also, fuel consumption is disregarded and so is the non-stationarity due to the temporal change in helicopter mass being eliminated.

Because the helicopter has six degrees of freedom, for simplification it is assumed that the motion can be separated into longitudinal and lateral motions and that they can be investigated independently. It should be noted that the mathematical model of the helicopter relates to the helicopter's forward motion at velocity W. A mathematical model that would incorporate all the motions of a helicopter, all together with takeoff and landing, would be far too complicated. The influence of resonance and vibration is also ignored. The blade throughback is also ignored in this paper, because if this was not the case the blade-angle velocity in the plane of rotation would no longer be constant. A separate study of the individual motions of blades is a great simplification, because there is an interdependency of all the blade motions. If the motions are not separated, then it is necessary to analyze the stability of all the motions of the blade.

The choice of the coordinate origin in the center of inertia makes it possible to eliminate certain moments of inertia so the Euler equations can be

Eulerjeve enačbe lahko poenostavijo. Opazovanje rotorja kot celote odstrani potrebo po preučevanju posameznih gibanj krakov. V tem je v veliko pomoč uvajanje osi rotorskega diska in osi v smeri vlečne sile rotorja.

Določanje aerodinamičnih odvodov je povezano z vrsto približkov. Treba je poudariti, da poleg predpostavk pri modeliranju, uporabljamo tudi matematične poenostavitve (na primer, izpuščanje zanemarljivo majhnih vrednosti iz enačb), ki jih ni mogoče prikazati v obliki predpostavke, ker je njihov pomen tesno povezan z določeno enačbo.

Kot izstopne značilnosti je mogoče določiti projekcije vektorja lege v nepremičnem koordinatnem sistemu, povezanem s tlemi namesto projekcij hitrosti helikopterja v premičnem koordinatnem sistemu. Ta problem bi se rešil s projiciranjem hitrosti helikopterja na nepremični koordinatni sistem, nakar bi se integrirale projekcije hitrosti po času z začetnimi pogoji.

Nadaljnja analiza matematičnega modela se lahko usmeri na raziskovanje dinamičnih in statičnih lastnosti in na določanje primernega krmarjenja, ki bi helikopterju zagotovilo zahtevano dinamično obnašanje.

simplified. Viewing the rotor as a whole eliminates the need for investigating the motion of an individual blade. This is made much simpler by the introduction of the rotor disc axis and the control axis.

The determination of the aerodynamic derivatives is related to a series of approximations. It should be noted that, besides assumptions in the modeling, mathematical simplifications were also made (for example, omitting small values in the equations) which could not have been derived in the form of an assumption due to their meaning, which is tightly related to a specific equation.

It is possible to determine projections of the position vector with respect to the non-moveable coordinate system tied to Earth instead of using projections of the helicopter's velocity with respect to a moveable coordinate system such as the exit characteristics. Projecting the helicopter velocity onto a non-moveable coordinate system and then integrating the velocity projections over time with the initial conditions may solve this problem.

A further analysis of the mathematical model can be made in order to investigate the dynamic and static properties, and to determine the control that would guarantee the object to execute the required dynamic behavior.

8 LITERATURA
8 REFERENCES

[1] W. Z. Stepniewski (1984) Rotary-wing aerodynamics, New York.
[2] M. Nenadović (1987) OAK – Elise i propeleri, Beograd.
[3] M. Nenadović (1978) OAK – Aeroprofili - I. deo, Beograd.
[4] M. Nenadović (1977) OAK – Opšti deo, Beograd.
[5] M. Nenadović (1982) Osnovi projektovanja i konstruisanja helikoptera, Beograd.
[6] A. R. S. Bromwell (1976) Helicopter dynamics, London.
[7] W. Johnson (1980) Helicopter theory, London.
[8] G. Saundres (1972) Teorija leta helikoptera [prevod sa engleskog], Beograd.
[9] Gesov & Myers (1952) Aerodynamics of the helicopter, New York.
[10] A. K. Martinova (1973) Teorija nesuščego vinta, Moskva.
[11] M. L. Mil' (1973) Vertoljoti, Moskva.

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