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STATE SYSTEM OF PLAIN RECTANGULAR COORDINATES (Draft proposal for Republic Slovenia)

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Abstract

For the state system of plain rectangular coordinates a draft proposal of Gauss-Krueger coordinates is given. Geometrically the Gauss-Krueger system is based on GKP widened meridian zones. To achieve greater use of coordinate records the change of coordinate origin is suggested. The UTM (Universal Transverse Mercator) should serve as a connection with Europe.
Keywords: *Gauss-Krueger projection, proposal, Slovenia, system of plain rectangular coordinates, UTM*

INTRODUCTION

The technical term „system of plain rectangular coordinates" contains surveying-cartographic projection and system of plain rectangular coordinates, which is based on projection as a geometrical basis. The article deals with:

- basic projection and system of plain rectangular coordinates for the territory of Republic Slovenia and,
- system for connecting the basic Slovene system with Europe.

Geographical position of Republic Slovenia

If for adjacent meridians and parallels the external margins of sheets TK 25 are taken, Slovenia then occupies the following geographical position:

$$\lambda_W = 13°22'30" \qquad \varphi_N = 46°52'30"$$
$$\lambda_E = 16°37'30" \qquad \varphi_S = 45°22'30"$$
$$\lambda_E - \lambda_W = 3°15' \qquad \varphi_N - \varphi_S = 1°30'$$

Projection position of Republic Slovenia

- The position in the Gauss-Krueger projection

Medium meridian of the system: $\lambda_o = 15°$
Width of the meridian zone: $3°15'$
$\lambda_o - \lambda_W = \lambda_E - \lambda_o = 1°37'30"$

- The position in the UTM system (Universal Transverse Mercator)

Medium meridian of the system: $\lambda_o = 15°$
Width of the meridian zone: $6°$
$\lambda_o - \lambda_1 = \lambda_2 - \lambda_o = 3°$

where $\lambda_1 = 12°$, $\lambda_2 = 18°$ are the adjoining meridians of the six-degree zone.
Zone number: 33

BASIC PROJECTION OF REPUBLIC SLOVENIA AND THE STATE SYSTEM OF PLAIN RECTANGULAR COORDINATES

The basic projection for all surveying calculations, cadastral and topographic survey and cartographing of the entire territory of Republic Slovenia is the Gauss-Krueger projection of the meridian zone 3°15' longitude. The fact is that $\lambda_o - \lambda_W = \lambda_E - \lambda_o = 1°37'30''$ still enables, that after scale reduction ($m_o = 0{,}9999$) the mapping keeps 1:10 000 accuracy on the entire territory.

The state system of plain rectangular coordinates is a system of Gauss-Krueger coordinates, which is geometrically based on Gauss-Krueger projection of the meridian zone. It is meant for uniform defining the position of points on the plane on the entire territory of Slovenia. The system is formed by two coordinate axes, intersecting under right angle in the origin of the coordinate system. The vertical axis or axis of abscissae (X-axis) is a projection of the medium meridian of the zone in GPK. The horizontal axis or axis of ordinates (Y-axis) is a projection of equator in GPK or according to the given specification: the axis, which is at $\varphi_o \neq 0°$ longitude the parallel projection of equator. The geographical coordinates of the origin are therefore:

$$\lambda_o = 15°,\ \lambda - \lambda_o = 0°,\ \varphi_o = 0°$$

or after the modification

$$\lambda_o = 15°,\ \lambda - \lambda_o = 0°,\ \varphi_o > 0°.$$

The rectangular coordinates of the origin are:

$y_o = 0{,}0$ m, conditionally 500 000 m
$x_o = 0{,}0$ m.

As opposed to geographical coordinates φ and λ on ellipsoid the system of plain rectangular Gauss-Krueger coordinates is isometric and equals the Descartes system of coordinates.

PROJECTION SYSTEM FOR CONNECTING WITH EUROPE

For location connecting on the territory of Republic Slovenia with Europe the UTM system should be used. It contains:

- Meridional Mercator (Gauss-Krueger) projection of 6° meridian zones, and
- UTM rectangular coordinate net, which is based on Meridional Mercator projection as a geometrical basis.

Note: Naturally, a direct connection with Europe is impossible without previous connection of astronomy-surveying nets or transformation by aid of mutual data.

General specifications for UTM

Zone margins: the UTM zones are limited by meridian n. 6° east or west of Greenwich, where is n = 0,1,2, ... 30.

Origin of geographical coordinates: $\lambda - \lambda_o = 0°$, $\varphi_o = 0°$, where λ_o is the longitude of the medium meridian zone.

Origin of rectangular coordinates:

y_o = 0,0 m, conditionally 500 000 m

x_o = 0,0 m, for northern hemisphere

Parallel margins of the system: 84°N, 80°S,

Zone marking: from 1-60, beginning by 180° W

Scale module on medium meridian: $m_o = 0{,}9996$

GAUSS-KRUEGER (MERIDIONAL MERCATOR) PROJECTION EQUATIONS

The most practical form of computer oriented equations is still a series of polynome approximations. Although there is a tendency to come to a so-called „closed" equations, which could be most effective for computer processing, the results are still unsatisfactory. The so-called American form of equations enables the demanded degree of accuracy for the zone up to ±4° of the medium meridian. By using narrower zones and at our latitudes some articles may be left out:

$$x = m_o \{B - B_o + N \operatorname{tg}\varphi [A^2/2 + (5 - T + 9C + 4C^2) A^4/24 + (61 - 58T + T^2 + 600C - 330e'^2) A^6/720] \} \quad (1)$$

$$y = m_o N [A + (1 - T + C) A^3/6 + (5 - 18T + T^2 + 72C - 58\, e'^2) A^5/120] \quad (2)$$

$$m = m_o [1 + (1 + C) A^2/2 + (5 - 4T + 42C + 13C^2 - 28e'^2) A^4/24 + (61 - 148T + 16T^2) A^6/720] \quad (3)$$

in which there are:

m_o = scale module on the medium meridian (0,9999 for GKP, 0,9996 for UTM)

$$e'^2 = e^2/(1 - e^2) \quad (4)$$

$$N = a/(1 - e^2 \sin^2\varphi)^{1/2} \quad (5)$$

$$A = (\lambda - \lambda_o) \cos\varphi, \text{ with } \lambda \text{ and } \lambda_o \text{ in radians.} \quad (6)$$

$$T = \operatorname{tg}^2\varphi \quad (7)$$

$$C = e'^2 \cos^2\varphi \quad (8)$$

$$B = a [(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...) \varphi - (3e^2/8 + 3e^4/32 + 45e^6/1024 + ...) \sin 2\varphi + (15e^4/256 + 45\, e^6/1024 + ...) \sin 4\varphi - (35e^6/3072 + ...) \sin 6\varphi + ...] \quad (9)$$

with φ in radians.

B is the distance along the medium meridian from equator to φ latitude.

B_o is B calculated for φ_o, width of the parallel, which intersects the medium meridian λ_o at the origin of y and x coordinates. Namely, there isn't always the case that the origin of coordinates in the Meridional Mercator projection would necessarily lie on the equator, but it is adapted to the area of the mapping. In the GPK case in our present use and six-degree UTM zones, $B_o = 0$.

The equation (3) for the m scale can be written also as a function of y and φ :

$$m = m_o [1 + (1 + e'^2 \cos^2 \varphi) y^2 / (2m_o^2 N^2)] \quad (3a)$$

Equations for the reverse task:

$$\varphi = \varphi_1 - (N_1 \operatorname{tg} \varphi_1/M_1) [D^2/2 - (5 + 3T_1 + 10C_1 - 4C_1^2 - 9 e'^2) D^4/24 + + (61 + 90T_1 + 298C_1 + 45T_1^2 - 252e'^2 - 3C_1^2) D^6/720] \quad (10)$$

$$\lambda = \lambda_o + [D - (1 + 2T_1 + C_1) D^3/6 + (5 - 2C_1 + 28T_1 - 3C_1^2 + 8e'^2 + + 24T_1^2) D^5/120]/\cos \varphi_1 \quad (11)$$

where the φ_1 is the point width on the medium meridian, which has the same value of abscissae x as the φ, λ point. The equation for calculation is as follows:

$$\varphi_1 = \mu + (3e_1/2 - 27 e_1^3/32 + ...) \sin 2\mu + (21e_1^2/16 - 55e_1^4/32 - ...) \sin 4\mu + + (151e_1^3/96 + ...) \sin 6\mu \quad (12)$$

in which where

$$e_1 = [1 - (1 - e^2)^{1/2}] / [1 + (1 - e^2)^{1/2}] \quad (13)$$

$$\mu = B/[a(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...)] \quad (14)$$

where

$$B = B_o + x/m_o \quad (15)$$

with B_o calculated by equation (9) for the given φ_o. If the origin of rectangular coordinates is at the point $\varphi_o = 0°, B_o = 0$.

$$e'^2 = e^2/(1 - e^2) \quad (4)$$

$$C_1 = e'^2 \cos^2 \varphi_1 \quad (16)$$

$$T_1 = \operatorname{tg}^2 \varphi_1 \quad (17)$$

$$N_1 = a/(1 - e^2 \sin^2 \varphi_1)^{1/2} \quad (18)$$

$$M_1 = a(1 - e^2)/(1 - e^2 \sin^2 \varphi_1)^{3/2} \quad (19)$$

$$D = y/(N_1 m_o) \quad (20)$$

Note: Except by the equation (12), by introducing the auxiliary, so-called rectified width φ_1, the width μ, can be calculated also by the equation (9) for the given x. The interaction procedure is described in reference (Borčić 1976).

ALLOWABLE GAUSS-KRUEGER ZONE WIDTH AND DEFORMATIONS

Under conditions $m_o = 0{,}9999$ and $d \leq 0{,}0001$, the latitudes of Republic Slovenia the following allowable widths $\lambda - \lambda_o$ can be obtained:

φ	45°22'30"	46°00'00"	46°52'30"
$\lambda - \lambda o$	1°37'43"	1°38'49"	1°40'25"

This table shows that in the zone 3°15' latitude the accuracy 1:10 000 is maintained. On new margin meridians $\lambda - \lambda_o = \pm 1°37'30"$ for the same latitudes the following values of m scale e.g. relative deformations d are obtained:

φ	45°22'30"	46°00'00"	46°52'30"
m	1,000 099	1,000 095	1,000 088
d	1 : 10 100	1 : 10 500	1 : 11 400

MODIFICATION OF RECTANGULAR COORDINATES

To obtain greater expediency of plain rectangular coordinates records, one of the following modifications is possible:

a) Since on the entire territory of Republic Slovenia all $\varphi > 45°$ the origin of the system for RPK listings is moved from the point $\lambda_o, \varphi_o = 0°$ to point $\lambda_o, \varphi_o = 45°$. The origin of abscissa is not measured from the equator but from the point, in which the parallel $\varphi_o = 45°$ intersects the projection of the medium meridian. Abscissae are calculated by equation (1), in which B_o is the length of the medium meridian from equator up to $\varphi_o = 45°$ latitude. The transition from old (previously calculated) into new abscissae is carried out by equation

$$x = x_t - m_o B_o,$$

where x_t is the old abscissa and $m_o B_o$ the calculated constant.

b) Due to the fact that in the present system all abscissae in Republic Slovenia are greater than 5 000 000 meters, the starting point of the system is moved to the point on the medium meridian, for which

$$B_o = 5\,000\,000/m_o,$$

which corresponds to the order of magnitude B_o for $\varphi_o \approx 45°09'$. With this order of magnitude B_o abscissae are calculated by equation (1) and the reverse task in the equation (15) is solved. The transition from old to new abscissae is done by equation (21), in which

$$m_o B_o = 5\,000\,000.$$

The modification a) is used in 22 USA states. As the surveying and topographic projection they use TM (GKP) and on it based system of plain rectangular UTM coordinates. E.g. for Arizona $\varphi_o = 31°00'$, Georgia 30°00', Idaho 41°40', etc. Although the modification a) – due to its very simple transition from the present into the modified system of Gauss-Krueger coordinates – has already been proposed also

to be used for Republic Slovenia (Peterca 1989), the modification b) is more suitable for Republic Slovenia. Unless the value m_oB_o in a) is a round number, in the case b) all already calculated abscissae are depreciated for 5 000 000 meters. As opposed to a), this change does not alter the position of the existing trigonometric sections and division on sheets from 1:10 000 to 1:500, for which the frames of sheets are lines of the rectangular net. Besides, in 1950 in the USA they have begun introducing a completely new system and so instead of the Polyconic projection they introduced the use of TM and the Lambert conformal conical projection and on these projections based systems of plain coordinates. In Slovenia there is not a question of changing the system but only its modification. Since the entire territory of Republic Slovenia would fall into one coordinate system e.g. one Gauss-Krueger zone, in ordinates value listings the number 5, which has been so far used to mark the zone number, could be left out.

Example of Marking Gauss-Krueger coordinates

The old mode:	y = 5 576 979,6	New mode:	576 979,6
	x = 6 132 590,1		132 590,1

Description of the rectangular net on topographic maps:

Old mode:	55 77	51 33
New mode:	5 77	1 33

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