UDK/*UDC*: 519.87:627.13(282)(497.6)
Izvirni znanstveni članek – *Original scientific paper*
DOI: 10.15292/acta.hydro.2024.10
Prejeto/*Received:* 29.01.2025
Sprejeto/*Accepted:* 04.03.2025
Objavljeno na spletu/*Published online:* 24.03.2025

ESTIMATING THE DEFORMATION OF A RIVER BED USING DIMENSIONLESS GEOMETRIC PARAMETERS AND MATHEMATICAL MODEL

OCENA DEFORMACIJE REČNE STRUGE Z UPORABO BREZDIMENZIJSKIH GEOMETRIJSKIH PARAMETROV IN MATEMATIČNEGA MODELA

Nerma Lazović[1], Mario Krzyk[2]

[1]Faculty of Civil Engineering, University of Sarajevo, Patriotske lige 30, 71000 Sarajevo, Bosnia and Herzegovina
[2]Fakulteta za gradbeništvo in geodezijo, Univerza v Ljubljani, Jamova cesta 2, 1000 Ljubljana, Slovenia

Abstract

The river channel cut into an alluvial substrate constantly adapts to changes in water flow and sediment transport. This paper presents a new approach to determine the magnitude of channel deformation based on the definition of dimensionless spatial parameters of channel deformation. The new approach's applicability is evaluated by experimental studies on the Željeznica River in the Sarajevo Field in central Bosnia and Herzegovina. The morphological changes of the channel over a period of ten years were analyzed using dimensionless parameters of the channel geometry. A numerical analysis of changes in the channel of the Željeznica River was carried out using a mathematical model and a selected equation for sediment transport. Sensitivity analysis of the model parameters for channel deformation and statistical reliability analysis of the numerical model showed a good agreement between the modeled and observed values of channel deformation parameters during the analyzed period.

Keywords: river morphology, dimensionless geometric parameters, river regime, sediment transport, channel deformation, HEC-RAS, Željeznica River, model reliability.

Izvleček

Struga reke, vrezana v aluvialno podlago, se nenehno prilagaja spremembam vodnega toka in transporta plavin. V prispevku je predstavljen nov pristop k določanju velikosti deformacije struge, ki temelji na opredelitvi brezdimenzijskih prostorskih parametrov deformacije struge. Ocena uporabnosti novega pristopa temelji na eksperimentalnih študijah na reki Željeznici na območju Sarajevskega polja v osrednjem delu Bosne in Hercegovine. Morfološke spremembe struge v desetletnem obdobju so analizirane na podlagi brezdimenzijskih parametrov geometrije struge. S pomočjo matematičnega modela in izbrane enačbe za transport plavin je izvedena numerična analiza sprememb v strugi reke Železnice. Analiza občutljivosti parametrov modela deformacije rečne struge in analiza statistične zanesljivosti numeričnega modela sta

[2] Stik / *Correspondence*: mario.krzyk@fgg.uni-lj.si

pokazali dobro ujemanje med modeliranimi in opazovanimi vrednostmi deformacije rečne struge v analiziranem obdobju.

Ključne besede: rečna morfologija, brezdimenzijski geometrijski parametri, rečni režim, transport plavin, deformacija struge, HEC-RAS, reka Željeznica, zanesljivost modela.

1. Introduction

Investigating the deformation of river channels, understanding the processes that influence deformation and assessing the stability of river banks have long been areas of research for geologists and engineers. Classical contributions laid the foundation for today's studies on and practices regarding river channel deformation and management. The first investigations began in the second half of the nineteenth century (Gilbert, 1877; du Boys, 1879; Davis, 1889; Lindley, 1919; Shields, 1936; Lane, 1937; Mackin, 1948; Leopold and Maddock, 1953; Schumm, 1969) (Singh, 2003). The theories of hydraulic geometry, which describe the relationships between river channel features and water flow, were significantly advanced by the work of Leopold and Maddock (Buffington, 2012). These theories help predict how changes in water flow and sediment load affect the geometric characteristics of river channels. The integration of geomorphology and hydraulic engineering has been crucial to the understanding of river processes. This interdisciplinary approach has become widely accepted in modern river engineering.

Rivers change their channel geometry (longitudinal profile, cross-sections and channel shape in horizontal projection) depending on the geological, hydrologic and hydraulic characteristics of the watercourses and the catchment area, as well as on the characteristics of the sediments (volume, material properties of the sediments, granulometric properties). Natural and anthropogenic factors influence fluvial processes and riparian morphology (Figure 1). Flow regime and sediment quality, along with valley characteristics, are among the most important natural factors determining boundary conditions. (Ibisate et al., 2011). Natural processes can proceed extremely slowly and be practically imperceptible to humans (Minh Hai et al., 2019, Simon, 1989). In contrast, human activities such as damming rivers, building reservoirs, constructing dikes, and dredging waterways can significantly and rapidly affect natural processes and trends. In these cases, the time scale for river adaptation is reduced (Minh Hai et al., 2019, Rinaldi and Simon, 1998, Zheng et al., 2019). The interaction between these parameters or processes leads to new river dynamics and forms (Lane, 1955).

The precipitation regime, the frequency and intensity of precipitation, and, in particular, extreme events and periods of drought determine runoff formation and erosion capacity, and thus sediment input and transport. The type and density of vegetation, soil type and land use depend on climatic conditions, which influence runoff dynamics and sediment movement and transport (Ibisate et al., 2011).

Natural streams that are cut into the alluvial bedrock undergo the following processes: deposition of bedload and suspended sediments at low discharge and destabilisation of the riverbed and the deposited suspended sediments by their reintegration into the flow at high discharge. The local hydrodynamic conditions of the stream and the characteristics of the sediments can cause various combinations of processes. Depending on the stream's current hydrologic condition, one of these processes may be more pronounced than the others, but always with the additional influence of other parameters that make analyzing the processes even more difficult.

Over time, the methodology for calculating the parameters that characterize the sediment transport has evolved. From the first, rather crude predictions based on empirical and semi-empirical equations, mathematical models are now being developed that should be able to predict the concentration of suspended sediments and the balance of load in space and time using basic differential equations. Recently, hydro-morphodynamic processes in river channels have been studied in detail using numerical models (Krzyk, 1997, Martínez-Aranda, 2019, Mahdizadeh and Sharifi, 2021, Stipić et al., 2022, Peng et al., 2022, Chang et al., 2024). Considering that the changes in the physical properties of sediments due to mechanical forces are negligible, sediments behave like conservative materials. However, they should be considered as non-conservative materials because their concentration varies spatially and temporally, depending on the conditions for the deposition or flushing of the material from the bottom and its displacement (Krzyk, 1997).

There are various calculation and measurement methods for determining sediment transport. Despite the availability of mathematical models, calculation and measurement devices, it is still difficult to accurately determine the amount of transported material and the changes in the configuration of a channel's bed. The calibration of larger and more accurate mathematical models also requires a range of input data that can only be obtained through more complex and accurate measurements. The development of suitable measurement techniques is therefore also important. Furthermore, the experience of other researchers with similar problems shows that the process of calibrating mathematical models is very time-consuming. In some cases, this process has taken more than ten years. Given the quality of the available measurement data on the hydrodynamic characteristics of flow, sediment transport, turbidity and bedload changes, it makes sense to select the complexity of a mathematical model. This makes it possible to avoid the use of parameters for which there is no validated data but for which more complex mathematical models are required.

Many researchers have endeavored to equate the morphological responses of rivers with the corresponding driving factors. Minh Hai et al. (2019) established an empirical model by evaluating the spatiotemporal long-term variations of morphological characteristics of the lower reaches of the Tedori River in Japan. The results showed that sediment removal and dam construction strongly influenced the morphological features of the lower Tedori River and that the river channel's vertical adjustment went through five phases.

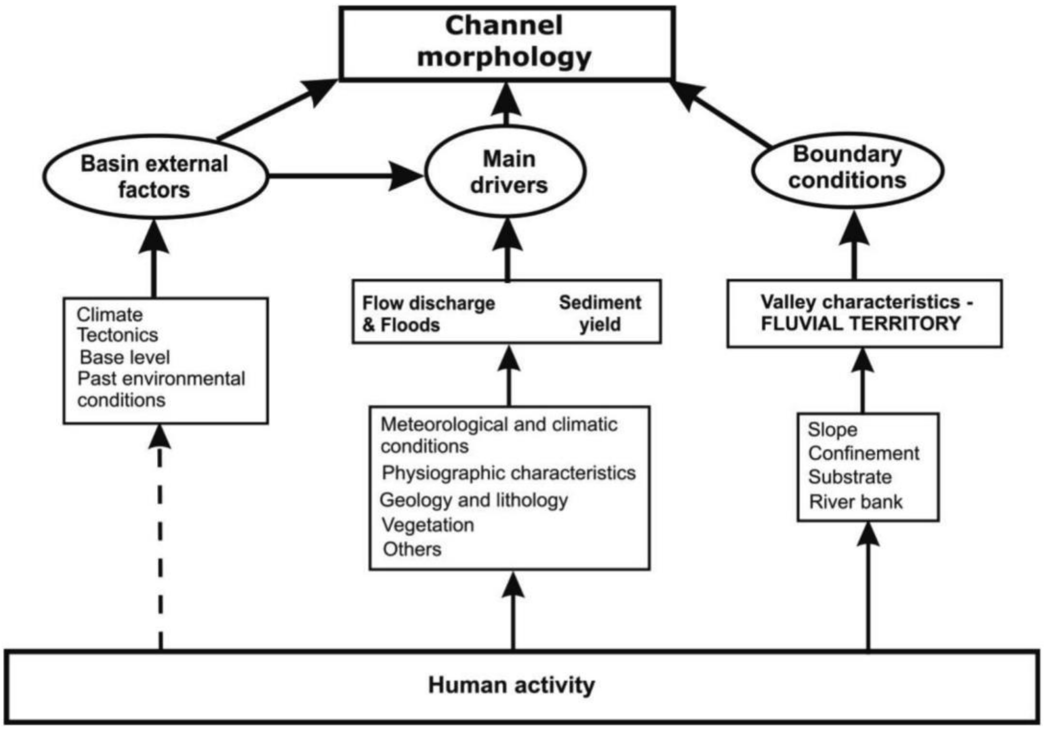


Figure 1: *The interaction of the main factors on the morphology of the riverbed (Ibisate et al., 2011).*

Slika 1: *Medsebojni vpliv glavnih dejavnikov na morfologijo rečne struge (Ibisate et al., 2011).*

Tang et al (Tang et al., 2021) analyzed the long-term (1977–2006) riverbed deformation in the Lower Atchafalaya River, the largest tributary of the Mississippi River. The riverbed is degraded in its upper part, which is mainly influenced by the frequency of floods. The riverbed grows larger mainly in the last 35 km of its course, although less sediment is delivered there. Zheng et al. (2019) used data from four decades (1976–2015) of systematic surveys of water discharge, sediment load, cross-sectional profiles, and water surface elevation. They showed that the backwater zone influences the development and discharge of the Qingshuigou Channel, a recent bulge in the Yellow River delta. Rahman et al. (2024) analyzed the characteristics of riverbed deformation, bank erosion, and channel changes in the lower Sangu River basin using a depth-averaged 2-D flow model in which suspended sediment transport dominates and flow characteristics are influenced by active tides. These studies have clarified the characteristics and mechanisms of morphodynamics in the lower Sangu catchment, particularly the potential of the combination of tides and floods to enhance riverbed deformation and associated bank shifts. Ma et al. (2022) investigated the morphological evolution characteristics based on annual comprehensive cross-sectional data and water and sediment data from the Xianyang, Lintong, and Huaxian hydrological stations in the lower Weihe River from 2006 to 2018, as well as the response to water and sediment fluctuations. The results show that large floods impact the channel’s cross-sectional morphology. The fluctuations in the meander elevation of the main channel in the lower Weihe River are closely correlated with changes in the inflowing sediment coefficient. In addition, human activities such as river development projects can also have a direct impact on the cross-sectional morphology.

Information on historical morphological changes is useful for understanding the evolutionary trends and condition of rivers. The integration of remote sensing and GIS techniques has contributed significantly to investigations into rivers’ morphological parameters by analyzing geospatial data (Beyene et al., 2023, Ibitoye, 2021, Momin et al., 2022). Langat et al. (2020) characterized the planform changes and floodplain dynamics of the Tana River system in Kenya between 1975 and 2017 using an integrated methodological framework of remote sensing and geographic information system (GIS). The river has undergone significant changes in its channel morphology over this period of 42 years, indicating a very active and meandering nature with a generally increasing tortuosity in the downstream direction. After the river regulation, which took place between 1981 and 1988, the annual lateral movement of the river decreased considerably. Andualem et al. (2024) developed a new framework for studying changes in overall channel morphology by using very high-resolution aerial imagery and a LiDAR-derived digital elevation model (DEM). By digitizing channel boundaries with ArcGIS Pro 3.0 and analyzing various morphological parameters as well as erosion and deposition patterns, they investigated the impact of urban expansion and infrastructure development on channel adjustments. Đorđević et al. (2023) investigated the relationship between the observed decline in water level and the incision of the riverbed recorded at the gauging stations along a length of around 300 km on the Middle Danube. The study found that in the absence of systematic monitoring of sediment transport, regularly surveyed cross-sections of the navigable river can be used to estimate the long-term averaged sediment transport, i.e. to infer the average rates of channel aggradation and degradation along the reach.

Considering the increasing frequency of flood events due to climate change and the exposure of watercourses to anthropogenic activities, understanding the magnitude of morphological changes in river channels is of great importance for effective water management and an invaluable tool for decision makers in the water management process.

The paper presents new dimensionless parameters for the geometric characteristics of the river channel using the example of the Željeznica River in the Sarajevo Field region in Bosnia and Herzegovina. Due to the nature of the Želježnica River, which has a certain torrential character, its high discharges threaten the surrounding settlements with flooding.

The Želježnica River is an important source for filling the underground reservoirs used for Sarajevo's water supply. Therefore, it is very important to understand the morphological changes of its riverbed.

2. Analysis of the changes in riverbed geometry

2.1 Riverbed spatial deformation

The analysis of morphological changes in river channels involves a comprehensive investigation of various parameters that influence channel dynamics. Parametric analysis focuses on quantifying aspects such as channel depth, width, and cross-sectional area to understand the riverbed's evolution over time. Deepening of the channel, often caused by increased flow velocities or sediment transport, can lead to significant changes in cross-sectional area and influence hydraulic conditions. In addition, changes in the river's horizontal alignment, such as meander migration or bank erosion, are crucial for assessing the river system's stability. Topographic surveys are becoming increasingly affordable and are possible with higher spatial resolution and over larger spatial extents. The digital elevation models (DEMs) created from such surveys are used to create DEM of Difference (DoD) maps and to estimate the net change in storage conditions for morphological sediment budgets.

2.2 Numerical modelling

The goal of river modelling is to predict a river's hydraulic behavior over a wide range of hydraulic conditions. Water flow and sediment transport in a natural river are part of a very complex natural process that is unstable, irregular, and highly turbulent, with characteristics that are constantly changing and varying in time and space. The hydraulic roughness of the river channel's wetted contour also varies, and there is a multiphase composition to the flow (biphasic: mixture of water and sediment, or triphasic: mixture of water, sediment, and air). Due to the geometric characteristics, the solution to the equations describing the flow of water and substances can often be simplified in practice. The most common simplifications are the two-dimensional (2D) approach, where the problem is considered in the vertical or horizontal plane (as a width- or depth-averaged model), and the one-dimensional (1D) approach (Krzyk, 1997, Krzyk and Četina, 2003).

Erosion processes in rivers are 3D phenomena and their intensity depends on the water's local flow velocities, which are homogeneous neither in the lateral nor in the vertical direction. The question is how this inhomogeneity affects the results of hydrodynamic and sediment transport models. Laboratory experiments were carried out at the Hydraulic Engineering Institute of the University of Innsbruck and then simulated with numerical 1D, 2D, and 3D models (Glock et al., 2019) to obtain measurement data from the laboratory experiment. A standardised, relative comparison of the models shows that, after successful calibration to measured water levels, the corresponding 2D/1D and 3D/1D ratios are almost unity, while the bed shear stresses in the 3D models are about 62-86% of the simulated 1D values and 90-100% in the case of 2D/1D. In the studies of morphological changes of the Željeznica river channel, a separate calibration procedure was performed for the hydrodynamic parameters and sediment transport. For this reason, the (1D) hydraulic mathematical model used can be justified. This approach has been proven for decades in numerical modelling of water and sediment flow for cases where the length of the reach is 20 times or more the channel width and where the transverse variations in flow velocity and water depth are not significant (Spasojević, 1996).

Deformation of river channels is a spatially three-dimensional phenomenon whose intensity largely depends on local velocities that are not homogeneous in either the transverse or vertical direction. However, since the main aim of this study was to develop a simple method that facilitates analysis of the general trend of riverbed deformation, which could be applied to a wider area and based on available data, the 1D hydraulic model was chosen.

In hydraulic terms, applying a 1D model is justified on longer river sections where the cross-section length is 20 or more times greater than the channel width and where transverse changes in flow velocity and water depth are not significant (Spasojević, 1996).

The flow velocity is the fundamental parameter for mass transport in the form of a solution or solid particles. Therefore, precise knowledge of the hydrodynamic parameters such as water velocity and water depth is essential. Only when the hydrodynamic situation has been accurately calculated and confirmed by measurements can the modelling of mass transport begin. Based on the measurements, we need to determine the actual values of the coefficients in the transport equation (turbulent diffusion coefficients) that describe the movement of sediments, as it is often difficult to determine the order of these coefficients' magnitude from the literature alone (Krzyk, 1997).

In the analyzed example of the Željeznica River section, a one-dimensional approach was used to solve the problem due to the river's geometric characteristics with a significantly greater length of the section compared to the width of the riverbed and the depth of the river. In linear (1D) hydraulic calculation models, the space is limited to the main axis of the river and the variation of the relevant parameters in the orthogonal direction to the river axis (i.e. in the river's cross-sectional profile) is ignored. The calculation results are determined in each cross-sectional profile of the river: water level, mean profile velocity, etc. Considering the higher flow resistance due to vegetation in inundations compared to the main riverbed, the concept of a complex riverbed cross-section is normally used for hydraulic calculation in line flow models. This concept means that the calculation is performed with three adjacent 1D flows: in the basic riverbed, to the left, and to the right inundation.

The numerical modelling was carried out using the HEC-RAS model (Chang, 1988). Two modules of the HEC-RAS model were used: the hydrodynamic module (HD) and the transport module (ST). In the HD module, the equations of one-dimensional (1D) unsteady flow in a non-prismatic channel, which describe the laws of conservation of mass and momentum (Saint-Venant equations), are solved numerically using an implicit finite difference scheme. The Preissmann scheme was used to calculate the wave propagation in a watercourse. The ST module uses the results of the HD module as a basis. The flow duration represents the duration of each constant flow value, which is further divided into calculation time steps, where for each time step the hydraulic volumes and sediment properties are calculated for each cross-section. Although the flow is constant over the time step of the flow duration, HEC-RAS calculates the channel geometry and hydrodynamic parameters after each time step. If the geometry changes rapidly and sudden erosion or sediment deposition occurs, the model may become unstable and the calculation time step must be shortened. The calculation time step is divided into calculations of the sediment transport capacity according to the granulometric properties of the sediments in the active layer. In one calculation step, the hydraulic properties and the transport potential are constant, but the transport capacities for different sediment grains vary, which leads to changes in the geometry of the cross-sections. The HEC-RAS software allows the user to select an empirical equation to calculate the sediment transport capacity. The equations available are Ackers–White, Engelund–Hansen, Laursen (Copeland), Meyer-Peter and Muller, Toffaletti, and Yang and Wilcock (Chang, 1988).

The mathematical modeling of the general deformation of the riverbed is based on the numerical solution of the system of four equations (1)–(4): the continuity and momentum equation, continuity equation for the sediment and selected equations for sediment transport with the given initial and boundary conditions (Chang, 1988).

$$\frac{\partial Z}{\partial t} + \frac{1}{B}\frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial Z}{\partial x} + g\,A\,I_e = 0 \tag{2}$$

$$(1-\lambda_0)\frac{\partial A_d}{\partial t} + \frac{\partial Q_v}{\partial x} = 0 \tag{3}$$

$$q_v = q_v(Z, Q) \tag{4}$$

The equations of this system are linked by the dependent variables $Z(x,t)$ and $Q(x,t)$ and the elevations of the bed $Z_d(x,t)$, which are implicit in the geometry of the profile and the surface $A_d(x,t)$, where:

$Z(x,t)$ and $Q(x,t)$ are the water level and the discharge in space and time,

$A(x,t)$ is the cross-sectional area, and

$A_d(x,t)$ is the cross-sectional deformation area.

The change in the area (A_d) in the time interval Δt is obtained by solving the continuity equation for the sediment:

$$(\Delta A_d)_i = \\ = -\frac{\Delta t}{1-\lambda_0} \cdot \frac{\left[\frac{Q_v^k + Q_v^{k+1}}{2}\right]_i - \left[\frac{Q_v^k + Q_v^{k+1}}{2}\right]_{i-i}}{\Delta x_{i-1}} \quad (5)$$

The type of deformation is determined by the sign of the calculated change:

$(\Delta A_d)_i > 0$ sedimentation and

$(\Delta A_d)_i < 0$ erosion.

In the mathematical model used, the system of equations is solved step by step: first, the hydrodynamic equations are solved to obtain the water depths and velocities of the analyzed flow, and then the sediment transport is calculated in all profiles on the same time scale using the chosen formula Qv = Qv (Z, Q). Finally, the Exner equation is applied to determine the change in profile geometry through the values of ΔAd (or Δzd). The results obtained describe the propagation of the water waves and the "sediment waves".

The results of the numerical calculations carried out with the mathematical model were used as the basis for analyzing the morphological changes of the river channel in the lower reaches of the Željeznica River. The deepening and widening of the riverbed were identified as changes to the riverbed in the section under consideration.

3. Case study

3.1 Location

On the lower course of the Željeznica River in the Sarajevo area, which is characterized by an unstable and changing riverbed, studies were carried out on the riverbed's deformation. The 10-year period from 2009 to 2019 was analyzed. The Sarajevo Field, located at the foot of the Igman and Bjelašnica mountains in central Bosnia and Herzegovina (Figure 2), plays an important role in Sarajevo's water supply. The Željeznica River is the main source enriching the underground water reservoir. Morphological changes in the Željeznica River influence the amount of water infiltrated into the ground. Vertical movements of the riverbed have the greatest impact (Spasojević, 1996, Lazović et al., 2023).

3.2 Hydrologic data

The closest gauging station to the Željeznica section under consideration is the Ilidža gauging station, which is located 200 m downstream from the area under investigation. Hourly water level observations were carried out at the Ilidža gauging station in the period from 2009 to 2019. The hourly discharge values were determined based on the established flow curves. These data were used for statistical data processing, in which the maximum, minimum, average daily and monthly discharges, and the flow duration curve were determined.

The average annual discharge of the Željeznica River at the Ilidža gaging station is 8.65 m^3/s for the period 2009–2019. The maximum discharge was measured on 01 December 2010 at 8 p.m. and amounted to 272.50 m^3/s.

3.3 Geodetic data

Geodetic surveys of the channel of the Željeznica River in the considered section were carried out in 2009, 2018, and 2019. The geodetic surveys were carried out along cross-sections, which were located in the same places each time. 36 cross-sections of the channel were created for the considered section with a maximum distance of about 60 m between them (Figure 3).

The morphological changes of the Željeznica riverbed in the area and period under consideration were analyzed by superimposing the same cross-sections from different years. With the help of AutoCAD, CAD – Earth and Civil 3D tools, information on the changes of the channel was obtained in all three spatial directions: in the horizontal plane, on the cross-sections, and on the longitudinal profile of the channel.

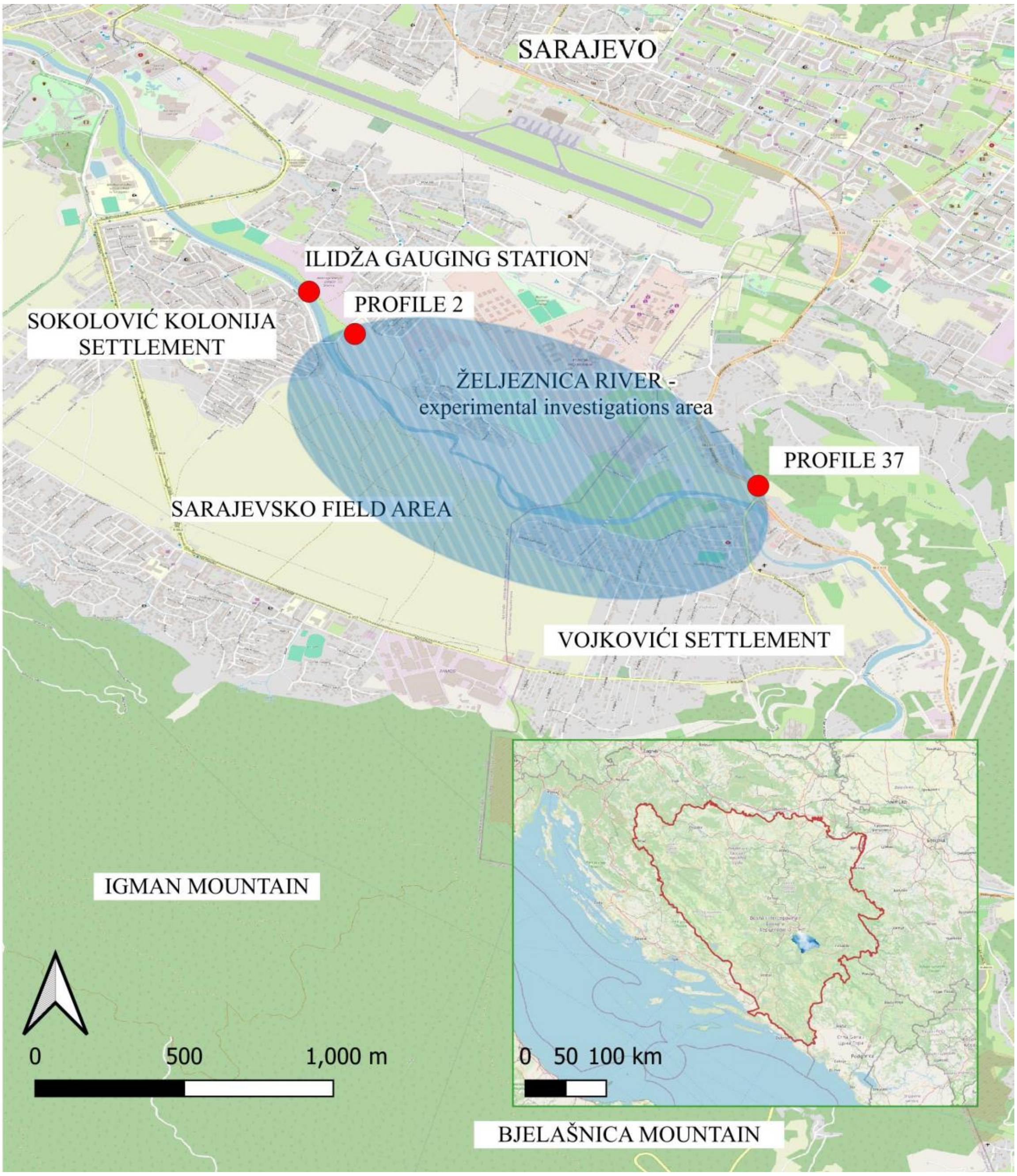


***Figure 2:** Location of experimental investigations area.*

***Slika 2:** Lokacija območja eksperimentalnih raziskav.*

The data on the configuration of the river channel at different time intervals form the basis for the evaluation of its deformation. The configuration of the Željeznica riverbed in the considered section was digitally represented by the Digital Terrain Model (DTM). It was created based on a Triangulated Irregular Network (TIN) created from the results of geodetic measurements. In order to assess the deformation of the Željeznica riverbed between 2009 and 2019, raster-based 1 m resolution digital terrain models (DTM) were created using surveyed geodetic points obtained from the riverbed terrain, which are shown in Figure 4. The AutoCAD – Civil 3D tool was used to process and analyze the changes in river morphology.

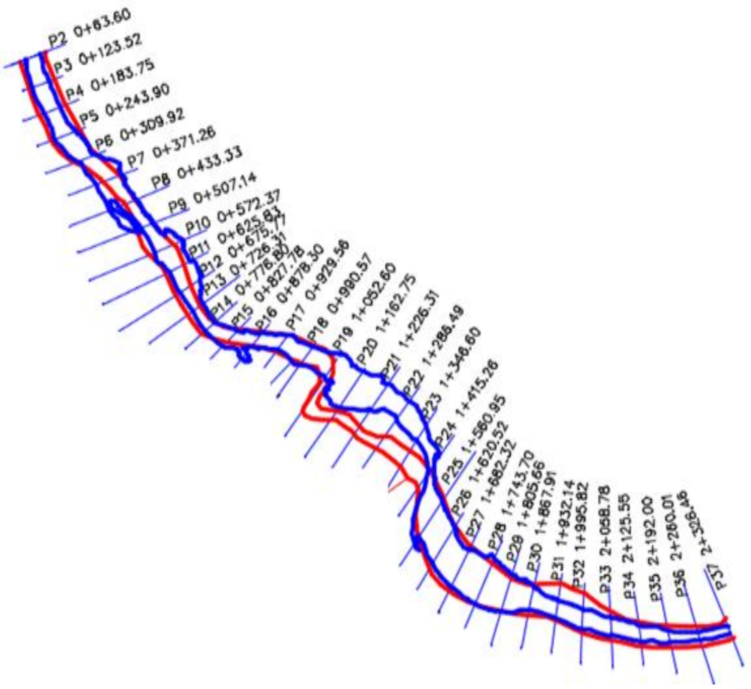
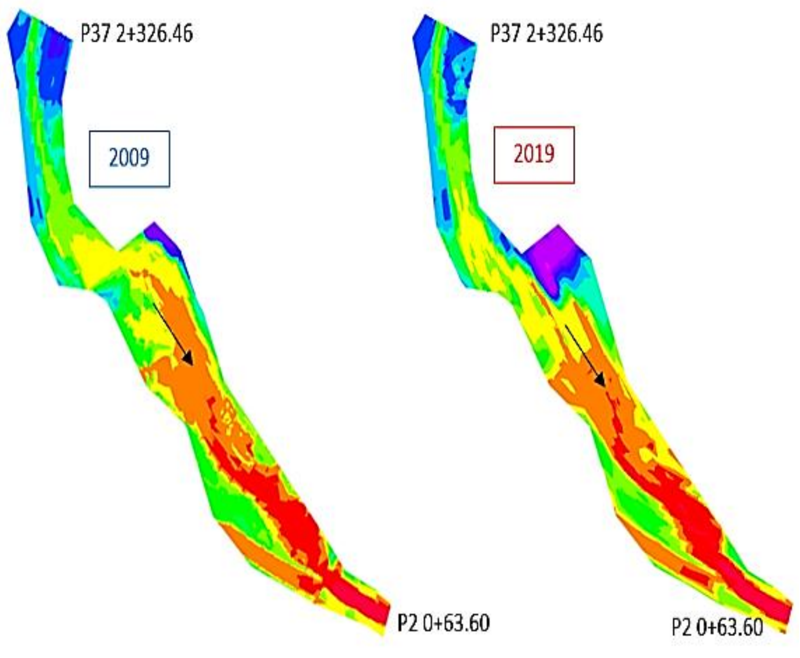


Figure 3: *The section of the Železnica River under investigation for the years 2009 (blue) and 2019 (red) with defined cross-sections.*

Slika 3: *Obravnavani odsek reke Železnice za leti 2009 (modra) in 2019 (rdeča) z označenimi prečnimi profili.*



Bottom level from – to (m asl)		Colour
500.00	501.50	
501.50	503.00	
503.00	504.50	
504.50	506.00	
506.00	507.50	
507.50	509.00	
509.00	510.50	
510.50	512.00	
512.00	513.50	
513.50	514.00	
514.00	516.50	
516.50	518.00	
518.00	519.50	
519.50	521.00	

Figure 4: *Digital terrain models (DTM) of the Železnica riverbed for 2009 and 2019.*

Slika 4: *Digitalna modela terena (DTM) struge reke Železnice za leti 2009 in 2019.*

3.4 Analysis of the spatial deformation of cross-section

Hydraulic geometry refers to the interrelationship between rivers' water discharge, sediment discharge, stream width, depth, velocity, and planform (Chang, 1988). In order to observe the morphological changes to the channel over time, three dimensionless parameters were defined to measure the deformation: 1. the change in channel bed height as deepening/increase in relation to the water depth in the channel ($\Delta z/h$), 2. the change in cross-sectional area of the channel ($A1/A2$), and 3. the horizontal displacement of the channel axis in relation to the channel width ($\Delta L/B$) (Lazović et al., 2023). The definition and graphical description of the spatial deformation elements of the river channel are shown in Figures 5a, 5b, and 5c.

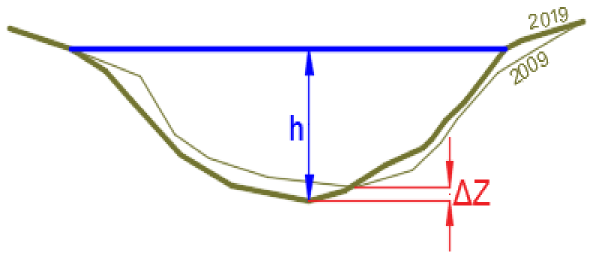


Figure 5a: *Rate of vertical movement of a riverbed bottom (Δz/h).*

Slika 5a: *Mera vertikalnega premika dna struge (Δz/h).*

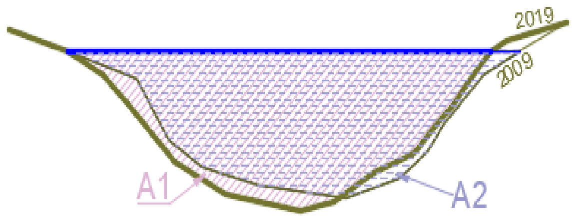


Figure 5b: *Rate of change of the cross-sectional area of a channel (A_1/A_2).*

Slika 5b: *Mera spremembe ploščine prečnega prereza struge (A_1/A_2).*

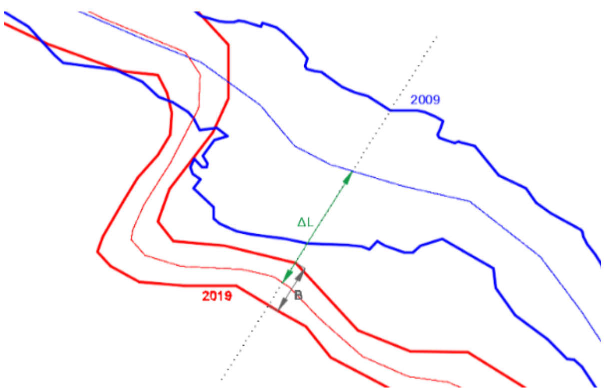


Figure 5c: *Rate of horizontal displacement of the riverbed axis (ΔL/B).*

Slika 5c: *Mera horizontalnega premika osi rečne struge (ΔL/B).*

4. Numerical modelling

After conducting field research on the morphological changes of the riverbed in the lower course of the Željeznica River, numerical investigations into the morphological changes to the riverbed were carried out. The results of the field investigations were used for the setup, input data, calibration, and verification of the numerical investigations. The mathematical model simulated the elevation changes to the riverbed bottom, i.e. the deepening and raising of the riverbed bottom.

Since the field investigations showed that in the period from 2009 to 2019 there was a horizontal shift of the axis of the Željeznica River in the section between profiles P19 and P24, the section from profile P2 (0+63.60) to profile P19 (1+052.60) was selected for the numerical modeling of the morphological changes of the riverbed. In the section P2–P19, there were neither significant changes in the geometry of the cross-sections nor horizontal shifts in the river axis during the period under consideration.

The calibration of the HEC-RAS model includes the calibration of the hydrodynamic model (HD) and the sediment transport model (ST). For the calibration of the hydrodynamic model, the water levels calculated with the HEC-RAS software were applied to the cross-sectional profiles of the sections under consideration and compared with the measured water levels of the automatic water level station in Ilidža. In addition, the level of the flood line Q = 102.82 m^3/s was used, which was registered on 14 May 2014 at 21:00. The calibration parameter of the HD model was the Manning's roughness coefficient, which takes into account the friction due to the absolute roughness of the channel bed as well as a number of other factors that contribute to the overall friction, such as: sediment formations, variable channel cross-section, structures in the channel, vegetation, and meander formation. The friction coefficient used can be represented as the sum of the partial friction coefficients (Jovanović, 2008):

$$n_g = \sum(n_{gi}) \cdot c_m \quad (6)$$

where:

n_{gi} is the partial coefficient of friction,

c_m is the coefficient of meandering.

The HEC-RAS Version 5.0 software allows the modelling of water and sediment transport using the formulae (1)–(4). When numerical simulations were carried out for the various empirical equations chosen for the sediment transport calculations, considerable discrepancies were found between the results. The model results and the modelled channel bottom were compared with the measured geodetic data of the channel bottom. The best agreement between the model and the natural conditions was found when using the empirical Ackers-White equation to calculate sediment transport. The general form of the transport equation for the Ackers-White function for a single grain size is expressed in formulae as follows (Ackers, 1973):

$$X = \frac{G_{gr} s d_s}{D\left(\frac{u_*}{V}\right)^n}$$

$$G_{gr} = C\left(\frac{F_{gr}}{A} - 1\right)^m \quad (7)$$

$$F_{gr} = \frac{u_*^n V^{1-n}}{\sqrt{g(s-1)d}\left[\sqrt{32}\log\left(\alpha d_s/D\right)\right]^{1-n}},$$

where:
X – sediment flux,
G_{gr} – dimensionless sediment transport rate,
s – relative sediment density,
d_s – mean particle diameter,
D – effective depth,
u_* – shear velocity,
V – mean channel velocity,
n – transition exponent, depending on sediment size,
α, C – coefficient,
F_{gr} – particle mobility parameter,
A – critical particle mobility parameter, and
m – exponent in the general function.

The results of the mathematical model of sediment transport are shown in the longitudinal section in Figure 6. The longitudinal sections for 2009 and 2019 are presented, as well as the results of the mathematical model simulating a 10-year period.

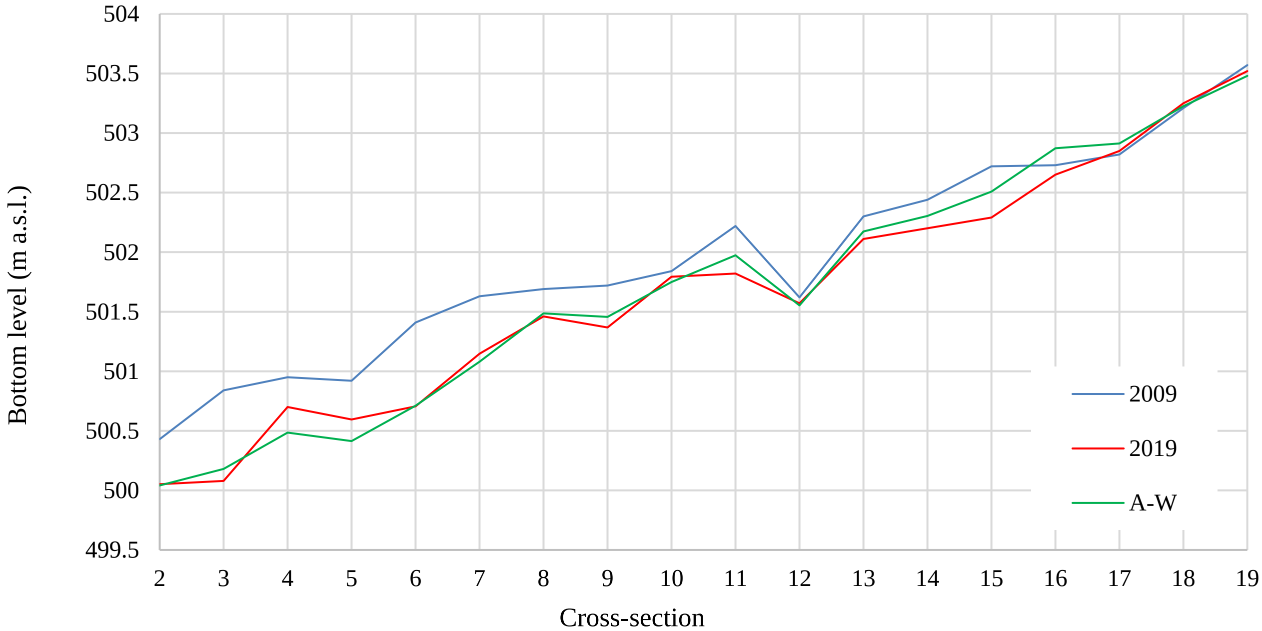


***Figure 6:** Longitudinal profile of the deepest points of the modelled section of the Železnica River using the empirical Ackers-White equation to calculate sediment transport (green) and the recorded channel point in 2009 (blue) and 2019 (red).*

***Slika 6:** Vzdolžni profil modeliranega odseka reke Železnice po najglobljih točkah prečnih profilov izračunanega z upoštevanjem Ackers-Whiteove enačbe za izračun transporta plavin (zelena) in posneta podolžna profila v letih 2009 (modra) in 2019 (rdeča).*

5. Sensitivity and reliability analysis of the mathematical model

The natural process of channel deformation is described mathematically by a system of equations that describe the flow of water and sediment. As part of the sensitivity analysis, the parameters of the selected empirical Ackers-White equation for the calculation of the sediment transport (ST model) were analyzed: the coefficients *n*, *A*, *m*, and *C* of the selected empirical Ackers-White equations for the prediction of sediment transport (ST model).

In the selected empirical Ackers-White equation, the coefficients *n*, *A*, *m*, and *C*, which are a function of the size of the mean grain size of the gravel ds, are determined for the calculation of bedload transport. By analyzing the local sensitivity of the coefficients *n*, *A*, m and *C*, it was determined which of the coefficients has the greatest variation for the different values of the mean grain size of the sediment, d_s.

In addition, a local sensitivity analysis of the empirical Ackers-White equation for the calculation of bedload transport to the variation of the mean sediment grain size d_s and to the other hydraulic and hydromorphological parameters of the river specified in this equation was carried out.

The results of the local sensitivity analysis (Figure 7) of the coefficients *n*, *A*, *m*, and *C* of the Ackers-White equation to the variation of the mean grain size d_s in the range of 0.05–1 mm are as follows:

n varies in the interval 0.23–0.96;
A varies in the interval 0.19–0.35;
m varies in the interval 1.96–7.42;
C varies in the interval 0.0004–65.80.
As can be seen from Figure 7, the coefficient *C* for different values of the mean sediment grain size d_s shows the greatest variation compared to the other coefficients. Since the coefficient *C* in the Ackers-White equation is directly proportional to the amount of bedload transport, it follows that bedload transport is equally 'sensitive' to a change in the input value of *C*, as can be seen in the Spider diagram in Figure 8.

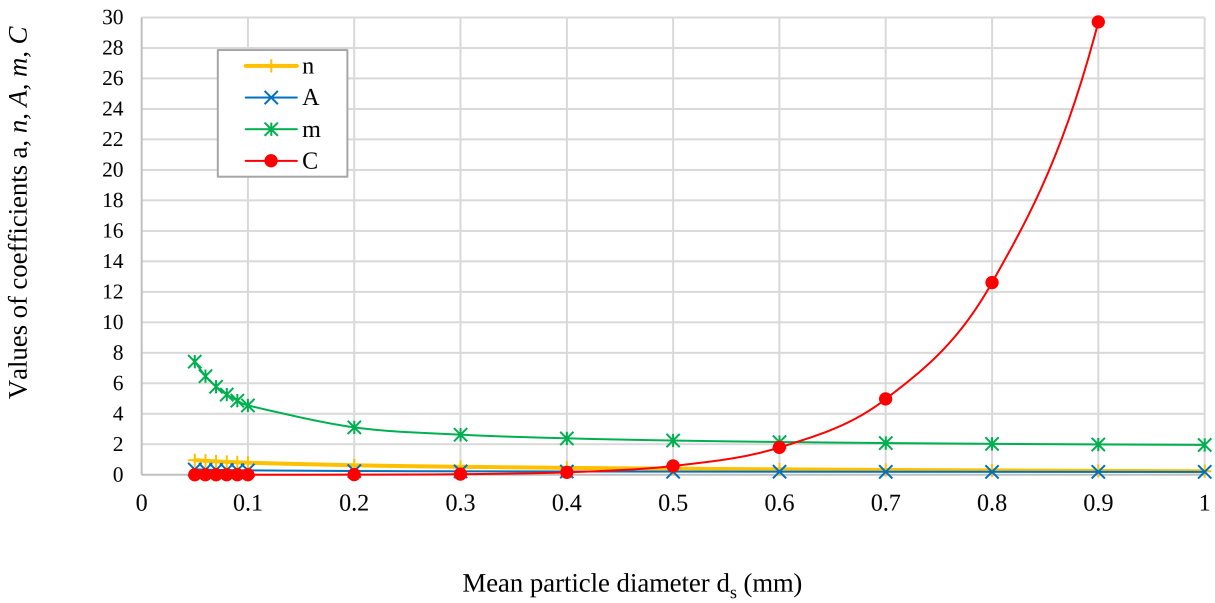


Figure 7: *Results of the local sensitivity analysis – variation of the coefficients n, A, m, and C of the Ackers-White equation for the values of the mean grain* d_s*.*

Slika 7: *Rezultati lokalne občutljivostne analize – spremembe koeficientov n, A, m in C Ackers-Whiteove enačbe za vrednosti srednjega zrna* d_s*.*

6. Results and discussion

Figure 6 shows that, along the P2–P19 section of the Željeznica River, the riverbed was lowered/deepened between 2009 and 2019. The largest deepening of the riverbed is observed on profile P6 (Δz = -0.7 m). The deepening of the river channel is the result of erosion processes caused by combined hydrological and hydraulic influences, especially high river flows.

By comparing the DTMs of the river channel from 2009 and 2019 and analyzing the results of the comparison, the magnitude of deformation of the river channel was determined at each cross-section of the considered section of the Željeznica River. Dimensionless spatial channel deformation parameters are introduced to comprehensively evaluate the geometric changes in the cross-sectional profiles of the river channel and to determine the magnitude of the channel deformation on this basis. For the analysis, geometric coefficients were used to express the ratios of geometric changes of each cross-section. The vertical bed displacements (Δz/h) of each cross-section of the considered section of the Željeznica River are shown in Figure 9 and modelled and measured deformation parameters A_1/A_2 of cross-sections (P2–P19) on the Željeznica River are shown in Figure 10.

It is obvious that the maximum vertical bed displacements are not correlated with the maximum changes in cross-sectional area. On average for all profiles, the cross-sectional area has not changed significantly during the period under consideration, but the bed of the river channel has deepened considerably. The horizontal displacements of the riverbed axis were also analyzed for the studied section of the Željeznica River. The results are shown in Figure 3. It can be observed that, in the cross-section profiles, where a pronounced horizontal displacement of the riverbed has taken place, a reduction in the width of the cross-section can be observed. This phenomenon can be interpreted by the fact that the river has incised a new channel, which requires a corresponding ability to erode the base material, which is achieved by the higher flow velocity in a narrower cross-section. The results of the local sensitivity analysis of bedload transport to changes in the river's hydraulic and hydromorphological parameters show a higher sensitivity of sediment transport to depth and rate of change of water and a lower sensitivity to changes in the energy gradient and hydraulic radius.

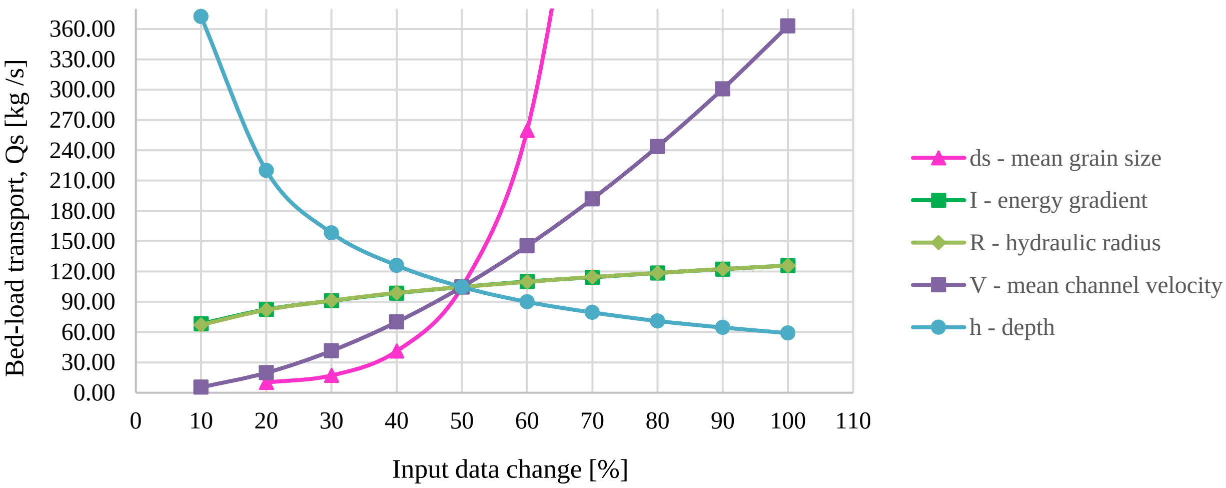


Figure 8: *Spider diagram – results of a local sensitivity analysis of bedload transport, calculated with the empirical equation of Ackers-White (analytical expression) as a function of the variation of the mean particle diameter, the hydraulic and hydromorphological parameters of the flow.*

Slika 8: *Pajkov diagram – rezultati lokalne občutljivostne analize transporta plavin, izračunane z empirično enačbo Ackers-White (analitični izraz) v odvisnosti od spremembe srednjega premera zrna ter hidravličnih in hidromorfoloških parametrov toka.*

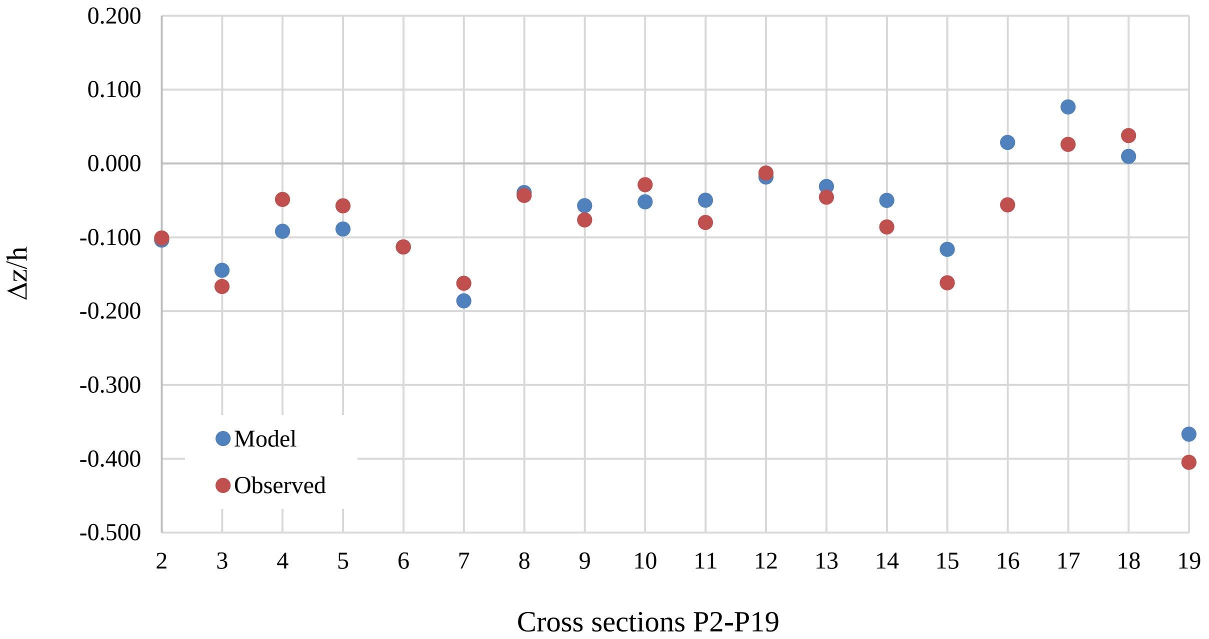


Figure 9: *Comparative presentation of modeled and measured deformation parameters Δz/h of cross-sections (P2–P19) on the Željeznica River.*

Slika 9: *Primerjalni prikaz modeliranih in izmerjenih deformacijskih parametrov Δz/h prerezov (P2–P19) na reki Željeznici.*

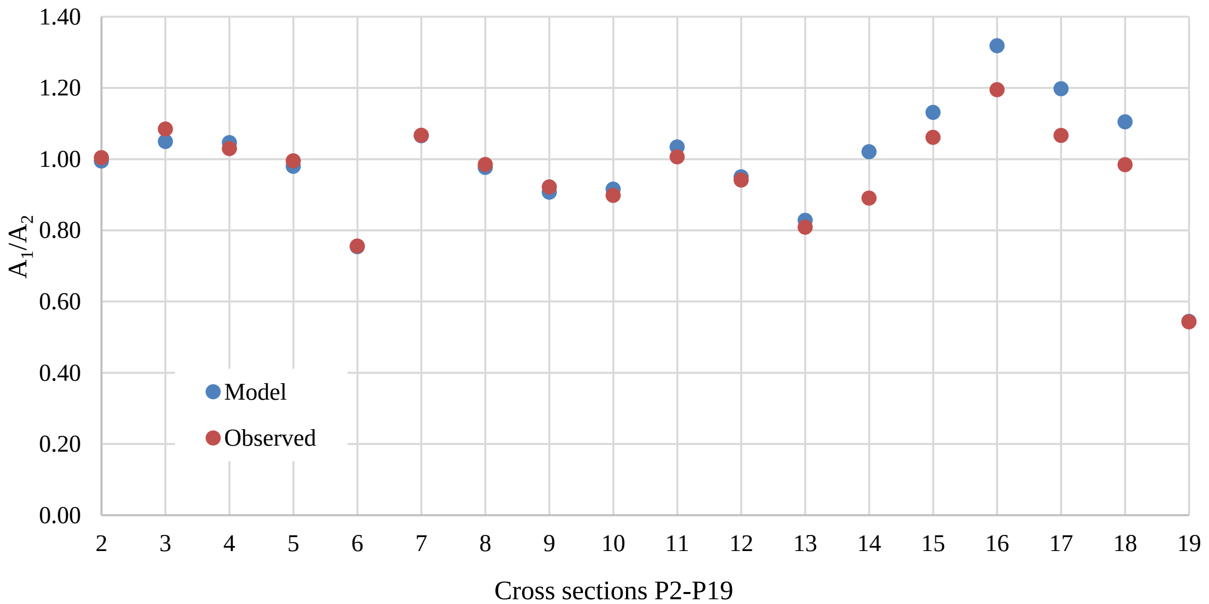


Figure 10: *Comparative presentation of modeled and measured deformation parameters A_1/A_2 of cross-sections (P2–P19) on the Željeznica River.*

Slika 10: *Primerjalni prikaz modeliranih in izmerjenih deformacijskih parametrov A_1/A_2 prerezov (P2–P19) na reki Željeznici.*

Analysis of the reliability of the calculation and measurement results for the Δz/h and A_1/A_2 parameters for all individual transverse profiles and for the entire stretch was performed using two statistical methods:

- mean absolute error (MAE) and

- the root mean square error (RMSE).

The formulas for calculation according to these methods are as follows:

$$MAE = N^{-1}\sum_{i=1}^{N}(X_m - X_o)^2$$

0 < MAE < 1

$$RMSE = \sqrt{N^{-1}\sum_{i=1}^{N}(X_m - X_o)^2}$$

0 < RMSE < 1

where: X_m – model value; X_o – observed value; N – number of observations.

A better model accuracy is represented by a MAE and RMSE value closer to 0.

The results of the analysis by indicator are shown in Table 1.

Table 1: *Accuracy indicators for the mathematical model.*

Preglednica 1: *Kazalniki točnosti matematičnega modela.*

Indicator	The value for the parameter $\Delta z/h$	The value for the parameter A_1/A_2
"MAE"	0.002	0.004
"RMSE"	0.034	0.063

7. Conclusions

This paper presents a method for determining and estimating the parameters that influence the magnitude of spatial deformation of a river channel. The focus is on identifying the elements of the river channel's spatial deformation based on geodetic data of cross-sections of the river channel and numerical analyses of the process of river channel change over the considered time period. A hydrodynamic 1D model and a model for sediment transport were used for the analysis, which enables the analysis of longer sections of the watercourse. To ensure the quality of the mathematical model results, the hydrodynamic model and subsequently the sediment transport model were calibrated. Three dimensionless geometric coefficients were introduced to allow a more transparent analysis of the changes in the channel's geometric characteristics. The relationships between the dimensionless coefficients were used to estimate the geometric change of the river channel over time caused by erosion or deposition of sediments and the displacement of the river channel in the horizontal plane. The proposed approach is an efficient method of determining the morphological changes to the river channel for various time periods on all geodetically surveyed cross-sections.

Numerical investigations based on simulations with different empirical sediment transport equations showed that the best agreement between the measured and modelled bed elevations in the considered river section of the Željeznica River was achieved when the Ackers-White equation was used to calculate the sediment transport. The empirical Ackers-White equation used to calculate the sediment transport represents the functional dependence of the hydraulic flow parameters (tangential stress and gradient of the energy line) and the sediment characteristics. The dependence was specified for the changing hydrodynamic parameters along the section under consideration (for different geometric properties of the cross-sections) and for different discharges. The knowledge of the spatial and temporal variations of the tangential stress and the gradient of the energy line is important for the assessment of the transport capacity and the prediction of the morphological changes of the river channel.

The calculated values of MAE and RMSE of the deformation model of the Željeznica River show a very good agreement between the measured and simulated values of deformation parameters, i.e. good model reliability.

In order to provide information on the possibility of generalizing the conclusions obtained with the proposed method of dimensionless geometric parameters to other watercourses, it is necessary to carry out similar analyses in further studies on watercourses with other hydrodynamic and morphological characteristics. Based on the analysis of the dimensionless geometric parameter values, the numerical modelling and the comparison of the results with the results of the geodetic measurements, the most important conclusions can be drawn:

1. For the studied section of the Željeznica River, the Ackers-White formula for estimating the total sediment transport based on the energy approach provides the best results of the sediment transport simulation.

2. The coefficient *C* for the different values of the mean sediment grain size ds shows the greatest variation compared to the other coefficients. Since the coefficient *C* in the Ackers-White formula is directly proportional to the amount of sediment transport, it follows that the sediment transport is linearly dependent on the coefficient C.

3. The maximum vertical bottom displacements are not related to the maximum changes in the wet cross-sectional area. On average for all profiles, the cross-sectional area did not change significantly during the period under consideration, but the bed of the river channel deepened considerably.

4. A decrease in the width of the cross-section can be observed in the cross-section profiles, where a pronounced horizontal displacement of the river course has taken place.

References

Ackers, P. W. W. (1973). Sediment transport: a new approach and analysis, *Journal of Hydraulics* 99(11), 2041-2060. https://doi.org/10.1061/JYCEAJ.0003791.

Andualem, T.G., Peters, S., Hewa, G.A., Myers, B.R., Boland, J., Pezzaniti, D. (2024). Channel morphological change monitoring using high-resolution LiDAR-derived DEM and multi-temporal imageries, *Science of The Total Environment* **921**. https://doi.org/10.1016/j.scitotenv.2024.171104.

Balzhieva, D. (2015): Analysis of morphological changes of the Danube on the basis of repeated river bed surveys. Doctoral dissertation. Technische Universitat Wien, 386 p., https://doi.org/10.34726/hss.2015.34329.

Beyene, A.M., Abate, M., Sinshaw, B.G., Belete, A.M., Chekole, B.Z. (2023). Anthropogenic amplification of geomorphic processes on fluvial channel morphology, case study in Gilgel Abay River Mouth; Lake Tana Sub Basin, Ethiopia, *Heliyon* **9(4)**, 14390. https://doi.org/10.1016/j.heliyon.2023.e14390.

Buffington J. M. (2012). "Changes in Channel Morphology Over Human Time Scales" in M. Church, P. M. Biron, A. G. Roy, Eds.,, *Gravel-Bed Rivers: Processes, Tools, Environments*. John Wiley & Sons, Ltd, 433-463, https://doi.org/10.1002/9781119952497.ch32.

Chang, H.H. (1988). *Fluvial Processes in River Engineering*. Krieger Publishing Company, Malabar, 432 p.

Chang, K.H., Wu, Y.T., Wang, C.H., Chang, T.J. (2024). A new 2D ESPH bedload sediment transport model for rapidly varied flows over mobile beds, *Journal of Hydrology* **634**, 131002. https://doi.org/10.1016/j.jhydrol.2024.131002.

Đorđević, D., Tamás, E.A., Mihajlović, L., Abonyi, C., Vujanović, A., Kalocsa, B. (2023). Estimation of Changes in Sediment Transport along the Free-Flowing Middle Danube River Reach, *Appl. Sci*. **13(18)**, 10513. https://doi.org/10.3390/app131810513.

Glock, K., Tritthart, M., Habersack, H. and Hauer, C. (2019). Comparison of Hydrodynamics Simulated by 1D, 2D and 3D Models Focusing on Bed Shear Stresses. *Water* **11(2)**, 226. https://doi.org/10.3390/w11020226.

Ibisate, A., Ollero, A., Diaz, E. (2011). Influence of catchment processes on fluvial morphology and river habitats, *Limnetica* **30(2)**, 169-182. https://doi.org/10.23818/limn.30.14.

Ibitoye, M. O. (2021). A remote sensing-based evaluation of channel morphological characteristics of part of lower river Niger, Nigeria, *SN Applied Science* 3(3), 340. https://doi.org/10.1007/s42452-021-04215-1.

Jovanović, M. (2008). *Regulacija reka - Rečna hidraulika i morfologija*, Građevinski fakultet Univerziteta u Beogradu, Beograd, p. 472 (in Serbian).

Krzyk, M. (1997). Two-dimensional mathematical model of convective-diffusion transport of concentration and suspended load in free surface flows, *Acta hydrotechnica* **15**, 5-100. https://doi.org/10.5545/sv-jme.2010.216.

Krzyk, M., Četina, M. (2003). A Two-Dimensional Mathematical Model of Suspended-Sediment Transport, *Journal of Mechanical Engineering* **49(3)**, 173-184.

Lane, E.W. (1955). The importance of fluvial morphology in hydraulic engineer. American Society of Civil Engineering Proceeding, *Journal of the Hydraulics* **81(745)**, 1-17.

Langat, P.K., Kumar, L., Koech, R., Ghosh, M.K. (2020). Characterisation of channel morphological pattern changes and flood corridor dynamics of the tropical Tana River fluvial systems, Kenya, *Jounal of African Earth Sciences* **163**, 103748. https://doi.org/10.1016/j.jafrearsci.2019.103748.

Lazović, N. (2021). Doprinos proučavanju opšte deformacije riječnog korita primjenom terenskih i numeričkih istraživanja (Doktorska disertacija). Građevinski fakultet Univerziteta u Sarajevu, p. 169., https://doi.org/10.13140/RG.2.2.11636.97922 (in Bosnian).

Lazović, N., Hadžić, E., Kalajdžisalihović, H., Krzyk, M., Šuvalija, S. (2023). "Determination of the Magnitude of Riverbed Spatial Deformation Caused by Hydrological-Hydraulic and Anthropogenic Influences" in N. Ademović, E. Mujčić, M. Mulić, J. Kevrić, Z. Akšamija, Eds. *Advanced Technologies, Systems, and Applications VII. IAT 2022. Lecture Notes in Networks and Systems.* Springer, Cham, 166-178. https://doi.org/10.1007/978-3-031-17697-5_14.

Ma, C., Qiu, D., Mu, X., Gao, P. (2022). Morphological Evolution Characteristics of River Cross-Sections in the Lower Weihe River and Their Response to Streamflow and Sediment Changes, *Water* **14**, 3419. https://doi.org/10.3390/w14213419.

Mahdizadeh, H., Sharifi, S. (2021). Coupled and splitting bedload sediment transport models based on a modified flux-wave approach, *International Journal of Sediment Research* **36(1)**, 38-49. https://doi.org/10.1016/j.ijsrc.2020.05.001.

Martínez-Aranda, S., Murillo, J., García-Navarro, P. (2019). A 1D numerical model for the simulation of unsteady and highly erosive flows in rivers, *Computers and Fluids* **181**, 8-34. https://doi.org/10.1016/j.compfluid.2019.01.011.

Minh Hai, D., Umeda, S., Yuhi, M. (2019). Morphological Changes of the Lower Tedori River, Japan, over 50 Years, *Water* **11(9)**, 1852. https://doi.org/10.3390/w11091852.

Momin, H., Biswas, R., Tamang, C. (2022). Morphological analysis and channel shifting of the Fulahar river in Malda district, West Bengal, India using remote sensing and GIS techniques, *GeoJournal*, **87**, 197–213. https://doi.org/10.1007/s10708-020-10248-7.

Peng, H., Lyu, B., Li, J., Sun, M., Li, W., Cao, Z. (2022). A new two-phase shallow water hydro-sediment-morphodynamic model based on the HLLC solver and the hybrid LTS/GMaTS approach, *Advances in Water Resources* **166**, 104254. https://doi.org/10.1016/j.advwatres.2022.104254.

Rahman, M.M., Harada, D., Egashira, S. (2024). Numerical Simulation of River Channel Change in the Suspended Sediment-Dominated Downstream Reach of the Sangu River, *Water* **16**, 1934. https://doi.org/10.3390/w16131934.

Rinaldi, M., Simon, A. (1998). Bed-level adjustments in the Arno River, central Italy, *Geomorphology* **22(1)**, 57-71. https://doi.org/10.1016/S0169-555X(97)00054-8.

Simon, A. (1989). A model of channel response in disturbed alluvial channels, *Earth Surface Processes and Landforms* **14**, 11–26. https://doi.org/10.1002/esp.3290140103.

Singh V. P. (2003). On the Theories of Hydraulic Geometry, *International Journal of Sediment Research* **18(3)**, 196-218

Spasojević, M. (1996). *Numerička hidraulika - otvoreni tokovi*. Građevinski fakultet Subotica, 151 p. (in Serbian).

Stipić, D., Budinski, Lj., Fabian, J. (2022). Sediment transport and morphological changes in shallow flows modelled with the lattice Boltzmann method, *Journal of Hydrology* **606**, 127472. https://doi.org/10.1016/j.jhydrol.2022.127472.

Tang, M., Jun Xu, Y., Xu, W., Wang, B., Cheng, H. (2021). Three-decadal erosion and deposition of channel bed in the Lower Atchafalaya River, the largest distributary of the Mississippi River. *Geomorphology* **380**, 107638. https://doi.org/10.1016/j.geomorph.2021.107638.

US Army Corps of Engineers, HEC-RAS - 2D Modeling User's Manual, version 5.0 (2016).

Zheng, S., Edmonds, D.A., Wu, B.S., Han, S.S. (2019). Backwater controls on the evolution and avulsion of the Qingshuigou channel on the Yellow River Delta, *Geomorphology* **333**, 137–151. https://doi.org/10.1016/j.geomorph.2019.02.032.