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Metoda robnih elementov v akustiki – primer ovrednotenja zvočnega polja enosmernega elektromotorja

The Boundary-Element Method in Acoustics – an Example of Evaluating the Sound Field of a DC Electric Motor

Martin Furlan - Miha Boltežar

V prispevku so predstavljeni osnovni prijemi za reševanje zunanjih akustičnih problemov z metodo robnih elementov (MRE) ter njena izvedba v programu za reševanje trirazsežnih problemov. Prikazana je tudi integracija programa v komercialni programski paket metode končnih elementov (MKE), ki osnovno rabi kot orodje za določitev strukturnega odziva oz. vzroka za zvočno polje, drugotno pa za pripravo in kasnejšo obdelavo akustičnega modela. Program MRE je ovrednoten na akustičnem problemu, pri katerem je poznana analitična rešitev. Sama overitev je izvedena s poudarkom na raziskavi vpliva gostote diskretizacije na natančnost izračuna zvočnega polja. Poleg tega je zaradi narave numeričnega problema oz. sistema linearnih enačb, ki pri tem nastane, ovrednotena tudi hitrost reševanja problema glede na izbiro postopka reševanja sistema linearnih enačb. Prikazana je tudi uporaba izvedenega programa MRE na dejanskem primeru. Na podlagi strukturnega odziva enosmernega elektromotorja, ki smo ga dobili kot posledico harmonskega vzbujanja magnetnih sil z MKE, smo ovrednotili zvočno polje v okolici enosmernega elektromotorja.

(Ključne besede: akustika, metode robnih elementov, hrup, dinamika struktur)

This paper presents a basic approach to solving exterior acoustic problems using the boundary-element method (BEM) and the development of a BEM program for solving three-dimensional (3D) problems. It also shows the integration of the developed BEM program into a finite-element method (FEM) program, which is primarily used to evaluate the structural response that generates a sound field, but can also be used to pre-process and post-process the acoustic model. The program was verified by using an acoustic problem for which the analytical solution was already known. The verification was performed by researching the influence of the discretization density on the accuracy of the numerically defined sound field. In addition to this, we also evaluated the time necessary to solve the problem and its system of linear equations, and related it to the method chosen for solving this system of linear equations. The program was applied to a real case where the sound field of a DC electric motor was calculated using the BEM. Based on the structural response of the DC electric motor that is the result of the harmonic excitation of magnetic forces, and was calculated with the FEM, we evaluated the sound field surrounding the DC electric motor.

(Keywords: acoustics, boundary-element method, noise, structural dynamics)

0 UVOD

Metoda robnih elementov (MRE) je orodje, ki se lahko uporablja tudi za ovrednotenje zvočnega polja kot posledice gibanja kompaktnih teles ali vibracij na njihovi površini. MRE je posebej učinkovita pri reševanju tako imenovanih zunanjih akustičnih problemov, to je sevanju zvoka v neomejenem zvočnem polju oz./ali prostem zvočnem polju. V takem primeru so druge metode, npr. metoda

0 INTRODUCTION

The boundary-element method (BEM) is a numerical tool that can be used for evaluating the sound field that results from the movement of compact bodies or due to the vibrations on their surfaces. The BEM is especially effective for solving so-called exterior acoustic problems, where the unbounded acoustic domain or free field is considered. In this case the finite-element method končnih elementov (MKE) ali metoda končnih razlik (MKR) zaradi potrebe po diskretizaciji celotnega obravnavanega območja numerično bistveno bolj zahtevne od MRE in zato manj primerne ali celo neuporabne. Poleg tega omenjene metode zahtevajo posebej pazljivo obravnavo vpliva neomejenosti zvočnega polja. Nasprotno pa se pri obravnavi takih problemov MRE ponuja sama po sebi, saj že v temelju odpravlja omenjeni problem in zahteva diskretizacijo le na površini telesa, kar zelo zmanjša čas za pripravo modela in za njegovo numerično reševanje.

V nadaljevanju so opisani osnovni prijemi za reševanje zunanjih akustičnih problemov z uporabo MRE ter izvedba MRE v programu za reševanje trirazsežnih problemov. Prikazana je tudi integracija izvedenega programa v programski paket MKE *ANSYS*, ki je namenjena za pripravo izhodnih podatkov in kasnejšo obdelavo rezultatov akustičnega modela. Izveden program je ovrednoten na akustičnem problemu, pri katerem je poznana analitična rešitev. Sama verifikacija je izvedena s poudarkom na raziskavi vpliva gostote diskretizacije na natančnost izračuna spremenljivk zvočnega polja.

(FEM), the difference method (DM) and other similar methods are numerically more demanding, due to the need for a discretization of the whole domain under consideration. In some cases these methods can also prove to be ineffective; moreover, they need special consideration because of the effect of the unbounded acoustic field. In contrast, the BEM itself solves this problem, meaning that only the boundary needs to be discretized, and this shortens the time necessary for preparing the model and solving the problem.

This paper describes the BEM for solving exterior acoustic problems and the development of the BEM program for solving three-dimensional (3D) problems. The integration of the developed BEM program into the FEM software *ANSYS*, which is used for pre-processing and post-processing of the acoustic BEM models, is also presented. The developed program is verified and the verification is based on an acoustic problem with a known exact solution. The verification gives special emphasis to the influence of the discretization on the accuracy.

1 DEFINICIJA PROBLEMA AKUSTIKE

Širjenje zvočnega valovanja skozi tekočino opisuje valovna enačba (1). Le-ta vsebuje eno samo akustično spremenljivko, zvočni tlak p, in velja za širjenje valovanja skozi poljuben posrednik, pri čemer je c hitrost širjenja valovanja v posredniku.

1 DEFINITION OF THE ACOUSTIC PROBLEM

The sound radiation through a fluid medium is described by the wave equation (1), which includes only one acoustic variable, the sound pressure p. The wave equation is valid for any acoustic medium, where c is the speed of sound in this medium.

$$\nabla^2 p(\mathbf{r},t) - \frac{1}{c^2}\ddot{p}(\mathbf{r},t) = 0 \qquad (1).$$

Valovna enačba (1) ima lahko različne rešitve, ki so odvisne od oblike valovnih čel v prostoru oz. oblike in karakteristike meje tekočine in porazdelitve vira zvoka. V večini primerov ne poznamo analitičnih rešitev zanjo, ker so viri in njegove meje pogosto zapletenih oblik ali celo neznane. Analitične rešitve obstajajo le za nekatere zelo preproste oblike zvočnih valov, to so ravni, krogelni ali cilindrični val. Če predpostavimo, da se zvočni tlak in hitrost gibanja delcev v posredniku spreminjata s časom harmonično:

The solution of the wave equation (1) depends on the distribution of the sound source and on the shape and the characteristics of the boundary of the acoustic medium, which all together define the acoustic waves. In most cases the exact analytical solution is unknown as both the source and the boundary have complicated or even unknown shapes. The exact analytical solution exists only for very simple sound waves, like planar, cylindrical or spherical waves. If we assume that the sound pressure and the velocity in the medium change harmonically with time:

$$p = \hat{p}e^{i\omega t}, \mathbf{v} = \hat{\mathbf{v}}e^{i\omega t} \qquad (2),$$

lahko valovno enačbo (1) zapišemo v skrčeni obliki,

the wave equation (1) can be formulated in a reduced form,

$$\nabla^2 \hat{p} + k^2 \hat{p} = 0 \qquad (3),$$

ki ne vsebuje več odvoda po času. Enačbo (3) imenujemo tudi Helmholtzova valovna enačba, kjer je k akustično valovno število in velja $k^2=\omega^2/c^2$. Z namenom poenostavitve pri numerični obravnavi akustičnih problemov vpeljemo pojem hitrostnega potenciala ϕ. Ta omogoča, da tako zvočni tlak kakor

that does not include a time derivative. Equation (3) is also known as the Helmholtz wave equation, where k is the acoustic wave number defined as $k^2=\omega^2/c^2$. When a numerical approach to the sound field is needed, the introduction of the velocity potential ϕ is convenient, as both the sound pressure and the hitrost izračunamo neposredno iz hitrostnega potenciala ϕ, ki je definiran z:

velocity can be calculated from it directly. The velocity potential ϕ is defined as:

$$\hat{p} = i\omega\rho\phi, \text{ kar da/that gives } \hat{\mathbf{v}} = \nabla\phi \quad (4).$$

Z uporabo hitrostnega potenciala lahko Helmholtzovo valovno enačbo zapišemo v obliki:

With the use of the velocity potential ϕ the Helmholtz wave equation can be reformulated as:

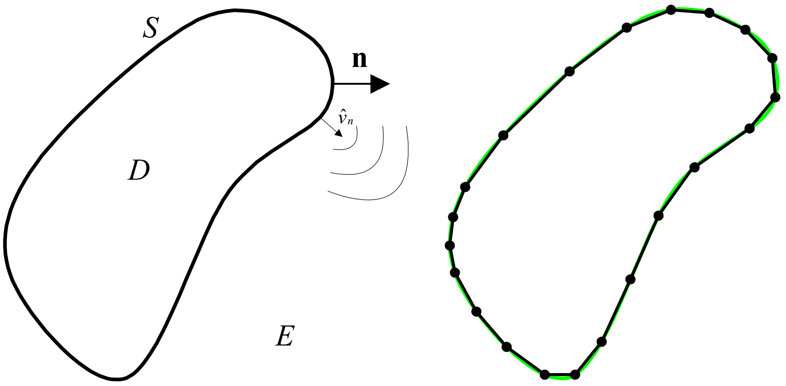
$$\nabla^2\phi + k^2\phi = 0 \quad (5).$$

1.1 Reševanje Helmholtzove valovne enačbe

1.1 Solving the Helmholtz wave equation

V splošnem lahko razdelimo akustične probleme kakor tudi reševanje akustične valovne enačbe na: *notranji akustični problem*, *zunanji akustični problem* ter *problem lastnih vrednosti*. V našem primeru se bomo omejili na reševanje zunanjega akustičnega problema. Slika 1 prikazuje osnovno obliko, ki je potrebna za definiranje takega problema. Na njej je prikazano neomejeno zunanje območje *E*, ki objema površino *S* z zunanjo normalo **n**, ter omejeno notranje območje *D*.

In general, acoustic problems and the solution of the wave equation can be divided into the following: *an interior acoustic problem, an exterior acoustic problem* and *an eigenvalue problem*. In our case we are limited to the exterior acoustic problems only. Figure 1 shows a definition of the acoustic problem for an object with an infinite exterior domain *E* that surrounds surface *S* with an outward normal **n** and an interior domain *D*.



S. 1. *Oblika zunanjega akustičnega problema (na levi) in njegova diskretizacija (na desni)*
Fig. 1. *Definition of the exterior acoustic problem (on the left) and its discretization (on the right)*

Ker gre v omenjenem problemu za akustično analizo neomejenega zvočnega polja, mora v oddaljenem zvočnem polju veljati Sommerfeldov sevalni pogoj ([1] do [4]), ki opisuje zmanjševanje zvočnega tlaka z oddaljenostjo od vira. Posledica Sommerfeldovega pogoja je, da v fizikalni rešitvi za oddaljeno zvočno polje dopušča le tako valovanje, ki se od vira oddaljuje:

The exterior acoustic problem is related to the unbounded medium and therefore the Sommerfeld radiation condition, which describes the relation between the sound pressure and the distance from the source, should be applied in the far field ([1] to [4]). The consequence of the Sommerfeld radiation condition is a physical solution to the sound field that is limited to a wave that travels away from the source:

$$\lim_{r\to\infty} r\left(\frac{\partial\hat{p}}{\partial r} - ik\hat{p}\right) = 0 \quad (6).$$

Rešitev Helmholtzove valovne enačbe je v našem primeru opredeljena z le Neumannovim robnim pogojem, ki opisuje gibanje delcev tekočine tik ob površini:

The solution of the Helmholtz wave equation of the investigated problem is in our case defined only by the Newmann boundary condition, which describes the velocity in the medium at the surface *S*:

$$\frac{\partial\phi}{\partial n} = \hat{v}_n, \quad \hat{v}_n = \mathbf{n}\cdot\hat{\mathbf{v}} \quad (7),$$

kjer je $\hat{v}_n$ hitrost točke na površini v smeri normale **n**. Osnovna rešitev skrčene oblike valovne enačbe (5), ki zadostuje Sommerfeldovemu sevalnemu pogoju, je Greenova funkcija:

where $\hat{v}_n$ is the velocity of the surface S in the direction of the surface's outward normal **n**. The fundamental solution of the reduced form of the wave equation (5) that satisfies the Sommerfeld radiation condition is the Green's function:

$$G_k(P,Q)=\frac{e^{ikr}}{4\pi r},\ r=\left|P-Q\right|,\ \nabla^2 G_k(P,Q)+k^2 G_k(P,Q)=-\delta(P,Q) \qquad (8),$$

kjer sta $\delta(P,Q)$ Diracova funkcija in r evklidska razdalja med točkama P in Q v zvočnem polju, tako v območju E kakor tudi na površini S.

where $\delta(P,Q)$ is the Dirac delta function, r is Euclidian distance between the points P and Q in the exterior domain E or on the surface S.

1.2 Helmholtzovi integralski operatorji

1.2 Helmholtz integral operators

V enačbah (9) do (12) so zapisane definicije integralskih operatorjev Helmholtzove valovne enačbe. Vsi navedeni operatorji so rešitve valovne enačbe (5) ob predpostavki zvezne porazdelitve vira z amplitudo $\phi(Q)$ na sevalni površini S ([2] in [3]).

The equations (9) to (12) define the Helmholtz integral operators. All the presented operators are the solution of the Helmholtz wave equation (5), where the distribution of the source on the surface S, with the amplitude of $\phi(Q)$, is considered to be continuous ([2] and [3]):

$$L_k\left[\phi\right](P)=\int_S G_k(P,Q)\phi(Q)dS \qquad (9)$$

$$M_k\left[\phi\right](P)=\int_S \frac{\partial G_k(P,Q)}{\partial n_Q}\phi(Q)dS \qquad (10)$$

$$M_k^T\left[\phi\right](P)=\int_S \frac{\partial G_k(P,Q)}{\partial n_P}\phi(Q)dS \qquad (11)$$

$$N_k\left[\phi\right](P)=\int_S \frac{\partial G_k(P,Q)}{\partial n_P \partial n_Q}\phi(Q)dS \qquad (12)$$

in operator istovetnosti:

and the identity operator:

$$I\left[\phi\right](P)=\phi(P) \qquad (13).$$

1.3 Integralska oblika Helmholtzove valovne enačbe

1.3 Integral formulation of the Helmholtz wave equation

Z upoštevanjem prvega Greenovega izreka na prostorninskem integralu Helmholtzove valovne enačbe po širšem zunanjem območju E, ki objema zvočni vir D, dobimo zapis enačbe kot integral na površini S, imenovan tudi *površinska Helmholtzova integralska enačba* ([2] in [3]):

Considering the Green's formula, the volume integral of the Helmholtz wave equation over an extended exterior domain E, that completely surrounds the sound source D, a new formulation, the so-called *surface Helmholtz integral equation*, is introduced as a surface integral on the surface S ([2] and [3]):

$$\int_S \left(\phi(Q)\frac{\partial G_k(P,Q)}{\partial n_Q}-\frac{\partial \phi(P,Q)}{\partial n_Q}G_k(P,Q)\right)dS=c(P)\phi(P) \qquad (14),$$

kjer je $c(P)$ faktor lege točke P, določen tudi z enačbo (15) ([2] in [3]):

where $c(P)$ is a jump term, which is defined by the position of the point P, see equation (15) ([2] and [3]).

$$c(P)=\begin{cases}1 & P\in E\\ \frac{1}{2} & P\in S\\ 0 & P\in D\end{cases} \qquad (15).$$

Za enačbo (15) mora veljati, da je površina S v točki P gladka. Površinsko Helmholtzovo integralsko enačbo (14) lahko zapišemo tudi v obliki z integralskimi operatorji ([2] in [3]):

Equation (15) should be valid if the surface S is to be smooth at the point P. In terms of the integral operators the surface Helmholtz integral equation (14) can be given by ([2] and [3]):

$$L_k \frac{\partial \phi}{\partial n} = \left[-c(P)I + M_k \right] \phi \quad (16).$$

Odvod površinske Helmholtzove integralske enačbe (14) po normali na površini S da nov izraz, enačba (17), imenovan *diferencirana površinska Helmholtzova integralska enačba*, ki ga uporabljamo pri upoštevanju robnih pogojev ([2] in [3]):

With the differentiation of the surface Helmholtz integral equation (14) with respect to the normal to the surface S, we get a new formulation, Equation (17), the so-called *differentiated surface Helmholtz integral equation*, which is used when boundary conditions need to be taken into account ([2] and [3]):

$$\int_S \left(\phi(Q) \frac{\partial^2 G_k(P,Q)}{\partial n_Q \partial n_P} - \frac{\partial \phi(P,Q)}{\partial n_Q} \frac{\partial G_k(P,Q)}{\partial n_P} \right) dS = c(P) \frac{\partial \phi(P)}{\partial n_P} \quad (17).$$

Tako kakor osnovno obliko površinske Helmholtzove integralske enačbe (14) lahko tudi njeno diferencirano obliko zapišemo z integralskimi operatorji ([2] in [3]):

Like the surface Helmholtz integral equation, its differentiated formulation, Equation (17), can also be written with the Helmholtz integral operators ([2] and [3]):

$$N_k \phi = \left[-c(P)I + M_k^T \right] \frac{\partial \phi}{\partial n} \quad (18).$$

Ker obe formulaciji Helmholtzove integralne enačbe (16) in (18) nimata enolične rešitve za določena akustična valovna števila, sta Burton in Miller vpeljala nov zapis ([1] in [3]), njuno linearno kombinacijo, ki izloči vpliv nestabilnih akustičnih valovnih števil:

As both formulations of the Helmholtz integral equations (16) and (18) do not have unique solutions for specific wave numbers, Burton and Miller introduced a new formulation ([1] and [3]). Their linear combination that eliminates the effect of instable wave numbers is:

$$\left\{ \alpha \left[-c(P)I + M_k \right] + \beta N_k \right\} \phi = \left\{ \alpha L_k + \beta \left[c(P)I + M_k^T \right] \right\} \frac{\partial \phi}{\partial n} \quad (19),$$

kjer je vrednost faktorjev α in β odvisna od valovnega števila, in sicer $\alpha=1$ ter $\beta=i/k$ za vrednosti $k \geq 1$ in ter $\beta=i$ za vrednosti $k<1$ ([1] in [3]).

where the values of the factors α and β depend on the wave number: $\alpha=1$ and $\beta=i/k$ for wave numbers $k \geq 1$ and $\beta=i$ for wave numbers $k<1$ ([1] and [3]).

1.4 Zapis problema MRE

Burton–Millerjev zapis Helmholtzove integralske enačbe (19) z integralskimi operatorji je osnova za postavitev MRE za reševanje zunanjih akustičnih problemov. Sprememba problema v zapis MRE temelji na diskretizaciji površine S z robnimi elementi (sl. 1). Površino S poenostavimo z diskretizirano površino, ki je sestavljena iz n površin robnih elementov. Podobno je treba diskretizirati tudi funkcijo hitrostnega potenciala ϕ z interpolacijsko funkcijo, ki pomeni njen ustrezek na diskretiziranem območju. V našem primeru je interpolacijska funkcija konstanta in ima po celotnem elementu vrednost 1. Ob vseh zgoraj navedenih predpostavkah lahko določimo približno vrednosti za integralske operatorje.

Reševanje zunanjega akustičnega problema je v splošnem sestavljeno iz dveh delov. Prvi del vključuje reševanje integralne enačbe (19) na območju S, kjer je robni pogoj definiran kot $\partial \phi(P)/\partial n_Q = \hat{v}_n(P)$ pri pogoju $P \in S$. Rešitev je hitrostni potencial $\phi(P)$ za točke P na površini S. Drugi del vključuje določitev hitrostnega potenciala $\phi(P)$ za točke P, ki ležijo v zunanjem območju E, in temelji na uporabi površinske Helmholtzove integralske enačbe (16).

1.4 Formulation of the BEM problem

The Burton–Miller formulation of the Helmholtz integral equation (19), written by integral operators, represents a basis for the BEM formulation for solving exterior acoustic problems. Its transformation into the BEM formulation is based on the discretization of the surface S with boundary elements, see Figure 1. The surface S is approximated by a discretized surface that consists of n boundary elements. Consequently, it is also necessary to discretize the velocity potential function ϕ by an interpolation function that represents its equivalent on the discretized surface. In our case a constant interpolation function that has a constant value of one over the entire boundary element was chosen. Taking into account all the above-mentioned assumptions, the approximation of the integral operators can be completed.

Generally, the solution of an exterior acoustic problem can be divided into two steps. The first step includes the solution of the integral equation (19) on the boundary S, where the boundary condition is defined as $\partial \phi(P)/\partial n_Q = \hat{v}_n(P)$ where $P \in S$. Its solution is the velocity potential $\phi(P)$ for points P on the surface S. The second step includes the calculation of the velocity potential $\phi(P)$ for points P that lie in the exterior domain E and is based on the usage of the surface Helmholtz integral equation (16).

Z uporabo kolokacijske metode in diskretizacije območja lahko diskretizirane Helmholtzove integralske operatorje, kot interakcije med i-to kolokacijsko točko z j-tim elementom, zapišemo v obliki matrik $\mathbf{L}_k$, $\mathbf{M}_k$, $\mathbf{M}_k^{\mathrm{T}}$ in $\mathbf{N}_k$ z izmero $n \times n$. Podobno lahko v vektorski obliki zapišemo tudi aproksimacijsko funkcijo za hitrostni potencial v kolokacijskih točkah robnih elementov Φ.

Z uvedbo kolokacijske metode na Burton-Millerjevem zapisu Helmholtzove integralske enačbe (19) dobimo matrično enačbo za rešitev akustičnega problema z MRE:

Using the collocation method, the discretized Helmholtz integral operators, as the interaction between the collocation point with an index i and the boundary element with an index j, can be written with the matrices $\mathbf{L}_k$, $\mathbf{M}_k$, $\mathbf{M}_k^{\mathrm{T}}$ and $\mathbf{N}_k$ that have the dimensions $n \times n$. Similarly, the velocity potential approximation can also be written as a vector Φ in the collocation points of the boundary elements.

Applying the collocation method to the Burton-Miller formulation of the Helmholtz integral formulation (19) we get a matrix equation that describes the BEM formulation of the acoustic problem:

$$\left\{\alpha\left[\mathbf{M}_k - \tfrac{1}{2}\mathbf{I}\right] + \beta\mathbf{N}_k\right\}\mathbf{\Phi} = \left\{\alpha\mathbf{L}_k + \beta\left[\mathbf{M}_k^T + \tfrac{1}{2}\mathbf{I}\right]\right\}\mathbf{v} \qquad (20),$$

kjer je $\mathbf{v}$ vektorski zapis robnih pogojev – normalnih hitrosti v kolokacijskih točkah robnih elementov, $\mathbf{v}=[\hat{v}_n(P_1),..., \hat{v}_n(P_n)]^T$. Rešitev matrične enačbe vodi k aproksimativni določitvi hitrostnega potenciala v kolokacijskih točkah robnih elementov, $\Phi=[\phi(P_1),...,\phi(P_n)]$, in pomeni prvi del reševanja zunanjega akustičnega problema. Fizikalna interpretacija tako dobljenega hitrostnega potenciala je zvočni tlak tik ob površini S, $\mathbf{p}=[(1/i\omega\rho)\phi(P_1),..., (1/i\omega\rho)\phi(P_n)]^T=(1/i\omega\rho)\,\Phi$. V drugem delu reševanja zunanjega akustičnega problema z MRE, ko imamo znano aproksimativno rešitev hitrostnega potenciala na diskretizirani površini, določimo aproksimativno rešitev hitrostnega potenciala zunanjih točk $\phi(P)$ za $P \in E$. Rešitev lahko izvedemo za poljubno število točk P, ki ležijo v zunanjem območju E. V ta namen uporabimo diskretiziran zapis enačbe (16) pri pogoju, da točke P ležijo v zunanjem območju E. Določitev hitrosti v točkah zunanjega območja lahko izvedemo z numeričnim odvajanjem hitrostnega potenciala ali pa neposredno z izračunom parcialnega odvoda hitrostnega potenciala po želenem smernem vektorju $\mathbf{n}$. V slednjem primeru uporabimo diskretizirano obliko diferencirane površinske Helmholtzove integralske enačbe (18).

where $\mathbf{v}$ is the vector of the boundary conditions – normal velocities in the collocation points of the boundary elements, $\mathbf{v}=[\hat{v}_n(P_1),..., \hat{v}_n(P_n)]^T$. The solution of this equation gives the approximation of the velocity potential in the collocation points of the boundary elements and represents the first step in the solution of the exterior acoustic problem. The physical interpretation of the calculated velocity potential is the sound pressure at the surface S, $\mathbf{p}=[(1/i\omega\rho)\phi(P_1),..., (1/i\omega\rho)\phi(P_n)]^T=(1/i\omega\rho)\,\Phi$. In the second step of the solution to the exterior acoustic problem with the BEM, the velocity potential $\phi(P)$, where $P \in E$, is calculated. This calculation is based on the previously calculated velocity potential on the discretized surface. The calculation of the velocity potential can be made for any number of points P that lie in the exterior domain E, using the discretized formulation of Equation (16), for which it is assumed that the point P lies in the exterior domain E. The velocities at the points of the exterior domain can be calculated either by numerical differentiation, or directly, by using the discretized formulation of the differentiated surface Helmholtz integral Equation (18), where the partial differentiation of the velocity potential with respect to the desired directional vector $\mathbf{n}$ is considered.

2 IZVEDBA MRE

Za reševanje splošnih trirazsežnih zunanjih akustičnih problemov z MRE smo v programskem paketu *DIGITAL Visual Fortran 6.0* razvili program, ki omogoča integracijo v programske pakete MKE za reševanje problemov iz strukturne dinamike, kakršen je npr. *ANSYS* ipd. Pri izračunu diskretnih Helmholtzovih integralskih operatorjev smo izhajali iz že izvedenih podprogramov avtorja Kirkupa [2]. Za diskretizacijo trirazsežnega problema smo uporabili trikotne elemente. Reševanje problema smo razdelili v dva dela.

V prvem delu reševanja problema MRE se opravi izračun diskretnih Helmholtzovih integralskih operatorjev, čemur sledi sestavljanje matrične enačbe (20) in nazadnje njeno reševanje. Z upoštevanjem razmerja

2 DEVELOPMENT OF THE BEM PROGRAM

To solve the 3D exterior acoustic problems with the BEM we developed a computer program, using *DIGITAL Visual Fortran 6.0*, that can be integrated into any FEM software, e.g. *ANSYS*, intended for solving structural dynamic problems. The evaluation of the discretized Helmholtz integral operators is made with Kirkup's [2] subroutines. The discretization of the 3D problem is made with triangular elements. The problem's solution procedure is divided into two steps.

In the first step the discrete Helmholtz integral operators are calculated, this is followed by the composition of the matrix equation (20), and finally the matrix equation is solved. Regarding the relation between the velocity potential and the sound med hitrostnim potencialom in zvočnim tlakom v enačbi (2) lahko enačbo (20) zapišemo v poenostavljeni obliki:

pressure, Equation (2), Equation (20) can be simplified in the following way:

$$\mathbf{H} \cdot \mathbf{p} = i\omega\rho\ \mathbf{G} \cdot \mathbf{v} \qquad (21).$$

Rešitev matrične enačbe (21) je zvočni tlak **p** na robu diskretizirane površine oz. v kolokacijskih točkah robnih elementov. V drugem delu reševanja se na podlagi poznavanja zvočnega tlaka **p** in robnih pogojev – normalnih hitrosti **v** na robu diskretizirane površine z diskretiziranimi oblikami Helmholtzovih integralskih enačb (16) in (18) določi zvočni tlak $\mathbf{p}_E=[(1/i\omega\rho)\phi(P_1),...,(1/i\omega\rho)\phi(P_m)]^T=(1/i\omega\rho)\Phi_E$ in hitrosti $\mathbf{v}_E=[\hat{v}_n(P_1),..., \hat{v}_n(P_m)]^T$ za množico točk zunanjega območja, ki ležijo v zunanjem območju *E*. Poenostavljen matrični zapis omenjenih enačb za določitev rešitev v zunanjem območju se tako glasi:

The solution of the matrix Equation (21) is the sound pressure **p** on the boundary of the discretized surface, exactly in the collocation points of the boundary elements. In the second step, the sound pressure $\mathbf{p}_E=[(1/i\omega\rho)\phi(P_1),...,(1/i\omega\rho)\phi(P_m)]^T=(1/i\omega\rho)\Phi_E$ and the velocity $\mathbf{v}_E=[\hat{v}_n(P_1),..., \hat{v}_n(P_m)]^T$ are calculated for the group of points that lie in the exterior domain *E*. This calculation is based on the previously calculated sound pressure **p,** on the boundary conditions – velocities **v** on the boundary – and on the formulation of the discretized Helmholtz integral Equations (16) and (18). The simplified formulation for the calculation of the sound field in the exterior domain is given by:

$$\mathbf{p}_E = \mathbf{h}^T \cdot \mathbf{p} - i\omega\rho\ \mathbf{g}^T \cdot \mathbf{v} \qquad (22),$$

$$\mathbf{v}_E = \mathbf{h}_v^T \cdot \mathbf{p} - i\omega\rho\ \mathbf{g}_v^T \cdot \mathbf{v} \qquad (23).$$

V nasprotju s prvim delom reševanja, pri katerem gre za iskanje rešitve na robu in je treba rešiti sistem linearnih enačb (21), je drugi del reševanja numerično manj zahteven, če ne gre za veliko število točk zunanjega območja. Tu imamo opraviti le z množenjem matrik in vektorjev, glej enačbi (22) in (23).

The first step, where a system of linear equations (21) is to be solved, is numerically more demanding than the second step, except if we have an enormous number of exterior points. However, the second step is numerically less demanding, as we only have to multiply matrices and vectors, see Equations (22) and (23).

2.1 Reševanje problema MRE

Glede na numerično zahtevnost tako zastavljenega zunanjega akustičnega problema MRE lahko njegovo reševanje, v smislu časa, potrebnega za izračun, razdelimo v tri sklope. Prva dva sklopa izhajata iz iskanja rešitve na robu, pri čemer prvi sklop vključuje ovrednotenje diskretnih Helmholtzovih integralskih operatorjev in sestavljanje matrične enačbe (20) oz. (21), drugi pa pomeni reševanje omenjene matrične enačbe. V zadnjem, tretjem sklopu reševanja problema MRE, se izvede ovrednotenje zvočnega polja za zunanje točke.

Sistem linearnih enačb, ki nastane pri sestavu matrične enačbe (21), lahko postane iz več vidikov numerično izredno zahteven, predvsem pa takrat, ko se število robnih elementov poveča prek določene meje. Vzrok za to je matrika **H**, ki se pojavi v enačbi (21) in ima lastnost, da je polna in nesimetrična – vsi njeni elementi so različni od nič in so kompleksna števila. Pri reševanju nastalega sistema linearnih enačb so običajne metode, ki se uporabljajo pri MKE, praktično neuporabne. Zato smo za reševanje problema MRE uporabili iterativne metode, izhajajoč iz fortranske knjižnice *PIM 2.2* [5]. Tako smo poleg osnovne Gaussove izločilne metode v izvedeni program MRE vključili še štiri iteracijske

2.1 Solution of the BEM problem

Considering the numerical pretentiousness of the presented exterior acoustic BEM problem its solution can be divided into three parts with regard to the time needed for the computation of the whole problem. The first two parts represent the solution on the boundary, where the first part includes the calculation of the discrete Helmholtz integral operators and their composition into the matrix Equation (20) or (21), while the second part represents the solution of the matrix equation. The third, and last, part includes the computation of the sound field for the points that lie in the exterior domain.

The system of linear equations that results from the composition of the matrix equation (21) can become very numerically demanding, especially when the number of boundary elements exceeds certain limits. The reason for this is the matrix **H,** which is presented in Equation (21). This matrix is typically unsymmetrical and dense – all its elements are non-zero complex values. When solving such a system of linear equations, classical methods, which are normally convenient for the FEM, are not practical. To solve our BEM problem we applied iterative methods, using the Fortran library *PIM 2.2* [5]. So, besides the Gauss elimination method the developed BEM program has metode [5], t.i. metoda konjugiranih gradientov z raznimi izpeljankami ter metoda posplošenih najmanjših residuumov (CGS, Bi-CGSTAB, RBi-CGSTAB in GMRES).

Čas, ki je bil potreben za reševanje sistema linearnih enačb, v odvisnosti od izbrane metode in obsežnosti problema, odvisnega od števila robnih elementov, je prikazan v preglednici 1. Poleg tega omenjena preglednica ponuja tudi primerjavo časa, potrebnega za sestavo matrične enačbe in za izračun zvočnega polja v 148 zunanjih točkah. Število zunanjih točk je bilo za različne obsežnosti problemov vedno enako. Primerjalna analiza je bila izvedena na osebnem računalniku s procesorjem Celeron 900MHz in pomnilnikom 384MB DRAM. Analizo smo izvedli na trirazsežnem problemu, ki je v nadaljevanju natančneje opisan.

four iterative methods included, the so-called »Conjugate-Gradients Squared« (CGS), »Stabilised Bi-Conjugate-Gradients« (Bi-CGSTAB), »Restarted Stabilised Bi-Conjugate-Gradients« (RBi-CGSTAB) and »Generalised Minimal Residual (GMRES)«.

The time needed to solve the system of linear equations regarding the solution method and the size of the problem, defined by the number of boundary elements, is given in Table 1. This table also shows the time needed, both for the composition of the matrix equation and for the calculation of the integral operator, and the time needed for the calculation of the sound field in 148 exterior points. The number of exterior points is the same for all the problem sizes. The analysis was performed on a personal computer with a Celeron 900MHz processor and 384MB of DRAM. The problem that was investigated is precisely described in the text that follows.

Preglednica 1. *Analiza porabe časa za reševanje problema MRE (v sekundah)*
Table 1. *Time needed for the solution of the BEM problem analysis (in seconds)*

Postopek / Method	Število robnih elementov / Number of boundary elements									
	36	144	224	480	642	1028	1228	1736	2654	3788
Gauss	0,070	0,070	0,401	7,951	11,09	58,72	105,6	575,4	2245,	7657,
CGS	0,000	0,070	0,180	0,851	1,592	4,587	7,120	16,59	42,80	128,9
Bi-CGSTAB	0,000	0,070	0,180	1,032	1,963	5,558	8,632	12,65	54,42	134,0
RBi-CGSTAB	0,010	0,300	0,951	5,638	10,45	34,30	53,98	124,2	354,2	1053,
GMRES	0,000	0,170	0,651	2,704	5,908	17,87	20,42	76,16	254,1	548,1
Sestav matrike / Matrix composit.	0,080	1,302	3,135	14,33	25,69	65,33	93,36	186,9	404,3	774,9
Zunanje reševanje / External solution	1,282	5,177	8,022	17,26	22,95	36,80	44,02	62,21	87,09	116,8

Iz podatkov v preglednici 1 lahko ugotovimo, da je čas, potreben za reševanje obsežnejših problemov, močno odvisen od izbire postopka reševanja sistema linearnih enačb. Pri reševanju problemov večjega obsega so iterativne metode tudi 60-krat in več hitrejše od običajne Gaussove izločilne metode, ki je za obsežnejše probleme praktično neuporabna. Za najhitrejšo izmed vseh metod se je izkazala metoda CGS.

Reševanje problema je še dodatno oteževalo dejstvo, da je matrika, ki je opisovala problem MRE, zasedala precejšen del pomnilnika. Elementi matrike so namreč kompleksna števila, določena z dvojno natančnostjo (COMPLEX*16), kar je k temu še dodatno pripomoglo. Npr. matrika reda 3788, ki v predstavljeni analizi opisuje najobsežnejši problem z enakim številom robnih elementov, zaseda v pomnilniku 224MB. Zaradi omejitve, ki izhaja iz operacijskih sistemov *Windows* in dovoljuje največjo uporabo pomnilnika 2GB, je reševanje problemov omejeno do približno desettisoč robnih elementov.

From Table 1 it is clear that the time needed to solve extensive problems mainly depends on the chosen method for solving the system of linear equations. When dealing with such problems, iterative methods can be up to 60 times faster than the classical Gauss elimination method, which is practically useless for problems that are more extensive. From among all four iterative methods, the fastest one was the CGS method.

Additional difficulties with solving BEM problems occur because of the reality that the matrix describing the BEM problem takes up a considerable amount of computer memory. The fact is that the matrix is dense and its elements are complex numbers, defined with double precision (COMPLEX*16). For example, a matrix with the size 3788, which in our case represents the biggest BEM problem with the same number of boundary elements, takes up 228MB of computer memory. Due to the limitation relating to the computer's operating system, *Windows,* which allows a 2GB-maximum memory, the solution of the BEM problem is limited to a maximum of approximately ten thousand boundary elements.

2.2 Vhodno-izhodni podatki

Podatke, ki se pojavljajo v interakciji z izvedenim programom MRE, delimo na vhodne in izhodne. Med prve uvrščamo informacije o položaju vozlišč, povezljivosti vozlišč v elemente, zunanjih normalah elementov in vektorjih hitrosti v vozliščih ter informacije o materialnih lastnostih posrednika, frekvenci ter obsežnosti problema. Drugi del vhodnih podatkov vsebuje informacije o mreži zunanjih točk, pri katerih je zajet zapis njihovih koordinat in povezljivost v končne elemente ter informacija o obsegu mreže zunanjih točk. Izhodni podatki vsebujejo rešitev problema na robu in v zunanjih točkah. Prvi del vsebuje informacije o vrednostih tlaka na robnih elementih ter vrednosti zvočne moči in sevalne učinkovitosti. Drugi del izhodnih podatkov vsebuje informacije o vrednostih tlaka in vektorjih hitrosti v zunanjih točkah. Vsi podatki so urejeno zapisani v ustreznih datotekah.

2.2 Input–output data

All the data that interact with the developed BEM program can be divided into input and output data. The input data include information about the nodes, their coordinates and connectivity into the elements, element outward normals, velocity vectors in the nodes, material data, frequency and information about the size of the problem. The second part of the input data contains information about the mesh of exterior points, nodes with their coordinates, and connectivity into finite elements, and information about the size of the exterior mesh. On the other hand, the output data contain information about the solution on the boundary and exterior points. The first part of the data includes the calculated complex values of the sound pressure for boundary elements, the sound power and the sound radiation factor. The second part of the output data contains information about the sound pressure and the velocity at exterior points. All data are available in formatted data files.

2.3 Verifikacija programa MRE

Za verifikacijo programa MRE smo pripravili trirazsežni verifikacijski model, za katerega je znana analitična rešitev. V našem primeru smo izbrali problem sevanja utripajoče krogle, ki ga lahko enakovredno opišemo z monopolnim točkastim virom. Valovna čela, ki pri takem valovanju nastajajo, imajo obliko krogel in se z zvočno hitrostjo enakomerno odmikajo od vira. Enačbi (24) in (25) sta matematični opis take oblike zvočnega valovanja za časovno spreminjanje zvočnega tlaka p in hitrosti v v prostoru:

2.3 Verification of the BEM program

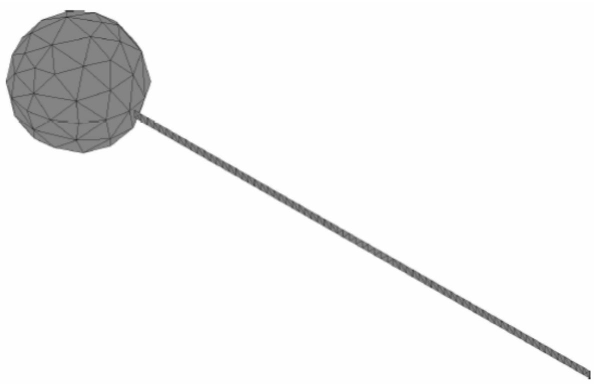
For the verification of the developed BEM program a 3D verification model, which has a known analytical solution, was prepared. In our case we chose the problem of a pulsating sphere. This kind of sound source can be described with a monopole point source that produces spherical waves travelling away from the source with the speed of sound. These sound waves are defined by the Equations (24) and (25), which represent the exact analytical solution of the sound pressure p and velocity v in time and space:

$$p(r,t) = \left(i\omega\rho/r\right)e^{i(\omega t+kr)} \tag{24}$$

$$v(r,t) = \left(ik-1/r\right)e^{i(\omega t+kr)} \tag{25}$$

Za izvedbo verifikacijskega modela je bilo treba problem krogelnega valovanja natančneje opredeliti. Predpostavili smo, da gre za zvočno valovanje, ki se širi s hitrostjo 344m/s skozi zrak

To prepare the verification model it was necessary to define its parameters more precisely. We assumed that we were dealing with sound radiation in the air at a frequency of 688 Hz, with the



Sl. 2. *Trirazsežni verifikacijski model*
Fig. 2. *3D verification model*

gostote 1,22kg/m^3 pri frekvenci 688Hz in ga oddaja krogla s polmerom 0,1m. Valovno število k, ki izhaja iz navedenih predpostavk, znaša 12,5664. Robne pogoje oz. hitrost v na površini krogle določimo iz enačbe (25). Njena vrednost pomeni amplitudo hitrosti nihanja površine krogle in je kompleksno število z vrednostjo (150,4–56,27i) m/s.

Za analizo vpliva gostote diskretizacije je bilo pripravljenih več različnih izvedenk verifikacijskega modela, tako da je bila dolžina robnega elementa določena na podlagi valovne dolžine, in sicer od največje priporočene velikosti robnega elementa [2] λ/6, padajoče v korakih, λ/10, λ/15 do λ/40. Na sliki 2 je prikazana izvedenka modela z robnimi elementi velikosti λ/10. V zunanji okolici krogle je definirano območje končnih elementov v obliki pasnice, namenjene za opazovanje rešitve zvočnega polja. Velikost končnih elementov je tu opredeljena z λ/40, kar omogoča bolj natančno opazovanje akustičnih parametrov v odvisnosti od oddaljenosti od zvočnega vira. Iz primerjave med analitično in numerično izračunanimi vrednostmi zvočnega tlaka in hitrosti v odvisnosti od razdalje do zvočnega vira (sl. 3) lahko ugotovimo, da so odstopanja v oddaljenem polju skoraj neopazna.

Pri analizi vpliva gostote diskretizacije smo opazovali nekaj osnovnih vrednosti zvočnega polja, ki smo jih primerjali z analitično izračunanimi vrednostmi in nato ocenili njihovo napako. Med opazovane vrednosti smo uvrstili zvočno moč ter zvočni tlak in hitrost v točki, oddaljeni 1m od središča krogle. Pri vrednostih zvočnega tlaka in hitrosti smo opazovali napako amplitude in faze. Rezultati analize so prikazani v preglednici 2, kjer poleg napak lahko opazujemo tudi število robnih elementov, potrebnih za diskretizacijo obravnavanega zvočnega vira, krogle s polmerom 0,1m.

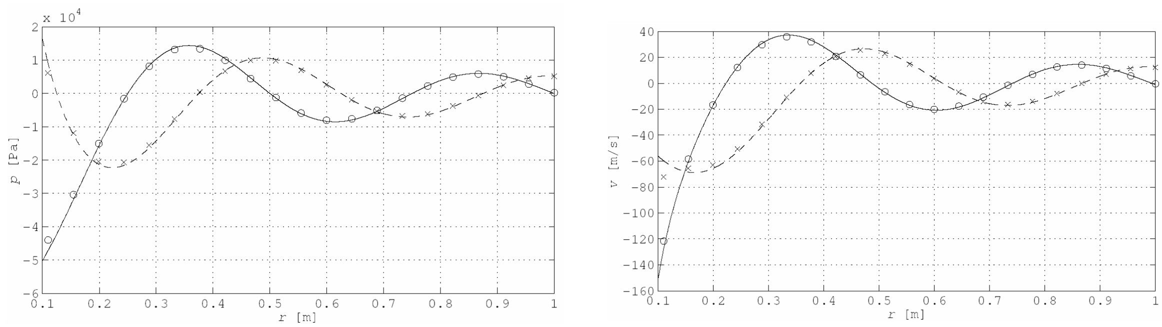
Napake, ki nastanejo pri numeričnem izračunu zvočnega polja z MRE, imajo tri glavne vire [3], v formulaciji zapisa MRE, numeričnem izračunu in diskretizaciji. Kljub temu, da izvedena

sound speed equal to 344 m/s and with an air density of 1.22 kg/m^3. The radius of the pulsating sphere is 0.1 m. Thus, the wave number k is 12.5664. The boundary conditions – velocity v is calculated directly from Equation (25). Its value represents the amplitude on the surface of the pulsating sphere and it is a complex value of (150.4–56.27i) m/s.

To analyse the influence of the discretization it was necessary to prepare more derivatives of the verification model, where each derivative had a different discretization or size of the boundary element. The size of the boundary element for each derivative is related to the wavelength, starting from the biggest recommended size of a boundary element of λ/6 [2] to smaller sizes of λ/10, λ/15 to λ/40, decreasing in steps. Figure 2 shows the derivative verification model with an element size of λ/10. In the exterior domain of the sphere there is also a strip-shaped mesh of finite elements that are intended for observing the solution of the sound field. The size of the exterior finite elements is defined by λ/40, which allows precise control of the numerically calculated acoustic parameters in terms of the distance from the sound source. From Figure 3 it is evident that the differences between the analytically and numerically calculated values are hardly noticeable in the far field.

In the analysis of the influence of the discretization only a few parameters of the sound field were controlled, and their errors are compared to the exact analytical values. Those acoustic parameters are the sound power, the sound pressure and the velocity at a distance of 1 m from the centre of the sphere. For the sound pressure and the velocity, both the phase and the amplitude errors were estimated. The results of the analysis are given in Table 2, where one can also find the number of boundary elements needed for the discretization of the sound source, i.e. the sphere with the radius of 0.1 m.

The errors that occur in the sound field, calculated by using the developed BEM program, have three major sources [3]: the formulation of the BEM, the numerical calculation and the discretization. Although this analysis



Sl. 3. *Primerjava analitično (*—— Re, - - - Im*) in numerično* (o Re, × Im*) izračunanih vrednosti zvočnega tlaka in hitrosti v odvisnosti od razdalje do zvočnega vira*

Fig. 3. *Comparison between analytically* (—— Re, - - - Im*) and numerically (*o Re, × Im*) calculated sound pressure and velocity as function of the distance from the sound source*

Preglednica 2. *Analiza napak pri izračunu akustičnih spremenljivk s programom MRE*
Table 2. *Error analysis of the acoustic parameters calculated by the BEM program*

Velikost elementov Element size	Število elementov Number of elements	Napaka moči Power error %	Napaka tlaka Sound pressure error Ampl. %	Napaka tlaka Sound pressure error Faza / Phase °	Napaka hitrosti Velocity error Ampl. %	Napaka hitrosti Velocity error Faza / Phase °
λ/6	36	-29,740	-20,047	-1,912	-20,034	-1,886
λ/10	144	-2,724	-2,903	-2,445	-2,901	-2,434
λ/15	224	1,329	-0,563	-3,174	-0,556	-3,161
λ/20	480	4,749	2,282	-3,687	2,284	-3,667
λ/25	642	5,836	3,231	-3,677	3,233	-3,657
λ/30	1028	7,367	4,689	-3,626	4,691	-3,607
λ/35	1228	7,920	5,289	-3,623	5,292	-3,603
λ/40	1763	8,833	6,461	-3,536	6,462	-3,543

analiza napak ne opredeljuje virov, lahko ob podrobnejšem pogledu v preglednico 2 sklepamo o njihovem viru. Pri velikosti robnih elementov λ/6 ali blizu tej vrednosti prevladuje napaka diskretizacije z negativnimi vrednostmi. Z zmanjševanjem velikosti elementov se napaka manjša in pri λ/15 ali več spremeni predznak ter postane pozitivna vendar manjša kot prej. Pozitivna vrednost napake se zvečuje neodvisno od numeričnega obsega oz. števila robnih elementov problema, vendar počasneje. Zaradi omenjenega sklepamo, da je v tem primeru vir napake v oblikovanju zapisa MRE in/ali numeričnem izračunu. Pri analizi napak je pomembno tudi spoznanje o vplivu frekvence. Ugotovili smo tudi, da se pri enaki gostoti diskretizacije in velikosti zvočnega vira napaka zmanjšuje z večanjem frekvence. To pomeni, da diskretizacija problema z velikostjo elementov λ/6 zagotavlja dovolj natančno reševanje problemov pri višjih frekvencah.

2.4 Integracija MRE v programski paket MKE

ANSYS je eden izmed programskih paketov za reševanje fizikalnih problemov z uporabo MKE. Odprtost omenjenega programa omogoča, da ga lahko izkoristimo tudi kot orodje za pripravo in obdelavo splošnih fizikalnih problemov, pri katerih diskretizacija modela temelji na mreženju (MKE, MRE, MKR idr.). Ker smo se v našem primeru pri reševanju akustičnih problemov omejili na tiste, ki izhajajo iz strukturnih nihanj, je bilo smiselno izkoristiti možnost reševanja problemov strukturne dinamike v programu *ANSYS* ter vanj integrirati izvedeno MRE za reševanje akustičnih problemov, ki je bistveno hitrejša od MKE.

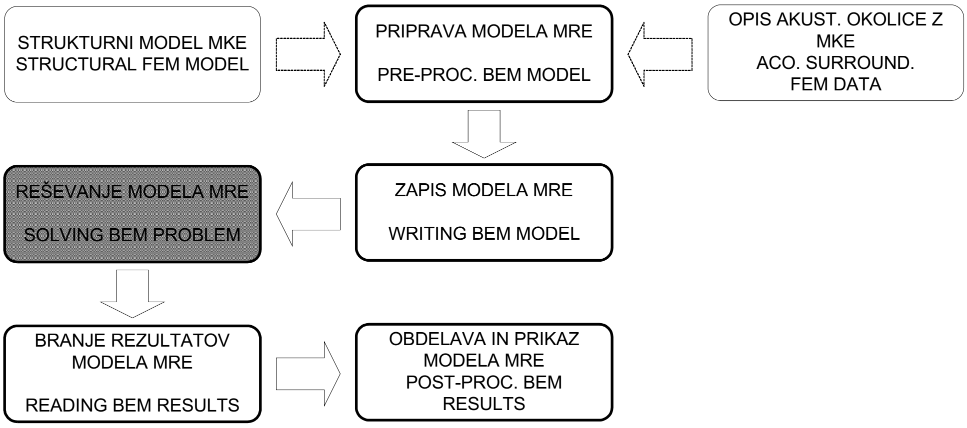
Za program MRE smo v *ANSYS*-ovem programskem okolju pripravili knjižnjico makro ukazov, ki so namenjeni za pripravo in obdelavo izhodno–vhodnih podatkov ter njihovo obdelavo.

does not specially define the source of the errors, closer inspection of the data in Table 2 can lead to the correct conclusion about the sources of the errors. As the size of the boundary element gets bigger or closer to λ/6, it is obvious that the error due to the discretization dominates with negative values. When decreasing the size of the boundary elements or increasing the discretization density, the error reduces, and at an element size of approximately λ/15 the error changes sign. A further reduction in the element size leads to an increase of the positive error, but its value is relatively small. For the error that occurs at smaller element sizes we cannot say if it is related to the discretization. On the contrary, this error has its source in the formulation of the BEM or/and in the numerical calculation. Analysing the error we came to an important conclusion about the frequency's influence on the accuracy. When we increase the frequency and keep the same discretization and the same size of the sound source, the error decreases. This means that the discretization that is defined with an element size of λ/6 also ensures sufficient accuracy for solving problems at higher frequencies.

2.4 Integration of the BEM program into the FEM software

ANSYS is one of the FEM software packages intended for solving general physical problems, and it can also be used as a tool for pre-processing and/or post-processing general physical models where the discretization is based on meshing (FEM, BEM, DM, etc.). In our case we limited the study to acoustic problems related to structural vibrations. Therefore, it was reasonable to use one of the FEM software packages, in our case *ANSYS*, to solve the structural dynamic problem and later on to integrate the developed BEM program into *ANSYS*. The fact is that the BEM is much faster for solving exterior acoustic problems than the FEM.

To allow the interaction between the BEM program and *ANSYS* it was necessary to develop a library of macro commands that allow one to pre-process, post-process and analyse the BEM input



Sl. 4. *Postopek izdelave in reševanja modela MRE*
Fig. 4. *Pre-processing, solving and post-processing the BEM model*

Omogočen je tudi prikaz in animacija akustičnih spremenljivk, kakor so zvočni tlak, vektorsko polje hitrosti in vektorsko polje zvočne intenzivnosti, kakor tudi prikaz ravni zvočnega tlaka ter zvočne intenzivnosti. Celoten postopek izdelave in reševanja modela MRE v okolju *ANSYS* je prikazan na sliki 4. Praktičen primer reševanja akustičnega problema z MRE pa je opisan v nadaljevanju za primer enosmernega elektromotorja.

and output data in the *ANSYS* environment. With the so-prepared macro commands it is also possible to animate the acoustic parameters, such as the instantaneous sound pressure and the velocity vector field and to display the sound pressure and the sound intensity level. Figure 4 shows the complete process of pre-processing, solving and post-processing of the BEM model in the *ANSYS* environment. A practical example of the acoustic BEM problem study is presented in the next paragraph.

3 OVREDNOTENJE ZVOČNEGA POLJA ELEKTROMOTORJA Z MRE

Prvi korak v pripravi modela MRE je bila izdelava diskretizirane zunanje površine, to je mreže robnih elementov. V našem primeru smo izhajali iz *ANSYS*-ovega strukturnega modela MKE enosmernega elektromotorja (sl. 5) in poznavanja njegovega harmonskega odziva, kot posledice vzbujanja z magnetnimi silami ([6] in [7]) (sl. 6). Na celotni zunanji površini strukturnega modela MKE smo izdelali mrežo trikotnih elementov, pri čemer je le-ta imela sklenjeno površino (sl. 7).

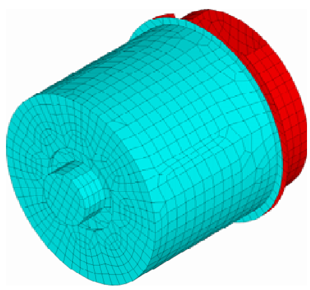
Zaradi poenostavitve prenosa robnih pogojev – hitrosti oz. vibracij iz strukturnega modela

3 SOUND-FIELD CALCULATION OF AN ELECTRIC MOTOR USING THE BEM

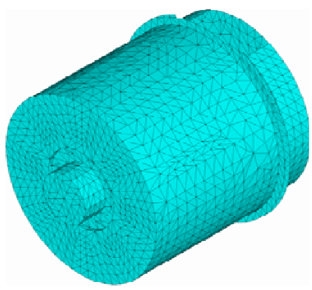
Building a mesh of boundary elements that covers the outer surface of the DC electric motor, shown in Fig. 5, was the first step in the procedure of the BEM model set-up. In our case we started from the *ANSYS* structural FEM model of the investigated electric motor, see Fig. 6, and from the previously FEM-calculated harmonic response due to the harmonic excitation of the magnetic forces. To build up a boundary-element mesh, the complete surface of the structural FEM model was covered by triangular elements that all together represented a closed surface, see Fig. 7.

Due to the simplification of the transfer of boundary conditions, velocities or vibrations from the

Sl. 5. *Trirazsežni model*
Fig. 5. *3D model*



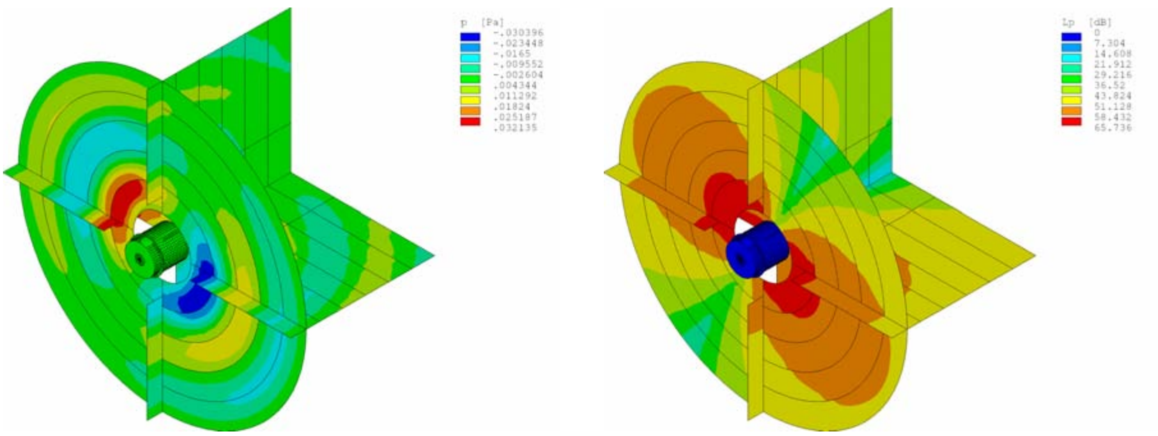
Sl. 6. *Strukturni model MKE*
Fig. 6. *Structural FEM model*



Sl. 7. *Model MRE*
Fig. 7. *BEM model*

na robne elemente, smo pri nastajanju mreže robnih elementov uporabili vozlišča in/ali elemente, ki določajo zunanjo površino strukturnega modela MKE. Ker pa je natančnost rešitve akustičnega problema odvisna od velikosti robnih elementov v celotni mreži, je treba pri snovanju strukturnega modela MKE že vnaprej upoštevati dejstvo, da morajo biti elementi na zunanji površini dovolj majhni. Drugi korak priprave modela MRE je izdelava mreže zunanjih točk. Za izdelavo mreže smo uporabili trikotne elemente, s katerimi smo zmrežili površine, na katerih smo želeli opazovati zvočno polje elektromotorja. Na podlagi pripravljenega modela MRE smo z izvedenim programom MRE poiskali njegovo rešitev in ovrednotili zvočno polje v okolici elektromotorja. Na sliki 8 so prikazane nekatere akustične spremenljivke, ki opisujejo zvočno polje v okolici enosmernega elektromotorja in so posledica strukturnega odziva osnovnega harmonika magnetnih sil. Prikazano zvočno polje, ki ga elektromotor oddaja in nastane zaradi delovanja osnovne harmonske komponente magnetnih sil, približno ustreza zvočnemu polju dipola [4].

structural FEM model into the BEM model, we used the nodes of the structural FEM model that lie on the exterior surfaces to define the boundary-element mesh. As the accuracy of the acoustical model depends on the size of the boundary elements, it was necessary to consider the mesh density of the structural FEM model in advance. Therefore, the finite elements of the structural model should be small enough. The second step in the process of the BEM model set-up was to prepare the mesh of the exterior points. The exterior mesh was built using triangular finite elements on the surfaces, where the sound field of the electric motor was to be controlled. Based on the so-prepared BEM model, the developed BEM program was used to calculate the sound field of the DC electric motor. Figure 8 shows two acoustic parameters that describe the sound field in the surroundings of the DC electric motor and that are the result of the structural response due to the basic harmonic component of the magnetic forces' excitation. The presented sound field radiated by the electric motor, due to the basic harmonic component of the magnetic forces, is similar to the acoustic dipole sound field [4].



Sl. 8. *Trenutni zvočni tlak (levo) ter raven zvočnega tlaka (desno)*
Fig. 8. *Instantaneous sound pressure (on the left) and sound pressure level (on the right)*

4 SKLEP

Prispevek opisuje reševanje zunanjih akustičnih problemov z MRE ter primer uporabe MRE za ovrednotenje zvočnega polja enosmernega elektromotorja, nastalega zaradi delovanja magnetnih sil. Predstavljena je tudi izvedba programa MRE za reševanje trirazsežnih problemov ter težave, na katere smo naleteli pri svojem delu ter integracija MRE v programski paket MKE – ANSYS. Izvedeni program smo ovrednotili s poudarkom na raziskavi vpliva gostote diskretizacije in identificirali vire napak. Glede na težave, ki se pojavijo pri reševanju problema MRE

4 CONCLUSION

This paper shows a basic approach to solving exterior acoustic problems using the BEM and a practical example of the calculation of an electric motor's sound field resulting from magnetic forces. Also presented is the development of the BEM program for solving 3D problems and its integration into a commercial FEM software package called *ANSYS*, with all the encountered difficulties. The developed BEM program was verified and a special emphasis was given to the influence of the discretization on the error. In addition, the sources of error were identified. Due to the difficulties that we came across when solving the system of linear equations oz. sistema linearnih enačb, ki pri tem nastane, smo raziskali vpliv izbire postopka reševanja sistema linearnih enačb glede na porabljeni čas.

of the BEM problem, different iterative methods were implemented into the BEM program and tested with regard to the time needed for the solution.

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