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Partial wave analysis of η photoproduction data with analyticity constraints*

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Abstract. We perform partial wave analysis of the η photoproduction on data. The obtained multipoles are consistent with the fixed-t analyticity and fixed-s analyticity. A fixed-t analyticity is imposed using Pietarinen expansion method. The invariant amplitudes obey the required crossing symmetry.

1 Introduction

A big problem in partial wave analyses are ambiguities of partial wave solutions. More than one set of partial waves describe equally well the experimental data. A first attempt to solve this problem was to require smoothness of partial waves as a function of energy. It was shown that this criteria was not enough to achieve a unique partial wave solution [1]. Furthermore, it was shown that more stringent constraints, based on the analytic properties of invariant amplitudes from Mandelstam hypothesis, should be taken into consideration. An efficient method for imposing the fixed-t analyticity on invariant amplitudes was proposed by E. Pietarinen [2–5] and was used in Karlsruhe-Helsinki partial wave analysis of πN scattering data KH80 [6–8]. In our partial wave analysis of η-photoproduction data we follow main ideas from Karlsruhe-Helsinki analysis. The method consists of two separate analyses: Fixed-t amplitude analysis (FT AA) and a single energy partial wave analysis (SE PWA). The two analyses are coupled in such a way that results from one are used as a constraint in another in an iterative procedure. The resulting partial waves (multipoles) describe experimental data adequately and are consistent with fixed-t and fixed-s analyticity as well.

2 Preparing experimental data for partial wave analysis

Our data base consists of the following experimental data:
— Differential cross sections at 120 energies in the range 710 MeV $\leq E_{lab} \leq$ 1395 MeV [9];

* Talk presented by J. Stahov

— Beam asymmetry Σ at 15 energies in the range 724 MeV – 1472 MeV [10];
— Target asymmetry T at 12 energies in the range 725 MeV – 1350 MeV [11];
— Double asymmetry F at 12 energies in the range 725 MeV – 1350 MeV [11].
In SE PWA experimental data are required at a predetermined set of energies. Experimental values of beam asymmetry, target asymmetry and double polarization asymmetry are interpolated to 113 energies, where data on differential cross sections are available. A spline fit method with $\chi^2/dp = 0.7$ (DP-number of data points) was used. FT AA requires experimental data at predetermined set of t values. Using the same method, data previously prepared for SE PWA were shifted to 40 t values in the range $t \in [-1.00\,\mathrm{GeV}^2, -0.05\,\mathrm{GeV}^2]$.

3 Fixed-t amplitude analysis

Following definition in Ref. [12], in description of η-meson photoproduction, we use crossing symmetric invariant amplitudes B_1, B_2, B_6, and B_8/ν. For a given value of variable t amplitudes are represented by two Pietarinen expansions in the form

$$F_k(\nu^2, t) = F_{kN}(\nu^2, t) + (1 + z_1) \sum_{i=1}^{N_1} b_{1i}^{(k)} z_1^i + (1 + z_2) \sum_{i=1}^{N_2} b_{2i}^{(k)} z_2^i, \qquad (1)$$

where F_k stands for invariant amplitudes B_k. F_{kN} are explicitly known nucleon pole contributions and s, u and $\nu = (s - u)/4m$ with the proton mass m are Mandelstam variables. The conformal variables z_1 and z_2 are defined as

$$z_1 = \frac{\alpha_1 - \sqrt{\nu_{th1}^2 - \nu^2}}{\alpha_1 + \sqrt{\nu_{th1}^2 - \nu^2}}, \qquad z_2 = \frac{\alpha_2 - \sqrt{\nu_{th2}^2 - \nu^2}}{\alpha_2 + \sqrt{\nu_{th2}^2 - \nu^2}}. \qquad (2)$$

ν_{th1} and ν_{th2} correspond to the π and η photoproduction thresholds ($\gamma p \to \pi^0 p$ and $\gamma p \to \eta p$). N_1 and N_2 are number of parameters in expansion (1) (in our applications $N_1, N_2 \approx 15$). α_1 and α_2 are parameters which determine distribution of points on a unit circle ($|z_1| = |z_2| = 1$). Coefficients $b_1^{(k)}$ and $b_2^{(k)}$ in expansion (1) are determined by minimizing a quadratic form

$$\chi^2 = \chi^2_{data} + \chi^2_{PW} + \Phi. \qquad (3)$$

The term χ^2_{data} is the standard exspression containing all the data at a fixed-t value

$$\chi^2_{data} = \sum_{D} \sum_{n=1}^{N_D} \frac{(D_n^{exp}(\nu^2, t) - D_n^{fit}(\nu^2, t))^2}{\Delta^2_{D_n}}, \qquad (4)$$

where D stands for measurable quantities ($\sigma_0 = d\sigma/d\Omega$, $\sigma_0 \cdot T$, $\sigma_0 \cdot F$, $\sigma_0 \cdot \Sigma$). The sum goes over all N_D available experimental values of measured quantities D for a given t value. D_n^{fit} are predicted values in terms of coefficients in expansion (1).

A second term χ^2_{PW} is also a usual χ^2 expression containing as "data" the helicity amplitudes calculated from the partial wave solution

$$\chi^2_{PW} = q \sum_{k=1}^{4} \sum_{i=1}^{N_D} \left\{ \frac{\left[\mathrm{Re}\, H_k^{fit}\left(t, \nu_i^2\right) - \mathrm{Re}\, H_k^{PW}\left(t, \nu_i^2\right)\right]^2}{(\varepsilon_R)^2_{ki}} + \frac{\left[\mathrm{Im}\, H_k^{fit}\left(t, \nu_i^2\right) - \mathrm{Im}\, H_k^{PW}\left(t, \nu_i^2\right)\right]^2}{(\varepsilon_I)^2_{ki}} \right\}. \qquad (5)$$

In the first iteration H_k^{PW} are calculated from an initial, already existing solution. In the subsequent iterations H_k^{PW} are calculated from partial waves obtained in the single energy partial wave analysis (SE PWA) of the same set of experimental data. The weight factor q and errors ε_{ki} are unknown. They are adjusted in such a way that $\chi^2_{data} \approx \chi^2_{PW}$. Φ is Pietarinen's penalty function in the form

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4, \qquad (6)$$

where Φ_k is defined as

$$\Phi_k = \lambda_{1k} \sum_{i=1}^{N_1} \left(b_{1i}^{(k)}\right)^2 (i+1)^3 + \lambda_{2k} \sum_{i=1}^{N_2} \left(b_{2i}^{(k)}\right)^2 (i+1)^3 .$$

$\lambda_{11}, \lambda_{21}, \ldots, \lambda_{14}, \lambda_{24}$ are weight factors determined according to the convergence test function method [5]. The final result of the fixed-t amplitude analysis consists of 40 sets of coefficients $b_1^{(k)}$ and $b_2^{(k)}$. The invariant amplitudes may be calculated at any c.m. energy W and scattering angle θ in the physical region. Helicity amplitudes are used as a constraint in a SE PWA. Helicity amplitudes in terms of invariant amplitudes are given in the Appendix.

3.1 Single energy partial wave analysis

In the single energy partial wave analysis we minimize a quadratic form:

$$\chi^2 = \chi^2_{data} + \chi^2_{FT}. \qquad (7)$$

χ^2_{data} is again a standard expression containing all the data at a given energy. For a given observable D, measured at N_D angles θ_i, contribution to the χ^2_{data} reads:

$$\left(\chi^2_{data}\right)_D = \sum_{i=1}^{N} \left[\frac{D_{exp}\left(\theta_i\right) - D_{fit}\left(\theta_i\right)}{\Delta_{Di}}\right]^2,$$

$$\chi^2_{data} = \sum_{D} \left(\chi^2_{data}\right)_D .$$

$D_{exp}\left(\theta_i\right)$ are experimental values of observable D with corresponding experimental errors Δ_{Di}. $D_{fit}\left(\theta_i\right)$ are values of observable D calculated from partial waves which are parameters in the fit. The second term χ^2_{FT} is also a usual χ^2

expression containing as "data" the helicity amplitudes H_k from the fixed-t amplitude analysis. It has the form

$$\chi^2_{FT} = \sum_{k=1}^{4}\sum_{i=1}^{Nc}\left\{\left[\frac{\mathrm{Re}\,H_k(\theta_i) - \mathrm{Re}\,H_k^{fit}(\theta_i)}{(\varepsilon_R)_{ki}}\right]^2 + \left[\frac{\mathrm{Im}\,H_k(\theta_i) - \mathrm{Im}\,H_k^{fit}(\theta_i)}{(\varepsilon_I)_{ki}}\right]^2\right\}.$$

The angles θ_i are calculated using the formula

$$\cos\theta_i = \frac{t_i - m_\eta^2 + 2k\omega}{2kq}, \quad \cos\theta_i \in [-1.00, +1.00], \tag{8}$$

where m_η, q, and ω are mass, c.m. momentum and c.m. energy of the η meson, and k is the c.m. momentum of the photon. N_c is the number of angles at which constraining amplitudes are given. Errors of real and imaginary parts (ε_R), (ε_I) are not determined. They are adjusted in such a way that $\chi^2_{data} \approx \chi^2_{FT}$. After performing SE PWA at predetermined N_D energies, the obtained partial wave values are used as a constraint in the fixed-t amplitude analysis. The "data" in the term χ^2_{PW} of (3) are to be calculated using these partial waves.

Our iterative procedure is shown in Fig 1.

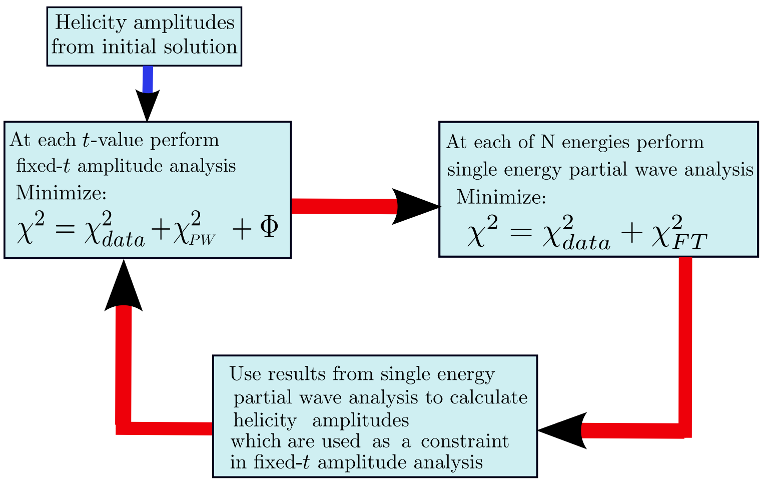

Fig. 1. (Color online) Iterative procedure in a combined single energy partial wave analysis and fixed-t amplitude analysis.

To make our analysis easier to follow, we give more details about important steps after preparing input data as described in section 2.

1. Take an initial solution (MAID [13] or Bonn-Gatchina [14, 15]) and calculate all four invariant amplitudes $B_i(W, t)$ at all t values and energies where input data are available.
2. Perform the Pietarinen expansion for all invariant amplitudes using equation (1) with conformal variables defined in formula (2).

3. Calculate helicity amplitudes from invariant amplitudes (see Appendix).
4. For all t values perform a non-linear fit of observables minimizing the quadratic form (3). As starting values of parameters $b_1^{(k)}$ and $b_2^{(k)}$ take coefficients obtained in step 2. Calculate term χ^2_{PW} using initial solution to calculate H_k^{PW}. This step completes the FT AA.
5. At a given energy W calculate helicity amplitudes $H_k(W, \cos\theta_i)$, where $\cos\theta_i$ are given by formula (8). Use coefficients $b_1^{(k)}$ and $b_2^{(k)}$ from FT AA for corresponding t-values.
6. Perform a non-linear SE PWA using helicity amplitudes obtained in step 5 as a constraint. As starting values for partial waves (multipoles) use the same initial solution as in step 1.
7. Use results from step 6 in step 1 and perform next iteration. Our preliminary results show that, depending on the strength of constraints, it is enough to perform 2-3 iterations to get a stable final solution.

In Fig. 2 fits of invariant amplitudes are shown at $t = -0.15$ GeV^2. Multipoles with $L \leq 3$, obtained after two iterations, are shown in Figs. 3 and 4. The Eta-Maid2015b solution was chosen as a starting solution in both analyses, FT AA and SE PWA.

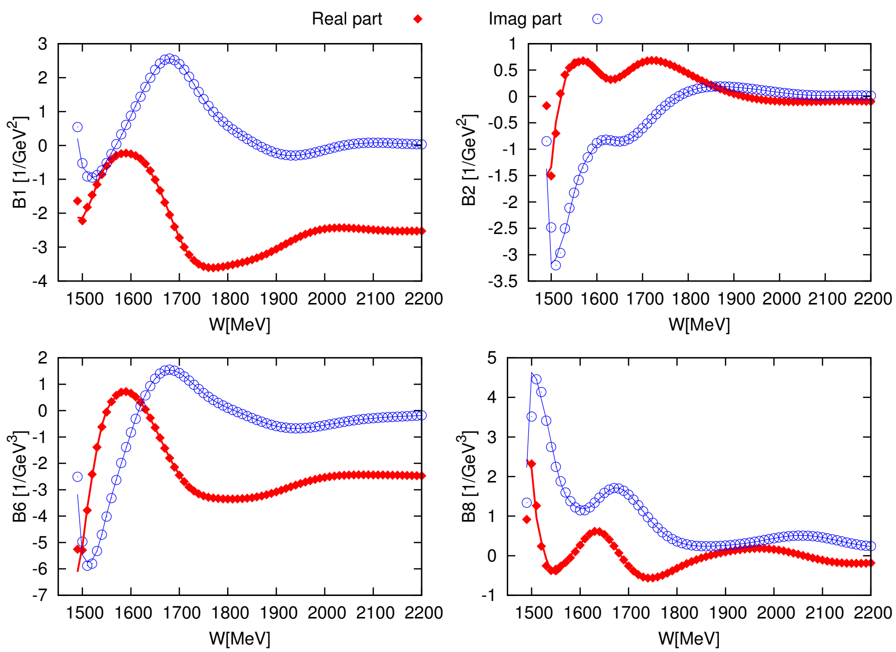

Fig. 2. (Color online) Red diamonds and blue circles show initial real and imaginary values of invariant amplitudes. As initial solution invariant amplitudes for $t = -0.15$ GeV^2 from etaMAID2015b [13] are used. The red and blue lines show the Pietarinen fits to real and imaginary parts of invariant amplitudes, respectively

.

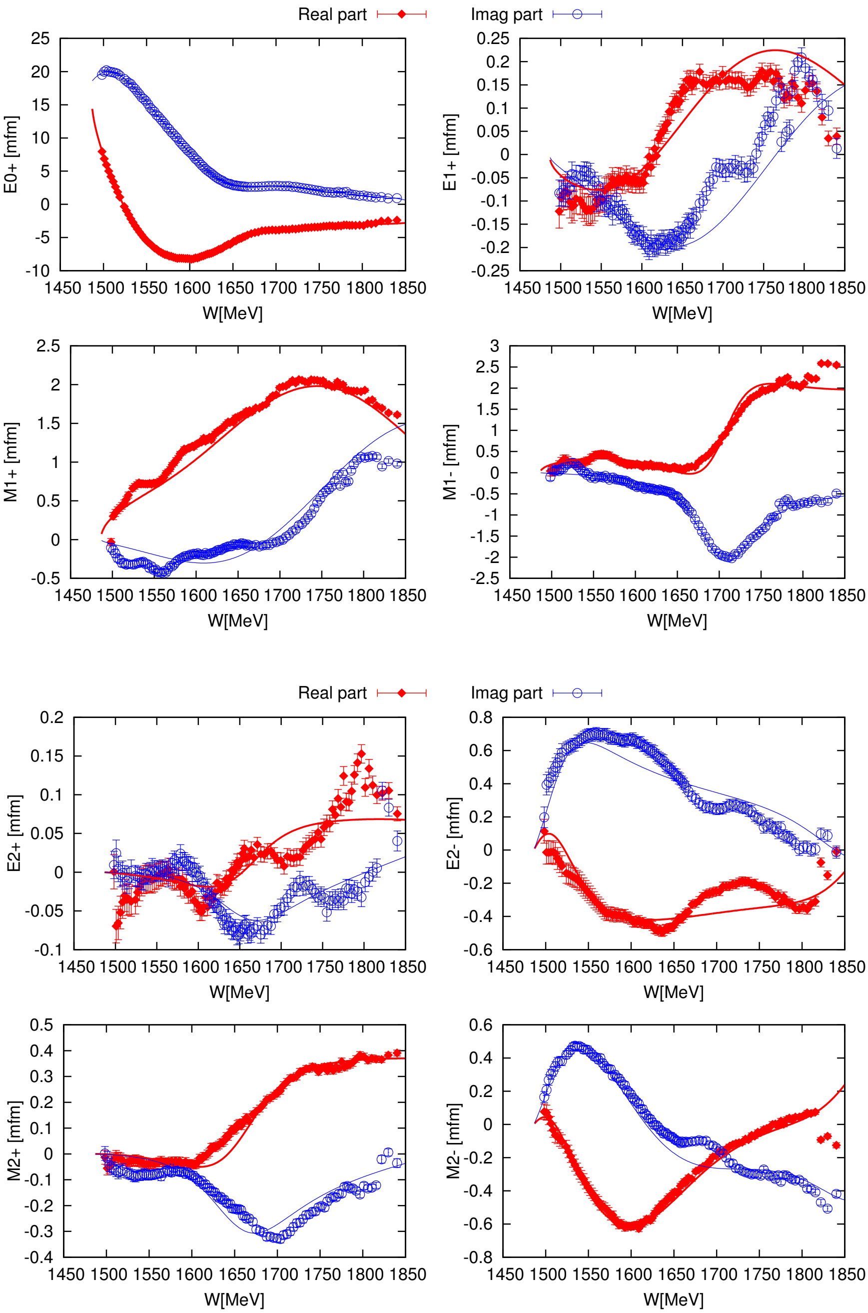

Fig. 3. (Color online) Real and imaginary parts of multipoles obtained from SE PWA in 2nd iteration are shown as red diamonds and blue circles. The initial solution etaMAID2015b is given as red and blue solid lines.

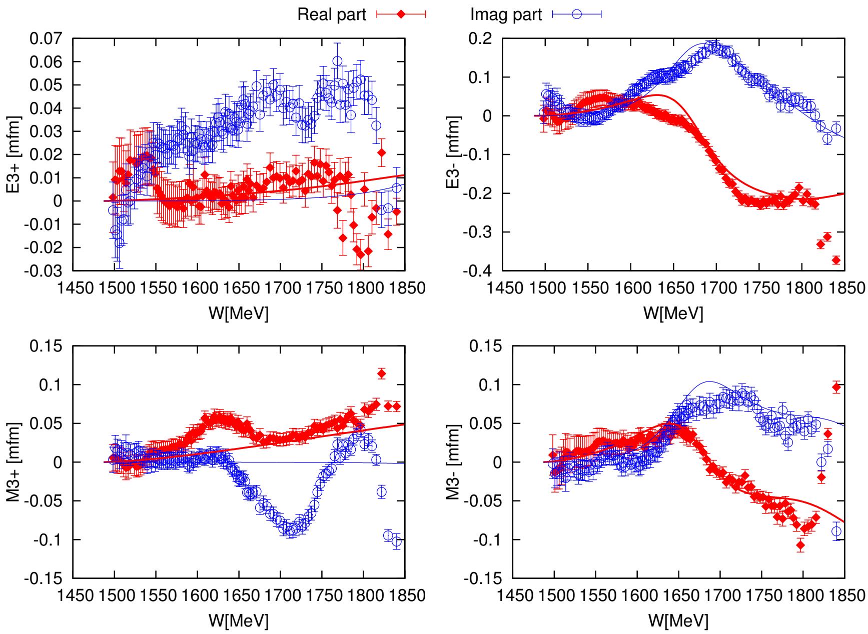

Fig. 4. [Continued from previous page.] Caption as in Fig. 3.

4 Conclusions

A SE PWA with fixed-t constraints has been performed and multipoles, consistent with crossing symmetry and fixed-t analyticity, have been obtained. The helicity amplitudes from fixed-t show good consistency with fixed-s analyticity. It implies that our amplitudes are consistent with both, fixed-t and fixed-s analyticity.

Acknowledgment

This work was supported in part by the Federal Ministry of Education and Science, Bosnia and Herzegovina, Grant No. 05-39-3545-1/14 and by the Deutsche Forschungsgemeinschaft, Collaborative Research Center 1044.

Appendix

A Multipole expansion of invariant amplitudes

In partial wave analysis of pseudoscalar meson photoproduction it is convenient to work with CGLN amplitudes [16] giving simple representations in terms of electric and magnetic multipoles and derivatives of Legendre polynomials

$$
\begin{aligned}
F_1 &= \sum_{l=0}^{\infty}[(lM_{l+} + E_{l+})P'_{l+1}(x) + ((l+1)M_{l+} + E_{l-})P'_{l-1}(x)],\\
F_2 &= \sum_{l=1}^{\infty}[(l+1)M_{l+} + lM_{l-}]P'_{l}(x),\\
F_3 &= \sum_{l=1}^{\infty}[(E_{l+} - M_{l+})P''_{l+1} + (E_{l-} + M_{l-})P''_{l-1}(x)],\\
F_4 &= \sum_{l=2}^{\infty}[M_{l+} - E_{l+} - M_{l-} - E_{l-}]P''_{l}(x).
\end{aligned}
\tag{A.1}
$$

Another common set of amplitudes are helicity amplitudes, which are linearly related to the CGLN amplitudes

$$
\begin{aligned}
H_1 &= -\frac{1}{\sqrt{2}}\sin\theta\cos\frac{\theta}{2}(F_3 + F_4),\\
H_2 &= \sqrt{2}\cos\frac{\theta}{2}[(F_2 - F_1) + \frac{1-\cos\theta}{2}(F_3 - F_4)],\\
H_3 &= \frac{1}{\sqrt{2}}\sin\theta\sin\frac{\theta}{2}(F_3 - F_4),\\
H_4 &= \sqrt{2}\sin\frac{\theta}{2}[(F_1 + F_2) + \frac{1+\cos\theta}{2}(F_3 + F_4)].
\end{aligned}
\tag{A.2}
$$

The relations between CGLN and invariant amplitudes are given by

$$
\begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix} = M \cdot \begin{pmatrix} B_1 \\ B_2 \\ B_6 \\ B_8 \end{pmatrix},
\tag{A.3}
$$

with the matrix M:

$$
M = \frac{1}{2W(s-m^2)}\begin{pmatrix}
\frac{(s-m^2)}{a_1} & -\frac{(s-m^2)}{a_2} & 0 & 0 \\
0 & 0 & -\frac{(t-m_\eta^2)(m-W)}{2a_3} & -\frac{(t-m_\eta^2)(m+W)}{2a_4} \\
-\frac{2(m+W)}{a_1} & \frac{2(m-W)}{a_2} & -\frac{(t-m_\eta^2)}{a_3} & -\frac{(t-m_\eta^2)}{a_4} \\
-\frac{(m+W)}{a_1} & \frac{(m-W)}{a_2} & -\frac{(s-u)}{2a_3} & -\frac{(s-u)}{2a_4}
\end{pmatrix}.
\tag{A.4}
$$

and

$$
\begin{aligned}
a_1 &= \frac{\sqrt{(E_1+m)(E_2+m)}}{8\pi W},\\
a_2 &= \frac{\sqrt{(E_1-m)(E_2-m)}}{8\pi W},\\
a_3 &= \frac{\sqrt{(E_1-m)(E_2-m)}(E_2+m)}{8\pi W} = a_2\cdot(E_2+m),
\end{aligned}
$$

$$a_4 = \frac{\sqrt{(E_1+m)(E_2+m)}(E_2-m)}{8\pi W} = a_1 \cdot (E_2 - m)\,,$$

$$s + t + u = \sum = 2m^2 + m_\eta^2, \qquad \nu = \frac{s-u}{4m}\,,$$

where E_1 and E_2 are c.m. energies of the incoming and outgoing nucleons and W is the total c.m. energy.

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Analiza delnih valov za podatke pri fotoprodukciji mezona η z upoštevanjem omejitev zaradi analitičnosti

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Izvedemo analizo delnih valov za podatke pri fotoprodukciji η. Dobljeni multipoli so v skladu z analitičnostjo pri fiksnem t in pri fiksnem s. Analitičnost pri fiksnem t zagotovimo s Pietarinenovo metodo. Invariantne amplitude ubogajo zahtevano navzkrižno simetrijo.

Napredek pri poznavanju sklopitev nevtrona

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Podajamo pregled prizadevanj skupine GW SAID za analizo fotoprodukcije pionov na nevtronski tarči. Razločitev izoskalarnih in izovektorskih elektromagnetnih sklopitev resonanc N^* in Δ^* zahteva primerljive in skladne podatke na protonski in na nevtronski tarči. Interakcija v končnem stanju igra kritično vlogo pri najsodobnejši analizi in izvrednotenju podatkov za proces $\gamma n \to \pi N$ pri eksperimentih z devteronsko tarčo. Ta je pomemben sestavni del tekočih programov v laboratorijih JLab, MAMI-C, SPring-8, CBELSA in ELPH.

Vzbujanje barionskih resonanc s fotoprodukcijo mezonov

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Spektroskopija lahkih hadronov je še vedno živahno področje v fiziki jedra in delcev. Celo 50 let po odkritju Roperjeve resonance in več kot 30 let po pionirskem delu Hoehlerja and Cutkoskyja je še veliko odprtih vprašanj glede barionskih resonanc. Danes je glavni vzbujevalni mehanizem fotoprodukcija in elektroprodukcija mezonov, merjena na elektronskih pospeševalnikih kot so MAMI, ELSA in JLab. V združenem prizadevanju izvrednotimo lege in jakosti polov iz parcialnih valov, dobljenih z analizo parcialnih valov pri nedavnih meritvah polarizacij ob uporabi analitičnih omejitev iz disperzijskih relacij pri fiksnem t. Poseben poudarek pri barionskih resonancah je na strukturi pola na različnih Riemannovih ploskvah.