Assessment of surface deformation with simultaneous adjustment with several epochs of leveling networks by using nD relative pedaloid

Ocena deformacij-s simultano izravnavo več terminskih izmer z nD relativnim pedaloidom

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Abstract: Relative error hyperellipsoid, 3D relative error pedaloid and 2D relative pedal curve are discussed.

Izvleček: V članku govori o 3D relativnem pedaloidu pogreškov in 2D relativni pedali.

Key words: Adjustment by parameter variation, nD relative error hyperellipsoid and hyperellipsoid, 3D relative error ellipsoid, 2D relative ellipse.
Ključne besede: Posredna izravnava, nD relativni hiperelipsoid pogreškov in hiperelipsoid, 3D relativni elipsoid pogreškov, 2D relativni elipsoid.

Introduction

Consequence of underground extraction of coal is surface alteration. Negative consequences of mining are reflected above all as ground deformation, field landslides, formation of lakes, climate changes due to alteration of landscape, influence on subterranean waters and thermal springs, seismic effects of subterranean blasting.

Ground subsidence is the most intensive above extraction fields but can also be observed on the edge fields. That is the reason for planning local observation networks, by which expanse of deformation can be determined. Observation of networks is important because of closeness of outbuildings and other buildings.

With simultaneous adjustment of several epochs of measurements, the field deformation can be determined.

Theoretical basis of adjustment by parameter variation in geodetic leveling network

Ultimate aim for adjustment of geodetic networks are point coordinates. Definitive

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or most probable point coordinates can not be obtained by direct mathematical processing of measured quantities (angles, lengths, height differences, etc.), they can be only determined by process of adjustment. This process is possible only if the number of measured data is greater then necessarily needed.

In the leveling network adjustment one point with known absolute height should be given (or assumed). This holds for adjustment by parameter (height coordinate) variation.

In the adjustment there are three types of quantities:

- given quantities (constant values, which don't change by adjustment),
- measured quantities,
- unknown quantities.

By adjustment unknown quantities are determined from given quantities through series of measured quantities on condition that the sum of squares of their residuals is minimal. With observation equation coefficients a_i, b_i,...u_i, and absolute terms f_i. Coefficients are partial derivatives of functional relation between given, measured and unknown quantities. Their values depend on configuration and size of network. Absolute terms can be symbolically expressed as f_i = *approximate – measured*. Approximate values are computed from approximate coordinates.
The adjustment is done considering:

$$\mathbf{v}^T \mathbf{Q}_{ll}^{-1} \mathbf{v} = \min \quad \text{or} \quad \mathbf{v}^T \mathbf{P} \mathbf{v} = \min \qquad \text{(a)}$$

v residuals,
$\mathbf{Q}_{ll}$ correlation matrix of measured quantities,
P weight matrix of measured quantities.

Observation equations can be written in matrix form:

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_i \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & \cdots & u_1 \\ a_2 & b_2 & \ldots & u_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_i & b_i & \cdots & u_i \end{bmatrix} * \begin{bmatrix} x \\ y \\ \vdots \\ t \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \end{bmatrix} \qquad \text{(b)}$$

Or shortly:

v vector of residuals,
A design matrix of observation equations,
x unknowns vector,
f vector of absolute terms.

Coefficient matrix of normal equations **N** reads:

$$\mathbf{v} = \mathbf{A} \cdot \mathbf{x} + \mathbf{f} \qquad \text{(c)}$$

$$\mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A} = \begin{bmatrix} [paa] & [pab] & \cdots & [pau] \\ [pba] & [pbb] & \cdots & [pbu] \\ \vdots & \vdots & \ddots & \vdots \\ [pua] & [pub] & \cdots & [puu] \end{bmatrix} \qquad \text{(d)}$$

$$\mathbf{P} = \mathbf{diag}\begin{bmatrix} p_1 & p_2 & \cdots & p_i \end{bmatrix} \qquad \text{(e)}$$

When measurements are of the same accuracy then $\mathbf{P} = p\mathbf{I}$, where **I** is unit matrix.
Vector of absolute terms of normal equations **n**:

$$\mathbf{n} = \mathbf{A}^{\mathrm{T}} \mathbf{P} \mathbf{f} = \begin{bmatrix} [paf] \\ [pbf] \\ \vdots \\ [puf] \end{bmatrix} = \mathbf{A}^{\mathrm{T}} p \mathbf{I} f = p \mathbf{A}^T f \qquad \text{(f)}$$

Vector of unknowns is:

$$\mathbf{x} = -\mathbf{N}^{-1}\mathbf{n} = -\left(\mathbf{A}^T\mathbf{PA}\right)^{-1}\mathbf{A}^T\mathbf{Pf} = -\mathbf{N}^{-1}\mathbf{A}^T\mathbf{Pf} \tag{g}$$

Then vector of residuals can be calculated:

$$\mathbf{v} = \mathbf{A}\cdot\mathbf{x} + \mathbf{f} \tag{h}$$

Relative error curve

Relative error curve does not depend on the network datum or coordinate origin.

nD relative hyperpedaloid and hyperellipsoid

$\underset{n,n}{\mathbf{Q}_{hyperellipsoid}}$ in equation (9) can be written as product of unit matrix **I**, matrix $Q_{\Delta_i\xi\Delta_j\xi}$:

$$\underset{n,n}{\mathbf{Q}_{hyperellipsoid}} = \underset{n,n}{\begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & Q_{\Delta_i\xi\Delta_j\xi} & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix}} = \underset{n,2n}{\begin{bmatrix} \underset{n,n}{\mathbf{I}} & -\underset{n,n}{\mathbf{I}} \end{bmatrix}} \underset{2n,2n}{\begin{bmatrix} \underset{n,n}{\mathbf{Q}_{\xi\xi SS}} & \underset{n,n}{\mathbf{Q}_{\xi\xi SV}} \\ \underset{n,n}{\mathbf{Q}_{\xi\xi SV}} & \underset{n,n}{\mathbf{Q}_{\xi\xi VV}} \end{bmatrix}} \underset{2n,2}{\begin{bmatrix} \underset{n,n}{\mathbf{I}} \\ -\underset{n,n}{\mathbf{I}} \end{bmatrix}} =$$

$$= \underset{n,2n}{\begin{bmatrix} \underset{n,n}{\mathbf{I}} & -\underset{n,n}{\mathbf{I}} \end{bmatrix}} \underset{2n,n}{\begin{bmatrix} \underset{n,n}{\mathbf{Q}_{\xi\xi SS}} & -\underset{n,n}{\mathbf{Q}_{\xi\xi SV}} \\ \underset{n,n}{\mathbf{Q}^T_{\xi\xi SV}} & -\underset{n,n}{\mathbf{Q}_{\xi\xi VV}} \end{bmatrix}} = \underset{n,n}{\begin{bmatrix} \underset{n,n}{\mathbf{Q}_{\xi\xi SS}} & -\underset{n,n}{\mathbf{Q}_{\xi\xi SV}} & -\underset{n,n}{\mathbf{Q}^T_{\xi\xi SV}} & +\underset{n,n}{\mathbf{Q}_{\xi\xi VV}} \end{bmatrix}} \tag{i}$$

But also:

$$\underset{n,n}{\mathbf{Q}_{hyperellipsoid}} = \underset{n,n}{\begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & Q_{\Delta_i\xi\Delta_j\xi} & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix}} = \underset{n,n}{\begin{bmatrix} \ddots & & \vdots & & \ddots \\ \cdots & Q_{{}_i\xi_S{}_j\xi_S} & -Q_{{}_i\xi_S{}_j\xi_V} & -Q_{{}_i\xi_S{}_j\xi_V} + Q_{{}_i\xi_V{}_j\xi_V} & \cdots \\ \ddots & & \vdots & & \ddots \end{bmatrix}} \tag{j}$$

or

$$Q_{\Delta_i\xi\Delta_j\xi} = Q_{{}_i\xi_S{}_j\xi_S} - Q_{{}_i\xi_S{}_j\xi_V} - Q_{{}_i\xi_S{}_j\xi_V} + Q_{{}_i\xi_V{}_j\xi_V} \tag{k}$$

And after addition:

$$Q_{\Delta_i\xi\Delta_j\xi} = Q_{{}_i\xi_S{}_j\xi_S} - 2Q_{{}_i\xi_S{}_j\xi_V} + Q_{{}_i\xi_V{}_j\xi_V} \tag{l}$$

3D relative pedaloid and ellipsoid

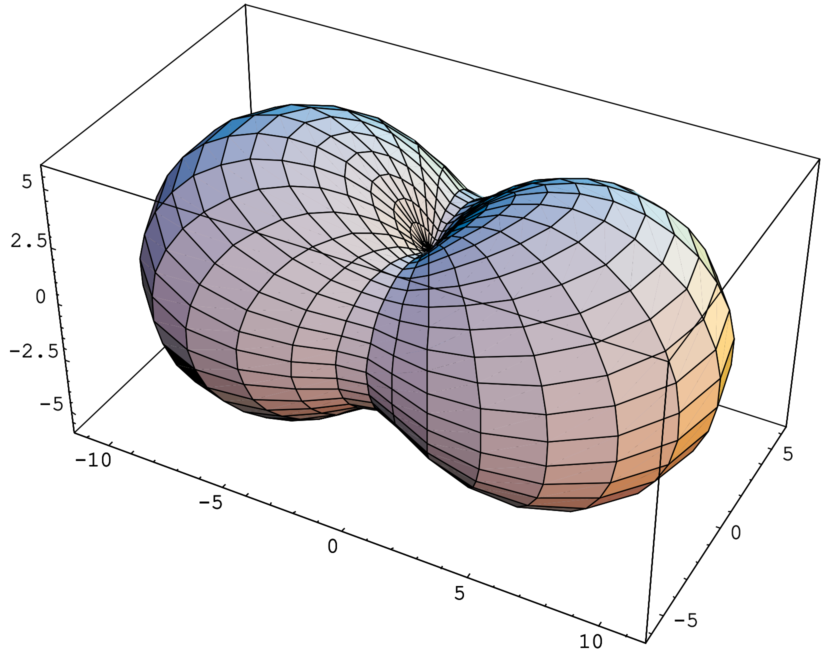

Figure 1. 3D relative ellipsoid
Slika 2. 3D relativni elipsoid

The elements of relative error ellipsoid $\mathbf{Q}_{ellipsoid\ SV}$ are linear combination of matrix **Q** elements:

$$\mathbf{Q}_{ellipsoid\ SV}=\begin{bmatrix} Q_{\Delta Z\Delta Z} & Q_{\Delta Z\Delta Y} & Q_{\Delta Z\Delta X} \\ Q_{\Delta Z\Delta Y} & Q_{\Delta Y\Delta Y} & Q_{\Delta Y\Delta X} \\ Q_{\Delta Z\Delta X} & Q_{\Delta Y\Delta X} & Q_{\Delta X\Delta X} \end{bmatrix}=$$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\begin{bmatrix} Q_{Z_SZ_S} & Q_{Z_SY_S} & Q_{Z_SX_S} \\ Q_{Z_SY_S} & Q_{Y_SY_S} & Q_{Y_SX_S} \\ Q_{Z_SX_S} & Q_{Y_SX_S} & Q_{X_SX_S} \end{bmatrix} & \begin{bmatrix} Q_{Z_SZ_V} & Q_{Z_SY_V} & Q_{Z_SX_V} \\ Q_{Y_SZ_V} & Q_{Y_SY_V} & Q_{Y_SX_V} \\ Q_{X_SZ_V} & Q_{X_SY_V} & Q_{X_SX_V} \end{bmatrix} \\ \begin{bmatrix} Q_{Z_SZ_V} & Q_{Y_SZ_V} & Q_{X_SZ_V} \\ Q_{Z_SY_V} & Q_{Y_SY_V} & Q_{X_SY_V} \\ Q_{Z_SX_V} & Q_{Y_SX_V} & Q_{X_SX_V} \end{bmatrix} & \begin{bmatrix} Q_{Z_VZ_V} & Q_{Z_VY_V} & Q_{Z_VX_V} \\ Q_{Z_VY_V} & Q_{Y_VY_V} & Q_{Y_SX_V} \\ Q_{Z_VX_V} & Q_{Y_VX_V} & Q_{X_VX_V} \end{bmatrix}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \\ -\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\end{bmatrix} \quad \text{(m)}$$

After right multiplication:

$$
\mathbf{Q}_{ellipsoid\ SV} = \begin{bmatrix} Q_{\Delta Z\Delta Z} & Q_{\Delta Z\Delta Y} & Q_{\Delta Z\Delta X} \\ Q_{\Delta Z\Delta Y} & Q_{\Delta Y\Delta Y} & Q_{\Delta Y\Delta X} \\ Q_{\Delta Z\Delta X} & Q_{\Delta Y\Delta X} & Q_{\Delta X\Delta X} \end{bmatrix} =
$$

$$
= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} Q_{Z_SZ_S} & Q_{Z_SY_S} & Q_{Z_SX_S} \\ Q_{Z_SY_S} & Q_{Y_SY_S} & Q_{Y_SX_S} \\ Q_{Z_SX_S} & Q_{Y_SX_S} & Q_{X_SX_S} \end{bmatrix} - \begin{bmatrix} Q_{Z_SZ_V} & Q_{Z_SY_V} & Q_{Z_SX_V} \\ Q_{Y_SZ_V} & Q_{Y_SY_V} & Q_{Y_SX_V} \\ Q_{X_SZ_V} & Q_{Y_SX_V} & Q_{X_SX_V} \end{bmatrix} \\ \begin{bmatrix} Q_{Z_SZ_V} & Q_{Y_SY_V} & Q_{X_SZ_V} \\ Q_{Z_SY_V} & Q_{Y_SZ_V} & Q_{X_SY_V} \\ Q_{Z_SX_V} & Q_{Y_SX_V} & Q_{X_SX_V} \end{bmatrix} - \begin{bmatrix} Q_{Z_VZ_V} & Q_{Z_VY_V} & Q_{Z_VX_V} \\ Q_{Z_VY_V} & Q_{Y_VY_V} & Q_{Y_VX_V} \\ Q_{Z_VX_V} & Q_{Y_VX_V} & Q_{X_VX_V} \end{bmatrix} \end{bmatrix} \tag{n}
$$

And after left multiplication:

$$
\mathbf{Q}_{ellipsoid\ SV} = \begin{bmatrix} Q_{\Delta Z\Delta Z} & Q_{\Delta Z\Delta Y} & Q_{\Delta Z\Delta X} \\ Q_{\Delta Z\Delta Y} & Q_{\Delta Y\Delta Y} & Q_{\Delta Y\Delta X} \\ Q_{\Delta Z\Delta X} & Q_{\Delta Y\Delta X} & Q_{\Delta X\Delta X} \end{bmatrix} = \begin{bmatrix} Q_{Z_SZ_S} & Q_{Z_SY_S} & Q_{Z_SX_S} \\ Q_{Z_SY_S} & Q_{Y_SY_S} & Q_{Y_SX_S} \\ Q_{Z_SX_S} & Q_{Y_SX_S} & Q_{X_SX_S} \end{bmatrix} -
$$

$$
- \begin{bmatrix} 2Q_{Z_SZ_V} & Q_{Z_SY_V} + Q_{Y_SZ_V} & Q_{Z_SX_V} + Q_{X_SZ_V} \\ Q_{Y_SZ_V} + Q_{Z_SY_V} & 2Q_{Y_SY_V} & Q_{Y_SX_V} + Q_{X_SY_V} \\ Q_{X_SZ_V} + Q_{Z_SX_V} & Q_{X_SY_V} + Q_{Y_SX_V} & 2Q_{X_SX_V} \end{bmatrix} + \begin{bmatrix} Q_{Z_VZ_V} & Q_{Z_VY_V} & Q_{Z_VX_V} \\ Q_{Z_VY_V} & Q_{Y_VY_V} & Q_{Y_VX_V} \\ Q_{Z_VX_V} & Q_{Y_VX_V} & Q_{X_VX_V} \end{bmatrix} \tag{o}
$$

Matrices in equation (15) are added up (or subtracted) and final value of matrix $\mathbf{Q}_{ellipsoid\ SV}$ is obtained:

$$
\mathbf{Q}_{ellipsoid\ SV} = \begin{bmatrix} Q_{\Delta Z\Delta Z} & Q_{\Delta Z\Delta Y} & Q_{\Delta Z\Delta X} \\ Q_{\Delta Z\Delta Y} & Q_{\Delta Y\Delta Y} & Q_{\Delta Y\Delta X} \\ Q_{\Delta Z\Delta X} & Q_{\Delta Y\Delta X} & Q_{\Delta X\Delta X} \end{bmatrix} =
$$

$$
= \begin{bmatrix} Q_{Z_SZ_S} - 2Q_{Z_SZ_V} + Q_{Z_VZ_V} & Q_{Z_SY_S} - Q_{Z_SY_V} - Q_{Y_SZ_V} + Q_{Z_VY_V} & Q_{Z_SX_S} - Q_{Z_SX_V} - Q_{X_SZ_V} + Q_{Z_VX_V} \\ Q_{Z_SY_S} - Q_{Z_SY_V} - Q_{Y_SZ_V} + Q_{Z_VY_V} & Q_{Y_SY_S} - 2Q_{Y_SY_V} + Q_{Y_VY_V} & Q_{Y_SX_S} - Q_{Y_SX_V} - Q_{X_SY_V} + Q_{Y_VX_V} \\ Q_{Z_SX_S} - Q_{Z_SX_V} - Q_{X_SZ_V} + Q_{Z_VX_V} & Q_{Y_SX_S} - Q_{Y_SX_V} - Q_{X_SY_V} + Q_{Y_VX_V} & Q_{X_SX_S} - 2Q_{X_SX_V} + Q_{X_VX_V} \end{bmatrix} \tag{p}
$$

2d relative error pedaloid and ellipse

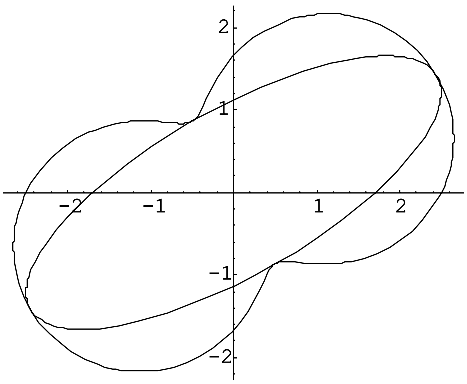

Figure 3. 2D relative pedaloid and ellipsoid
Slika 4. 2D relativni pedaloid in elipsoid

By the analogy of relative ellipsoid, relative error ellipse is:

$$\mathbf{Q}_{ellipse\ SV} = \begin{bmatrix} Q_{\Delta Y \Delta Y} & Q_{\Delta Y \Delta X} \\ Q_{\Delta Y \Delta X} & Q_{\Delta X \Delta X} \end{bmatrix} = $$
$$= \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} Q_{Y_S Y_S} & Q_{Y_S X_S} & Q_{Y_S Y_V} & Q_{Y_S X_V} \\ Q_{Y_S X_S} & Q_{X_S X_S} & Q_{X_S Y_V} & Q_{X_S X_V} \\ Q_{Y_S Y_V} & Q_{X_S Y_V} & Q_{Y_V Y_V} & Q_{Y_V X_V} \\ Q_{Y_S X_V} & Q_{X_S X_V} & Q_{Y_V X_V} & Q_{X_V X_V} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \text{(q)}$$

$$\mathbf{Q}_{ellipse\ SV} = \begin{bmatrix} Q_{\Delta Y \Delta Y} & Q_{\Delta Y \Delta X} \\ Q_{\Delta Y \Delta X} & Q_{\Delta X \Delta X} \end{bmatrix} = $$
$$= \begin{bmatrix} Q_{Y_S Y_S} - Q_{Y_S Y_V} & Q_{Y_S X_S} - Q_{X_S Y_V} & Q_{Y_S Y_V} - Q_{Y_V Y_V} & Q_{Y_S X_V} - Q_{Y_V X_V} \\ Q_{Y_S X_S} - Q_{Y_S X_V} & Q_{X_S X_S} - Q_{X_S X_V} & Q_{X_S Y_V} - Q_{Y_V X_V} & Q_{X_S X_V} - Q_{X_V X_V} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \text{(r)}$$

$$\mathbf{Q}_{ellipse\ SV} = \begin{bmatrix} Q_{\Delta Y \Delta Y} & Q_{\Delta Y \Delta X} \\ Q_{\Delta Y \Delta X} & Q_{\Delta X \Delta X} \end{bmatrix} = $$
$$= \begin{bmatrix} Q_{Y_S Y_S} - 2Q_{Y_S Y_V} + Q_{Y_V Y_V} & Q_{Y_S X_S} - Q_{X_S Y_V} - Q_{Y_S X_V} + Q_{Y_V X_V} \\ Q_{Y_S X_S} - Q_{Y_S X_V} - Q_{X_S Y_V} + Q_{Y_V X_V} & Q_{X_S X_S} - 2Q_{X_S X_V} + Q_{X_V X_V} \end{bmatrix} \qquad \text{(s)}$$

For given point:

$$\mathbf{Q}_{ellipse\ SV} = \begin{bmatrix} Q_{\Delta Y \Delta Y} & Q_{\Delta Y \Delta X} \\ Q_{\Delta Y \Delta X} & Q_{\Delta X \Delta X} \end{bmatrix} = \begin{bmatrix} Q_{Y_V Y_V} & Q_{Y_V X_V} \\ Q_{Y_V X_V} & Q_{X_V X_V} \end{bmatrix} \quad \text{(t)}$$

Summary

Multi epochs adjustment by parameter variation is simple enough, besides that the points subsidence is calculated directly. In this article nD hyperpedaloid and hyperellipsoid, error ellipse and error ellipsoid were formulated.

Their execution was shown. Their characteristic is that they do not depend on coordinate origin.

Reference

Mihajlović, K., Vračarić, K., Geodezija III. *Građevinski fakultet Beograd, Beograd*, 1985.

Mihajlović, K., Vračarić, K., Geodezija I. *Građevinski fakultet Beograd, Beograd*, 1989.

Mihajlović, K., Geodezija II (2. deo), *Naučna knjiga, Beograd*, 1978.

Mihajlović, K., Geodezija: Izravnanje geodetskih mreža. *Građevinski fakultet Beograd, Beograd*, 1992.

Todorović, R. T., Uvod v rudarsko škodo II. *Naravoslovnotehniška fakulteta, Oddelek za geotehnologijo in rudarstvo, Ljubljana,* 1996.

Todorović, R. T., O relativni elipsi pogreškov. *Rudarsko-metalurški zbornik.* 1994: 41, 169-174.

Zaključek

Ocena deformacij s simultano izravnavo več terminskih izmer z nD relativnim pedaloidom

Posredna izravnava več terminskih izmer je dokaj enostavna, poleg tega pa se ugrezke izračunava neposredno. V članku so predstavljeni nD hiperpedaloid in hiperelipsoid, elipsa pogreškov in elipsoid pogreškov. Prikazana je bila njihova izpeljava. Zanje je značilno, da so neodvisni od koordinatnega izhodišča.