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# Drilling process optimization by using fuzzy-based multi-response surface methodology

**Boyacı, A.İ.**[a]**, Hatipoğlu, T.**[a] **, Balcı, E.**[b,*]

[a]Kocaeli University Engineering Faculty Industrial Engineering Department, Umuttepe Campus, Kocaeli, Turkey
[b]Kocaeli University Engineering Faculty Mechanical Engineering Department, Umuttepe Campus, Kocaeli, Turkey

**A B S T R A C T**

In this study, a fuzzy mathematical model is developed using a multi-response surface methodology with fuzzy logic to optimize all response variables simultaneously. The model has the flexibility to weight the response factors depending on the decision maker's choices. The model has been applied to the drilling process using a high speed steel drill bit on PVC samples in an upright drill. The aim of the study is to minimize surface roughness and cutting forces. The input variables and their experiment intervals are determined as cutting speed (360-1080 rev/min), feed rate (0.10-0.30 mm), and material thickness (15-45 mm). Surface roughness, radial force-X and radial force-Y are chosen as response variables. According to the experiments and statistical analysis, the optimum levels of cutting speed, feed rate, and material thickness were calculated as 1068 rev/min, 0.1195 mm, and 21.33 mm respectively.



**A R T I C L E I N F O**

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**Corresponding author:*
erhan.balci@kocaeli.edu.tr
(Balcı, E.)

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## 1. Introduction

Recently the usage of polymers has increased dramatically. Polyvinyl chloride (PVC) is the second most commonly used thermoplastic [1]. PVC is used in a variety of products such as pipes, profiles, cable insulation, packaging and bottling [2]. This wide usage of PVC makes its surface properties important after machining processes. Drilling is one of the most important machining processes [3]. In the drilling process, the cutting speed, feed rate and drill bit may affect surface roughness and cutting forces [4, 5]. In this study, PVC samples with different heights were drilled at different cutting speeds and feed rates. During drilling, cutting forces were measured with the help of a dynamometer. In this way, the relationship between cutting parameters and surface roughness was discovered and statistically analyzed.

Fuzzy logic is an approach used to formalize the uncertain or approximate reasoning of human capacity. This method is applied to make decisions as a human being for approximately reasoning and judgement in ambiguity. In fuzzy logic, the fact is adjacent. In this way, the reasoning is called interpolative reasoning. Interpolation between the binary extremes of correct and incorrect operation is represented by the ability of fuzzy logic to include partial truths [6]. It is seen that there are both variations of repetitive experiments and numerical data which cause the complexity and the imprecision. In order to overcome this uncertainty and intricacy, the fuzzy logic method, which addresses approximate reasoning, is adopted. In this study, a multi-response surface methodology achieved by applying fuzzy logic in machining is presented. Hence a more robust model has been developed.

The main aim of the study is to determine the optimum factor levels for surface roughness, radial-X, and radial-Y forces. The input variables are cutting speed, feed rate, and material thickness.

Several methods have been developed to understand the effects of cutting parameters on surface roughness and cutting forces in the machining process [7-9]. Response surface methodology (RSM) is an empirical statistical technique, which can be employed to study the interactions between factors and optimize operating parameters [10]. Response surface design is useful when several process parameters potentially influence the quality properties of the product, which is called a response [11]. Also, when many factors and interactions affect the desired responses for a given process, RSM is an effective technique for evaluating the process parameters with the least number of experiments [12]. Using a second-order response surface design, we can gather data, estimate the mathematical second-order relationship between the response variables and process parameters, and obtain the optimal conditions of process parameters. The Box–Behnken design is an efficient three-level design for fitting second-order models [13].

This paper is constructed as follows; the second section includes details about the fuzzy approach and the proposed methodology. An illustrative example of a multi-response surface methodology attained by applying fuzzy logic in machining is presented in the third section. Finally, the results are provided and discussed.

## 2. Used methods

In this study, multi-response surface methodology with fuzzy logic is used to consider variation in repetitive experiments. In next section, the developed model by using fuzzy numbers is presented.

### 2.1 Fuzzy logic

In the fuzzy logic approach, factors and criteria can be classified without any concrete limitation. Fuzzy logic is very useful in the definition and solution of uncertain and ambiguous real-life problems. Fuzzy sets are defined with membership functions. The membership function of a fuzzy set $A$ is shown by $\mu A(x)$ and the membership of a factor is defined with a number between 0 and 1. If factor x certainly belongs to set $A$, then $\mu A(x) = 1$, or if not, then $\mu A(x) = 0$. A larger membership value shows that the degree of belonging to set $A$ is greater for factor $x$.

Triangular fuzzy numbers can be used to facilitate arithmetical operations. A triangular fuzzy number $(\tilde{A})$ is represented by three certain numbers $(l \leq m \leq u)$ and the membership function is defined according to these numbers. The membership function of a triangular fuzzy number is:

$$\mu A(x) \; = \begin{cases} \frac{X-l}{m-l}, & l \leq x \geq m \\ \frac{u-x}{u-m}, & m \leq x \geq u \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

With a fuzzy number $(\tilde{A})$ that is represented by $(l, m, u), m, l,$ and u show respectively the possible value of the fuzzy number, the lower and upper limits, i.e. the sphere of fuzziness [14].

### 2.2 Proposed methodology

In multi-response surface optimization studies, when the experiments are done with replicates, only the mean and variance of the collected data are taken into consideration to acquire optimum factor levels. But a great deal of unexpected noise might exist in experiments [15]. Thus, uncertainties associated with the predicted responses should be incorporated into the methodology to acquire more reliable solutions. Because some data are usually neglected in these problems, in this paper, we cover it by using Triangular Fuzzy Numbers for considering the mean and variance of data simultaneously to obtain more robust results. Using the desirability function approach, an attempt is made to simultaneously optimize different response variables. The algorithm of the proposed methodology which consists of 8 steps is shown in Fig. 1.

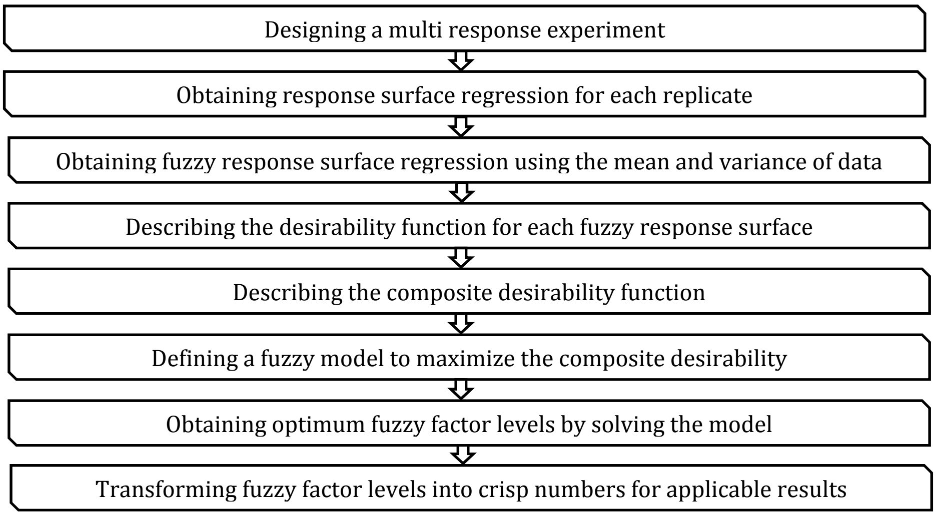


**Fig. 1** Algorithm of the proposed methodology

_Step 1:_ A multi response experiment is designed which is related to a process including more than one response with replicates shown in Table 1, where $x_{ij}$ is the $j^{\text{th}}$ ($j$ = 1, 2,..., $J$) factor level value and $y_{kir}$ is $k^{\text{th}}$ $(k = 1, 2, \dots, K)$ response value for the $r^{\text{th}}$ $(r = 1, 2, \dots, R)$ replicates in $i^{\text{th}}$ $(i = 1, 2, \dots, N)$ experiment respectively.

**Table 1** Results of experiments for a multi response process

| Run order | Factor levels | | | Responses | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $X_{i1}$ | … | $X_{iJ}$ | $Y_1$ | | | … | $Y_K$ | | |
| | | | | $y_{1i1}$ | … | $y_{1iR}$ | … | $y_{Ki1}$ | … | $y_{KiR}$ |
| 1 | $x_{11}$ | … | $x_{1J}$ | $y_{111}$ | … | $y_{11R}$ | … | $y_{K11}$ | … | $y_{K1R}$ |
| … | … | … | … | … | … | … | … | … | … | … |
| $N$ | $x_{N1}$ | … | $x_{NJ}$ | $y_{1N1}$ | … | $y_{1NR}$ | … | $y_{KN1}$ | … | $y_{KNR}$ |

_Step 2:_ The response surface regression for $r^{\text{th}}$ replicates is

$$\mu A Y_k^r = \beta_0^r + \sum_{j=1}^{J} \beta_j^r X_j + \sum_{j=1}^{J} \beta_{jj}^r X_j^2 + \sum_{i<p}^{J} \sum^{J} \beta_{ip}^r X_i X_P + \mathcal{E} \tag{2}$$

where $Y_r^k$ represents the $r^{\text{th}}$ response surface regression model for the $k^{\text{th}}$ response variable which is obtained based on experimental data and $\mathcal{E}$ is noise or error observed in the response value. $\beta_0^r$ is a model constant and $\beta_j^r$, $\beta_{jj}$, $\beta_{ip}$ are coefficients for the main effects, square effects and interaction effects of factors, respectively.

_Step 3:_ The fuzzy response surface regression can be expressed as:

$$\tilde{Y}_k = \tilde{\beta}_0 + \sum_{j=1}^{J} \tilde{\beta}_j X_j + \sum_{j=1}^{J} \tilde{\beta}_{jj} X_j^2 + \sum_{i<p}^{J} \sum^{J} \tilde{\beta}_{ip} X_i X_P + \mathcal{E} \tag{3}$$

where $\tilde{Y}_k$ express the fuzzy response surface regression model for the $k^{\text{th}}$ response variable. $\tilde{\beta}_0$, $\tilde{\beta}_j$, $\tilde{\beta}_{jj}$ and $\tilde{\beta}_{ip}$ fuzzy coefficients are calculated by applying the following procedure. Let $\beta_j^r = (\beta_j^1, \beta_j^2, \dots, \beta_j^R)$ be crisp values, mean and standard deviation of $\beta_j^1, \beta_j^2, \dots, \beta_j^R$ are calculated as follows:

- $\beta_j^m$ = Mean $(\beta_j^1, \beta_j^2, \dots, \beta_j^R)$
- $\beta_j^l$ = Mean $(\beta_j^1, \beta_j^2, \dots, \beta_j^R)$ – Standard Deviation $(\beta_j^1, \beta_j^2, \dots, \beta_j^R)$
- $\beta_j^u$ = Mean $(\beta_j^1, \beta_j^2, \dots, \beta_j^R)$ + Standard Deviation $(\beta_j^1, \beta_j^2, \dots, \beta_j^R)$
- Thus, fuzzy regression coefficients obtained as $\tilde{\beta}_j = (\beta_j^l, \beta_j^m, \beta_j^u)$

_Step 4:_ The most typical approach for the optimization of multiple responses is the desirability function technique introduced by [9]. The desirability function technique is to first convert each response $(Y_k)$ into an individual desirability value $(d_k)$, where $0 \leq d_k \leq 1$. Then the design factors are chosen to maximize the overall desirability value using the composite desirability function. The value of $d_k$ increases as the corresponding response approaches its goal or target [15]. Reference [16] introduced the desirability function for The Larger-The-Better, The Smaller-The-Better, Nominal-The-Best responses.

If $\tilde{d}_k = (d_k^l,\ d_k^m,\ d_k^u) = d_k^{l,m,u}$ and $\tilde{Y}_k = (Y_k^l,\ Y_k^m,\ Y_k^u) = Y_k^{l,m,u}$ we have the individual desirability functions as follows:

$$d_k^{l,m,u} = \begin{cases} 0, & Y_k^{l,m,u} < L^{l,m,u} \\ \left(\dfrac{Y_k^{l,m,u} - L^{l,m,u}}{U^{l,m,u} - L^{l,m,u}}\right)^s, & L^{l,m,u} \le Y_k^{l,m,u} \le U^{l,m,u} \\ 1, & Y_k^{l,m,u} > U^{l,m,u} \end{cases} \tag{4}$$

$$d_k^{l,m,u} = \begin{cases} 1, & Y_k^{l,m,u} < L^{l,m,u} \\ \left(\frac{U^{l,m,u} - Y_k^{l,m,u}}{U^{l,m,u} - L^{l,m,u}}\right)^t, & L^{l,m,u} \le Y_k^{l,m,u} \le U^{l,m,u} \\ 0, & Y_k^{l,m,u} > U^{l,m,u} \end{cases} \tag{5}$$

$$d_k^{l,m,u} = \begin{cases} 0, & Y_k^{l,m,u} < L^{l,m,u} \\ \left(\frac{Y_k^{l,m,u} - L^{l,m,u}}{T^{l,m,u} - L^{l,m,u}}\right)^a, & L^{l,m,u} \le Y_k^{l,m,u} \le T^{l,m,u} \\ \left(\frac{U^{l,m,u} - Y_k^{l,m,u}}{U^{l,m,u} - T^{l,m,u}}\right)^b, & T^{l,m,u} \le Y_k^{l,m,u} \le U^{l,m,u} \\ 0, & Y_k^{l,m,u} > U^{l,m,u} \end{cases} \tag{6}$$

*Step 5:* Derringer and Suich described the composite desirability function as:

$$D_{DS} = \left[d_1(\hat{Y}_1) * d_2(\hat{Y}_2) * \ldots * d_m(\hat{Y}_m)\right]^{\frac{1}{m}} \tag{7}$$

where there are $m$ responses. References [17-19] introduced some weighted composite desirability functions. Beside these, [20] and [21] proposed a 'Maximin' approach for the composite desirability function.

*Step 6:* The individual desirability functions for The Larger-The-Better, The Smaller-The-Better and Nominal-The-Best response variables can be modelled respectively as follows:

$$\begin{gathered} Max\ d_k^{l,m,u} \\ \alpha^{l,m,u} = \\ \left(\frac{Y_k^{l,m,u} - L^{l,m,u}}{U^{l,m,u} - L^{l,m,u}}\right)^s \\ d^{l,m,u} \le \alpha^{l,m,u} \\ d^{l,m,u} \le 1 \\ L^{l,m,u} \le Y_k^{l,m,u} \\ X \in [-1, 1] \end{gathered} \tag{8}$$

$$\begin{gathered} Max\ d_k^{l,m,u} \\ \alpha^{l,m,u} = \\ \left(\frac{U^{l,m,u} - Y_k^{l,m,u}}{U^{l,m,u} - L^{l,m,u}}\right)^t \\ d^{l,m,u} \le \alpha^{l,m,u} \\ d^{l,m,u} \le 1 \\ Y_k^{l,m,u} \le U^{l,m,u} \\ X \in [-1, 1] \end{gathered} \tag{9}$$

$$\begin{gathered} Max\ d_k^{l,m,u} \\ \alpha^{l,m,u} = \left(\frac{Y_k^{l,m,u} - L^{l,m,u}}{T_r - L^{l,m,u}}\right)^a \\ \beta^{l,m,u} = \left(\frac{U^{l,m,u} - Y_k^{l,m,u}}{U^{l,m,u} - T_r}\right)^b \\ d^{l,m,u} \le \alpha^{l,m,u} \\ d^{l,m,u} \le \beta^{l,m,u} \\ L^{l,m,u} \le Y_k^{l,m,u} \le U^{l,m,u} \\ X \in [-1, 1] \end{gathered} \tag{10}$$

For the objective function of the final model, one of the composite desirability functions defined above can be used to maximize the overall desirability value. The final model is categorized into 3 types of models ($l, m$ and $u$) and these models are solved separately.

*Step 7:* The optimum factor levels are obtained as $X_l^* = (x_1^{l^*}, x_2^{l^*}, \ldots, x_J^{l^*})$, $X_m^* = (x_1^{m^*}, x_2^{m^*}, \ldots, x_J^{m^*})$, $X_u^* = (x_1^{u^*}, x_2^{u^*}, \ldots, x_J^{u^*})$ where $J$ is the number of factors taken into consideration. Hence, optimum fuzzy factor levels are obtained as follows:

$$\tilde{X}^* = (\tilde{x}_1^*, \tilde{x}_2^*, \dots, \tilde{x}_J^*) = \left((x_1^{l^*}, x_1^{m^*}, x_1^{u^*}), (x_2^{l^*}, x_2^{m^*}, x_2^{u^*}), \dots, (x_J^{l^*}, x_J^{m^*}, x_J^{u^*})\right) \tag{11}$$

Step 8: The defuzzification process can be performed using many different methods [22]. In this study, the center of area which is the most commonly used defuzzification method calculates the centroid of the area under the membership function [23].

## 3. Application

### 3.1 Experimental data

The proposed methodology was applied to the drilling process of PVC material to determine the optimum processing conditions that yield minimum surface roughness for the material and the minimum radial forces for the machine being used.

PVC samples with 30 mm diameter and 15 mm, 30 mm, 45 mm thickness were drilled in upright drill with high speed steel drill bit. Hardness of PVC is 80 Shore D and its tensile strength is 52 MPa. The cutting speed ($x_1$), feed rate ($x_2$), material thickness ($x_3$) were independent factors investigated with respect to surface roughness ($Y_1$) in μm radial force-X ($Y_2$) in N, and radial force-Y ($Y_3$) in N as response variables. The experiments were designed according to the Box-Behnken design with these three factors each at three different levels and conducted on an automatic drill machine.

For each factor combination, three replications were made to take into consideration the variation factor. Real factor levels and their counterpart coded levels which are used for calculation are given in Table 2. Cutting parameter intervals are chosen as they are used in polymer industry.

**Table 2** Factor levels and their coded value

| Factors | Coded level value | | |
|---|---|---|---|
| | Low (-1) | Medium (0) | High (+1) |
| Cutting speed ($x_1$) | 360 rev/min | 720 rev/min | 1080 rev/min |
| Feed rate ($x_2$) | 0.10 mm | 0.20 mm | 0.30 mm |
| Material thickness ($x_3$) | 15 mm | 30 mm | 45 mm |

The results of the experiments are given in Table 3. For each response and each replication, second order polynomial models were developed using multiple linear regression analysis. Analysis of variance (ANOVA) was performed to check the adequacy and accuracy of the fitted models. MINITAB 17 software was used for statistical analysis.

**Table 3** Box-Behnken design with the experimental values of the response variables

| Run order | Factor levels | | | Surf. roughness (μm) | | | Radial force-Y (N) | | | Radial force-X (N) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | R. 1 | R. 2 | R. 3 | R. 1 | R. 2 | R. 3 | R. 1 | R. 2 | R. 3 |
| 1 | -1 | 0 | 1 | 1.8 | 1.83 | 1.78 | 4.3 | 4.5 | 4.62 | 4.2 | 4.4 | 4.3 |
| 2 | 0 | -1 | 1 | 1.11 | 1.182 | 1.08 | 3.4 | 3.75 | 3.65 | 3.1 | 3.3 | 3.22 |
| 3 | -1 | -1 | 0 | 1.57 | 1.62 | 1.54 | 3.7 | 3.94 | 4.09 | 3.3 | 3.6 | 3.5 |
| 4 | 1 | -1 | 0 | 0.87 | 0.92 | 0.85 | 3 | 3.3 | 3.45 | 3.2 | 3.4 | 3.3 |
| 5 | 0 | 0 | 0 | 1.19 | 1.24 | 1.16 | 3.7 | 3.95 | 4.1 | 3.8 | 4 | 3.9 |
| 6 | -1 | 0 | -1 | 1.57 | 1.62 | 1.54 | 3.8 | 4.1 | 4.25 | 3.9 | 4.1 | 4.15 |
| 7 | -1 | 1 | 0 | 2.06 | 2.15 | 2.03 | 4.8 | 5.1 | 5.25 | 4.9 | 5 | 5.08 |
| 8 | 0 | 0 | 0 | 1.16 | 1.21 | 1.13 | 3.4 | 3.8 | 3.95 | 3.7 | 3.9 | 3.8 |
| 9 | 1 | 0 | -1 | 0.68 | 0.73 | 0.75 | 3.05 | 3.35 | 3.45 | 3.4 | 3.6 | 3.5 |
| 10 | 1 | 0 | 1 | 0.96 | 0.95 | 0.93 | 3.7 | 4 | 4.15 | 3.5 | 3.7 | 3.6 |
| 11 | 0 | 1 | -1 | 1.34 | 1.39 | 1.31 | 4.2 | 4.5 | 4.65 | 4.1 | 4.3 | 4.2 |
| 12 | 0 | -1 | -1 | 1.09 | 1.14 | 1.15 | 3.1 | 3.4 | 3.55 | 3 | 3.2 | 3.1 |
| 13 | 0 | 1 | 1 | 1.66 | 1.7 | 1.63 | 5.5 | 5.65 | 5.75 | 4.3 | 4.6 | 4.37 |
| 14 | 1 | 1 | 0 | 1.04 | 1.09 | 1.01 | 4.1 | 4.4 | 4.55 | 3.9 | 4.15 | 4 |
| 15 | 0 | 0 | 0 | 1.17 | 1.22 | 1.14 | 3.6 | 3.9 | 3.95 | 3.8 | 4 | 3.9 |

For each response variable, 3 response surface regressions based on 3 replicates were obtained as follows:

$$Y_1^1 = 1.172 + 0.106x_3 - 0.431x_1 + 0.183x_2 + 0.082x_1x_1 + 0.130x_2x_2 + 0.075x_3x_2 - 0.080x_1x_2 \quad (12)$$

$$Y_1^2 = 1.156 + 0.084x_3 - 0.419x_1 + 0.170x_2 + 0.084x_1x_1 + 0.127x_2x_2 + 0.098x_3x_2 - 0.083x_1x_2 \quad (13)$$

$$\mathrm{Y}_1^3 = 1.213 + 0.098\mathrm{x}_3 - 0.441\mathrm{x}_1 + 0.184\mathrm{x}_2 + 0.077x_1x_1 + 0.147\, x_2x_2 + 0.067x_3x_2 - 0.090x_1x_2 \quad (14)$$

$$Y_2^1 = 3.746 + 0.088x_3 - 0.288x_1 + 0.575x_2 - 0.106x_3x_3 + 0.094x_1x_1 - 0.225x_1x_2 \quad (15)$$

$$Y_2^2 = 3.958 + 0.100x_3 - 0.281x_1 + 0.569x_2 - 0.101\, x_3x_3 + 0.087x_1x_1 - 0.163x_1x_2 \quad (16)$$

$$Y_2^3 = 3.848 + 0.068x_3 - 0.329x_1 + 0.566x_2 - 0.111\, x_3x_3 + 0.136x_1x_1 - 0.220x_1x_2 \quad (17)$$

$$Y_3^1 = 3.650 + 0.344x_3 - 0.344x_1 + 0.675x_2 + 0.325x_2x_2 + 0.250x_3x_2 \quad (18)$$

$$Y_3^2 = 3.943 + 0.319x_3 - 0.324x_1 + 0.658x_2 + 0.312x_2x_2 + 0.200x_3x_2 \quad (19)$$

$$Y_3^3 = 4.067 + 0.284x_3 - 0.326x_1 + 0.683x_2 + 0.300x_2x_2 + 0.250x_3x_2 \quad (20)$$

The adjusted R-Sq of these regression models are 99.2 %, 99.1 %, 99.7 %, 98.7 %, 98.1 %, 99.4 %, 93.1 %, 95.1 %, 95.1 % respectively.

Applying the proposed procedure (Step 3) to Eqs. 12 to 20 fuzzy response surface for each response are obtained as follows:

$$\tilde{Y}_1 = (1.156, 1.180, 1.204) + (0.087, 0.096, 0.105)x_3 + (-0.440, -0.430, -0.421)x_1 + (0.173, 0.179, 0.185)x_2 + (0.078, 0.081, 0.084)x_1x_1 + (0.126, 0.135, 0.144)x_2x_2 + (0.067, 0.080, 0.093)x_3x_2 + (0.088, -0.084, -0.080)x_1x_2 \quad (21)$$

$$\tilde{Y}_2 = (3.764, 3.851, 3.937) + (0.072, 0.085, 0{,}098)x_3 + (-0.320, -0.299, -278)x_1 + (0.566, 0.570, 0{,}574)x_2 + (-0.110, -0.106, -0.102)x_3x_3 + (0.084, 0.106, 0.128)x_1x_1 + (-0.231, -0.203, -0.174)\, x_1x_2 \quad (22)$$

$$\tilde{Y}_3 = (3.712, 3.887, 4.062) + (0.391, 0.315, 0.340)x_3 + (-0.340, -0.331, -0.322)\, x_1 + (0.661, 0.672, 0.682)x_2 + (0.302, 0.313, 0.323)x_2x_2 + (0.210, 0.233, 0.257)\, x_3x_2 \quad (23)$$

Then, individual desirability functions were described for each response variable. In this study, since all of the response variables were desired to be minimized, The Smaller-The-Better type desirability function was used. So individual desirability functions were obtained as follows,

$$d_1^{l,m,u} = \begin{cases} 1 & , Y_1^{l,m,u} < (0.68, 0.68, 0.68) \\ \left(\dfrac{(2.15, 2.15, 2.15) - Y_1^{l,m,u}}{(2.15, 2.15, 2.15) - (0.68, 0.68, 0.68)}\right) & , (0.68, 0.68, 0.68) \le Y_1^{l,m,u} \le (2.15, 2.15, 2.15) \\ 0 & , Y_1^{l,m,u} > (2.15, 2.15, 2.15) \end{cases} \quad (24)$$

$$d_2^{l,m,u} = \begin{cases} 1 & , \quad Y_2^{l,m,u} < (3, 3, 3) \\ \left(\dfrac{(5.75, 5.75, 5.75) - Y_2^{l,m,u}}{(5.75, 5.75, 5.75) - (3, 3, 3)}\right) & , \quad (3, 3, 3) \le Y_2^{l,m,u} \le (5.75, 5.75, 5.75) \\ 0 & , \quad Y_2^{l,m,u} > (5.75, 5.75, 5.75) \end{cases} \quad (25)$$

$$d_3^{l,m,u} = \begin{cases} 1 & , \quad Y_3^{l,m,u} < (3, 3, 3) \\ \left(\dfrac{(5.08, 5.08, 5.08) - Y_3^{l,m,u}}{(5.08, 5.08, 5.08) - (3, 3, 3)}\right) & , \quad (3, 3, 3) \le Y_3^{l,m,u} \le (5.08, 5.08, 5.08) \\ 0 & , \quad Y_3^{l,m,u} > (5.08, 5.08, 5.08) \end{cases} \quad (26)$$

The 'maximin' approach was used for obtaining the composite desirability value and following the fuzzy model constructed for determining the optimum fuzzy factor levels.

$$\begin{aligned}
&Max\ \left(Min\left(d_1^{l,m,u}, d_2^{l,m,u}, d_3^{l,m,u},\right)\right)\\
&\alpha_k^{l,m,u} = \left(\frac{U_k^{l,m,u} - Y_k^{l,m,u}}{U_k^{l,m,u} - L_k^{l,m,u}}\right), \qquad \text{for } k = 1,2,3\\
&d_k^{l,m,u} \leq \alpha_k^{l,m,u}, \qquad \text{for } k = 1,2,3\\
&d_k^{l,m,u} \leq 1, \qquad \text{for } k = 1,2,3\\
&Y_k^{l,m,u} \leq U_k^{l,m,u}, \qquad \text{for } k = 1,2,3\\
&X \in [-1, 1]
\end{aligned} \tag{27}$$

The final model was categorized into 3 types of models ($l, m$ and $u$) and these models were solved separately.

Model $l$:

$$\begin{aligned}
&Max\ \left(Min\left(d_1^l, d_2^l, d_3^l,\right)\right)\\
&\alpha_1^l = \left(\frac{2.15 - Y_1^l}{2.15 - 0.68}\right)\\
&\alpha_2^l = \left(\frac{5.75 - Y_2^l}{5.75 - 3}\right)\\
&\alpha_3^l = \left(\frac{5.08 - Y_3^l}{5.08 - 3}\right)\\
&d_k^l \leq \alpha_k^l, \qquad \text{for } k = 1,2,3\\
&d_k^l \leq 1, \qquad \text{for } k = 1,2,3\\
&Y_1^l \leq 2.15\\
&Y_2^l \leq 5.75\\
&Y_3^l \leq 5.08\\
&X \in [-1, 1]
\end{aligned} \tag{28}$$

Model $m$:

$$\begin{aligned}
&Max\ \left(Min\left(d_1^m, d_2^m, d_3^m,\right)\right)\\
&\alpha_1^m = \left(\frac{2.15 - Y_1^m}{2.15 - 0.68}\right)\\
&\alpha_2^m = \left(\frac{5.75 - Y_2^m}{5.75 - 3}\right)\\
&\alpha_3^m = \left(\frac{5.08 - Y_3^m}{5.08 - 3}\right)\\
&d_k^m \leq \alpha_k^m, \qquad \text{for } k = 1,2,3\\
&d_k^m \leq 1, \qquad \text{for } k = 1,2,3\\
&Y_1^m \leq 2.15\\
&Y_2^m \leq 5.75\\
&Y_3^m \leq 5.08\\
&X \in [-1, 1]
\end{aligned} \tag{29}$$

Model $u$:

$$\begin{aligned}
&Max\ \left(Min\left(d_1^u, d_2^u, d_3^u,\right)\right)\\
&\alpha_1^u = \left(\frac{2.15 - Y_1^u}{2.15 - 0.68}\right)\\
&\alpha_2^u = \left(\frac{5.75 - Y_2^u}{5.75 - 3}\right)
\end{aligned} \tag{30}$$

$$\alpha_3^u = \left(\frac{5.08 - Y_3^u}{5.08 - 3}\right)$$
$$d_k^u \leq \alpha_k^u, \quad \text{for } k = 1,2,3$$
$$d_k^u \leq 1, \quad \text{for } k = 1,2,3$$
$$Y_1^u \leq 2.15$$
$$Y_2^u \leq 5.75$$
$$Y_3^u \leq 5.08$$
$$X \in [-1, 1]$$

### 3.2 Results and discussion

After solving the model developed in subsection 3.1 for $l, m$ and $u$ separately, the optimum factor levels were obtained as:

$$\begin{aligned} X_l^* &= (0.900, -0.706, -0.528) \\ X_m^* &= (1.00, -0.887, -0.159) \\ X_u^* &= (1.00, -0.823, -1.00) \end{aligned} \tag{31}$$

So the optimum fuzzy factor levels in coded value were calculated as:

$$\begin{aligned} \tilde{X}_1^* &= (0.900, 1.00, 1.00) \\ \tilde{X}_2^* &= (-0.887, -0.823, -0.706) \\ \tilde{X}_3^* &= (-1.00, -0.528, -0.159) \end{aligned} \tag{32}$$

After translating the coded values, the real optimum fuzzy factor levels were obtained as follows:

$$\begin{aligned} \tilde{X}_1^* &= (1044, 1080, 1080) \\ \tilde{X}_2^* &= (0.1113, 0.1177, 0.1294) \\ \tilde{X}_3^* &= (15, 22, 27) \end{aligned} \tag{33}$$

Applying Equation 18 to the optimum fuzzy factor levels, optimum defuzzified factor levels were acquired as follows:

$$X_1^* = 1068, \;\; X_2^* = 0.1195, \;\; X_3^* = 21.33 \tag{34}$$

Contour plots for each response variable with respect to cutting speed (CS), feed rate (FR) and material type (MT) are given in Fig. 4, Fig. 5, and Fig. 6 respectively.

It can be understood from Fig. 2 that the increase in cutting speed decreases surface roughness strongly but feed rate and material thickness have a straight relationship with surface roughness and the degree of these relationships is not strong as with cutting speed. It is clear from the figure that the feed rate factor has a stronger effect when the material thickness factor value is high.

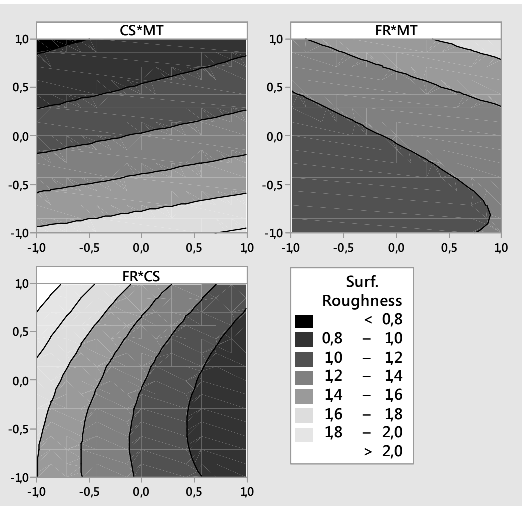


**Fig. 2** Contour plots for surface roughness response variable

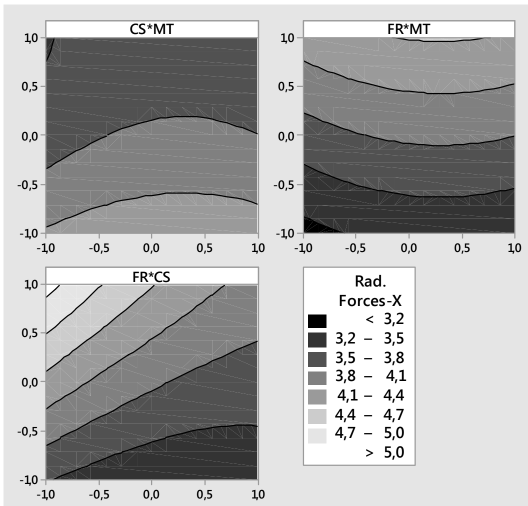


**Fig. 3** Contour plots for radial force-X response variable

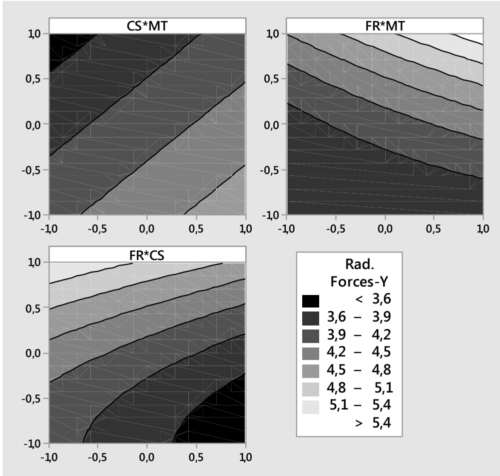


**Fig. 4** Contour plots for radial force-Y response variable

From Figs. 3 and 4 it can be expressed that cutting speed has an inverse relationship with radial forces X and Y in the drilling process. It is clear that material thickness does not have a strong effect on radial forces but it is slightly stronger for affecting radial force-Y than radial force-X. Aside from this, the middle level of the material thickness resulted in less radial force-X. When it comes to the feed rate factor, it has the same effect as in the surface roughness response. The feed rate factor has a stronger effect on both responses when the cutting speed factor value is low.

## 4. Conclusion

In this paper, a methodology was proposed for multi response surface optimization based on the desirability function and the fuzzy approach. The proposed approach takes into account variance along with mean and optimizes both of them simultaneously by applying fuzzy set theory to overcome variation in repetitive experiments. The methodology has the capability of weighting response variables thanks to the composite desirability function.

PVC is widely used in polymer industry. Optimisation of drilling process of PVC is quite important. The optimization procedure was also applied in the drilling process of PVC material to determine optimum drilling parameters (cutting speed, feed rate, material thickness) with the objective of minimizing material surface roughness and radial forces. For this purpose, a number of machining experiments based on the Box-Behnken design were carried out in order to collect surface roughness and radial forces values. For each factor combination, experiments were replicated three times to handle variance and obtain more robust results by using the fuzzy approach. In this application study, second order response surface models were developed for each replication to predict surface roughness and radial forces values in the drilling process of PVC material. Then, the coefficients of the fuzzy response surface model were calculated for each response as described. Finally, a fuzzy model was constructed for maximizing the composite desirability. Upon solving this model and defuzzifying the optimum fuzzy factor levels, the optimum levels were obtained as 1068 rev/min, 0.1195 mm, 21.33 mm for cutting speed, feed rate, and material thickness respectively. The predicted optimum machining process conditions were validated with an experimental measurement. According to experimental results, when cutting speed increases, surface roughness decreases significantly. However, feed rate and material thickness have little effect on surface roughness. Also, cutting speed has a strong effect on radial forces X and Y while material thickness doesn't affect cutting forces strongly.

In future studies, the proposed methodology could be applied to other machining problems such as tool life, dimensional errors, etc. as well. In addition to this, other fuzzy logic approaches such as fuzzy inference system and fuzzy multi criteria decision making could be used, especially for weighting response variables according to decision makers.

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