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On a conjecture about the ratio of Wiener index in iterated line graphs*

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Abstract

Let G be a graph. Denote by $W(G)$ its Wiener index and denote by $L^i(G)$ its i-iterated line graph. Dobrynin and Mel'nikov proposed to estimate the extremal values for the ratio $R_k(G) = W(L^k(G))/W(G)$ for $k \geq 1$. Motivated by this we study the ratio for higher k's. We prove that among all trees on n vertices the path P_n has the smallest value of this ratio for $k \geq 3$. We conjecture that this holds also for $k = 2$, and even more, for the class of all connected graphs on n vertices. Moreover, we conjecture that the maximum value of the ratio is obtained for the complete graph.

Keywords: Wiener index, line graph, tree, iterated line graph.

Math. Subj. Class.: 05C12, 05C05, 05C76

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O domnevi v zvezi z razmerjem Wienerjevega indeksa v iteriranih povezavnih grafih*

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Povzetek

Naj bo G graf. Označimo z $W(G)$ njegov Wienerjev indeks, z $L^i(G)$ pa njegov i-iterirani povezavni graf. Dobrynin in Mel'nikov sta predlagala, da se oceni ekstremno vrednost razmerja $R_k(G) = W(L^k(G))/W(G)$ za $k \geq 1$. Motivirani s tem raziskujemo razmerje za višje k-je. Dokažemo, da ima med vsemi drevesi na n vozliščih pot P_n najmanjšo vrednost tega razmerja za $k \geq 3$. Domnevamo, da to velja tudi za $k = 2$, in še več, za razred vseh povezanih grafov na n vozliščih. Poleg tega domnevamo, da je največja vrednost tega razmerja dosežena pri polnem grafu.

Ključne besede: Wienerjev indeks, povezavni graf, drevo, iterirani povezavni graf.

Math. Subj. Class.: 05C12, 05C05, 05C76

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