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Modelling Moving Feasts Determined by the Islamic Calendar: Application to Macroeconomic Tunisian Time Series

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Abstract

National and religious events always influence economic activity. Islamic events also influence production and consumption. Moreover, these Islamic feasts move over time, depending on the Hegirian calendar, which is based on the lunar cycles, even though, some Islamic countries use officially the Gregorian calendar. The lunar calendar is shorter than the Gregorian calendar, which is based on the cycles of the Earth revolution. Consequently, every year the dates of religious events change in the official calendar creating moving events. Tunisia offers a good example of this phenomenon. Twelve relevant series are analysed and five feasts are considered in our work. Modelling the effect of moving holidays improves the quality of the final adjustment. Removing Islamic feasts from time series is crucial to have better forecasting and comparison results. We adopt an approach initially developed by Bell and Hillmer (1983) to analyse the Easter effect. Since the effect is not the same, we consider three regressors for before, during, and after the holiday for each feast. For model selection and determining the number of regressors and their interval length, two methods are used: the F-adjusted Akaike's information criterion and a criterion based on forecast errors. The empirical results confirm our model selection for all the macroeconomic time series considered except for the exports and the broad money which are not affected by the religious feasts.

1 Introduction

In a time series analysis, unadjusted series may be decomposed into four unobservable components: the trend-cycle component, the calendar component, the seasonal component and finally the irregular component which includes all the

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remaining effects. The seasonal component is the regular movements observed in quarterly and monthly time series during a twelve-month period. For example, before Christmas retail sales increase and during vacation periods, the industrial activity falls down. In addition to the seasonal effect, another type of variation which is also linked to the calendar can be observed. This is the trading day effect. Trading day effects occur when a series is affected by the different day-of-week compositions of the same month in different years. For "flow" data which is obtained by summing daily figures, this effect arises because of the importance of the number of such days in the month. For example, a monthly time series of retail sales would be affected by the number of Saturdays in each month[3].
There are other kinds of variations in the calendar component, which may have an influence on time series. There are moving holidays or feasts such as Easter in the Christian calendar, the Lunar New Year in the Chinese Lunar Calendar, and Ramadan in the Islamic Calendar. Holiday effects arise from holidays whose dates vary over time (for example Easter falls in either March or April depending on the year). The influence of moving feasts on the economic activity is considerable.

Every country has specific holidays depending on its religion. The holidays and feasts are often defined by the country's official calendar. The most famous calendar is the Gregorian calendar which is based on the solar system. However, some countries such as Muslim ones use the lunar calendar to determine their feasts. Also countries with a large Chinese population are strongly affected by lunar calendar defined holidays[4].

The Islamic calendar is a strictly lunar-calendar. It contains 12 months that are based on the motion of the moon. And because 12 lunar months equal only 12^29.53 = 354.36 days, the Islamic calendar is consistently shorter than a tropical year, and therefore it shifts with respect to the Christian calendar.

The Islamic calendar is the official calendar in countries around the Gulf, especially Saudi Arabia. But other Muslim countries use the Gregorian calendar for civil purposes and only turn to the Islamic calendar for religious purposes such as the Maghreb countries.

Our work consists in modelling moving holidays' effects in order to have a better seasonal decomposition. Removing Islamic feasts from time series is very important for both forecasting and comparison purposes. An improper seasonal adjustment distorts national forecasting results and influences the policy decision related to employment, production, consumption and the economic activity in general. It also complicates data comparison between countries which do not have the same feasts and events. Removing the calendar effect from time series allow the comparison of data between two months or quarters for which these patterns are different. Also seasonal and moving holidays' effects on non-adjusted or original data complicate the comparisons over time using these data, especially for

[3] See (Cleveland and Grupe, 1983).
[4] See (Lin and Liu, 2002).

the most recent period. Consequently, convenient seasonally adjusted data are always used in economic modelling and cyclical analysis.

There are five religious events in Tunisia and the other Islamic countries. The holy month of Ramadan, the feast of Ramadan also called 'Eid Al-Fitr', the feast of the sacrifice, also referred to as 'Eid Al-Idha', the birthday of the Prophet Muhammad, also called 'Al Mawled' and finally the Islamic New Year which is the first day of the lunar new year. The last two feasts are less important than the other events. Our objective is to find out if the economic activity is affected by all of these moving holidays.

This paper uses the (Bell and Hillmer, 1983) regressor for each holiday, and defines the effects before, during and after the holiday. The length and the number of the intervals are determined by a model selection procedure. We analyse the effect of moving holidays on twelve economic time series in Tunisia.

The paper is organized as follows: Section 2 describes briefly the methodology used for seasonal adjustment and also introduces various diagnostic checking. Section 3 discusses the approach of modelling holidays effects. Section 4 provides empirical results and section 5 concludes.

2 Methodology

The ARIMA models are, in theory, the most general class of models for forecasting a time series which can be stationarized by transformations such as differencing and logging. The acronym ARIMA stands for "Auto-Regressive Integrated Moving Average."

2.1 General model

The ARIMA models, as discussed by (Box and Jenkins, 1976), are frequently used for seasonal time series. A general Seasonal ARIMA model (SARIMA $(p,d,q)(P,D,Q)_S$) for a time series z_t can be written

$$\mathrm{f(B)F}(B^s)(1 - B)^d(1 - B^s)^D z_t = q(B)Q(B^s)e_t \tag{2.1}$$

Where

B is the backshift operator ($Bz_t = z_{t-1}$),

s is the seasonal period,

p, d, and q are integers greater than or equal to zero and refer to the order of the autoregressive, integrated, and moving average non seasonal parts of the model respectively, P, D, and Q are integers greater than or equal to zero and refer to the order of the autoregressive, integrated, and moving average seasonal parts of the model respectively,

$f(B) = (1 - f B - ... - f_p B^p)$ is the non seasonal autoregressive (AR) operator,

$F(B^s) = (1 - F_1 B^s - ... - F_P B^{Ps})$ is the seasonal AR operator,

$q(B) = (1 - qB - ... - 0_q B^q)$ is the non seasonal moving average (MA) operator,

$0(B^s) = (1 - \quad B^s - ... - 0_Q B^{Qs})$ is the seasonal MA operator,

and t h e e are independent identically distributed *(iid),* with mean zero and variance s^2 (white noise).

The $(1 - B)^d(1 - B^s)^D$ implies non seasonal differencing of order d and seasonal differencing of order D. If $d = D = 0$(no differencing), it is common to replace z_t in (1.1) by deviations from its mean, that is, by $z_t - m$ where $m = E[z_t]$.

A useful extension of SARIMA models results from the use of a time varying mean function modelled via linear regression effects. More explicitly, suppose we write a linear regression equation for a time series y_t as:

$$y = \underset{i}{X} ßx_t + zt \qquad (2.2)$$

where y_t is the (dependent) time series, x_u are regression variables observed concurrently with y_t (trading day effect, leap year, fixed seasonal effect...), $ß_t$ are regression parameters, and $z_t = y_t - \underset{i}{X} ß \, i^x \, a$, the time series of regression errors, that is assumed to follow the SARIMA model (2.1). Modelling z_t as SARIMA addresses the fundamental problem with applying standard regression methodology to time series data, which is that standard regression, assumes that the regression errors (z_t in (2.2)) are uncorrelated over time. In fact, for time series data, the errors in (2.2) will usually be auto correlated, and, moreover, will often require differencing. Assuming z_t is uncorrelated in such cases will typically lead to grossly invalid results.

The expressions (2.1) and (2.2) taken together define the general regARIMA model (linear regression model with ARIMA time series errors).

Combining (2.1) and (2.2), the model can be written in a single equation as:

$$f(B)F(B^s)(1 - B)^d(1 - B^s)^D(yt - \underset{i}{\quad}) = 0(B)0(B^s)e_t \qquad (2.3)$$

This regARIMA model (2.3) can be thought of either as generalizing the seasonal ARIMA model (2.1) to allow for a regression mean function ($\hat{}ß_i\hat{}_{it}$), or as generalizing the regression model (2.2) to allow the errors z_t to follow the seasonal ARIMA model (2.1). In any case, notice that the regARIMA model implies first that the regression effects are subtracted from y_t to get the zero mean series z_t, then the error series z_t is differenced to get a stationary series, say w_t, which is then assumed to follow the stationary ARMA model, $f(B)F(B^s)w = q(B)0(B^s)e$.

Another way to write the regARIMA model (1.3) is:

$$(1 - B)^d(1 - B^s)^D y_t = \sum_i \beta_i (1 - B)^d(1 - B^s)^D x_u + w \quad (2.4)$$

where w_t follows the stationary ARMA model just given. Equation (2.4) emphasizes that the regression variables x_{it} in the regARIMA model, as well as the series y_t are differenced by the ARIMA model differencing operator $(1 - B)^d(1 - B^s)^D$.

Notice that the regARIMA model as written in (2.3) assumes that the regression variables x_{it} affect the dependent series y_t only at current time points, i.e., model (2.3) does not explicitly provide for lagged regression effects such as βx_{it-l}. Lagged effects can be included by the seasonal adjustment program used in this paper, however, by reading in appropriate user-defined lagged regression variables.

2.2 X-12-ARIMA Program and diagnostic checking

The X-12-ARIMA is the most recent outcome of a research program on seasonal adjustment which has been undertaken by the Census Bureau since the 1950's (Findley et al., 1998). It is an enhanced version of the X-11 Variant of the Census Method II seasonal adjustment program (Shinkin et al., 1967). The X-12-ARIMA package can be retrieved at http://www.census.gov/pub/ts/x12a/final/pc/. To get a graphic analysis, the program should be run in the graphic mode using SAS, which is an integrated system of software products, provided by SAS Institute (Rodriguez, 2004).

The seasonal adjustment procedure requires two steps. Firstly, a regARIMA model is defined for the time series or its logarithms. An Akaike's information criterion test for the transformed and untransformed series decides of the use of an additive or a multiplicative adjustment. The model is used to preadjust the series for holiday, seasonal and trading day effects and for forecasting and backcasting. Secondly, the regression error, which is the output of the first step, is implemented into X-12 for seasonal adjustment. The adjusted series are decomposed into trend, seasonal and irregular components. The whole procedure of seasonal adjustment is illustrated by (Findley et al., 1990) and also described in (Ladiray and Quennenville, 2001).

The major improvements in X-12-ARIMA are new modelling capabilities using regARIMA models. The seasonal adjustment module received new options including: the sliding spans diagnostics procedures, new options for seasonal filters, a table of the trading day factors by type of the day etc.

Diagnostic checking of a regARIMA model is performed through various analyses of the residuals from model estimation, the objective being to check if the true residuals appear to be white noise (i.i.d.), $N(0, s^2)$[5].

To check for autocorrelation, X-12-ARIMA can produce Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) of the residuals, along with (Ljung and Box, 1978) summary Q-statistics. The ACF and PACF of these residuals can then be examined in the usual way to identify the AR and the MA orders of the regression error term in the regARIMA model.

An important aspect of diagnostic checking of time series model is outlier detection. The X-12-ARIMA also provides for automatic detection of additive outliers (AO), temporary change outliers (TC) and level shifts (LS). In brief, this approach involves computing ^-statistics for the significance of each outlier type at each time point, searching through these ^-statistics for significant outlier(s), and adding the corresponding AO, LS, or TC regression variable(s) to the model.

Other measures of diagnostic checking are the M1-M11 quality control statistics (see details in Table 15). These can be calculated for short time series, something impossible for the current stability diagnostics of the X-12-ARIMA. If all eleven statistics fail, the adjustment is unacceptable. If some fail and others do not, we can not conclude. One statistic cannot cause the adjustment to be rejected; rather it must be a composite effect of all the statistics. A quality control statistic Q was developed that is a weighted sum of the eleven statistics. Each statistic was assigned a weight according to its relative importance to the overall quality of the adjustment. There are some important additional diagnostics in the X-12-ARIMA; spectrum estimates for the presence of seasonal and trading day effects, and the sliding spans and revision history diagnostics of the stability of seasonal adjustments.

Spectrum estimation is used to detect the presence of seasonal and calendar effects. The period that defines seasonal effects is one year. Thus, in monthly time series, seasonal effects can be discovered through the existence of prominent spectrum peaks at any of the frequencies $k/12$ cycles per month, $1 < k < 6$. In quarterly time series, the relevant frequencies are 1/4 and 1/2 cycles per quarter.

Sliding span analysis described in (Findley and Monsell, 1986), provide summary statistics for the different outcomes obtained by running the program on up to four overlapping subspans of the series. For each month, these diagnostics analyse the difference the largest and smallest adjustments of the month datum obtained from the different spans. They also analyse the largest and smallest estimates of month-to-month changes and of other statistics of interest. It was shown how they complement diagnostics for *(i)* determining if a series can be adjusted adequally, *(ii)* for deciding between direct and indirect adjustments of an aggregate series, and *(iii)* for confirming option choices such as the length chosen

[5] Normality of the residuals is not needed for large sample of estimation and inference results; it is most important for validity of prediction intervals produced in forecasting.

for the seasonal filter. There are two most important sliding span statistics which are used to check the stability of the seasonal adjustment; the seasonal factors (A %), and the month-to-month changes in the seasonal adjustment (MM %), see the (X-12-ARIMA reference manual of the Bureau of Census, 2002) for more details.

3 Holiday regressors

To model Easter effects, (Bell and Hillmer, 1983) introduced a simple type of regressor that has proven to be versatile for modelling effects of a variety of moving holidays. They suppose that the holiday affects the economy for an interval of t days length, over which the effect is the same for each day. With t denoting the number of days in month *t* that belong to the interval, the value in month *t* of the holiday regressor *H(t, t)* associated with this interval is defined to be the proportion of the interval contained within the month,

$$H(\mathrm{t}, \mathrm{t}) = \frac{\mathrm{t}}{T} \quad (3.1)$$

While a single regressor, before the holiday, is sufficient for modelling the Easter effect in the United States[6], several might be needed to model a more complicated effect, either in the way that a step function can be used to approximate a non constant function, or to model the situation in which there are different intervals over which the effect of the holiday is different.

(Lin and Liu, 2002) modelled the case of the Chinese New Year, where the economic activity surges before the holiday, stops during the holiday and slowly accelerates after the holiday. They used three regressors, $H_1(\mathrm{w}, \mathrm{t})$, $H_2(\mathrm{t}, \mathrm{t})$, $H_3(w, t)$ respectively before, during and after the holiday. They take the same interval *w* before and after the holiday.

To model the Islamic feasts effect, we need three regressors for each holiday. We generalize the Lin and Liu approach. We consider that the effect length (number of days) is not necessarily the same before and after the holiday. We note t the effect length before the holiday, t_2 the fixed length of the holiday and t_3 the effect length after the holiday. The model (2.4) becomes

$$\mathrm{f}(B)F(B^s)(1 - \mathrm{B})^{\mathrm{d}}(1 - B^s)^{\mathrm{D}}(\mathrm{y} - \underset{i}{Y^\wedge}\beta x^* - \underset{i=1}{\mathrm{X}}\, aH(ti, \mathrm{t})) = q\,\mathrm{B})0(B^s)e_t \quad (3.2)$$

We can approximately fix t_i for some time series, taking into account the country customs and the performance of the economic activity sectors. However,

[6] See (Findley and Soukup, 2000)

the true value is generally unknown. There are two methods for model comparison[7]. The first is using the Akaike Information Criterion Corrected *(AICC),* proposed by (Hurvich and Tsai, 1989). It is a modification (F-corrected) of the classical Akaike Information Criterion *(AIC).* Both measures are used for choosing between nested econometric models. The second method is a graphic method that compares the out-of-sample forecast performance.

The *AICC,* is defined as,

$$AICC = -2Loglikelihood + 2p[1 - (p+1)/(T - 12D - d)]^{-1} \tag{3.3}$$

where p is the number of estimated parameters, D the order of seasonal differencing, d the order of regular differencing and T the series length. The model with the smallest AICC value is preferred.

Let N_0 be a number less than $(N-h)$ which is large enough that the data y_t, $1 < t < N_0$ can be expected to yield reasonable estimates of the model's coefficients. For each t in $N_0 < t < N-h$, let $y_{t+h/1}$ denotes the forecast of y_{t+h} obtained by estimating the regARIMA model using only the data $y_s, 1 < s < t$, and by using this estimated model to forecast h-steps from time t. The h-step out-of-sample forecast of Y_{t+h} is defined as $Y_{t+h/1} = f^{-1}(y_{t+h/1})$ and the associated forecast error, $e_{t+h/1} = Y_{t+h} - Y_{t+h/1}$. With $y_t = f(Y_t)$ the transformed series, N the series length and $1 < N_0 < N-h$ denoting a number of observations larger enough for model coefficient estimation,

$$yt = f(Y) = \begin{cases} Log(Yt) \\ I + (Y? - 1)/i, i * 0 \end{cases} \tag{3.4}$$

We choose $y_t = Y_t$ if we have an additive model, $y_t = Log(Y_t)$ for a multiplicative model and $y_t = ? + (Y? - 1) / \wedge 0$ for a Box-Cox model.

Consider the accumulating sums of squared out-of-sample forecast errors,

$$SSh, m = \sum_{t=N0} e?+h/1, \qquad M = N,,..., N-h. \tag{3.5}$$

The weighted differences $SS\wedge M - SSf_h'M$ from two competing models, defined by,

[7] See (Findley and Soukup, 2001)

$$_{h,M} = \frac{CC(1) \_ CC(2)}{S S \hat{} / (N \_ h \_ N_0 Y}, \quad _0 \quad \pounds M \pounds N\_h \; ; \qquad (3.6)$$

can be used to compare the forecasting performance of two competing models over the time interval $N_0 \pounds M \pounds N\_h$. For example, over an interval of M values where $S^{l\ 2}_{h\ M}$ is persistently decreasing, the h-step forecast errors from the first model are persistently smaller in magnitude, i.e., better[8].

This method has two major drawbacks: it compares models two at a time and it can be inconclusive: neither model may have persistently smaller forecast errors than the other over the interval for which forecast errors are obtained.

4 Empirical results

To study the impact of the religious events on the Tunisian economy, we consider twelve monthly time series to analyse: Exports, imports, new jobs filled per month, energy production index, textile clothing and leather importation, which may be, highly affected by Eid Al-Fitr's feast. Textile clothing and leather production Index is also interesting to analyse. We also parse money supply which is represented by four time series M1, M2, M3 and M4. Finally, we discuss the TUNINDEX and the BVMT time series that correspond to the Tunisian stock indexes. Data is collected from the National Statistics Institute (Tunis, Tunisia) and the analysis period is from January 1986 to November 2006. A summary of the regARIMA modelling results is displayed in Table 16.

4.1 Imports

Tunisia imports about 1.5 times more than it exports. Imported products include food, textile and clothing, new technology etc. The year-to-year original series (in Figure 1) show that Tunisian imports are increasing since 1986. We notice that peaks and troughs move throughout years. This is not the only effect of the seasonality, but we think that it is the effect of moving holidays. Imports are strongly affected by the religious events. For example, Ramadan is a month which changes consumers' habits. As they fast during the day, after sunset, when they are allowed to eat, people consume much more than they used to consume before Ramadan. The needs of the population (especially food) are bigger than in any other month. So imports increase drastically before and during Ramadan. For example, in 2006, the first of Ramadan coincided with September the 23^{rd} in the

[8] For more details, see (Findley et al., 1998).

Gregorian calendar. In the graphic there is a peak in September that corresponds to the holy month imports.

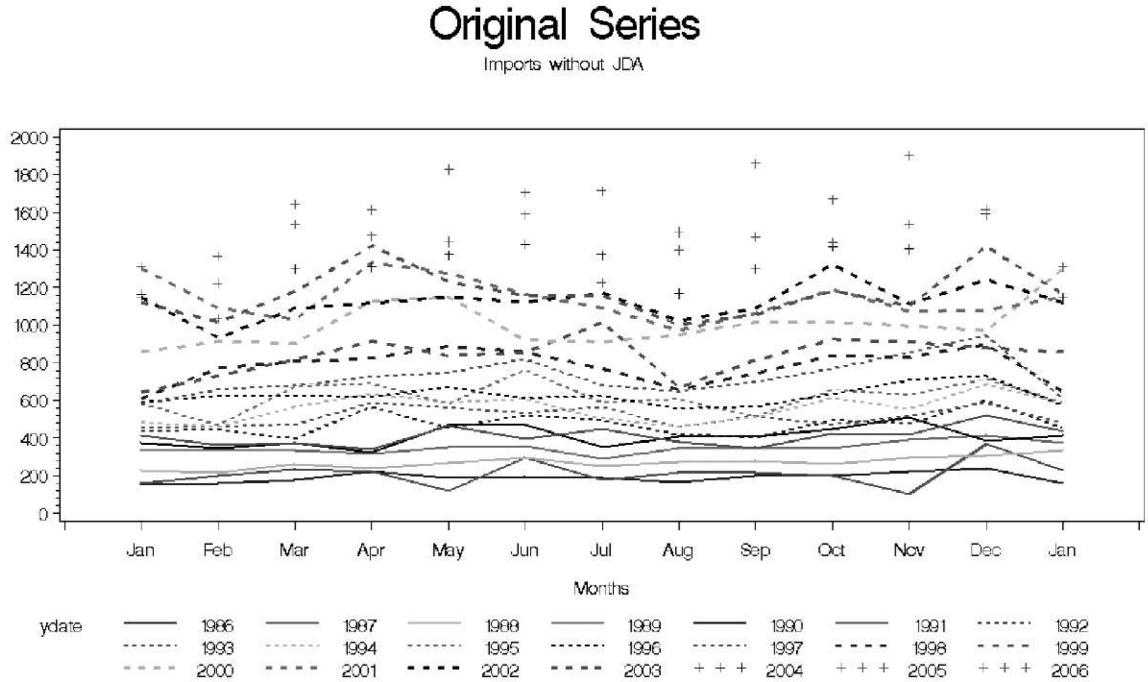

Figure 1: The year-to-year original series.

Since we have three regressors for each holiday, we want to determine the length of the intervals of the effect before, throughout and after the holiday. We then compute, the AICC for models with different values of Tj, r_2 and t_3. We also consider models with 5 holidays, 4 holidays... and only one holiday. In Table 1, we only illustrate three models, the most interesting ones. In model 1, we do not introduce holiday regressors; in model 2, all the intervals t before and after the holidays are equal to 15 days; this is the best symmetric model (where $t = t_3$) found after the AICC tests. And in model 3, we consider different values for t_1 and t_3. In this model, the New Year's regressors are not significant. We delete them from the model ($t_1 = t_3 = 0$). Note that every event has the same interval t_2 in the three models. Ramadan lasts 30 days (or 29 days), Eid Al-Fitr and Eid Al-Idha last 2 days each and both Mawled and the Islamic New Year (INY) last only one day.

The model is much better with the regressors; see the results in the Table 1 below:

Table 1: AICC test for model comparison.

Model 1	**Model 2**	**Model 3**
Model without holiday effect	Ramadan: $t_1 = 15$; $t_2 = 30$; $t_3 = 15$	Ramadan: $= 41$; $T2 = 29$; $t_3 = 19$
	Eid Al-Fitr: $t_1 = 15$; $t_2 = 2$; $t_3 = 15$	Eid Al-Fitr: $t_1 = 41$; $T2 = 2$; $t_3 = 11$
	Eid Al-Idha: $t_1 = 15$; $t_2 = 2$; $t_3 = 15$	Eid Al-Idha: $t = 41$; $T2 = 2$; $t_3 = 4$
	Mawled: $t_1 = 15$; $t_2 = 1$; $t_3 = 15$	Mawled: $t_1 = 14$; $t_2 = 1$; $t_3 = 6$
	INY: $t = 15$; $t_2 = 1$; $t_3 = 15$	INY: $t = t_3 = 0$
AICC = 2770	AICC = 2687	AICC = 2626

We choose the third model, not only because it has the smallest AICC but also because the length of the intervals in the third case is more logical; in fact, we think that the effect before Ramadan is bigger than 15 days. The Islamic New Year in model 3 does not have a big influence on the imports as compared to the other Islamic feasts.

We use the AICC to decide if it is necessary to transform the time series. The AICC (with aicdiff = -2.00) prefers no transformation. Additive seasonal adjustment is then performed. The model chosen is a Seasonal ARIMA (0,1,2)(0,1,1). Table 2 shows that trading day and holidays regressors are accepted as expected. The holiday-factors are significant in the regARIMA regression; this confirms the importance of the impact of moving holidays.

Table 2: Chi-squared Tests for Groups of Regressors.

Regression Effect	DF	Chi-Square	P-Value
Trading Day	6	21.12	0.00
User-defined	12	79.50	0.00
Combined Trading Day and Leap Year Regressors	7	21.34	0.00

Tests for stable and moving seasonality are presented in Table 3, 4 and 5. Monitoring and quality assessments statistics are shown in Table 15; we look for M7 and Q statistics less than 1.0. These diagnostics help us decide if X-12-ARIMA can adequately adjust the series. M7 = 0.72 and Q = 0.7; the adjustment is accepted.

Table 3: Test for the presence of seasonality assuming stability.

	Sum of Squares	**Dgrs of freedom**	**Mean Square**	**F-Value**
Between months	586915	11	53355	17.51**
Residual	728055	239	3046	
Total	1314970	250		

**Seasonality present at the 0.1 per cent level.

Table 4: Moving Seasonality Test.

	Sum of Squares	**Dgrs of freedom**	**Mean Square**	**F-Value**
Between Years	104305	19	5489	3.75**
Error	305568	209	1462	

**Moving seasonality present at the one percent level.

Table 5: Summary of tests for stable and moving seasonality for each span.

	Span1	**Span2**	**Span3**	**Span4**
Stable seasonality	12.22	18.33	17.76	17.72
Moving seasonality	0.72	0.94	0.52	1.28
M7	0.57	0.52	0.49	0.55
Identifiable seasonality	yes	yes	yes	yes

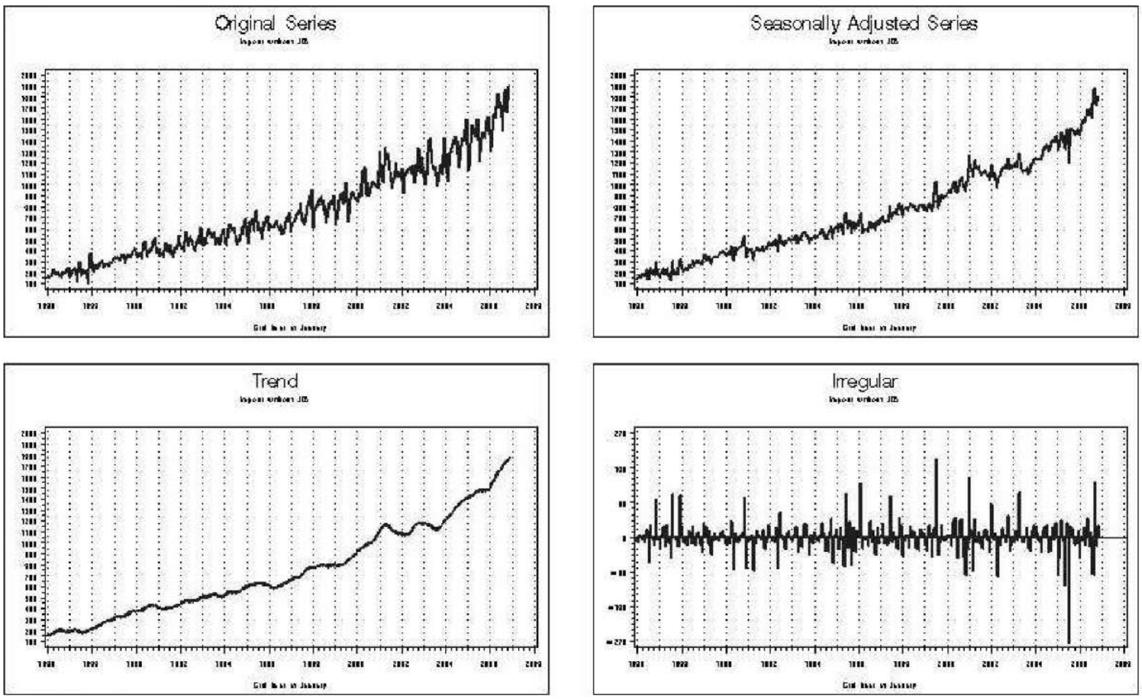

Figure 2: Seasonal, trend, irregular components and original series.

Seasonal, trend, irregular components and original series are shown in Figure 2. The holiday, seasonal and combined factors are reported in Figure 3.

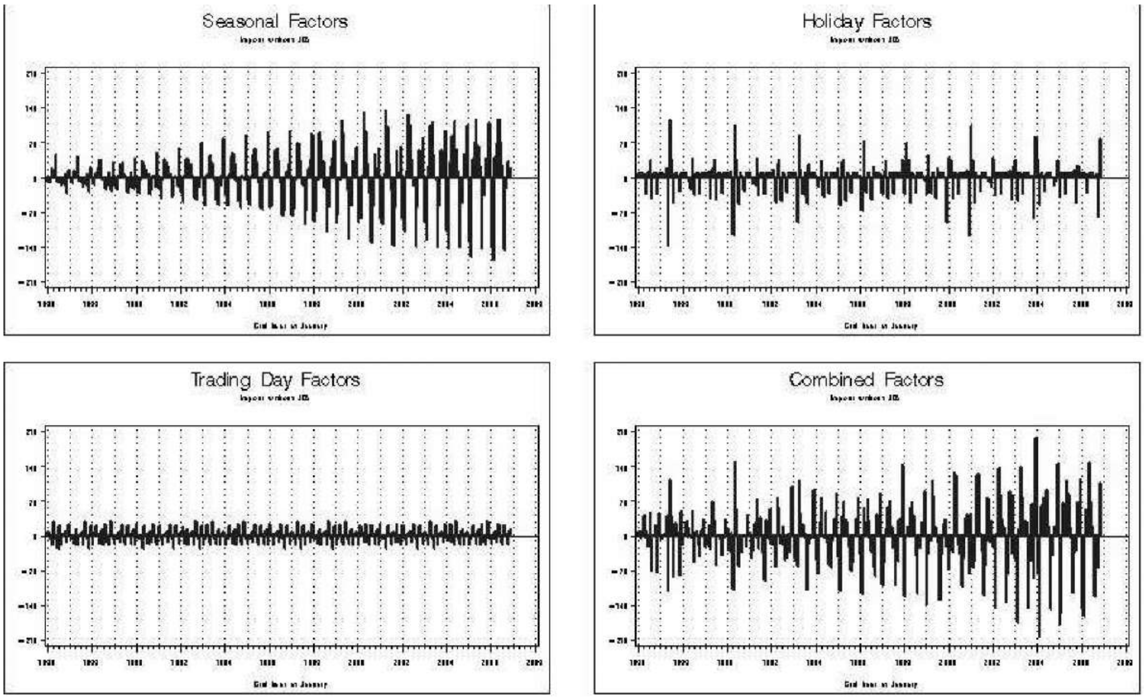

Figure 3: The holiday, seasonal and combined factors.

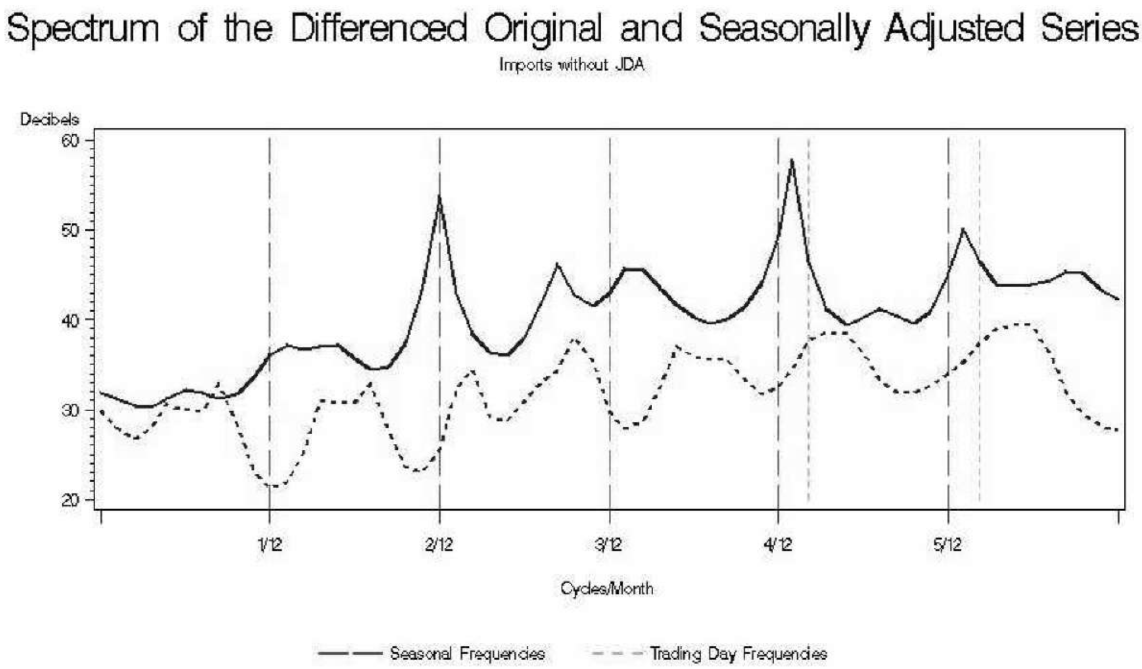

Figure 4: The spectrum for the original and adjusted series.

From the figures, we conclude that; *(i)* holiday factor is smaller than seasonal factor, *(ii)* while the magnitude of seasonal factor is increasing; the holiday factor is almost stable. It can be explained by the effect of the Islamic events that has always been as important as nowadays. The spectrum for the original and adjusted series is reported in Figure 4.

Vertical lines identify the amplitudes at seasonal and trading day frequencies. (Cleveland and Devlin, 1980) identified the trading day frequencies of the spectrum as the frequencies most likely to have spectral peaks if a flow series has a trading day component. The figure shows that the peak at seasonal frequency for the original series is removed for the adjusted series.

We also compare the difference of seasonal factors; Figure 5 shows that without holiday adjustment the seasonal factors are falsely extended or contracted in January and February.

The test for the presence of residual seasonality gives the following results: There is no evidence of residual seasonality in the entire series at the 1 per cent level (F = 0.36); there is no evidence of residual seasonality in the last 3 years at the 1 per cent level (F = 0.25).

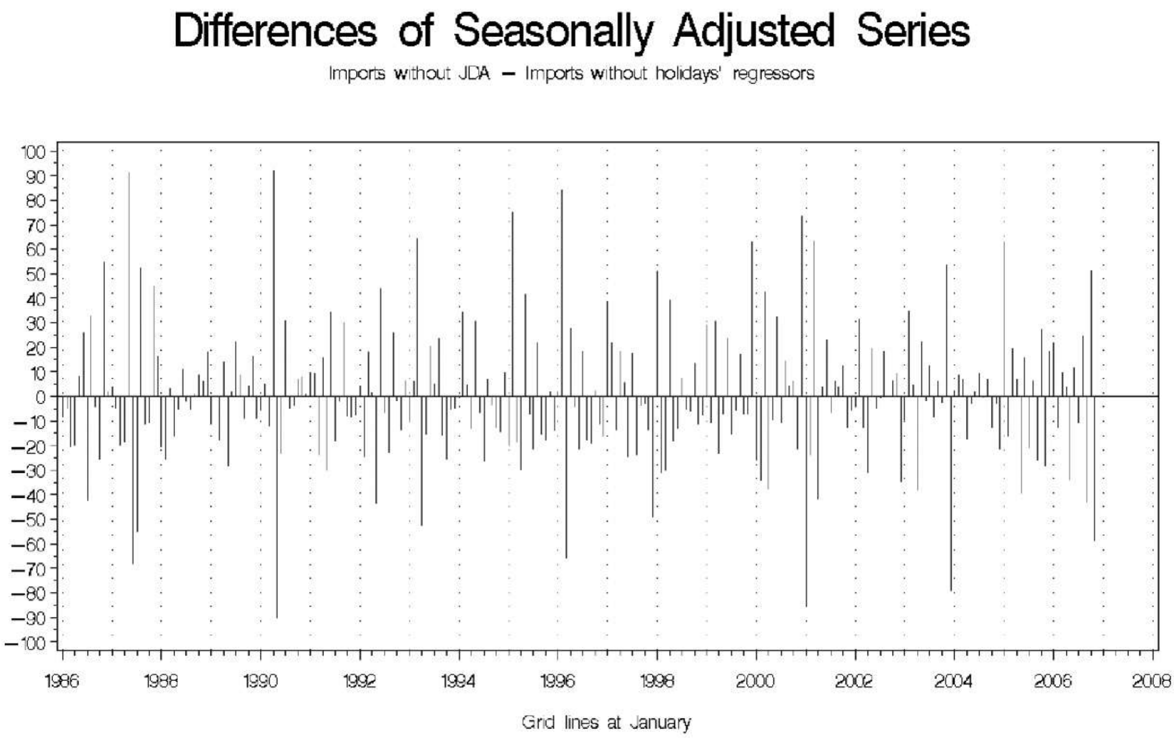

Figure 5: The seasonal factors without holiday adjustment.

4.2 Exports

Since 1986, exports in Tunisia have been increasing over the years, (see Figure 6). In fact the government is encouraging the companies to export their products by suppressing taxes on raw material.

The question is: do national and religious events affect the exports in Tunisia? As people consume more throughout feasts and holidays, one may think that some products, which used to be exported, are kept in the local market to satisfy the needs of the population.

We compute the AICC criterion for different models with different numbers of holidays and different lengths of the intervals before and after the holidays. The AICC obtained is always bigger than the value found when we do not consider the holiday-regressors, (AICC=2680). We conclude that the model without holiday-regressors is the best, and therefore exports in Tunisia are not really affected by religious events.

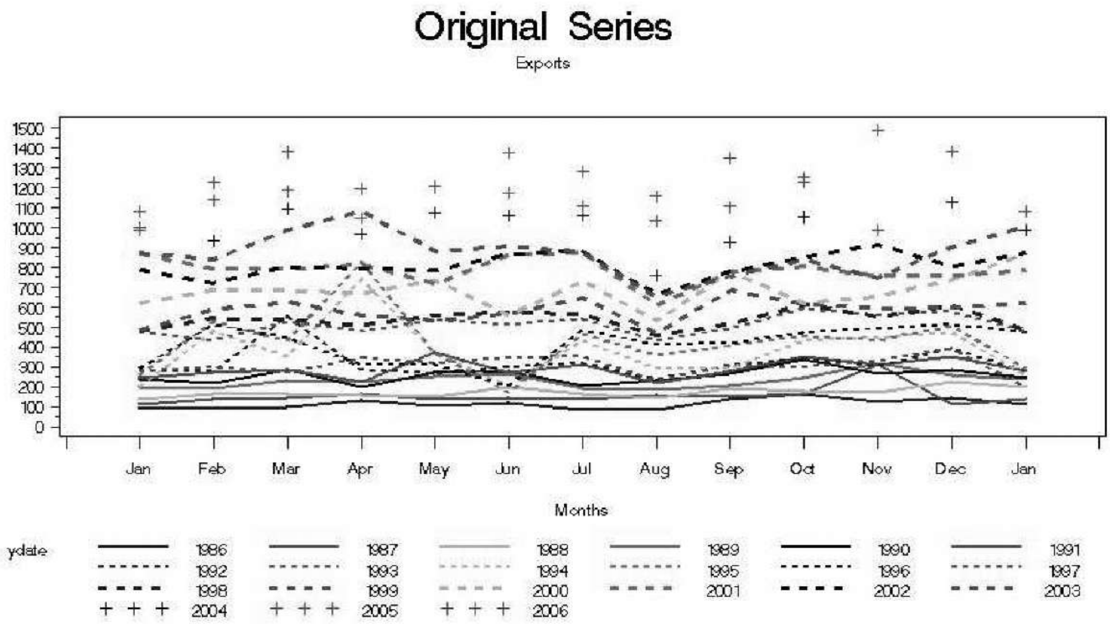

Figure 6: Exports in Tunisia since 1986.

What we find here invalidates our first thoughts. In fact, Tunisia exports an important part of its production of olive oil and citrus fruits as well as other products. The exporters have to respect the agreements in terms of time and quantity otherwise they have to pay penalties. Even if the population's needs increase drastically around the religious feasts, the country continues to export as in normal periods. The extra need is imported. A huge need of hard currency also explains this strategy. The example of olive oil and citrus fruits is mentioned because these are high quality products and a source of hard currency. The latter is used to import other products and to pay foreign debts.

Tunisia wants to preserve and even to increase its part of the external market, so an increase of the consumption affects the imports but not the exports.

4.3 New jobs filled

Figure 7 is a year-to-year plot of a monthly time series that represents the number of jobs filled every month In Tunisia. Before Eid Al-Fitr, retail sales grow up and consequently recruits increase in the trade sector. Eid Al-Fitr affects the number of jobs filled which increase before the Eid and decrease after the end of the feast, more details is given in section 4.4.

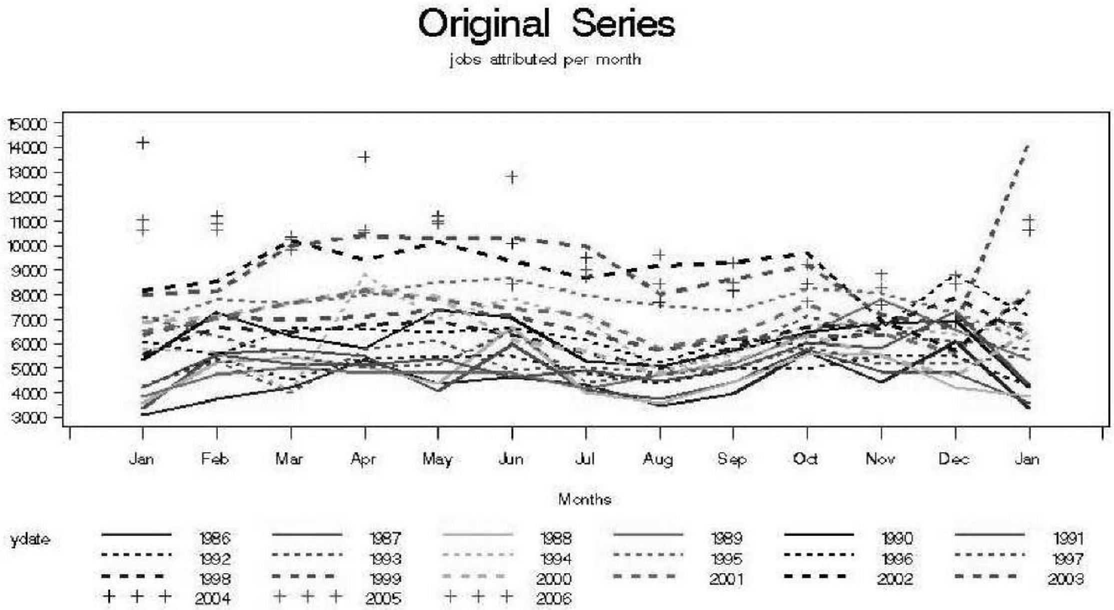

Figure 7: Year-to-year plot of a monthly time series.

The AICC comparison shows that the model with only one feast regressors (Eid Al-Fitr) with T_1= 30 and T_3= 7 gives the smallest AICC. In Table 6, are tabulated the results of only three models.

Table 6: AICC test for model comparison.

Model 1	Model 2	Model 3
Model without holiday effect	Ramadan: t = 15 ; $t_2 = 30$; $t_3 = 15$ Eid Al-Fitr: t_1 = 15 1 2 = 2; t = 15	Eid Al-Fitr: = 30 ; t_2 = 2; t_3 = 7
AICC =3946	AICC = 3934	AICC = 3928

The out-of-sample forecast comparison gives the same results. It is a graphic method which consists of drawing the differences of the accumulated sums of squared forecast errors between the competing models for forecast leads of interest (lead 1 and 12, see Figure 8). If the aspect of the accumulated differences is predominantly upward, then the forecast errors are larger for the first model and the second model is preferred. In Figure 8, four models are compared two at a time: model 1, 2, 3 of Table 6, and model 4 with $T_1 = T_3 = 15$ for before and after Eid Al-Fitr. In Figure 8a we compare model 3 and model 1. And in Figure 8b, we compare model 3 and model 2. In both cases, the graph goes down in 1-step and 12-step forecasting, then model 3 is better. Figure 8c compares model 3 and model 4, and shows that the graph goes up before 1999 (which means that model 4 is better), and goes down after 2003 (which means that model 3 is better for the recent period), so we choose model 3[9].

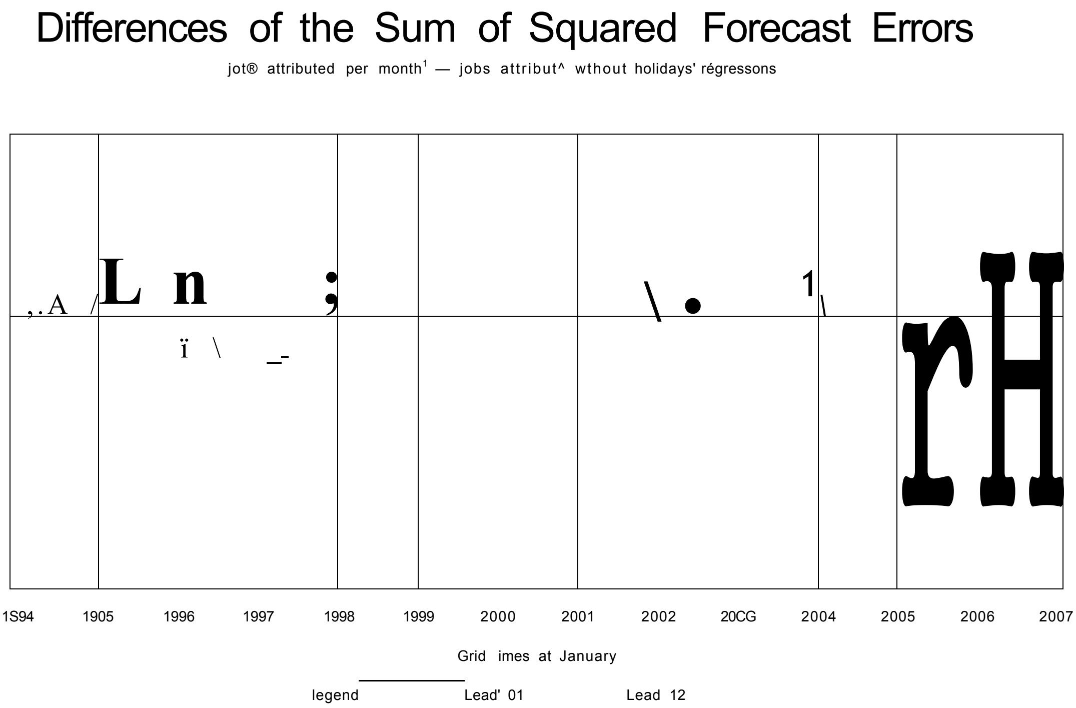

Figure 8a: Comparison of model 3 and model 1.

[9] For more information on forecast error plots, see (Findley and Soukup, 2000) and (Findley et al., 2005).

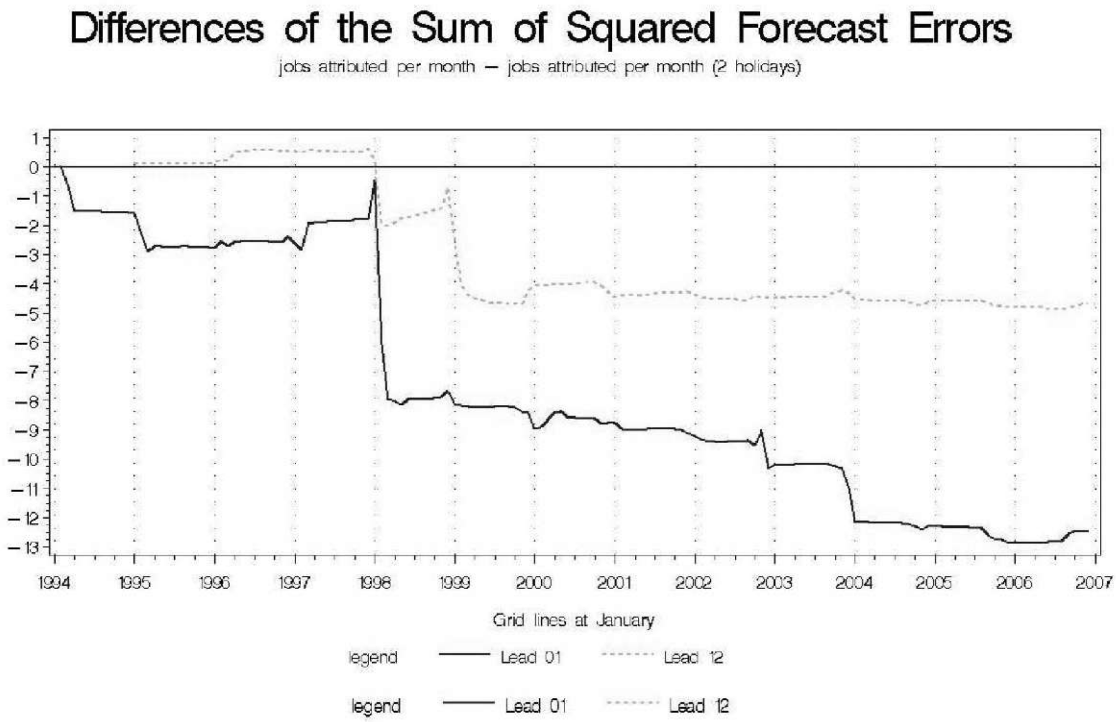

Figure 8b: Comparison of model 3 and model 2.

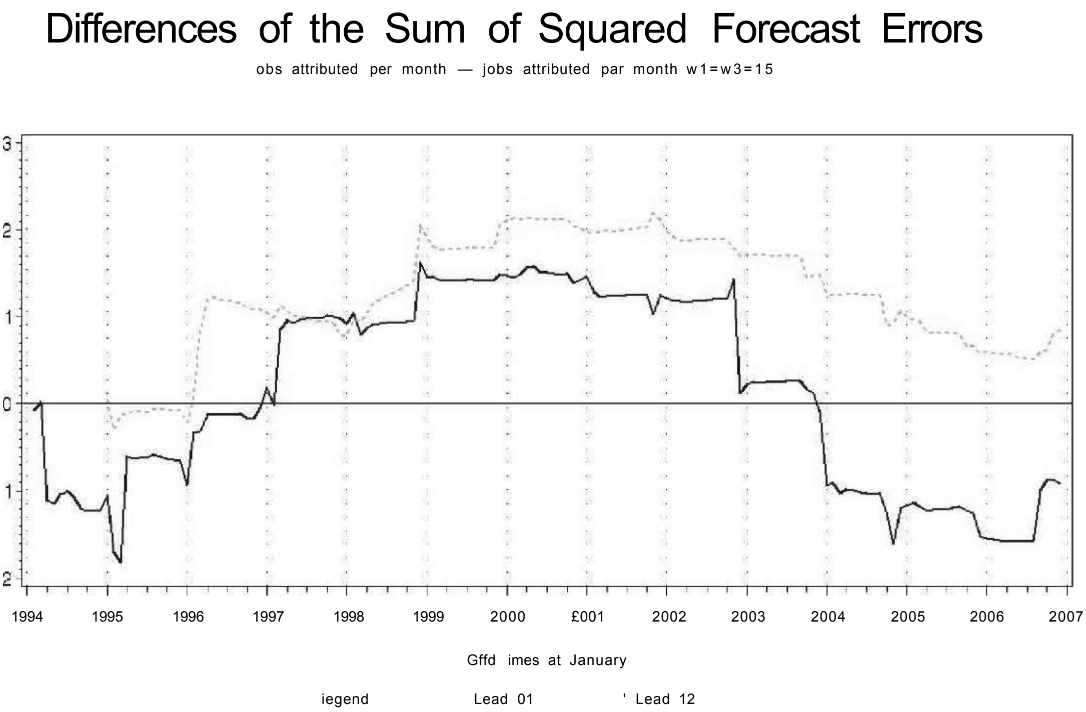

Figure 8c: Comparison of model 3 and model 4.

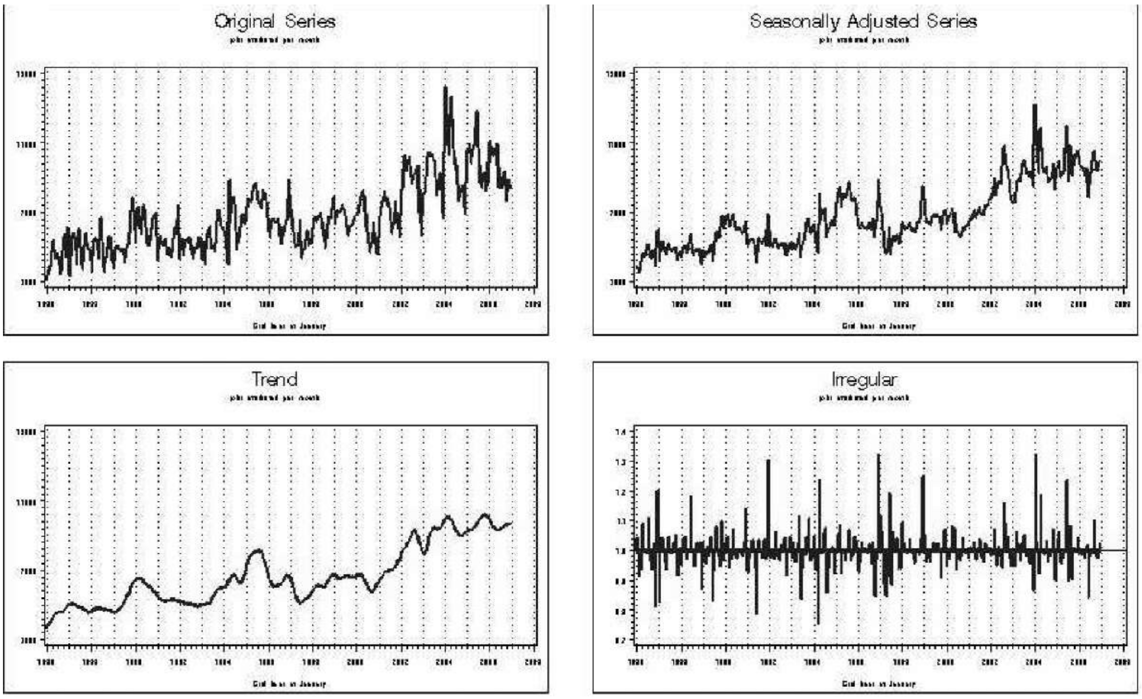

Figure 9: Seasonal adjusted series, the irregular and the trend components.

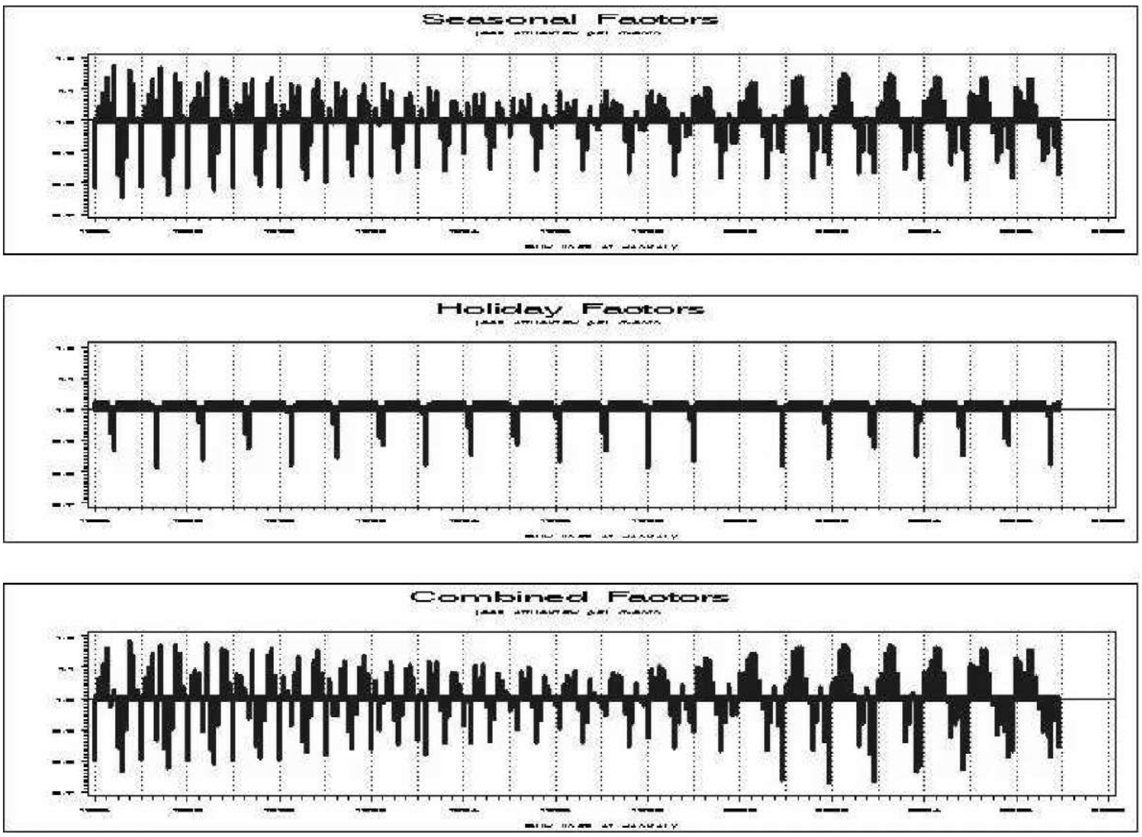

Figure 10: Seasonal holiday and combined factors.

Both methods show that model 3 is better; during a month before Eid Al-Fitr, stores and shops' owners as well as the bakers recruit new employees because of the big number of customers and the huge growth of sales. This effect is removed by the procedure of seasonal adjustment on filled jobs time series; see the final seasonal adjusted series, the irregular and the trend components in Figure 9.

M7 and Q (Table 15) are less than 1, the adjustment is declared to be acceptable. The holiday factor is regular and steady while the seasonal factor decreases until 1994, remains steady till 1997 then increases after 1998, (see Figure 10). The spectrum of the differenced original and seasonally adjusted series is displayed in Figure 11.

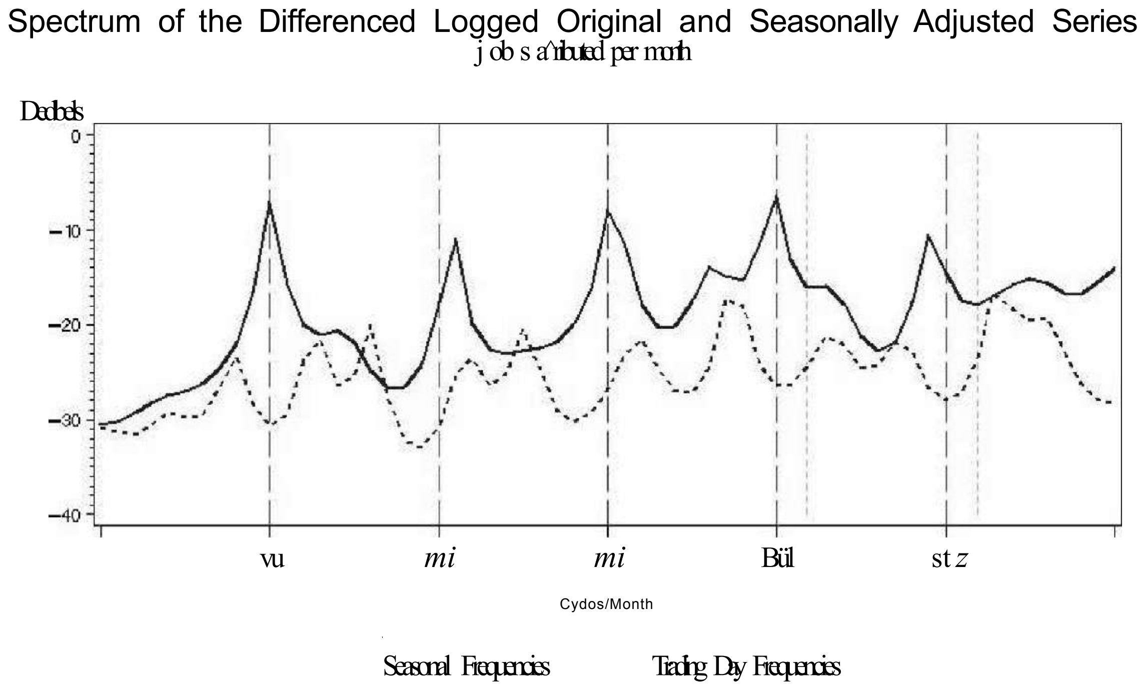

Figure 11: The spectrum of the differenced original and seasonally adjusted series.

From the test for the presence of residual seasonality we conclude that there is no evidence of residual seasonality in the entire series at the 1% level (F = 0.39). No evidence of residual seasonality in the last 3 years neither at the 1% level (F = 0.50) nor at the 5%level.

Sliding spans analysis permits to check the stability of our seasonal adjustment. There are four spans with January 1996, the first month of the first span. In Table 7, we show the percentages of months flagged as unstable for model 1 and model 3. Unstable adjustments can be the unavoidable result of the presence of highly variable seasonal or trend movements in the series being adjusted. The figures are too high but the values corresponding to the model 3 are smaller than model 1[10]. We conclude that the seasonal adjustment is more stable and better in the case of the model with holiday regressors.

Table 7: Percentage of months flagged as unstable.

Series	Seasonal Factors (A %)	Month-to-Month Changes in SA Series (MM %)
Model 1: No holiday factor	47.2	47.7
Model 3: With holiday factor	39.8	33.6

4.4 Textile, clothing and leather importation

In Tunisia, the Eid Al-Fitr feast highly influences the textile, clothing and leather importation. Eid Al-Fitr is a very long-waited event. It is a two-days holiday (and three days for pupils and students) that marks the end of Ramadan. Buying new clothes in Eid Al-Fitr is a religious act. It is highly approved that Muslims look good throughout the year but especially in Eid Al-Fitr. Those, who can not afford to buy new clothing, are only ask to put clean and good looking ones.

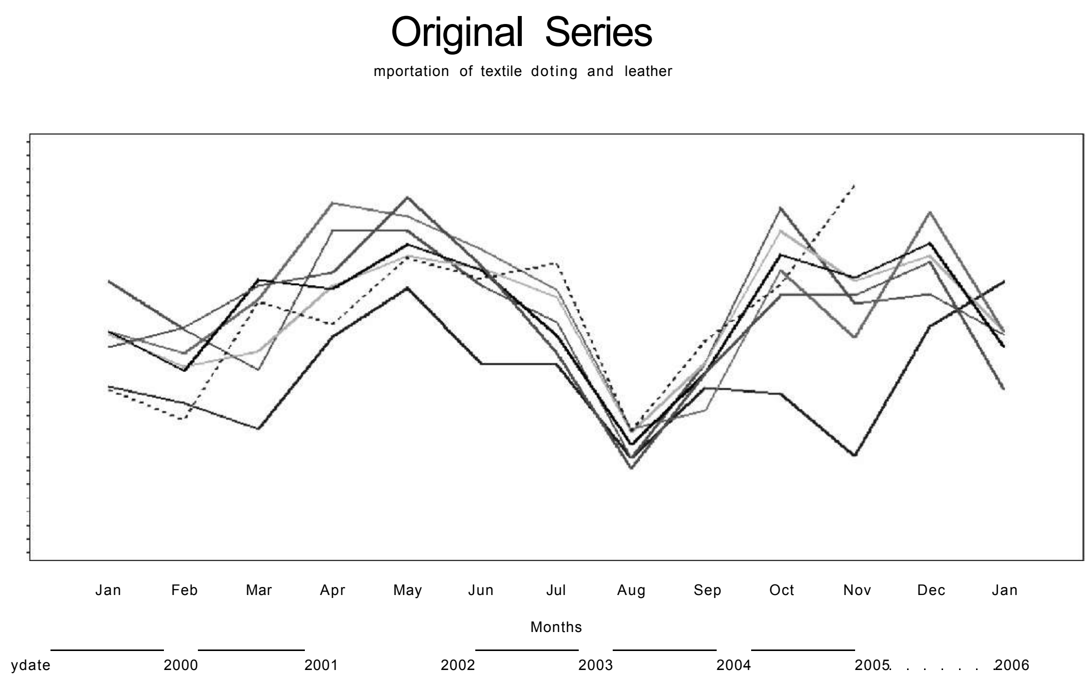

Figure 12: The year-to-year original series.

The year-to-year original series, in Figure 12, show that imports increase before the holiday. The trough in August corresponds to the end of summer where

[10] Recommended limits for percentages: A%: 15% is too high and 25% is much too high. MM%: 30% is too high and 40% is much too high

people stop buying light clothes. Imports grow up again before the new school term.

To analyse the effect of Eid Al-Fitr holiday on the textile importation time series, we first write the model with all the holiday regressors. The smallest value of the AICC is found when we add only the Eid Al-Fitr regressors. In Table 8 we put the model without the regressors and the model with the smallest AICC.

Table 8: AICC test for model comparison.

Model 1	**Model 2**
Model without holidays regressors	Model with only Aid el fitr regressors T = 20 ; Tj = 2; T_3 = 13
AICC = 693	AICC = 686

The interval effect before the holiday is about 20 days and 13 days after the holiday. Textile, clothing and leather importations increase before Eid Al-Fitr.

The quality assessment statistics accept the adjustment (see Table 15). In Table 9, there is a summary of sliding span analysis.

Table 9: Summary of tests for stable and moving seasonality for each span.

	Span 1	**Span 2**
Stable seasonality	24.10	45.22
Moving seasonality	0.35	0.43
M7	0.41	0.30
Identifiable seasonality	yes	yes

We show the original and the adjusted series, trend and irregular components in Figure 13. The holiday factor is smaller than the seasonal factor; both have a steady magnitude; (see Figure 14). The final adjustment removed the peak at seasonal frequencies (the vertical lines in Figure 15).

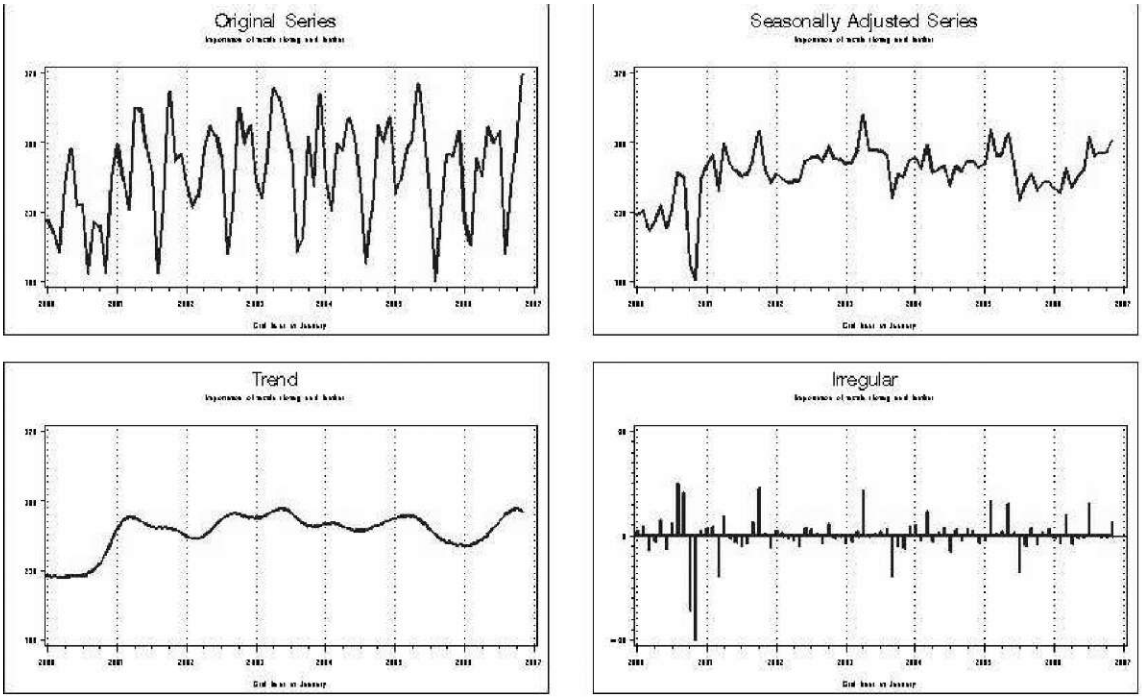

Figure 13: The original and the adjusted series, trend and irregular components.

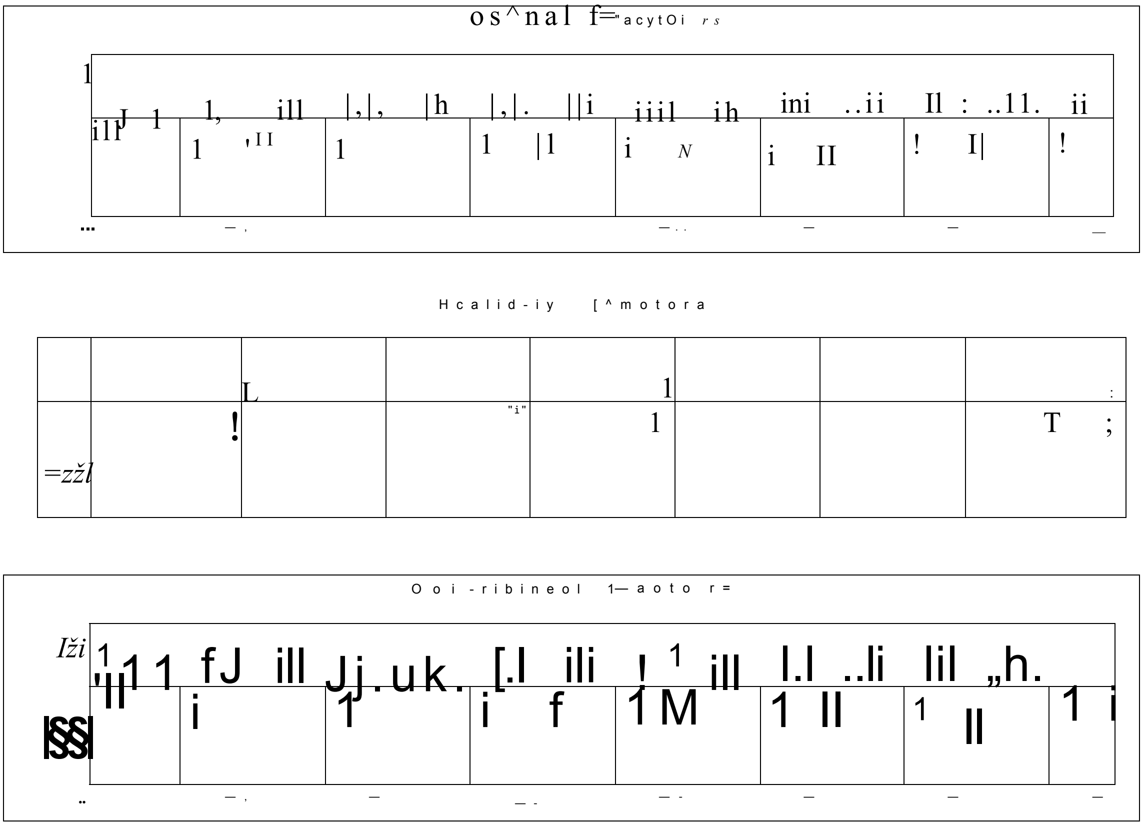

Figure 14: Holiday factors, seasonal factors and combined factors.

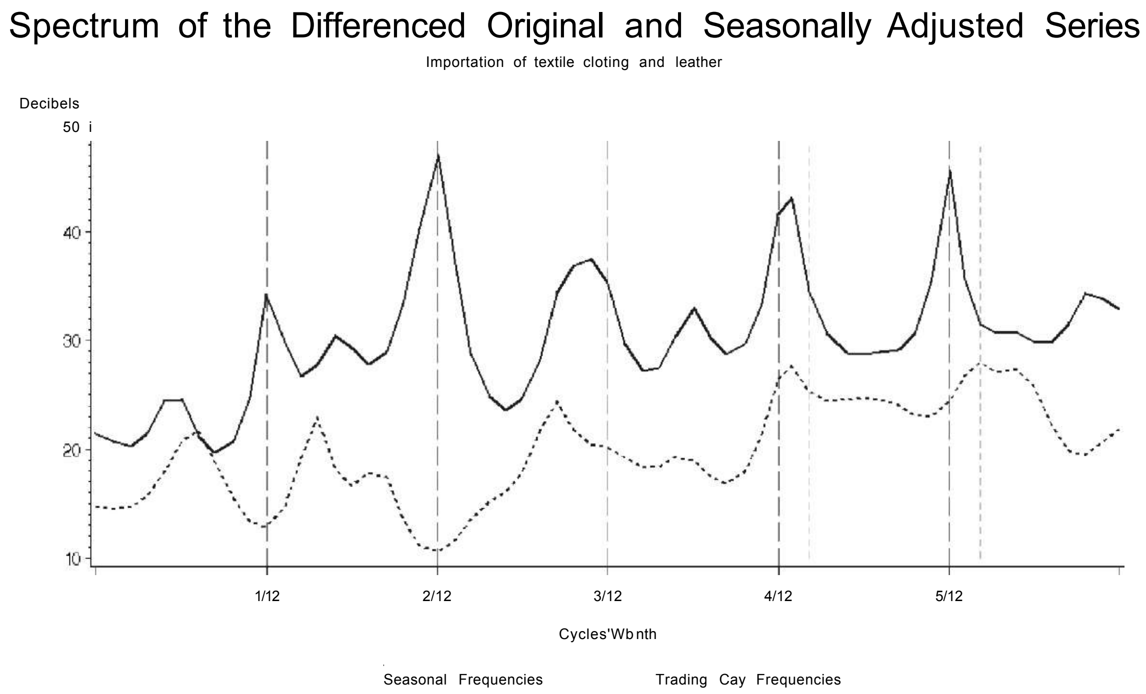

Figure 15: The final adjustment.

4.5 Textile, clothing, and leather production index

The textile and clothing sector includes finishing of textiles, manufacture of made-up textile articles, (except apparel), and manufacture of other textiles (codes 17.3, 17.4 and 17.5 according to the statistical classification of economic activities in the European community). This is one of the most important sectors of the Tunisian economy. It records over 14% growth per annum. It is the most exporting and job creating sector of the manufacturing industry. This is due to the clothing branch which supplies 85% of the jobs in the sector. More than 2000 firms operate in the textile and clothing industry, 80% are totally exporting enterprises, and 57% are foreign. Tunisia is the sixth supplier of Europe and the second supplier of France in this domain. The partnership with the European Union is established thanks to many Tunisian advantages and qualifications; in fact Tunisia is geographically close to its main trading partners: France, Italy and Germany. It also provides short delivery, cheap labour costs, skill and qualification, well-developed infrastructure. The country is attractive as well for its political stability.
The footwear and leather sector is also one of the main sectors in the manufacturing industry. The exports exceed 60% of the production. The main client is Italy with 40% of total exports, and then comes France with 38% followed by Germany (10%).

This brief description of 'textile and clothing' and 'footwear and leather' sectors shows that the bulk of the production is exported, so we are not sure that the effect of the religious events on the whole time series is significant. We proceed as before, by computing the AICC criterion for model without holiday's regressors and model with only Eid Al-Fitr regressors. In Table 10, only two models are represented. The model with 15 day-holiday regressors fits with the data more than the others; it also beats the model without the regressors. Sliding span statistics are put in Table 11. From the quality assessment statistics in Table 15, we conclude that the adjustment is acceptable. See Figures 16, 17, 18 and 19 for graphics.

Table10: AICC test for model comparison.

Model 1	**Model 2**
Model without holidays regressors	Model with only Eid Al-Fitr regressors
	$t = 15$; $t_2 = 2$; $t_3 = 15$
AICC = 913	AICC = 886

Table 11: Summary of tests for stable and moving seasonality for each span.

	Span 1	**Span 2**
Stable seasonality	44.44	19.88
Moving seasonality	2.76	1.35
M7	0.41	0.53
Identifiable seasonality	yes	yes

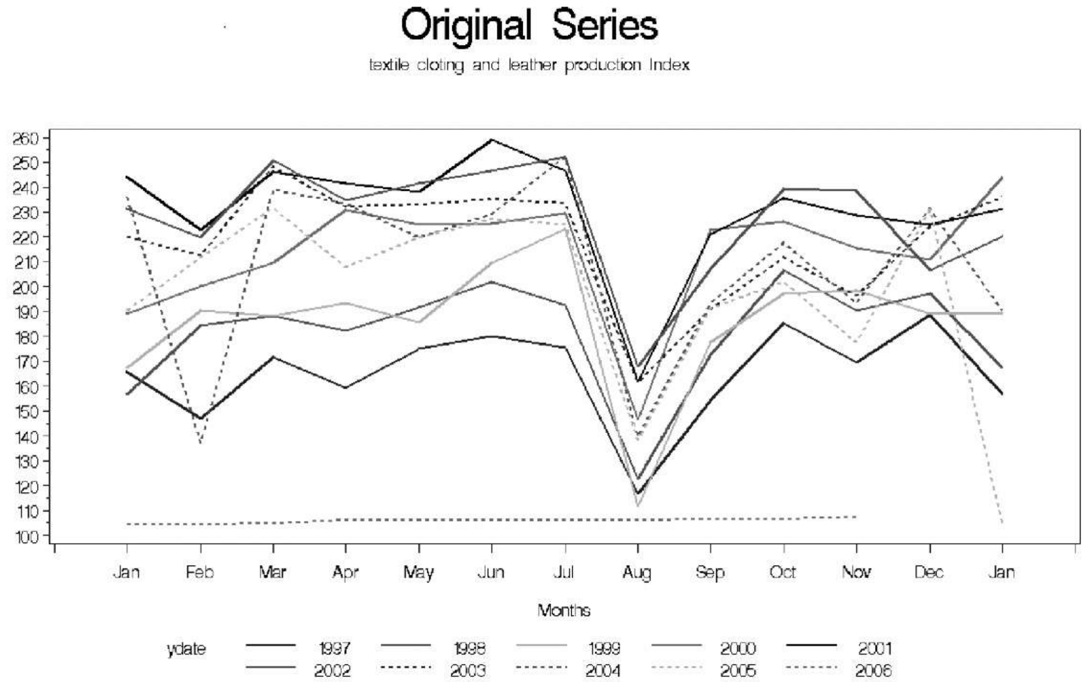

Figure 16: Original series.

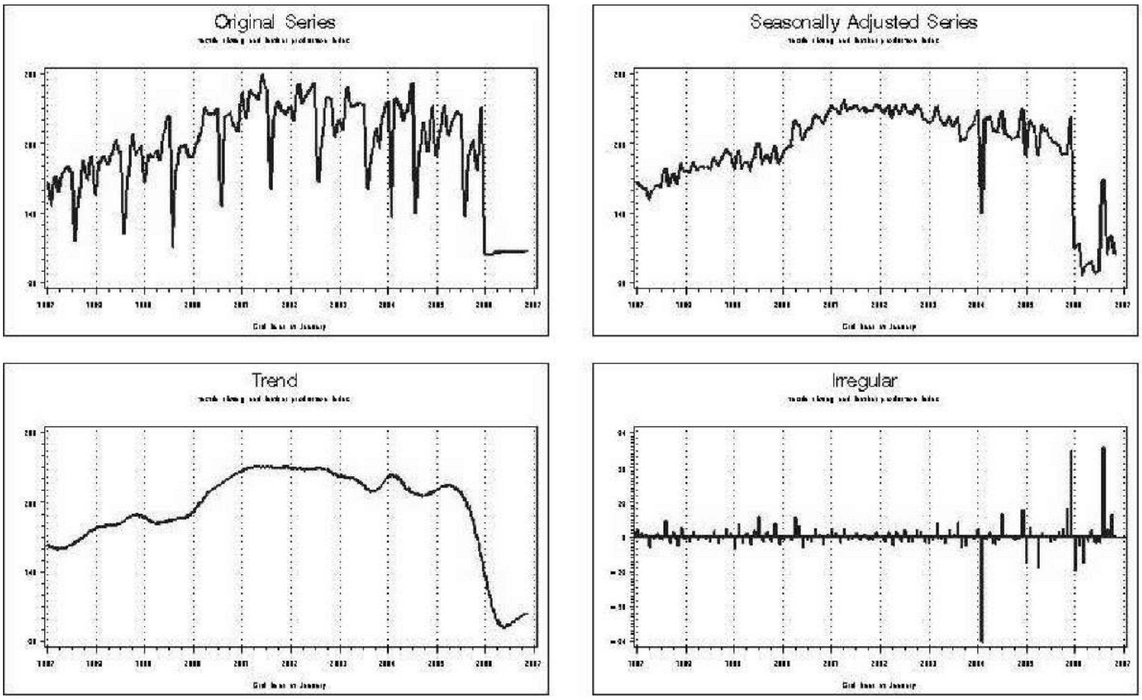

Figure 17: Original series, seasonally adjusted series, trend and irregular components.

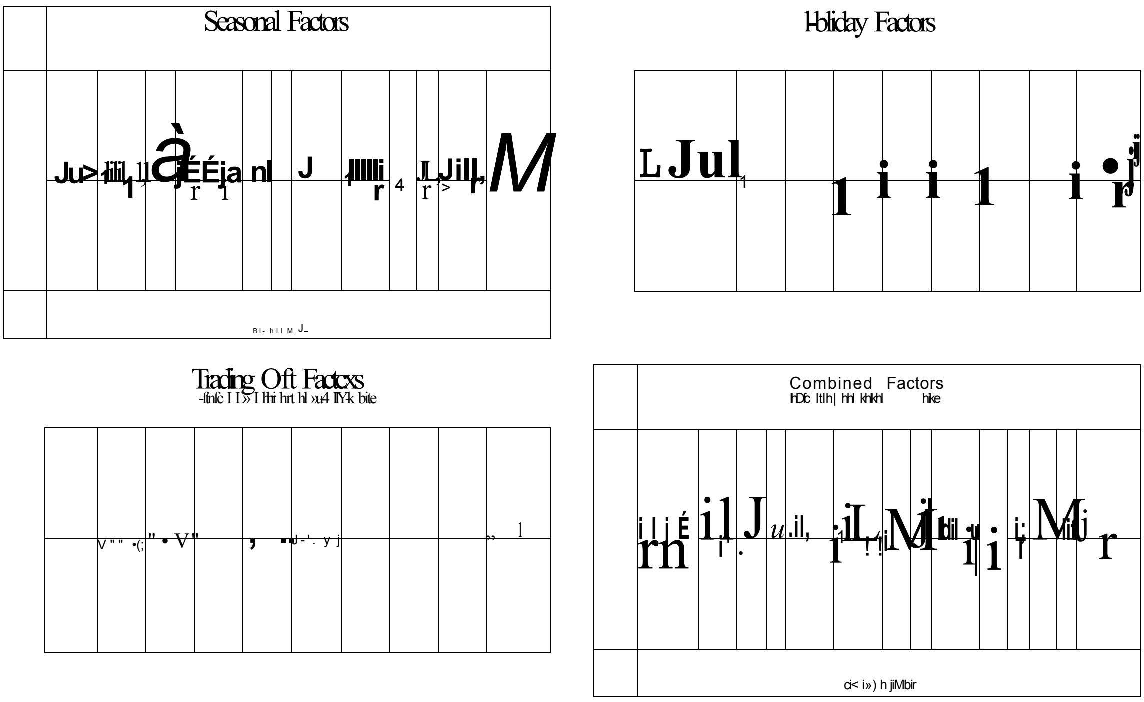

Figure 18: Seasonal factors, holiday factors, trading day factors and combined factors.

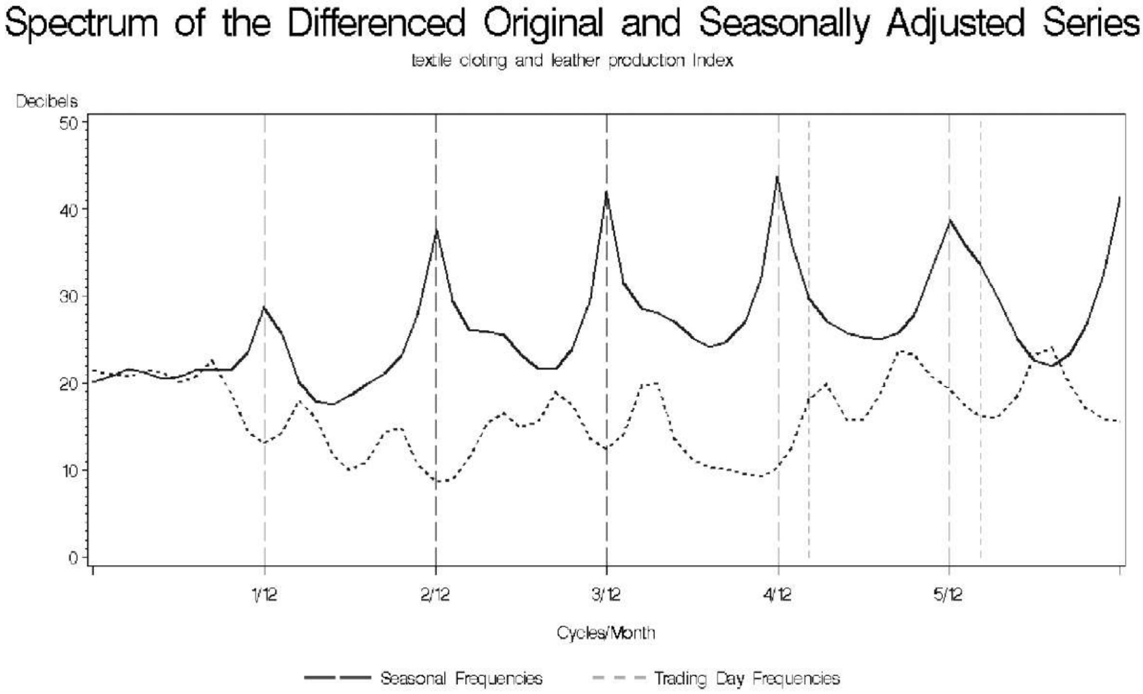

Figure 19: Spectrum of the differenced original and seasonally adjusted series.

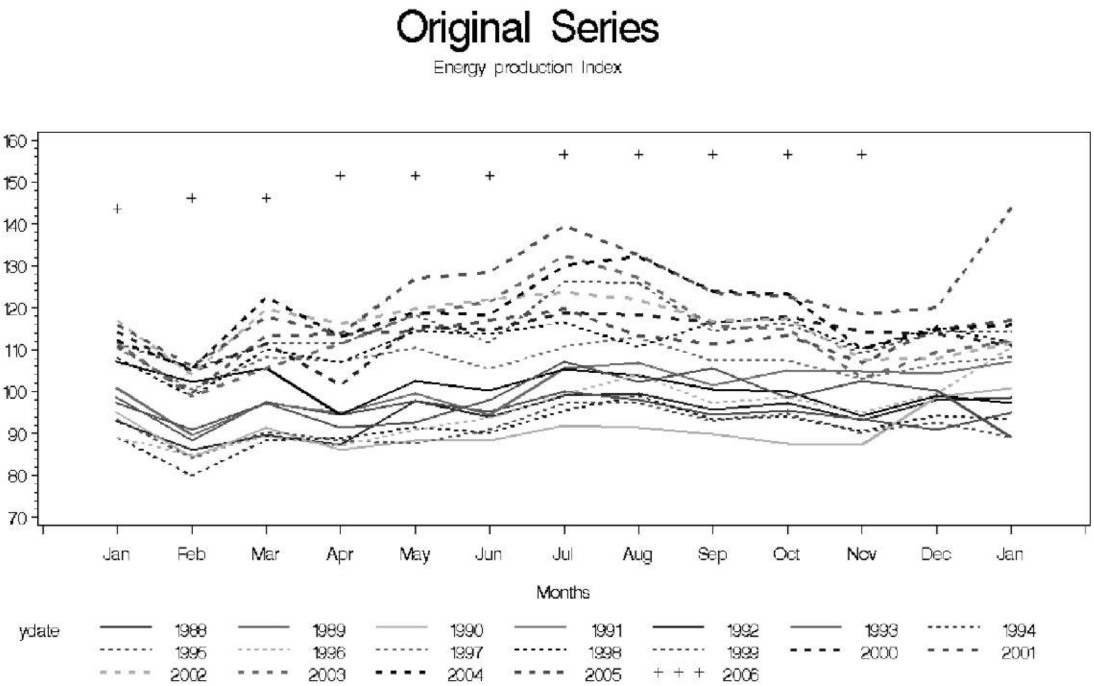

Figure 20: Original series.

The textile, clothing and leather production index time series is affected by Eid Al-Fitr feast. The behaviour of a small number of firms affects the whole time series. Even though the majority of the production is exported, the large demand of the local market around Eid Al-Fitr encourages the firms, which do not export their products, to produce more. Thus, compagnies respect their contractual obligations with their foreign partners and at the same time the local market is supplied with products intended to be consumed locally and also by the imported products.

4.6 Energies production index

Energy production index includes water, electricity, gas and oil production. Holidays and feasts strongly affect this index. During Islamic feasts, firms, schools and offices are closed. During Ramadan, the working day is shorter than in the other months. Workers finish their jobs at 3 p.m. Students' schedule also changes and becomes shorter. The consumption of energy, which is tied and correlated with the production, drops during these events and rises before and after. Year-to-year original time series is displayed in Figure 20.

To model this effect, we use all the Islamic holidays' regressors and we compare the model without and with the regressors. The AICC test shows clearly that the holidays' regressors improve the quality of adjustment. A model with an interval of 15 days before and after each holiday gives the best results, (see Table 12).

Table 12: AICC test for model comparison.

Model 1	**Model 2**
Model without holidays regressors	Model with 15 holidays regressors where $t = t_3 = 15$
AICC = 1235	AICC = 1217

Model 2 fits well with the data; the quality of the seasonal adjustment in model 2 is better than in model 1. Original and seasonally adjusted series, trend and irregular components are shown in Figure 21. We put holiday, seasonal and combined factors in Figure 22. The spectrum for the original and adjusted series is reported in Figure 23. Identifiable seasonality is also tested for each span in Table 13.

From the test for the presence of residual seasonality; we conclude that there is no residual seasonality in the entire series at the 1% (F = 0.22).

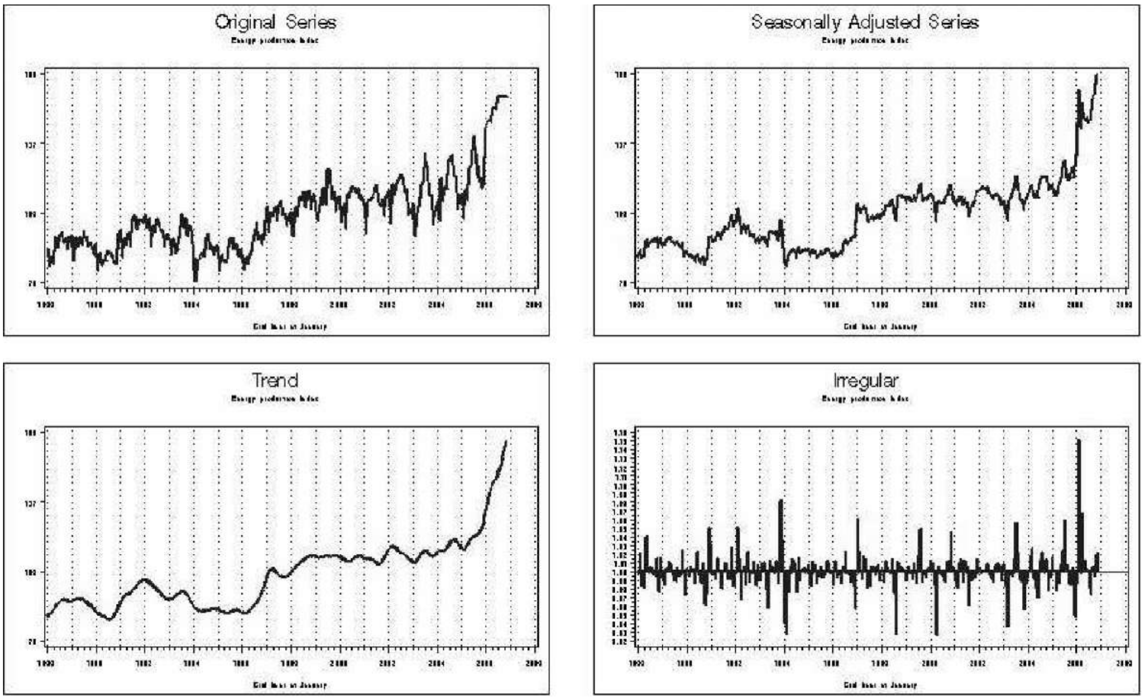

Figure 21: Original and seasonally adjusted series, trend and irregular components.

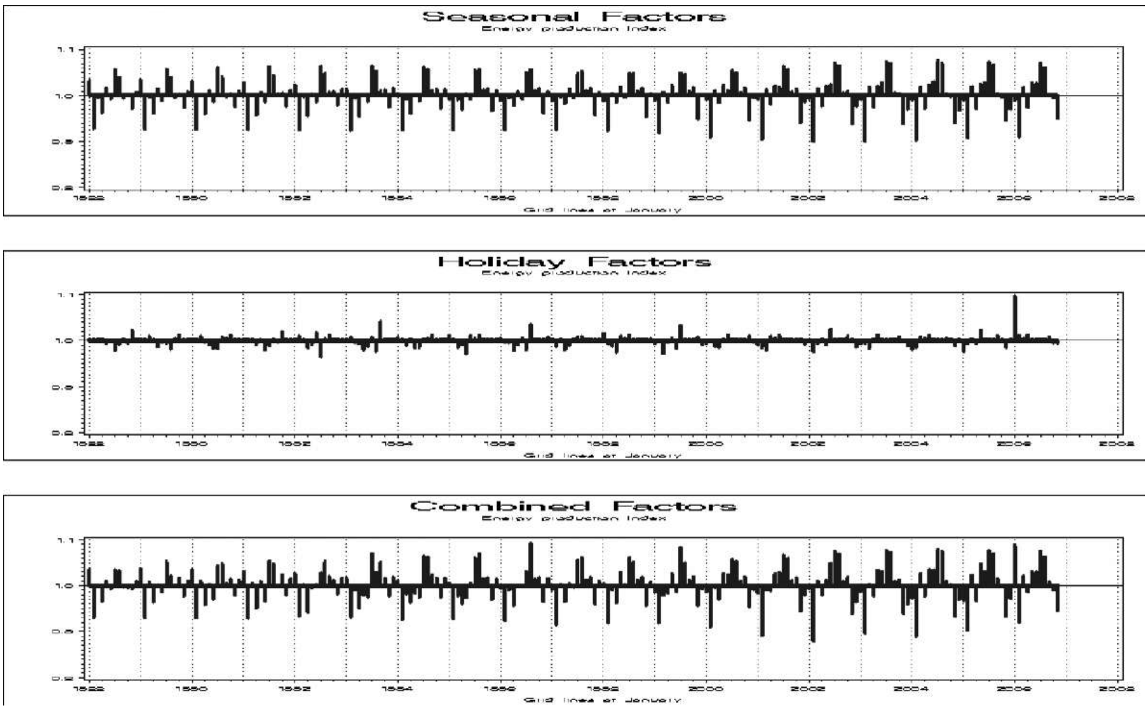

Figure 22: Holiday, seasonal and combined factors.

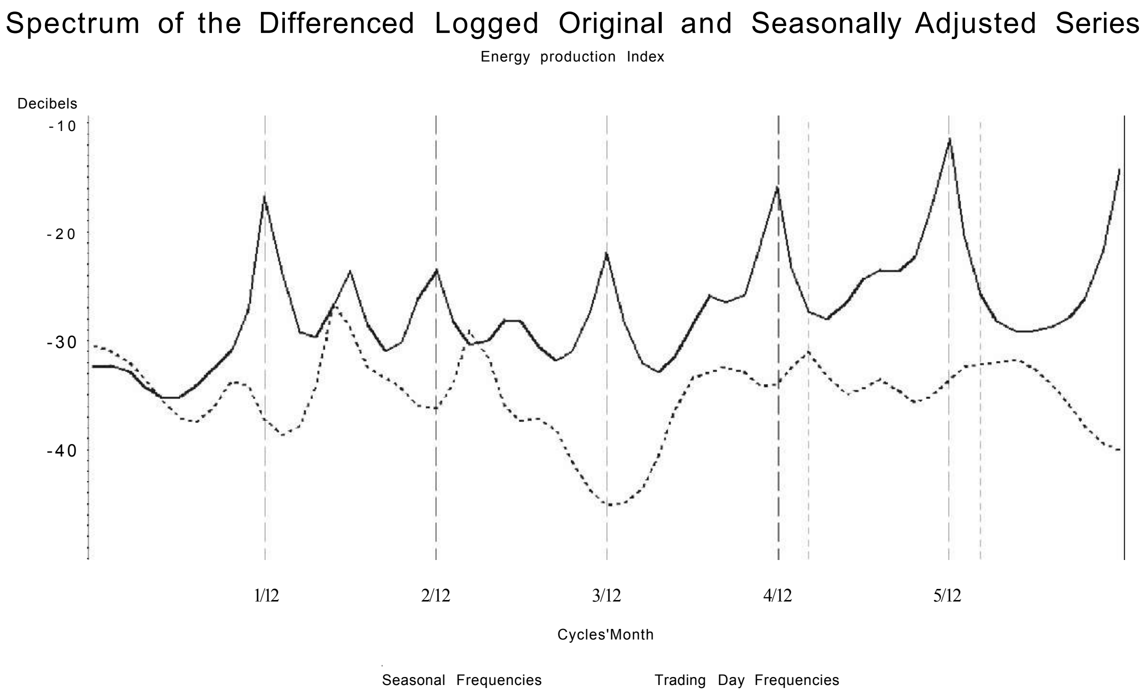

Figure 23: Spectrum for the original and adjusted series.

Table13: Summary of tests for stable and moving seasonality for each span.

	Span1	Span2	Span3	Span4
Stable seasonality	27.61	25.81	30.81	21.72
Moving seasonality	0.72	2.40	2.59	2.62
M7	0.41	0.52	0.49	0.58
Identifiable seasonality	yes	yes	yes	yes

Then, we check the stability of our seasonal adjustment by sliding spans. A and MM values above the threshold of 3% are considered stable. The results are listed in Table 14.

Table 14: Test for the seasonal adjustment stability.

Seasonal factors (A %)	Month-to-month changes in SA (MM %)
0% (0/119)	0% (0/118)

4.7 Money supply

Narrow money: Narrow money M1, is the money in public circulation comprising banknotes and coins plus business and household call deposits in the commercial banking system. Narrow money's fluctuations are more pronounced around Islamic feasts. The time series is quite stable in general, (see Figure 24).

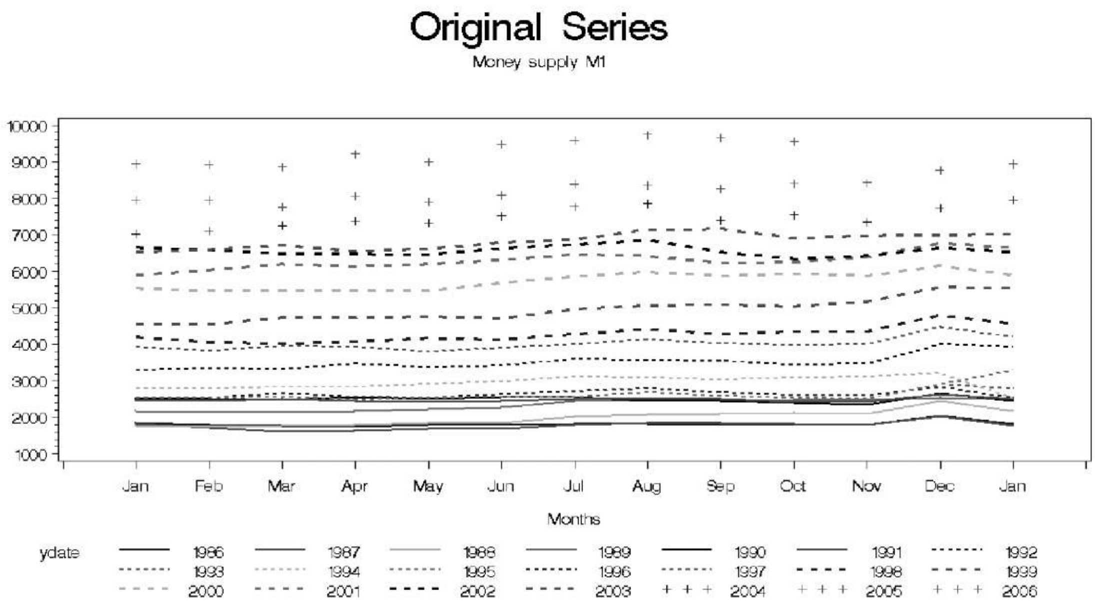

Figure 24: Original series.

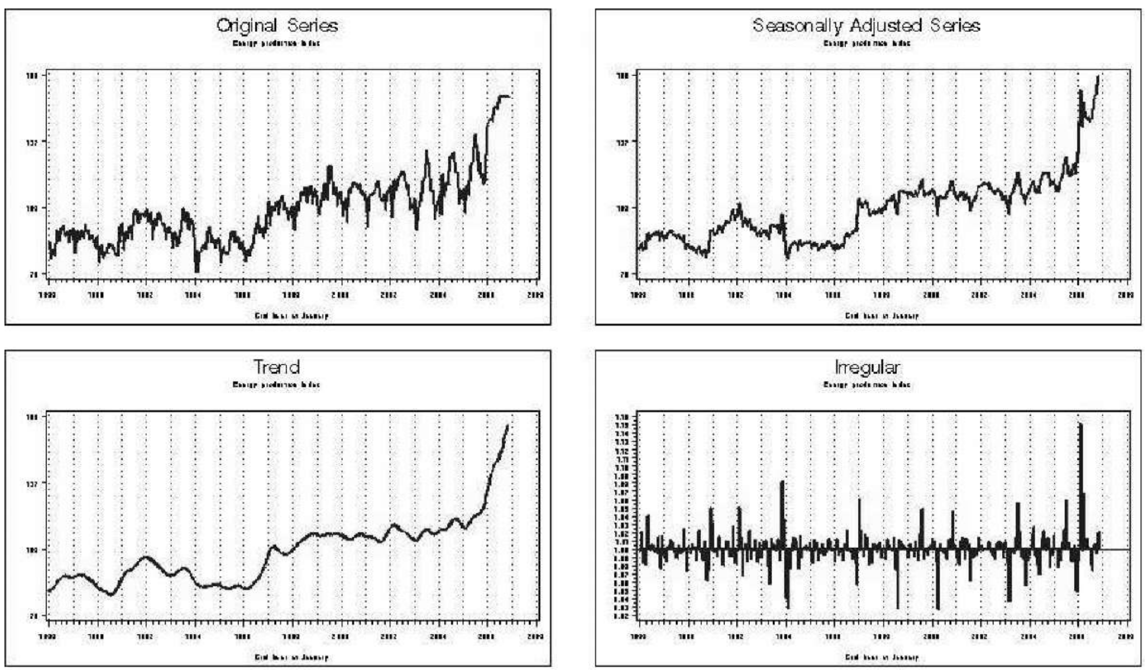

Figure 25: Original series, seasonally adjusted series, trend and irregular components.

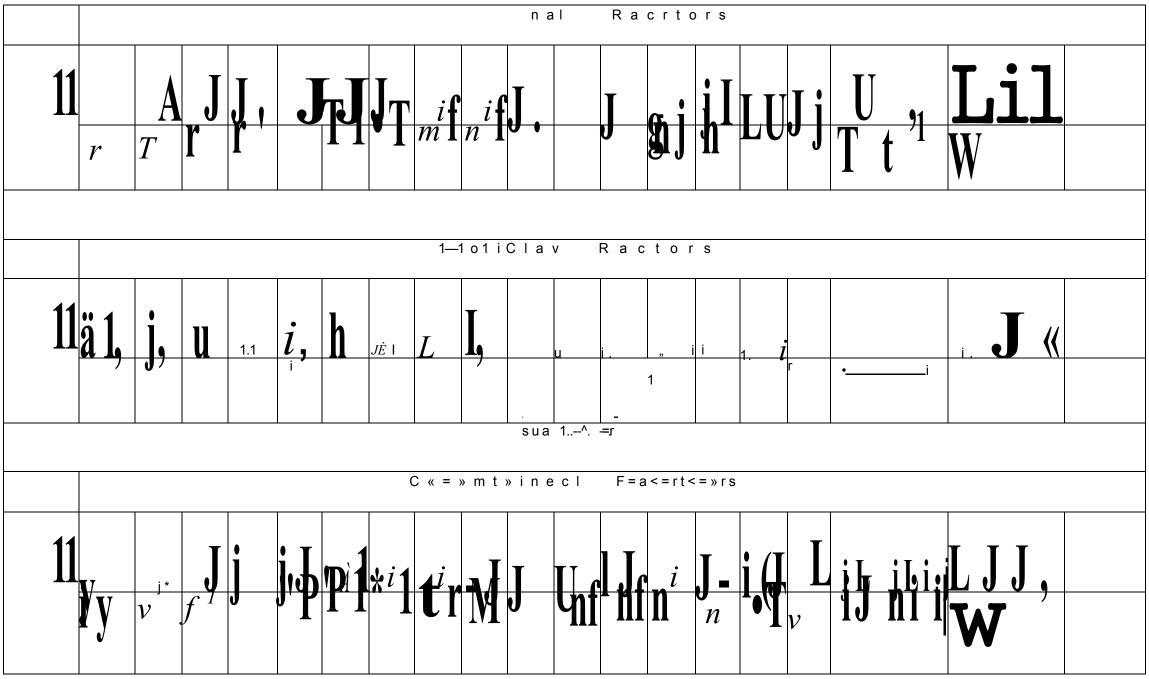

Figure 26: Seasonal factors, holiday factors and combined factors.

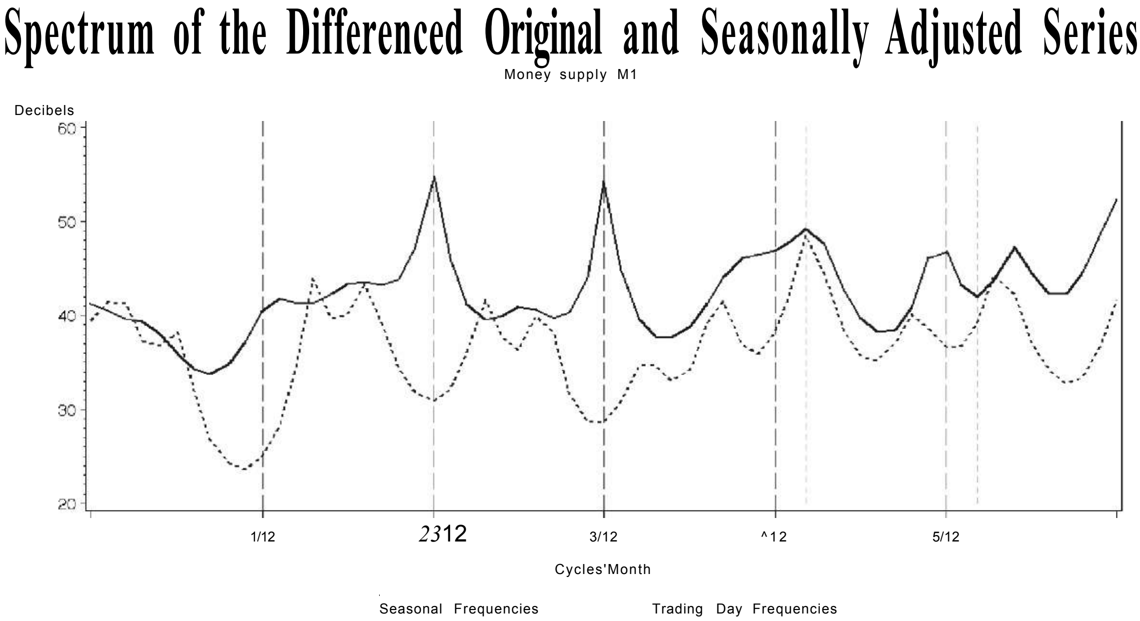

Figure 27: Spectrum of the differenced original and seasonally adjusted series.

We test several models with different intervals, and the best one is the model with 7 day-holiday-factor for Ramadan, Eid Al-Fitr and Eid Al-Idha. We have 9 regressors. The AICC criterion computed for this model is 2926 versus 2950 for the model without holidays' regressors; see Figure 25, 26 and 27 for graphics. The test for the presence of residual seasonality indicates that there is no residual seasonality in the entire series at 1% level.

Broad money: M2, M3 and M4 are not affected by the holidays' regressors and this is due to their nature. The four monetary aggregates fit into each other; Broad money includes M1, plus saving and small time deposits, overnight repos[11] at commercial banks and non-institutional money market accounts. That means that only the money in circulation is affected by religious events. In fact, the deposits and savings are not affected by such events because these funds holders, which are in general wealthy, do not use them for small expenditures.

The holidays effects become smaller and smaller while adding the other components to M1 and they are rejected in the regression procedure for M2, M3 and M4. So the seasonal adjustment is done without holiday regressors. The original and adjusted series are shown in Figure 28.

The impact of the Islamic events on narrow money is important but it is negligible on broad money.

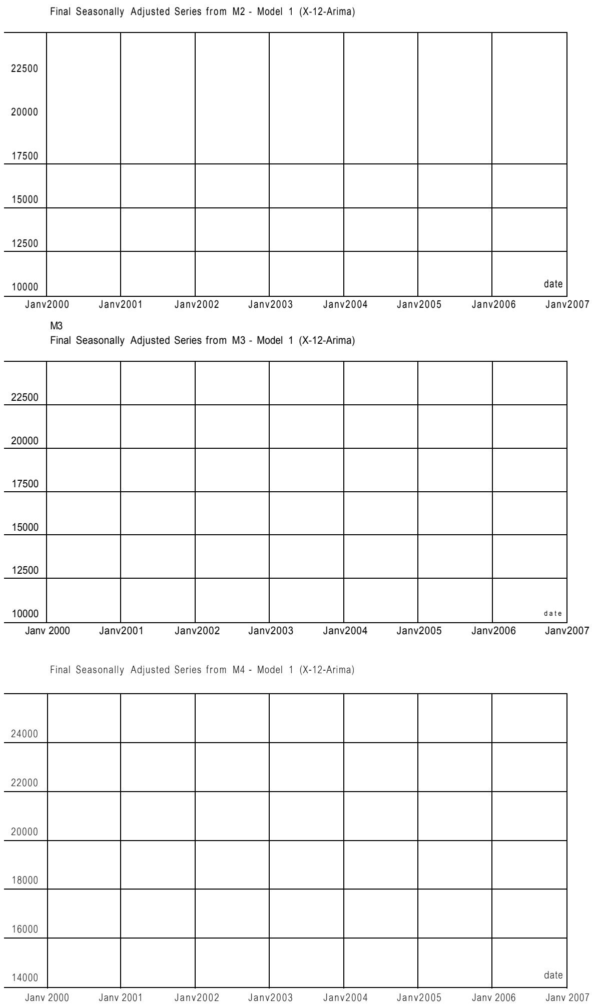

Figure 28: Original and adjusted series.

[11] Are Repurchase Agreements which are negotiated or renegotiated (rolled over) for one day periods. They are a form of borrowing/lending.

4.8 Tunis Stock Exchange: TUNINDEX and BVMT

The Tunis Stock Exchange TUNINDEX is a capitalization weighted index containing equities from the Tunis Stock Exchange. This index is open to all listed companies with minimum period of quotation of six months. The index was launched on December 31, 1997 with an initial base level of 1000.

The Tunis Stock Exchange BVMT Index is a price-weighted index containing equities from the Tunis Stock Exchange. Only stocks with a frequency of quotation of sixty percent or more are selected. The index was launched at the end of September 1990 with a base value of 100.

Holiday regressors are rejected for both financial time series. Islamic events have no influence on the Tunis Stock Exchange. We report the original and the adjusted series in Figure 29 below.

Figure 29: Original and adjusted series.

Table 15: Monitoring and Quality Assessment Statistics.

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	Q
Imports	0.88	0.49	1.11	0.11	094	0.11	0.72	1.05	0.60	0.37	1.31	0.7
Textile Imports	0.12	0.23	0.63	0.61	1.46	0.58	0.39	0.66	0.63	0.75	0.74	0.57
Textile prod. Index	0.42	0.10	0.44	0.17	0.56	0.55	0.23	0.33	0.18	0.34	0.27	0.33
Energy prod. Index	0.91	0.30	0.53	0.98	0.62	0.52	0.42	0.71	0.29	0.85	0.76	0.58
Jobs fulfilled	1.08	0.75	0.89	0.25	1.00	0.15	0.65	1.12	0.56	0.81	0.63	0.73
Money supply: M1	0.39	0.02	0.00	0.48	0.19	0.03	0.48	0.96	0.60	1.42	1.35	0.39

All the measures above are in the range of 0 to 3 with an acceptance region of 0 to 1.

M1: The relative contribution of the irregular over three-months span.

M2: The relative contribution of the irregular component to the variance of the stationary portion of the series.

M3: The amount of month-to-month change in the irregular component as compared to the amount of month-to-month change in the trend-cycle.

M4: The amount of autocorrelation in the irregular as described by the average duration of run.

M5: The number of months it takes the change in the trend-cycle to surpass the amount of change in the irregular.

M6: The amount of year to year change in the irregular as compared to the amount of year to year change in the seasonal

M7: The amount of moving seasonality present relative to the amount of stable seasonality.

M8: The size of the fluctuations in the seasonal component throughout the whole series.

M9: The average linear movement in the seasonal component throughout the whole series.

M10: Same as 8, calculated for recent years only.

M11: Same as 9, calculated for recent years only.

Q: The overall quality assessment statistic. It is a weighted average of M1-M11. Values from 1.0 to 1.2 may be accepted if other diagnostics indicate suitable adjustment quality.

Table 16: Summary of regARIMA modeling results.

Variable	Span	Model	% Forecast* Error
Imports	1986.1-2006.11	Additive, (0,1,2)(0,1,1) Outliers: AO2005.JU1 Trading day regressors : accepted Holiday regressors : accepted AICC :2726	5%
Jobs fulfilled	1986.1-2007.1	Multiplicative(0,1,2)(0,1,1) Trading day regressors : rej ected Holiday regressors : accepted AICC :3928	12.04%
Textile, clothing and leather Imports	2000.1-2006.11	Additive,(0,1,2)(0,1,1) Trading day regressors : rej ected Holiday regressors : accepted AICC :686	9.74%
Textile, clothing and leather Production Index	1997.1-2006.11	Additive,(0,1, 1)(0,1,1) Outliers: AO2004.Feb AO2005.Dec AO2006.Aug Trading day regressors: accepted Holiday regressors: accepted AICC:886	34.64%
Energy Production Index	1988.1-2006.11	Log, Multiplicative, (0,1,2)(0,1,1) Outliers: AO2006.Feb Trading day regressors: rej ected Holiday regressors: accepted AICC: 1217	8.27%
M1	1986.1-2006.11	Additive, (2,1,2)(0,1,1) Outliers: AO1995.Jan Trading day regressors : rej ected Holiday regressors : accepted AICC :2925	1.97%

* Average absolute percentage error in out-of-sample forecasts for the last three years. Percentage errors have the advantage of being scale independent, so they are frequently used to compare forecast performance between different data series. The tabulated values correspond to the chosen model; they are all smaller than the value corresponding to the model without holiday regressors.

5 Conclusion

Islamic events have an important influence on economic activity in Tunisia. Many time series can be studied to evaluate this impact. Ten macroeconomic and two financial time series were analysed in this paper.

The results confirm, as expected, that some of these time series, such as the energy production index, are affected by all the Islamic events. Others, such as M1, are influenced only by the three most important Islamic feasts: Ramadan, Eid Al-Fitr and Eid Al-Idha. There are also some series that are strongly affected by only one feast because the latter is an uncoupling factor of an increase or a decrease in the activity. Examples are textile imports and production, which are influenced by Eid Al-Fitr. Other time series may be studied to analyse the effect of Al Mawled and Eid Al-Idha. They are respectively dry-fruits' imports and sheep sales. Unemployment rates may be interesting to analyse since these fall especially before Eid Al-Fitr. This is due to the increase of small stores and stands during Ramadan and before the feast. In our work, we studied the new jobs filled time series because there is no monthly data for the unemployment rate.

Broad money, Tunindex and BVMT are not affected by moving holidays. Al Mawled and the Islamic New Year are also religious events but their effect on the economy is less important than are the other Islamic feasts. Al Mawled, the birthday of the prophet, is celebrated by preparing a special dish, served with dry fruits. The Islamic New Year is also celebrated by preparing some special food. During these holidays, people visit each other, some of them also buy presents or new clothes but it is not a general behaviour like Eid Al-Fitr. The impact of Al Mawled and the Islamic New Year on economic activity is different from the impact of Eid Al-Fitr and Eid Al-Idha. They may be considered like any other national holiday.

The regressors introduced in the model are divided into regressors before the holiday, regressors during the holiday and regressors after the holiday. For some time series, a symmetric model (with $t = t_3$) is selected. For the other time series, a model with different intervals (t_1 ^r_3)gives the best result. The effect length is not the same for all time series. This is logical because there are series which are more affected by Ramadan, for example, than the others series. The effect length of Ramadan in the imports time series is about 40 days before and 13 days after the end of the month. To meet the needs of the population, the country imports food and clothes sufficiently in advance. For the exports there is no effect.

Even if holiday effects are important; they stay smaller than seasonal effects which are increasing over time. Holiday effects have a stable magnitude; this can be explained by an unchanged behaviour of the population in the Islamic feasts over time. This proves that these events have always had the same importance.

Time series, for which the holiday regressors were accepted, show an improved seasonal decomposition. The effect of moving holidays is controlled by adding the appropriate regressors. The information about the number of regressors and the effect length may be known in advance but it is confirmed by an AICC test or with the use of the accumulated forecast error. A better seasonal adjusted data gives better forecasting results and allows comparison between Muslim and non-Muslim countries.

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