



# Single energy partial wave analyses on eta photoproduction – pseudo data

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**Abstract.** We perform partial wave analysis of the eta photoproduction data. In an iterative procedure fixed-t amplitude analysis and a conventional single energy partial wave analysis are combined in such a way that output from one analysis is used as a constraint in another. To demonstrate the modus operandi of our method it is applied on a well defined, complete set of pseudo data generated within EtaMAID15 model.

## 1 Introduction

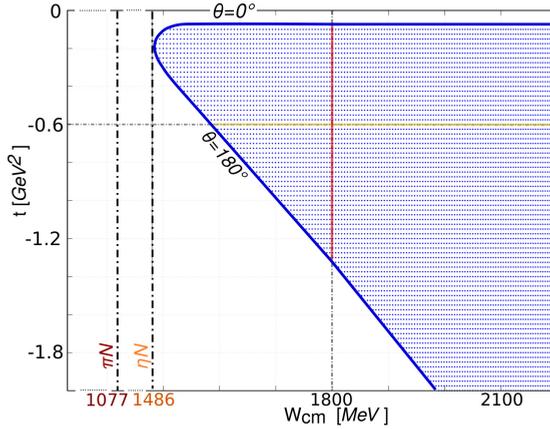
Single energy partial wave analysis (SE PWA) is a standard method used to obtain partial waves from scattering data at a given energy. Invariant amplitudes, reconstructed from partial waves by means of corresponding partial wave expansions obey a fixed-s analyticity required in Mandelstam hypothesis. It is quite general that at a given energy many different partial wave solutions equally well describe the data. The fit to the data at one energy “does not know” which solution was obtained in independent SE PWA at another, even neighboring energies. This poses a problem of finding a unique partial wave solution as a function of energy. To solve this problem and to achieve continuity of partial wave solution in energy, one has to impose some additional constraints on partial wave solutions. The aim of this paper is to demonstrate a method which imposes analyticity of invariant scattering amplitudes at fixed values of Mandelstam variable  $t$  in addition to analyticity at fixed  $s$ -value which is already achieved by partial wave expansion. In our method SE PWA and a fixed-t amplitude analysis (Ft AA) are coupled together in an iterative procedure in such a way that output from one analysis serves as a constraint in another. Detailed description of formalism and the method is given in refs. [2], [2]. Here we demonstrate how the method works. As an input we use the eta photoproduction pseudo data constructed from theoretical model EtaMAID-2015 [3]. Applying our method, we reproduced partial waves from a model which was used to generate the data fitted. This proves uniqueness of partial wave solution obtained applying our method.

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\* Talk presented by H. Osmanović

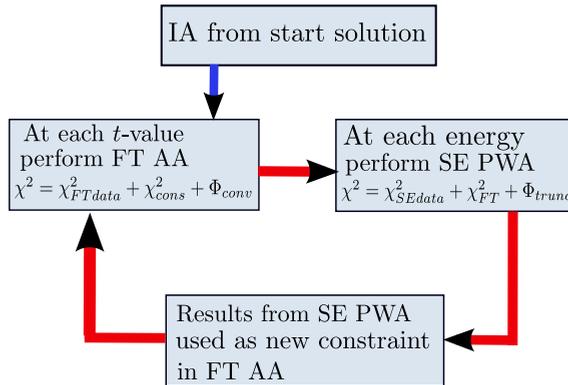
## 2 Method and results

To prove uniqueness of solution obtained by use of our method, we generated a complete set of observables in the eta photoproduction process:  $\{\sigma_0, \check{\Sigma}, \check{\Upsilon}, \check{\rho}, \check{F}, \check{G}, \check{C}_{x'}, \check{O}_{x'}\}$  [4,5]. To apply our method we need data at two different kinematical grids: energy -  $t$  ( $W, t$ ) to be used in the Ft AA, and energy - scattering angle theta grid to be used in SE PWA. Our pseudo data sets are generated at 140 energies inside the physical region, each at 50  $t$ -values with artificially small errors of 0.1%.  $W$ - $t$  kinematical grid is shown in Fig. 1. Yellow line shows the data used in the



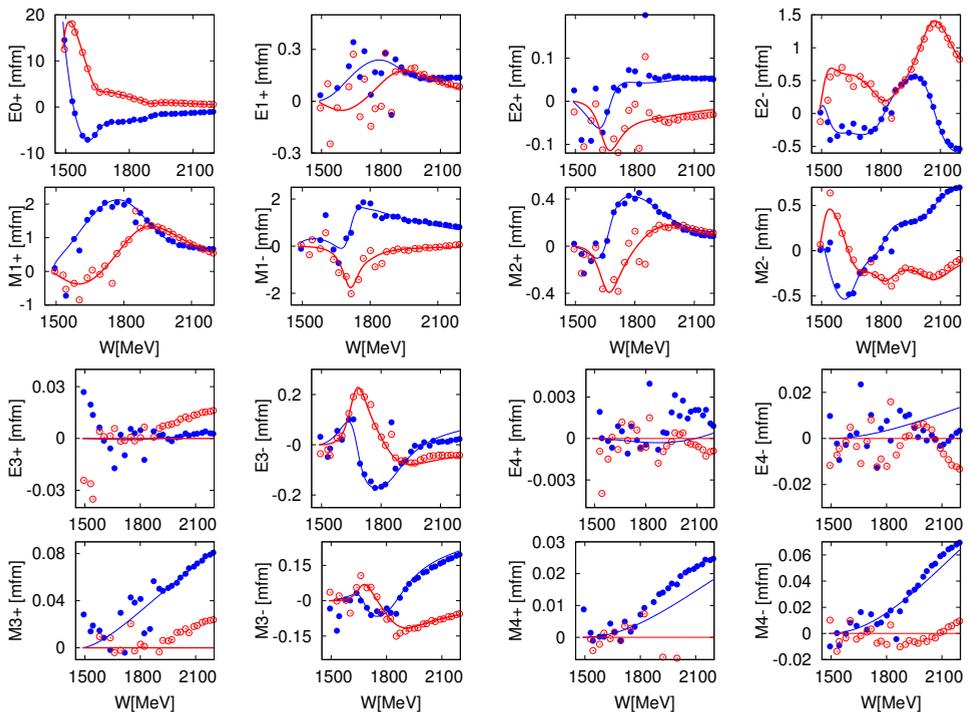
**Fig. 1.** (Color online) ( $W_{cm}, t$ ) diagram for  $\eta$  photoproduction. Points represent pseudo-data generated by EtaMAID2015a model in physical range. Yellow line symbolizes fixed- $t$  analysis, and red line symbolizes fixed- $s$  (SE) analysis.

Ft AA ( $t = -0.6\text{GeV}^2$ ), while the data along red line ( $W = 1800\text{MeV}$ ) are used in the SE PWA. Iterative procedure in our method is shown in Fig. 2.  $\chi^2_{SEdata}$  and



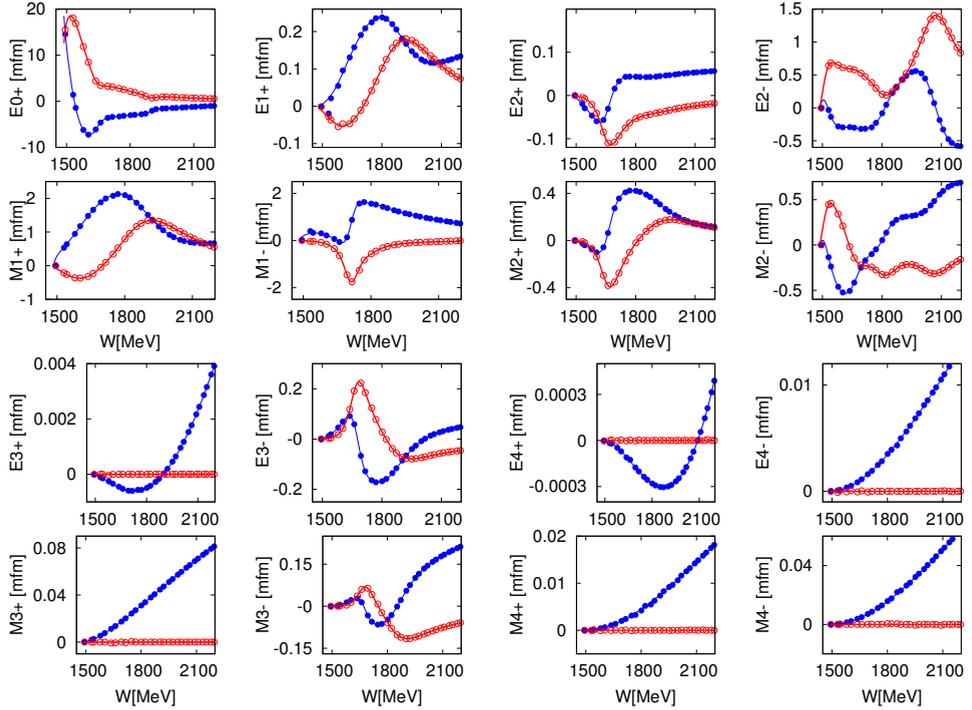
**Fig. 2.** Iterative procedure in a combined single energy partial wave analysis and fixed- $t$  amplitude analysis.

$\chi_{\text{FTdata}}^2$  are standard quadratic forms used in fitting the data,  $\Phi_{\text{conv}}$  is convergence test function which is integral part of Pietarinen expansion method used in Ft AA [6–10], while  $\Phi_{\text{trunc}}$  makes a soft cut off of higher multipoles at lower energies in SE PWA (for technical details see refs [2], [2]). The two analyses, SE PWA and Ft AA, are coupled by terms  $\chi_{\text{Ft}}^2$  and  $\chi_{\text{SE}}^2$  which measure deviations of values of invariant amplitudes obtained in SE PWA from corresponding ones obtained in Ft AA and vice versa. After several iterations, usually not more than three, results from both analyses agree reasonably well. Figure 3 and Figure 4 show importance of constraint from Ft AA in obtaining a unique partial wave solution in SE PWA. In Figure 3 are shown partial waves obtained in unconstrained SE



**Fig. 3.** (Color online) The result of on unconstrained single-energy fit described in the text. The blue and red points show the real and imaginary parts of the multipoles obtained in the fit compared to the “true” multipoles from the underlying EtaMAID-2015 model (blue and red solid lines).

PWA. Even if a complete set of data with small errors is used in analysis, unique solution is not obtained- input partial waves solution from which the data is generated is not reconstructed. Figure 4 shows results of PWA using our method with the same input data after two iterations. Starting solution is reconstructed with a high accuracy.



**Fig. 4.** (Color online) Real (blue) and imaginary (red) parts of electric and magnetic multipoles up to  $L = 4$ . The points are the result of the analytically constrained single-energy fit to the pseudo data and are compared to the multipoles of the underlying EtaMAID-2015 model, shown as solid lines.

### 3 Conclusions

In order to achieve unique and continuous solution in energy, additional constraint in an partial wave analysis is needed. It is shown that a unique solution may be obtained using only analytic properties of invariant scattering amplitudes at fixed values of Mandelstam variables  $s$  and  $t$  as constraint.

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