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A Petri net model for the integration of purchasing, production and packaging using Kanban system

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A B S T R A C T

A new generic deterministically timed Petri net (PN) model was developed for the integration of purchasing, production, and packaging using the Kanban system. Firstly, the individual building blocks of the model were developed which are then combined together to obtain the overall integrated PN model. This model allows the modeling of an integrated production system configuration for determining the optimal Work-In-Process (WIP), lead-time, station's utilization, and rate of production of the system. Each station can have multiple identical servers. The model is solved first by initial marking and then by optimal marking using LINGO software. The machining server circuit with the largest cycle time determines the bottleneck station, as the cycle time of this circuit merely represents the capacity of the corresponding station. Elementary circuits with cycle times greater than the cycle time of the machining server circuit are selected for optimization. These circuits result in constraints. The objective of optimization is to ensure the WIP minimum corresponds to the maximum throughput. The maximum throughput with the minimum WIP is formulated as a linear programming problem. The model can be used for designing, evaluating, and optimizing the layout of an integrated production system. This model could be extended using Fuzzy PN, Coloured PN, or Queuing PN.

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1. Introduction

The birth of modern manufacturing dates to the age of the industrial revolution which started in the middle of the eighteenth century in England and propagated in Europe and in North America. It was due primarily to the invention of the steam engine and the subsequent consequence of the ability to produce products for consumption by mechanization. From a social and economic perspective, it resulted in a significant improvement in wealth and in the standard of living [1]. To respond rapidly to the highly volatile market, the emerging reconfigurable manufacturing systems have brought forward two challenging issues, namely how to build a rapid formal model of an initial manufacturing configuration and how to yield the target model from the existing one along with manufacturing configuration changes [2]. Modeling and performance analysis of manufacturing systems helps decision makers at higher levels to conduct an economic feasibility analysis for diversification and/or modification of the system. Manufacturing systems design is a complex phenomenon, which is concerned with the selection from a wide variety of available system configurations and control strategy alternatives in the light of several criteria (flexibility, quality, productivity, costs etc.), many of which are difficult to quantify [3].

PNs have proved to be effective graphical, mathematical, and simulation tool for discrete event systems. From the design perspective, PNs provide many advantages in modeling, performance evaluation, and qualitative analysis of FMS. However, with the growth in the complexity of modern industrial, and communication systems, PNs were found inadequate to address the problems of uncertainty and imprecision in data. This gave rise to combination of Fuzzy logic, Object-oriented approach, and Queueing theory with PNs and new tools emerged with the names of Fuzzy PNs, Object-oriented PNs, Queueing PNs, and Colored PNs. Object-oriented PNs have been used for performance modeling of a Flexible manufacturing system (FMS) and efficient production control implementation [4]. To minimize makespan for FMSs, deadlock-free scheduling algorithm is used. In the reachability graph of the PN, scheduling is performed as a heuristic search where the search process is guided by a heuristic function [5]. Based on timed PN model of FMS with the goal of minimizing makespan, a hybrid heuristic search approach is presented which combines dynamic search window with best-first algorithm and backtracking search [6]. Generic hybrid PN model combined with the lowest-makespan-cut is used for job shop scheduling problems in mold manufacturing in order to minimize the makespan of the mold part manufacture schedule. The integration of the PN model and the lowest-makespan-cut algorithm can help to improve the production efficiency [7]. A method of modeling parallel processing flows, sharing limited number of resources in FMSs is presented. A new class of PN called parallel process net with resources is introduced for modeling such FMSs. Parallel process net with resources has the capacity to model the more complex resource-sharing among parallel manufacturing processes [8]. Stochastic PNs, together with fuzzy set theory, are used for modeling FMS to represent both stochastic variability and imprecision [9].

1.1 Motivation

The design and operation of modern industrial systems require modeling and analysis in order to select optimal design alternative and operational policy. PNs are graphical and mathematical modeling techniques developed as effective modeling tools for concurrent system operations [10]. Kanban cards pass through a series of events that can easily be included in the PN model. The kanban card in Just-In-Time (JIT) manufacturing has a lot of similarity with a token in a place, so it looks very attractive to model it as such in a PN. Another attractive aspect of PNs is that they can be used both as a simulation model and as an analytic mathematical model. In production system, the WIP is controlled by the number of pallets in the system. In JIT, the number of kanban cards controls the WIP. This is a strong motivation to model and operate production systems with JIT [11]. Furthermore, a PN can model deterministic or stochastic processing times and also presents the assembly sequencing of parts in a clear fashion.

1.2 Flexible manufacturing system

Manufacturing is the economic term for making goods and services available to satisfy human wants. The manufacturing system is an arrangement of physical elements characterized by measurable parameters. A flexible manufacturing concept has emerged due to the progress in manufacturing technology [12]. The entire company is often referred to as the enterprise or the production system [13]. FMS is defined as automated manufacturing system consisting of multi-functional machines interconnected by a material handling system [14]. It is an important tool for increasing the production efficiency and reducing the total production time [15]. Due to constant fluctuations in market demands, FMS is given great importance to improve competitiveness [16].

1.3 Inspection

In order to produce components that meet the design criteria, manufacturing companies have to ensure the components they produce meet the required dimensional and accuracy standards. Inspection is an important aspect of quality control. It ensures what is being manufactured will meet design specifications [13]. It helps to control the quality of products by fixing the sources of defects immediately after they are detected. It is useful for any industry that wants to improve productivity, reduce end-line defects (that impair performance), and save time and efforts of final inspection. In fact, any quality control system is based on measurements performed on pre-selected key quality characteristics. Perhaps the most apparent aspect of this link is conformance testing, which should guarantee products functionality. However, any measurement comes with a cost [17].

1.4 Flexible assembly system

Flexible assembly systems (FASs) are "assembly FMSs" [18]. An FAS often consists of a number of robots, material-handling devices, part feeders, computers, storage units, and communication networks etc. Key indicators of flexibility in a FAS are its capability of handling jobs of varying batch volume, varying assembly plans, varying products, and control adaptability etc. FAS has the capability of producing small and medium-sized products and avoids many disadvantages encountered with fixed assembly systems [19]. An FAS maximizes production rate and increases resource utilization by avoiding unnecessary job transfers within the factory. However, to realize the full benefits of FASs, one has to consider their modeling and simulation to investigate problems relating to design, performance optimization, and line control.

1.5 Packaging

In today's stiff competition in every product along with increasing consumer demand, it becomes imperative for companies to explore ways to improve their productivity in terms of maintaining safety, using sustainable packaging materials, implementing flexible and standardized technology, and adopting proven management principles [20]. Packaging can be described as a coordinated system of preparing goods for transport, warehousing, logistics, sale, and end use. Packaging protects, preserves, transports, informs, and sells. In many countries, it is fully integrated into government, business, institutional, industrial, and personal use [21]. Product packaging is as important as one of the market principle including prices, products, places, and promotions. Product packaging adds to the reputation of the manufacturer and also helps to create the brand’s image [22]. Packaging attributes are considered to have an influence on consumer purchasing decisions [23]. Packaging is an important function, enclosing materials and products for distribution and movement [24]. Some of the recent innovations in the field of packaging include development of passive and active packaging, intelligent packaging, and interactive packaging [25].

1.6 Petri nets

A PN is a bipartite directed graph consisting of four primitive elements (i.e., tokens, places, transitions, and arcs) along with rules that govern their operation. Tokens (represented by dots) are conceptual entities used to represent objects moving in an abstract network. Places (represented by circles) show the states of the objects. Places may represent resources such as machines, AGVs, computer code, or parts in a buffer. The existence of a token in a place represents the availability of a resource. Transitions (denoted by bars) represent activities. Places and Transitions together represent conditions and precedence relations in the system’s operation. A transition fires provided there is at least one token in each of the input places of the transition. The places and transitions are connected by directed arcs that represent the sequence of operations [11]. When the specified time is deterministic, the PN is called deterministic timed PN. When a probability distribution to the firing time is assigned, the PN is called a stochastic timed PN. Various colors can be used to distinguish part-types. However, the size of the colored PN model is smaller compared to the PN model due to grouping the places and transitions of PN model into the colored PN model. The use of kanban in PN controls the rate of production according to the demand of a market. Another advantage of kanban in PN is that it provides a better coordination between the upstream and the downstream activities in a manufacturing organization.

This paper presents a new generic deterministic timed PN model for the integration of purchasing, production, and packaging using Kanban system. A production system refers to the total company and includes within it the manufacturing system. The production system includes the manufacturing system, transportation system, inspection system, and assembly system. The subsystems are integrated together to get an overall integrated PN model. The model can be used for the design and performance evaluation of the system using kanban. The model can be developed using colored PNs. The model can be extended for rejection of parts (if any) during inspection.

The remainder of the paper is structured as follows: section 2 describes Petri net modeling of building blocks. It is followed by development of the proposed Petri net model for integration of purchasing, production, and packaging in section 3. An example/case study is presented in section 4, where the proposed PN model is applied to a ball bearing. Section 5 gives results, discussion, and managerial implications. Conclusion remarks are given in section 6.

2. Petri net modeling of building blocks

As mentioned earlier, first the building blocks of the overall PN model are developed. These building blocks are then combined together to get the PN model for integration of purchasing, production, and packaging. The physical meanings of the symbols used in the PN model are given in Table 1.

Table 1 Symbols and their meanings in the PN model

Symbols	Meaning in the PN model
K_B	Kanban transition for buying/purchasing
K_M	Kanban transition for movement/transportation
K_P	Kanban transition for production/machining
K_I	Kanban transition for inspection
K_A	Kanban transition for assembly
K_{Pa}	Kanban transition for packaging
K_R	Kanban transition for feedback for replenishment
T_B	Processing transition for buying/purchasing
T_T	Processing transition for transportation
T_P	Processing transition for production/machining
T_I	Processing transition for inspection
T_A	Processing transition for assembly
T_{Pa}	Processing transition for packaging
MC	Place for move card
BC	Place for buying/purchasing card
PC	Place for production/machining card
IC	Place for inspection card
AC	Place for assembly card
PaC	Place for packaging card
P_{MA}	Place for material availability
P_B	Place for buying/purchasing associated with buying transition
P_M	Place for movement/transportation associated with move kanban transition
P_p	Place for production associated with production/machining transition
P_T	Place for transportation associated with transportation transition
P_I	Place for inspection associated with inspection transition
P_A	Place for assembly associated with assembly transition
P_{Pa}	Place for packaging associated with packaging transition
P_R	Place for replenishment associated with kanban transition for replenishment
S_B	Place for buying/purchasing server
S_T	Place for movement/transportation server
S_P	Place for production/machining server
S_I	Place for inspection server
S_A	Place for assembly server
S_{Pa}	Place for packaging server
●	Token showing availability of material, part, subassembly, or final assembly

2.1 Petri net model of flexible manufacturing system

Production sequence for a typical part with a feedback for replenishment is shown in Fig. 1. Fig. 2 shows the closed PN model for this production sequence, using kanban.

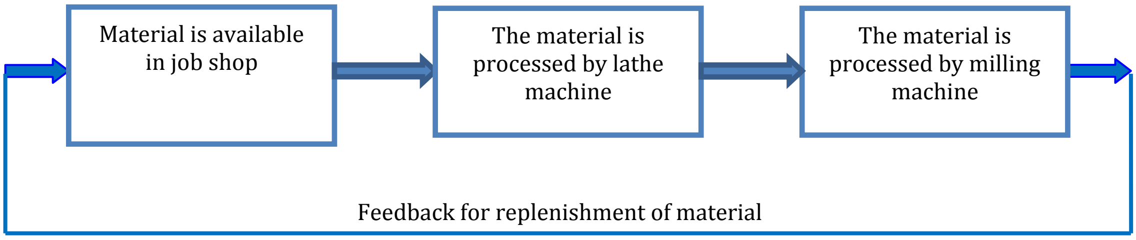

Fig. 1 Production sequence for a typical part

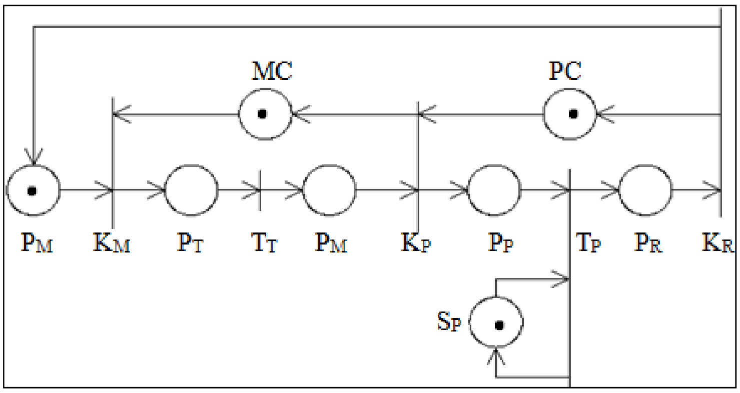

Fig. 2 PN model for Flexible Manufacturing System (FMS)

The availability of a token each in place P_M and place MC enables the kanban transition K_M to fire. After firing of K_M, tokens from places P_M and MC are taken and a token is added to place P_T which an output place for transition K_M. Now, the transition T_T gets enabled and therefore fires. After firing, the token from the input place of transition T_T is withdrawn and a token is added to its output place P_M. Transition K_P gets enabled and fires. Because each of its input places P_M and PC has got a token. After firing, the tokens from places P_M and PC are removed and token is added to each of its output places P_P and MC. The production transition T_P gets enabled because of availability of a token in each of its input places P_P and S_P. After firing of transition T_P, tokens are taken from its input places P_P and S_P and a token is added to each of its output places P_R and S_P. This makes the transition K_R enabled. Firing of K_R removes token from its input places P_R and adds a token to its output place P_M for repetitive manufacturing.

2.2 Petri net model of flexible assembly system

After manufacturing of individual parts, the next stage is their assembly. Fig. 3 shows the PN model for FAS of two parts. The presence of a token each in place P_1 and place P_2 shows the availability of part 1 and part 2 for assembly. The kanban transitions K_{M1} and K_{M2} attach move cards MC_1 and MC_2 to parts P_1 and P_2. After getting authorization for movement, the parts are moved after firing of transportation transitions T_{T1} and T_{T2}. Kanban transitions K_{A1} and K_{A2} detach move cards MC_1 and MC_2 from the two parts and attach assembly cards AC_1 and AC_2 to these parts. Firing of assembly transition T_A assembles the two parts together. The final assembly is shown by place P_R. Firing of transition K_R signals authorization for replenishment of individual parts, in places P_1 and P_2, for cyclic assembly of parts.

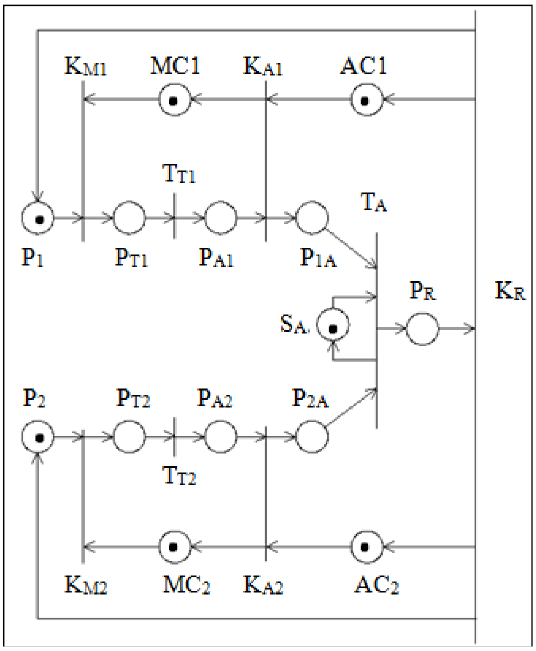

Fig. 3 PN model for Flexible Assembly System (FAS)

2.3 Petri net model of flexible purchasing system

Purchasing is the management of acquisition process. It consists of deciding which suppliers to use, whether to buy locally or centrally, and negotiating contracts. Storage, conversion, and distribution are of strategic importance at the start of the purchasing. It must satisfy the firm's long-term supply needs and support production capabilities of the firm. This task is crucial for every organization, whether it is retailer, service provider, or manufacturer.

Flexible purchasing system (FPS) is defined as a system in which raw material will be purchased by the manufacturing organization and will be supplied by the supplier only when it is required in order to avoid the unnecessary storage of the material in the organization. Purchasing is made flexible by the use of kanban. It will allow supplier to supply material to the manufacturing organization as and when required by the organization. Similarly, the system will allow the organization to demand for the purchase of raw material only when it is required. Thus, the system establishes a good coordination between the supplier and the manufacturing organization. This coordination is established with the help of kanban card called buying card (BC) in our case. Thus, BC will show authorization for buying (purchasing) of raw material.

Fig. 4 shows a closed PN model for FPS of raw material. The token in place P_{MA} shows the availability of the raw material in the market. The availability of the tokens in places P_{MA} and BC enables the kanban transition K_B to fire. Firing of the kanban transition K_B authorizes buying the material from the supplier. Firing of transition K_B takes tokens from its input places P_{MA} and BC and adds a token to its output place P_B. The buying transition T_B gets enabled because a token is available at each of its input places P_B and S_B. Firing of transition T_B withdraws tokens from its input palces and adds a token to its output place P_M. The kanban transition K_M now gets enabled in the presence of a token in each of its input places P_M and MC. Transition K_M shows authorization to detach buying card BC and attach move card MC. After firing of kanban transition K_M, tokens from places P_M and MC are removed and a token is added to each of the places P_T and BC. The presence of a token in place P_T shows that the material is ready to be moved from the supplier to the warehouse. The transportation transition T_T gets enabled and therefore fires. Transition T_T shows the actual transportation time of the material from the supplier to the warehouse. After firing of transition T_T, kanban transition K_R gets enabled and fires to signal authorization for buying material.

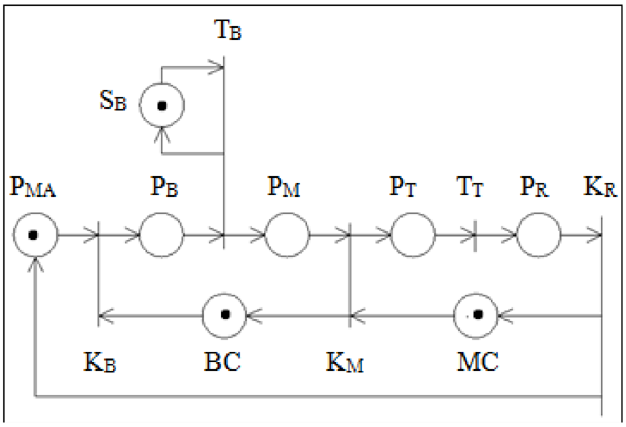

Fig. 4 PN model of Flexible Purchasing System (FPS)

Similarly, building blocks can also be produced, for other functional areas as well, that will be seen in the overall PN model.

3. Development of the proposed Petri net model for integration of purchasing, production, and packaging

Fig. 5 shows a schematic of integration of purchasing, production, and packaging. Initially, raw material is purchased. It is then transported for manufacturing of individual parts. When the parts are manufactured, they are transported for inspection. After meeting the required specifications during inspection, the parts are transported for assembly. Once the final assembly is produced, the product is transported for packaging. After packaging is done, a feedback is given for purchase of raw material for repetitive production. Thus, the key functional areas are working in coordination with each other in the system.

Fig. 6 shows the proposed PN model for the integration of purchasing, production, and packaging. Fig. 7 shows the model with initial marking. Presence of a token in place P_{MA} shows availability of raw material for purchasing. Presence of a token in place BC shows authorization for buying raw material. Kanban transition K_B gets enabled and therefore fires due to availability of a token in each of its input places PM_A and BC. With firing of buying transition T_B raw material is purchased. It is transported to the shop floor, for manufacturing of parts, after firing of transportation transition T_{T1}. Firing of production transitions T_{P1} and T_{P2} manufacture the two parts, which are transported for inspection after firing of transportation transitions T_{T2} and T_{T3}. When inspection transitions T_{I1} and T_{I2} enable and fire, the two parts are inspected and become ready for transportation for assembly. Assembly of the two individual parts, represented by place P_{M6}, takes place when assembly transition T_{A1} gets enabled and fires. Availability of a token in place P_{3T7} shows a third part available for assembly with the subassembly to give a higher subassembly. It is transported for assembly when transportation transition T_{T7} gets fired. When assembly transition T_{A2} gets enabled and fires, this third part is assembled with the subassembly and a higher subassembly represented by place P_{M8} is obtained. A fourth part, represented by a token in place P_{4T9}, is transported for assembly with the higher subassembly after firing of transportation transition T_{T9}. When assembly transition T_{A3} enables and fires, this part is assembled with the higher subassembly to give a final assembly. This final assembly is represented by place P_{M10}. After firing of transportation transition T_{T10}, this final assembly is transported for packaging. When the packaging transition T_{Pa} enables and fires, the packaging operation takes place. With this, the cycle completes, the replenishment transition K_R enables and therefore gets fired. Firing of K_R gives a feedback to the system for buying raw material and replenishment of new individual parts in the input places P_{MA}, P_{3T7} and P_{4T9}. It gives repetitive production.

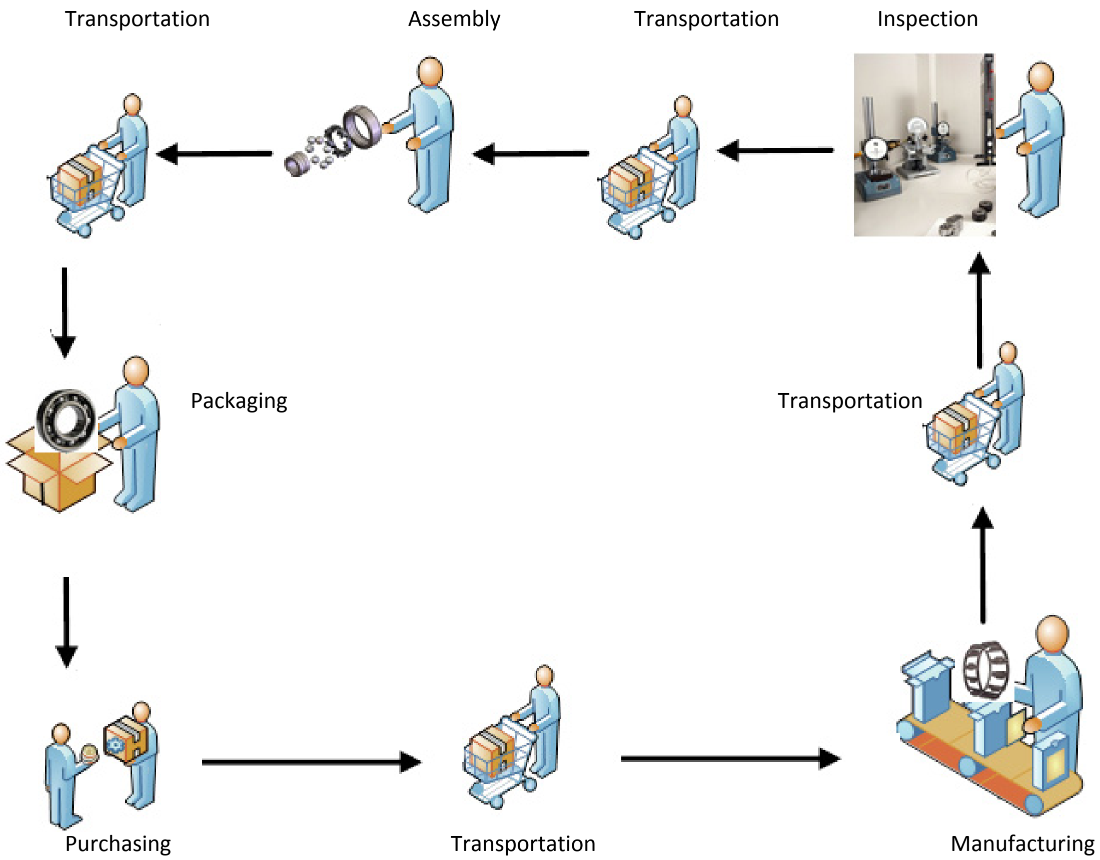

Fig. 5 Schematic of integration of purchasing, production, and packaging

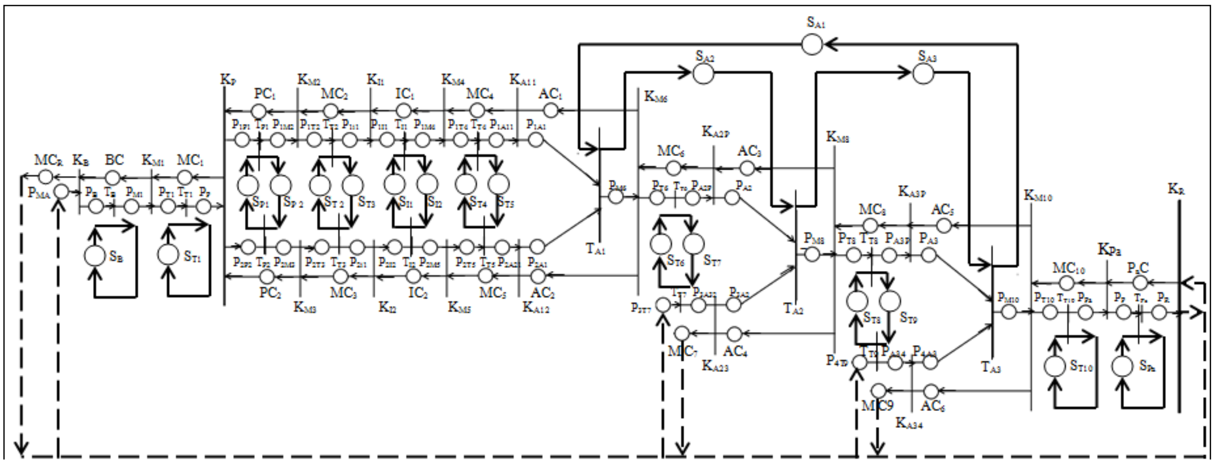

Fig. 6 Petri net model for integration of purchasing, production, and packaging

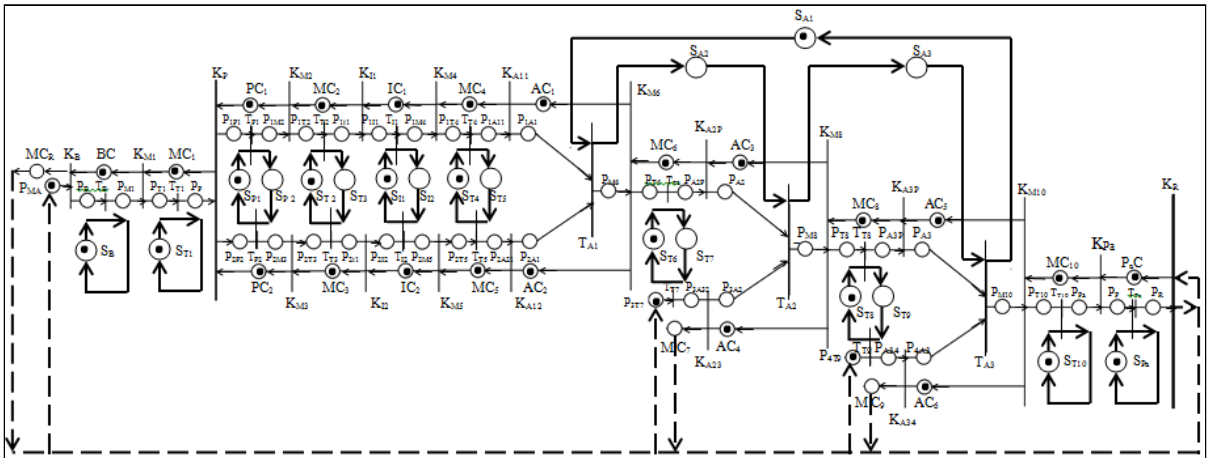

Fig. 7 Petri net model for integration of purchasing, production, and packaging, after initial marking

3.1 Initial marking of the Petri net model

The PN model shown in Fig. 6 and 7 is strongly connected, as there exists a directed path from any node to any other node. The PN is an event graph or decision-free PN because each place has exactly one input transition and one output transition. The initial marking is determined in such a way that the number of tokens in each elementary circuit is at least equal to the maximum arc weight of that elementary circuit. This condition guarantees the liveness of the PN. Physically, this corresponds to a deadlock free FMS. The objective function can be formulated as linear programming problem as follows:

$$\text{MIN} \sum_{i,j} \left(P_i M_j + P_i P_j + P_i T_j + P_i I_j + P_i A_j + P_i Pa_j \right) \tag{1}$$

subject to

$$M(\gamma) \geq w, \qquad \forall \gamma, \tag{2}$$

$$\forall \; P_i P_j, P_i M_j, P_i I_j, P_i A_j, P_i Pa_j \in IN_0 \tag{3}$$

$$\forall \; BC_i, MC_i, PC_i, IC_i, P_a C_i, S_{Bi}, S_{Pi}, S_{Ti}, S_{Ii}, S_{Ai}, S_{Pai} \in IN_0 \tag{4}$$

where $M(\gamma)$ is the sum of tokens in the places of circuit γ. Place names represent the number of tokens in the places which belong to that circuit. Some simple rule to do this can be given: all the parts/kanban circuits should contain one token. The rule of thumb is to put a token in a kanban place instead of a part place because it does not increase WIP.

The total transition time $\tau(\gamma)$ in each elementary circuit γ is determined as the sum of the transition firing times in that elementary circuit:

$$\tau(\gamma) = \sum_{i=1}^{m} \tau(t_i) \tag{5}$$

The total number of tokens $M(\gamma)$ in each elementary circuit γ is obtained as the sum of the number of tokens in that circuit:

$$M(\gamma) = \sum_{j=1}^{n} \left(\frac{M_0(p_j)}{w} \right) \tag{6}$$

where M_0 stands for initial marking. The cycle time $C(\gamma)$ of each elementary circuit is the ratio between the total transition time of the circuit $\tau(\gamma)$ and the total number of tokens $M(\gamma)$ in that circuit:

$$C(\gamma) = \frac{\tau(\gamma)}{M(\gamma)} \tag{7}$$

3.2 Optimal marking of the Petri net model

Let $C(\gamma_c)$ be the largest cycle time of an elementary circuit. This elementary circuit will be called a critical circuit. The cycle time in steady-state is given by the maximum cycle time taken over all elementary circuits. Increasing the number of tokens in each elementary circuit reduces the cycle time of the elementary circuits. The machining server circuit (also called machining sequencing circuit) with the largest cycle time will limit the maximum throughput. In other words, this station will be the bottleneck station. It is possible to increase the number of tokens in non-server circuits in such a way that the machining server circuit becomes the critical circuit. Hence, the objective is keeping WIP minimum corresponding to the maximum throughput. The maximum throughput with minimum WIP is formulated as a linear programming problem:

$$\mathrm{MIN} \sum_{i,j} \left(P_i M_j + \; P_i P_j + P_i T_j + P_i I_j + P_i A_j + P_i Pa_j \right) \tag{8}$$

where i = 1, 2, 3,… shows the number of part in the model, whereas j = 1, 2, 3,… shows the number of the activity/operation (manufacturing, movement, and assembly etc.). The place names stand for the number of tokens in that place subject to:

$$C(\gamma) \leq C(\gamma_c) \tag{9}$$

where

$$\gamma \in \{\gamma : \; C(\gamma) \geq C(\gamma_c)\} \tag{10}$$

Elementary circuits with cycle times greater than the cycle time of the machining server circuit are selected for optimization, since the cycle time of the machining server circuit merely represents the capacity of the corresponding station.

3.3 Calculation of station utilization and lead time for the Petri net model

The utilization U_j of each station j can be calculated as the ratio of the cycle time of the server circuit j and the cycle time of the critical circuit.

$$U_j = \frac{C(\gamma_{sj})}{C(\gamma_c)} \tag{11}$$

where, γ_{sj} represents the server circuit for station j. The lead time LT can be determined by using Little's law**.** The WIP and critical cycle time are known so the lead time can be calculated:

$$LT_i = C(\gamma_c) \sum_{i,j} \left(P_i M_j + P_i P_j + P_i T_j + P_i I_j + P_i A_j + P_i Pa_j \right) \tag{12}$$

The number of kanban cards in kanban places is determined by the following formula:

$$N_{kc} = \sum (P_i + KC_i) \tag{13}$$

where N_{kc} shows the number of kanban cards. The part places P_i stand for the number of kanban cards attached with parts, subassemblies, or final assembly. The kanban places KC_i show the number of kanban cards in the kanban places.

4. Case study

The proposed PN model is applied to a ball bearing. The designation of the ball bearing is SKF TAM 6208. The bearing consists of four parts: outer race, inner race, balls, and cage. The two races are almost made in the same way, i.e. by CNC lath machines. First, the inner race is inserted inside the outer race, with some eccentricity. Then balls are inserted between the two races. At the final stage of the assembly, the cages are riveted on both sides to equally distribute the balls around the inner race, and lock the balls between the races. The assembly sequence of the ball bearing is shown in Fig. 8.

The INA (Integrated network analyzer) software by Starke is a tool package supporting the analysis of Place/Transition Nets (PNs) and Colored PNs (*http://www2.informatik.hu-berlin.de*) and is used to determine the elementary circuits in the PN model. The elementary circuits given by INA, their cycle times, and initial marking are shown in Table 2.

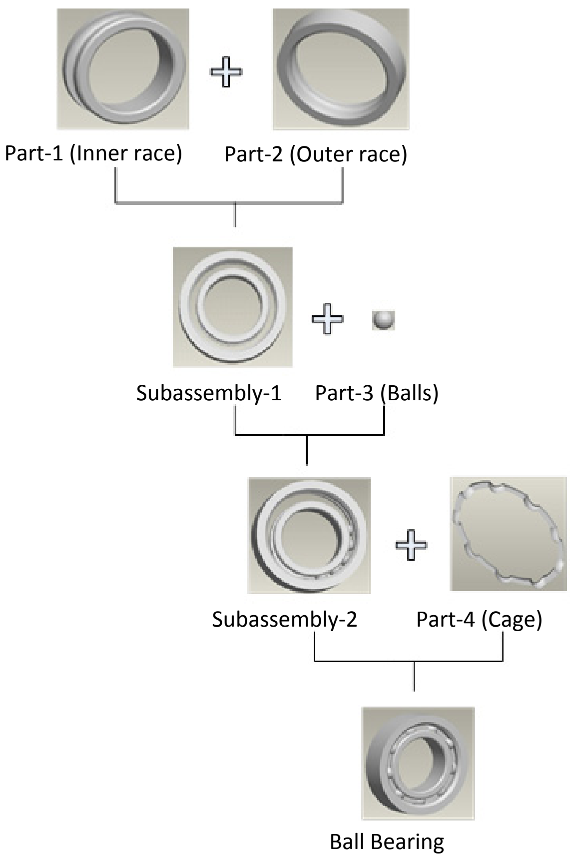

Fig. 8 Assembly sequence of a ball bearing

Table 2 Elementary circuits and their corresponding cycle times for PN model shown in Fig. 7 (a split window is shown)

TM	0	0	0	2	0	0	1	0	0	20	0	0	20			
TRAN	KR	KB	KB	TB	KM1	KM1	TT1	KP	KP	TP1	KM2	KP	TP2			
TOK	0	1	1	0	0	1	0	0	1	0	0	1	0			
PLACE	MCR	PMA	BC	PB	PM1	MC1	PT1	PP	PC1	P1P1	P1M2	PC2	P2P2			
No.	1	2	3	4	5	6	7	8	9	10	11	12	13	M(g)	t(g)	C(g)
1	0	1	0	1	1	0	1	1	0	1	0	0	0	1	61	61
2	0	1	0	1	1	0	1	1	0	1	0	0	0	1	60	60
3	0	1	0	1	1	0	1	1	0	1	0	0	0	1	60	60
4	0	1	0	1	1	0	1	1	0	1	0	0	0	1	59	59
5	0	1	0	1	1	0	1	1	0	1	0	0	0	1	59	59
6	0	1	0	1	1	0	1	1	0	1	0	0	0	1	59	59
7	0	1	0	1	1	0	1	1	0	1	0	0	0	1	58	58
8	0	1	0	1	1	0	1	1	0	1	0	0	0	1	58	58
9	0	1	0	1	1	0	1	1	0	1	0	0	0	1	57	57
10	0	1	0	1	1	0	1	1	0	1	0	0	0	1	55	55
11	0	1	0	1	1	0	1	1	0	1	0	0	0	1	54	54
12	0	1	0	1	1	0	1	1	0	1	0	0	0	1	53	53
13	0	1	0	1	1	0	1	1	0	1	0	0	0	1	52	52
14	0	1	0	1	1	0	1	1	0	1	1	0	0	1	43	43
15	0	1	0	1	1	0	1	1	0	1	1	0	0	1	42	42
16	0	1	0	1	1	0	1	1	0	1	1	0	0	1	42	42
17	0	1	0	1	1	0	1	1	0	1	1	0	0	1	42	42

After initial marking, the next step is to optimize the PN for keeping WIP minimum corresponding to the maximum throughput. For this purpose, the machining server circuit is made the bottleneck station because the cycle time of this circuit merely represents the capacity of the corresponding station. To do this, all those part circuits are considered of which cycle time is greater than the cycle time of the machining server circuit, after the initial marking stage. These part circuits result in constraints. Total twenty seven (27) elementary circuits appear as constraints. The cycle time of these circuits is made equal to or lower than the cycle time of the machining server circuit by putting more tokens in these circuits. LINGO is used for optimization of the PN model. The objective function for the system optimization based on equation (8) is as follows.

MIN: PMA + PB + PM1 + PT1 + PP + P1P1 + P1M2 + P2P2 + P2M3 + P1T2 + P1i1 + P2T3 + P2i1 + P1I1 + P1M4 + P2I2 + P2M5 + P1T4 + P1A11 + P2T5 + P2A21 + P1A1 + P2A1 + PM6 + PT6 + PA2P + P3T7 + P3A32 + PA2 + P3A2 + PM8 + PT8 + PA3P + P4T9 + PA34 + PA3 + P4A3 + PM10 + PT10 + PPa + PP + PR

Each place name represents the number of tokens in that place.

5. Results, discussion, and managerial implications

Total twenty seven (27) elementary circuits appear as constraints. The tokens to be added to the part circuits can be determined by dividing the cycle time of the part circuit with the cycle time of the critical circuit. The number of tokens to be added to the part circuits should be greater than or equal to 1.525, 1.5, 1.475, 1.45, 1.425, 1.375, 1.35, 1.325, 1.3, 1.075, 1.05, and 1.025. After optimization, the performance measures are calculated for the system as shown in Tables 3 and 4. Table 3 shows the total WIP in the system of four (4), the cycle time and lead time of 40 time unit, the throughput (or rate of production) of 0.025 product per unit time, and the total cycle time of the system of 65 time unit. Table 4 shows the optimal values of machine utilization for each station, calculated by dividing the cycle time of a corresponding station by the critical cycle time of the system. This table shows 100 % utilization for the machining station while that of all other stations is lower than 100 %, the minimum being for purchasing station. It is because the machining server circuit with the largest cycle time determines the bottleneck station. The production is bounded by the utilization of this bottleneck station. Table 5 shows the number of kanban cards in the system as follow: buying card is 1, productions cards are 2, move cards are

13, inspection cards are 2, assembly cards are 6, and packaging card is 1. Total number of kanban cards is twenty five (25).

Using the model, there can be a better coordination among all the functional areas involved in the system. The model can also provide managers a better coordination both with the suppliers and the end users. It can help them in coordination and cooperation of the enterprise's overall operation. This coordination will lead to JIT activities in the system. It will result in minimum WIP, less lead time, more throughput, and better product quality. Managers can choose among desired performance measures in order to achieve production management and control. The determination of the total WIP, total number of stations in the system, and the number of servers at each station will help in factory floor management. It will result in greater production efficiency along with ease of supervision.

Table 3 Performance measures for the system

WIP	Cycle time (time unit)	Lead time (time unit)	Throughput (product/time unit)	Total cycle time of system (time unit)
4	40	40	0.025	65

Table 4 Stations utilization in the system

Station's name	Station's utilization
Purchasing station	(2/40) × 100 = 5 %
Machining station	(40/40) × 100 = 100 %
Inspection station	(4/40) × 100 = 10 %
Assembly station	(6/40) × 100 = 15 %
Packaging station	(3/40) × 100 = 7.5 %

Table 5 Number of kanban cards in the system given by Lingo

Purchasing cards	Production cards	Move cards	Inspection cards	Assembly cards	Packaging cards
1	2	13	2	6	1
Total kanban cards					**25**

6. Conclusion

A generic deterministic PN model for the integration of purchasing, production, and packaging is developed in a pull environment using kanban. The performance evaluation of the PN model is based on solution of a linear programming problem. The optimization of the PN model is influenced by the utilization of the bottleneck station. The PN model gives minimum WIP for maximum production rate. Minimum WIP leads to less lead time. Also, the PN model provides a better coordination among the supplier, production manager, quality assurance unit, assembly manager, and packaging, and the end user. Because of this better coordination, JIT activities will take place. In future work, the model can be developed using Fuzzy PN, Colored PN, or Queuing PN.

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