A combined zone-LP and simulated annealing algorithm for unequal-area facility layout problem

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A B S T R A C T

Facility layout problem (FLP) is one of well-known NP-hard problems and has been demonstrated to be useful in enhancing the productivity of manufacturing systems in practice. This paper focuses on the unequal-area FLP (UA-FLP) whose goal is to locate departments with different areas within a given facility so as to minimize the total material handling cost. A novel approach, which we call a combined zone-linear programming (zone-LP) and simulated annealing algorithm, is developed for solving the UA-FLP. The zone-LP approach is a layout construction technique for the unequal-area departments and consists of two phases. In the first phase, a zoning algorithm is implemented to determine the relative positions between the departments. In this algorithm, for the sake of problem simplification and computational efficiency, each department is treated as a rectangle with an allowable aspect ratio and the area of the facility is assumed to be unbounded. In the second phase, by using the relative positions obtained in the first phase as input, a linear programming (LP) model is developed to identify the exact locations and dimensions of departments within the facility with specified sizes while satisfying their maximum aspect ratio requirement and the shape constraints. We also design a simulated annealing algorithm to improve the placing sequence. Finally, our computational results suggest that our proposed algorithm is efficient compared with the best existing approach in the literature.

1. Introduction

As the competition among the global marketplaces increases rapidly, the provision of high-quality products or services at low cost to customers becomes more and more crucial for companies to capture the mass market. Facilities planning "determines how an activity's tangible fixed assets best support achieving the activity objectives" [1] and thus plays an important role in the cost reduction and productivity gain in industrial firms.

Facility planning consists of facilities location, facility system design, facility layout design and handling system design. As a crucial part of facilities planning, facility layout design is to arrange departments interacting with each other within a facility so as to minimize the total material handling cost. The layout design of a facility significantly impacts manufacturing cost and the productivity and efficiency of the system. According to [1], material handling cost ac-

counts for 20 % to 50 % of the total manufacturing cost; an effective planning of facilities can reduce such cost by at least 10 % to 30 %. It is essential to design an efficient layout before manufacturing system design since future rearrangement of departments can result in a considerable expense. For these reasons, facility layout problem (FLP) has been studied extensively for over three decades [2-10].

There have been numerous studies adopting mathematical programming approaches to obtain optimal solutions for FLP. Montreuil [11] proposed the first mixed integer programming (MIP) model for solving the UA-FLP. However, their proposed MIP model can be solved for the problems with no more than six departments. Meller et al. [12] improved the formulation of the MIP model of Montreuil [11] by introducing a tightened department area constraint and several classes of valid inequalities. By their enhancement, the number of departments of the solvable problems increased to eight. Motivated by Meller et al. [12], Sherali et al. [13] further improved the accuracy for approximating the department areas by using a polyhedral outer approximation scheme. Moreover, Sherali et al. [13] investigated the effectiveness of the classes of valid inequalities proposed by Meller et al. [12] and reported that the MIP model was solved most efficiently by incorporating only two inequalities among them into the formulation. With this finding, the problems with up to nine departments could be optimally solved. Recently, Meller et al. [14] developed a new mathematical formulation for UA-FLP based on a sequence-pair representation. In this formulation, the feasible region of the problem was tightened such that the number of departments of the solvable problems has become eleven.

Exact solution methods such as mathematical programming approaches are not applicable to obtain optimal solutions for large-scale problems since FLP is NP-hard [2]. Hence, heuristic approaches based on MIP models have been designed to construct layout solutions of high quality efficiently [15-17]. One of the well-known algorithms for solving FLP is based on the use of a slicing tree [18, 19], which is a binary tree used to form a subdivision of a rectangle by recursive computations. In the slicing tree, each inner node contains a guillotine cut operator (which cuts the area horizontally or vertically), and each leaf node represents a department. A layout can be generated according to a slicing tree by applying a set of guillotine cuts. Liu and Meller [20] developed a sequence-pair approach, where a pair of sequences is used to determine the relative locations between departments. By employing those relative locations, a reduced MIP model is utilized to construct the layout solutions. Both of above method are primarily proposed in the VLSI (very large Scale Integration) design. Similarly, Bozer and Wang [21] presented a graph-pair technique to fix the relative locations between departments and then a linear program (LP) is solved to obtain the layout solutions.

This paper proposes a novel approach, which we call a combined zone-LP and simulated annealing (SA) algorithm, to solve the problem efficiently. The zone-LP approach is a layout construction technique for the UA-FLP and consists of two phases. In the first phase, the zone algorithm is implemented to determine the relative positions between departments. In this algorithm, for the sake of problem simplification and computational efficiency, each department is considered to be a rectangle with an allowable aspect ratio and the area of the facility is assumed to be unbounded. In the second phase, by using the relative positions obtained in the first phase as input, an LP is solved to determine the exact locations and dimensions of departments within the facility with specified sizes while satisfying their maximum aspect ratio requirement and the shape constraints. Furthermore, we also develop an SA algorithm to improve the solution quality. Finally, we conduct a computational study to examine the performance of our proposed algorithm and compare it with other existing solution approaches in the literature.

Our paper is organized as follows. In the next section, we will describe the problem of UA-FLP. Section 3 will present our proposed solution methodology. In Section 4, we conduct computational experiments to study the performance of our proposed algorithm. Section 5 concludes our paper.

2. Problem description

In UA-FLP, the areas of the departments are given and their dimensions are limited by a maximum aspect ratio (MAR), which is defined as the largest allowable ratio of the longest side to the

shortest side of the department. The aspect ratio of a department ranges from 1/MAR to MAR. Furthermore, material flow quantities and the location of input/output (I/O) points are given. In the problem, the I/O points are assumed to be located at the centroids of departments. During the manufacturing process, materials are transferred between I/O points for each pair of departments. The objective is to minimize the total travel distance (TTD), which is the sum product of the distances between departments and their material flow quantities. For simplicity, the distance between departments is defined as the rectilinear distance between the input point and the output point.

Table 1 provides an example of a setting of six departments with their required areas (a_m) and maximum aspect ratio (MAR_m) and the material flow quantities between each pair of the departments.

Table 1 An example of a setting of six departments with their required areas (a_m) and maximum aspect ratio (MAR_m) and the material flow quantities between each pair of the departments

| m | Material flow quantities | | | | | | a_m | MAR_m |
	1	2	3	4	5	6		
1	—	4	2	5	7	1	16	4
2	—	—	4	3	3	4	16	4
3	—	—	—	2	2	6	25	4
4	—	—	—	—	7	4	36	4
5	—	—	—	—	—	3	36	4
6	—	—	—	—	—	—	9	4

3. Solution methodology

It is well-known that UA-FLP is NP-hard. For this reason, we develop a heuristic algorithm to solve UA-FLP of large size. One complicating factor of UA-FLP is to determine the relative positions between the departments subject to the condition that their areas do not overlap. To circumvent this difficulty, we propose a solution methodology that integrates the zone-LP approach and an SA algorithm to obtain good-quality solutions. The zone-LP approach is a newly proposed layout construction technique based on an improved zone algorithm and the solution for an LP. When an initial solution is obtained from the previous stage, SA is employed to search within the solution space and acts to avoid the search procedure being trapped at a local optimum.

3.1 Zone-LP approach

The zone-LP approach consists of two phases. In the first phase, for the simplification of the problem and more efficient computations, we consider the shape of each department as a rectangle with an allowable aspect ratio. As an example, as shown in Fig. 1, the six departments in Table 1 are treated as squares. Given the allowable aspect ratios of the departments, we develop an improved zone algorithm to determine their relative positions. The zone concept for UA-FLP is first introduced by Xiao et al. [22] to reduce its computational complexity and has been shown to be computationally efficient. In the second phase, by using the relative positions obtained in the first phase as input, we solve an LP to obtain the exact locations of the departments within the facility, and their dimensions subject to their shape constraints are satisfied.
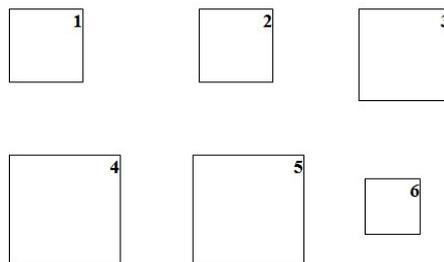

Fig. 1 Six departments to be located within the facility where each of them is of a fixed aspect ratio

3.2 Improved zone algorithm

The improved zone algorithm locates departments one by one according to a sequence such that a unique layout is generated. Given an initial sequence for locating the departments within the facility, say [1-4-2-5-3-6], we begin the algorithm by placing the first department in the sequence (i.e. Department 1) at the center of the facility. Then, four zones that are respectively on the left, above, on the right, and below this department can be identified. As an example, the four zones are shaded in the graphs of Fig. 2. The next step is to determine the zone that the following department in the sequence (i.e. Department 4) to be located. We first determine the optimal locations of this department within the four zones by using a median point (MP) method. The department will then be located at the best zone that results in the minimum total distance. Once the department is located within the facility, the set of zones will be updated for the determination of the location of the next department in the sequence. That procedure is repeated until all departments are located. For more detail about zone updating procedure, we refer the reader to Xiao et al. [22].
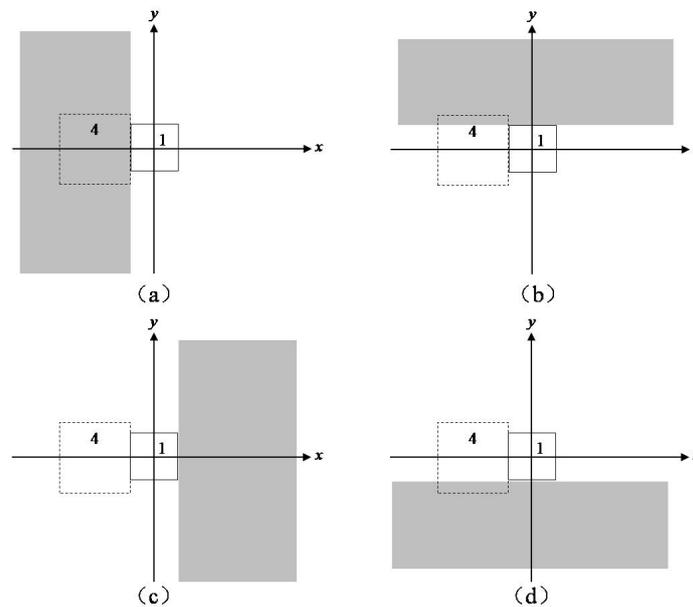

Fig 2 The four zones (shaded in grey) for the possible location of the department in our example

To generalize the expressions of the zones, for $i = 1, \dots, N$, let F_i be the i-th department in the sequence and let $Z(i) = \{z_1{}^i, \dots, z_j{}^i, \dots, z_J{}^i\}$ be the set of zones that F_i can be located, where j is the index of zone. Note that $Z^{(1)}$ consists of only one zone that is the entire rectangle facility. For each zone of F_j, $z_j{}^i$ is represented by $\left\{\left(x^i{}_{j1}, y^i{}_{j1}\right), \left(x^i{}_{j2}, y^i{}_{j2}\right)\right\}$, where $(x^i{}_{j1}, y^i{}_{j1})$ and $(x^i{}_{j2}, y^i{}_{j2})$ are the coordinates of the bottom left corner and the top right corner of the zone, respectively. For step of determining the location of the department, the MP method is applied to obtain the optimal position of F_i within each zone in $Z^{(i)}$. In this MP method, an ideal location of the centroid of F_i is first identified in such a way that the sum of rectilinear distances weighted by the flow quantities between F_i and the departments that have previously been located, denoted by the set B, is minimized. The objective function is formulated as follows:

Minimize
$$TTD_i = \sum_{n \epsilon B} f_{mn}(|x_i - x_n| + |y_i - y_n|) \tag{1}$$

To determine the optimal location, (x_i^*, y_i^*), the objective function Eq. 1 is reformulated as Equations Eq. 2 and Eq. 3. Therefore, the values of x_i^* and y_i^* can be calculated separately. In Equations Eq. 2 and Eq. 3, $TTD_i(x)$ and $TTD_i(y)$ are piecewise linear and convex functions, where all x_n (y_n), for $n \epsilon B$, are arranged in increasing order, i.e. $x_1' \leq x_t' \leq \cdots \leq x_{|B|}'$ $(y_1' \leq y_t' \leq \cdots \leq y_{|B|}')$. The flow quantities f_{mn} associated with x_n (y_n), for $n \epsilon B$, are also sorted by

$\left[w_1^x, w_t^x, \ldots, w_{|B|}^x\right]$ ($\left[w_1^y, w_t^y, \ldots, w_{|B|}^y\right]$). To determine the values of x_i^* and y_i^*, we make use of Property 1.

Minimize $\quad TTD_i(x) = \sum_{n \in B} f_{mn}|x_i - x_n| = w_1^x|x_i - x_1'| - w_2^x|x_i - x_2'| + \cdots + w_{|B|}^x|x_i - x_{|B|}'|$ (2)

Minimize $\quad TTD_i(y) = \sum_{n \in B} f_{mn}|y_i - y_n| = w_1^y|y_i - y_1'| - w_2^y|y_i - y_2'| + \cdots + w_{|B|}^y|y_i - y_{|B|}'|$ (3)

Property 1. Suppose that x_i^* is the MP and x_c' satisfies the median condition, i.e. $\sum_{t=1}^{c-1} w_t^x \leq \sum_{t=1}^{|B|} w_t^x /2$ and $\sum_{t=c+1}^{|B|} w_t^x \leq \sum_{t=1}^{|B|} w_t^x /2$. Then $x_i^* = x_c'$. Similarly, if y_i^* MP and y_c', satisfies the MP condition $\sum_{t=1}^{c-1} w_t^y \leq \sum_{t=1}^{|B|} w_t^y /2$ and $\sum_{t=c+1}^{|B|} w_t^y \leq \sum_{t=1}^{|B|} w_t^y /2$ then, $y_i^* = y_c'$.

By Property 1, the optimal location of F_i (i.e. the optimal solution for formulation Eq. 1) can be determined by searching the MP such that both of the total weights summing over those before the MP in the sequence and summering over those after the MP in the sequence is no more than the total weight summing over all the departments. In other words, the MP is the first point where the partial sum of weights from the first department in the sequence to itself is no less than half of the total weights of all departments. Let $W_x(t)$, $W_y(t)$ be the partial sums of the weights at point summing from the first department to the t-th department in the sequence in x-direction and y-direction respectively, and W be the total weights summing over all departments. With the sequence [1-4-2-5-3-6] for locating the departments within the facility in the previous example, we illustrate in Fig. 3 for how the aforementioned procedure can determine the MP. In Fig. 3, the locations of four departments have previously been determined. The TTD between F_5, i.e. Department 3 and the four departments that have been located in x-direction is: $TTD_5(x) = 2|x_5 - (-5)| + 2|x_5 - 0| + 4|x_5 - 0| + 2|x_5 - 7|$. In this example, $W = 11$, $W_x(1) = 2$, $W_x(2) = 2 + 2 = 4$, $W_x(3) = 2 + 2 + 4 = 8 > W/2$. Hence, the MP is $x_3' = 0$ and $x_5^* = x_3' = 0$. Similarly, $TTD_5(y) = 2|y_5 - 0| + 2|y_5 - 0| + 2|y_5 - 0| + 4|y_5 - 4|$. And $W_y(1) = 2$, $W_y(2) = 2 + 2 = 4$, $W_y(3) = 2 + 2 + 2 = 6 > W/2$. Hence, the MP in y-direction is $y_3' = 0$ and $y_5^* = y_3' = 0$. Therefore, the optimal location of Department 3 is (0, 0), which is marked by the dashed rectangle in Fig. 3.
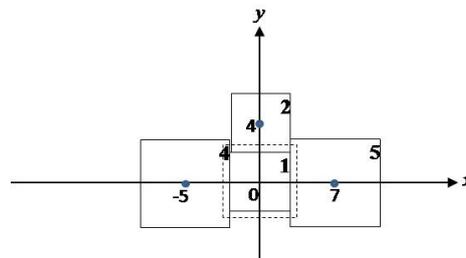

Fig. 3 Optimal location of Department 3 (represented by a dotted square)

Nevertheless, this optimal location determined for Department 3, in fact, is infeasible because it overlaps with the other departments as shown in in Fig. 3. In this case, we have to determine a new location for Department 3 to continue our UA-FLP procedure. To this end, we determine the location in each zone for Department 3 such that the location is closest to the previously determined optimal location in both x-direction and y-direction. The rationale is that such position can minimize TTD for each zone. For example, the best positions for Department 3 in those considered zones are shown in Fig. 4(a)-4(f), which are respectively the closest locations to the optimal location that was previously determined. The department is to be located at the best position that attains the minimum TTD_{ij} among those zones. In this example, Department 3 has the same TTD_{ij} at the positions as shown in Fig. 4(b) and Fig. 4(d). Thus, this tie is broken by randomly picking one of the two positions for locating Department 3. The location of the last department in the sequence of this example is also determined in the same way. Fig. 5 shows the final layout of the facility.
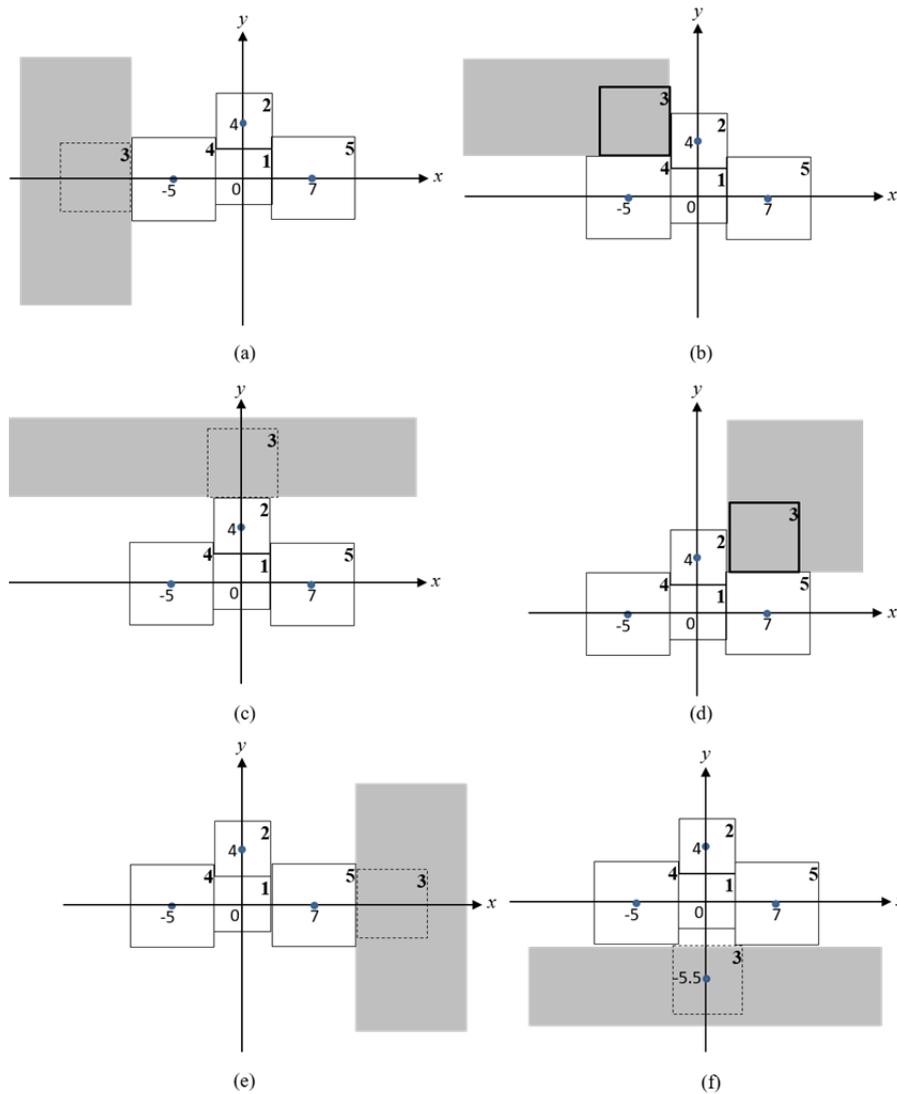
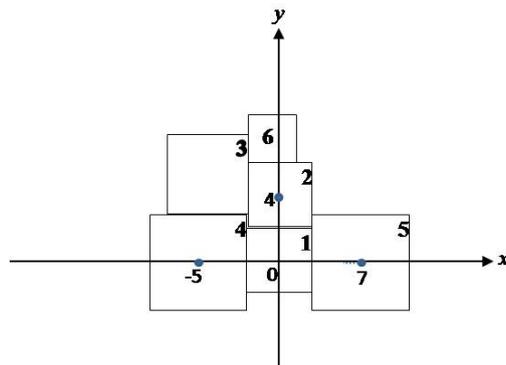**Fig. 4** Closest location in each zone to the optimal location

Fig. 5 Layout of the six departments of fixed shapes

To determine if the department can be located at the intended position, we also have to consider the dimension of each zone z_j^i which can be measured by $w_j^i = x_{j2}^i - x_{j1}^i$ and $h_j^i = y_{j2}^i - y_{j1}^i$, where w_j^i and h_j^i are respectively the width and the height of the zone. Let d_{wm} and d_{hm} be the width and the height of the department to be located, respectively. If a zone satisfies the conditions that $w_j^i \geq d_{wm}$ and $h_j^i \geq d_{hm}$, it is feasible to locate Department m in this zone; otherwise, check if the subsequent zone can accommodate Department m. The procedure of the modified zone algorithm is summarized as follows:

Step 1. Set $i = 1$. The centroid of F_i is placed at $(0, 0)$ in the coordinate system.

Step 2. Set $i = i + 1$. Create the zone set, $Z^{(i)} = \{z_j^i | j = 1, \dots, J\}$, by using the zone updating procedure. Determine the optimal location, (x_i^*, y_i^*), for F_i. Let TTD_i^* be the minimum TTD_{ij} among the zones contained in the zone set. Set $j = 1, TTD_i^* = +\infty$.

Step 3. If $x_{j1}^i + d_{wm}/2 \leq x_i^* \leq x_{j2}^i - d_{wm}/2$ and $y_{j1}^i + d_{hm}/2 \leq y_i^* \leq y_{j2}^i - d_{hm}/2$ (i.e. F_i with its centroid located at the optimal location can fit entirely within zone z_j^i), go to Step 3.1; otherwise, go to Step 3.2.

    <u>Step 3.1</u> F_i is optimally located at (x_i^*, y_i^*). Go to Step 5.

    <u>Step 3.2</u> If $w_j^i \geq d_{wm}$ and $w_j^i \geq d_{hm}$, locate D_i at the closest possible position in z_j^i to the optimal location. Calculate TTD_{ij}. If $TTD_{ij} \leq TTD_i^*$, then $TTD_i^* = TTD_{ij}$; else, TTD_i^* keeps unchanged.

    <u>Step 3.3</u> Set $j = j + 1$. If $j \leq J$, go to Step 3; otherwise, go to Step 4.

Step 4. F_i is placed in the zone whose TTD_{ij} attains TTD_i^*, go to Step 5.

Step 5. If $i \leq N$, go to Step 2; otherwise, all the departments have already been located within the facility. Terminate the procedure.

3.3 Linear programming (LP)

With facility layout determined by the improved zoning algorithm in the first phase, the relative positions of each pair of departments can be obtained. By using this information, an LP model is formulated to optimize the dimensions of departments within a specified facility while satisfying the physical constraints imposed on their dimensions. The additional notations used in this formulation are listed as follows:

W	Width of the facility on the floor plan
H	Height of the facility on the floor plan
N	Total number of departments to be located
m, n	Indices used to represent the departments, $m = 1, \dots, N$ and $n = 1, \dots, N$.
f_{mn}	Material flow quantity from Department m to Department n ($m \neq n$).
a_m	Area requirement of department m
α_m	Maximum aspect ratio of department m
ub_m^x, lb_m^x	Upper and lower bound of the length of Department m in the x-direction
ub_m^y, lb_m^y	Upper and lower bound of the length of Department m in the y-direction
r_{mn}, s_{mn}	Indicators to denote if Department m is on the right/below Department n respectively. More specifically, $r_{mn} = 1$ if Department m is on the left of Department n, and 0 otherwise; $s_{mn} = 1$ if Department m is to below Department n, and 0 otherwise
$\overline{x}$	The value of the tangential support point for the polyhedral outer approximation to the area constraints, $lb^x m /2 \leq \overline{x} \leq ub^x m/2$
Δ	Number of tangential support points, $\Delta \geq 2$
C	penalty for the violation of the floor boundary condition, $C = \sum_m \sum_{n>m} f_{mn}(W + H)$
w_m, h_m	Half of width and height of Department m
$v^x m, v^y m$	Amount of violation of the floor boundary condition for Department m in the x-direction and the y-direction, respectively

Our LP model to determine the dimensions of the departments is derived from the MIP formulated by Sherali et al. [13] and presented below:

Minimize
$$TTD = \sum_m \sum_{n>m} f_{mn}(d_{mn}^x + d_{mn}^y) + C\left(\sum_m v_m^x + \sum_m v_m^y\right) \tag{4}$$

subject to

$$d_{mn}^x \geq x_m - x_n \quad \forall m < n \tag{5}$$

$$d_{mn}^x \geq x_n - x_m \quad \forall m < n \tag{6}$$

$$d_{mn}^y \geq y_m - y_n \quad \forall m < n \tag{7}$$

$$d_{mn}^y \geq y_n - y_m \quad \forall m < n \tag{8}$$

$$w_m \leq x_m \leq W - w_m + v_m^x \quad \forall m \tag{9}$$

$$h_m \leq y_m \leq H - h_m + v_m^y \quad \forall m \tag{10}$$

$$lb_m^x \leq 2w_m \leq ub_m^x \quad \forall m \tag{11}$$

$$lb_m^y \leq 2h_m \leq ub_m^y \quad \forall m \tag{12}$$

$$x_m + w_m \leq x_n - w_n \quad \forall m, n, m \neq n: r_{mn} = 1 \tag{13}$$

$$y_m + h_m \leq y_n - h_n \quad \forall m, n, m \neq n: s_{mn} = 1 \tag{14}$$

$$a_m w_m + 4\overline{x}_{mp}^2 h_m \geq 2a_m \overline{x}_{mp} \quad \forall m, p \tag{15}$$

$$\overline{x}_{mp} = lb_m^x/2 + \frac{p}{\Delta - 1}(ub_m^x/2 - lb_m^x/2) \quad \forall m, p = 0, \dots, \Delta - 1 \tag{16}$$

$$x_m, y_m, w_m, h_m, v_m^x, v_m^y \geq 0 \quad \forall m \tag{17}$$

$$d_{mn}^x, d_{mn}^y \geq 0 \quad \forall m < n \tag{18}$$

Objective function Eq. 4 consists of two terms: *TTD* between departments and the penalty for violation of the floor boundary condition. Constraints Eq. 5 to Eq. 8 determine the *TTD* between departments. Constraints Eq. 9 and Eq. 10 measure the violation of the floor boundary condition (in x-direction and y-direction, respectively), if it is infeasible to locate all the departments within the facility. Constraints Eq. 11 and Eq. 12 impose the bounds on the width and height for each department, where the upper bounds $ub_m^x = \min\{W, \sqrt{\alpha_m a_m}\}$, $ub_m^y = \min\{H, \sqrt{\alpha_m a_m}\}$, and lower bounds $lb_m^x = a_m/ub_m^y$, $lb_m^y = a_m/ub_m^x$. Constraints Eq. 13 and Eq. 14 ensure that the departments do not overlap with each other. Constraints Eq. 15 is the polyhedral outer approximation for the area function, $a_i = 4w_m h_m$, with the number of tangential support points equal to Δ. The value of the tangential support points $\overline{x}$ is given in Eq. (16).

As an example, with the relative positions of the departments obtained by the improved zone algorithm (as shown in Fig. 5), the final facility layout for locating the six departments with the exact dimensions can now be obtained by the LP. Their positions and dimensions are shown in Fig. 6.
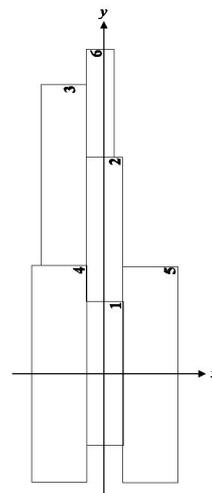

Fig. 6 Layout solution of the six departments with their dimensions determined

3.4 SA algorithm

With the facility layout solution obtained by the zone-LP approach, we implement an SA algorithm to improve the quality of the solution. The SA algorithm that we adopt is the same as the procedure proposed in Xiao et al. [22]. For the detail of the algorithm, we refer the reader to Xiao et al. [22].

4. Computational experiments

In this section, we conduct computational experiments to evaluate the performance of our proposed solution methodology for UA-FLP – an integrated method of zone-LP approach and SA algorithm. We use a set of widely tested instances in the literature to examine the efficiency of the solution methodology. There are five instances of different problem sizes included in the computational experiments: Problems O7 (with 7 departments), O8 (with 8 departments) and O9 (with 9 departments) from Meller et al. [12] and Two largest instances SC30 (with 30 departments) and SC35 (with 35 departments) from Liu and Meller [20].

The combined zone-LP and SA algorithm is run ten times for each computational instance. In the first phase of zone-LP algorithm, the dimensions of departments are fixed with the MAR because all departments tend to have a narrower rectangular shape to get a lower TTD. The parameter combination used in SA was $T = 200°$, $R = 0.8$, and $L = 1000$ from Xiao et al. [21]. The results on the computational performance of our proposed approach are shown in Table 2. We present the averages and the standard deviations of the costs to examine the robustness of our proposed method. The computational results suggest that the proposed algorithm appears to be quite robust. For the three small-sized problems (O7, O8, O9), the standard deviation is quite small, ranging from 0 to 0.57 (less than 0.5 % of the average). Although the magnitude of the standard deviation becomes large for the large-sized problems (SC30, SC35), its relative value is still small (less than 3 % of the average). For the computational efficiency, our proposed solution methodology can obtain the final solution within a reasonable time frame (less than 1 minute for the small-sized problems and less than 4 minutes for the large-sized problems). Given that facility layout planning is not a daily practice (is usually determined for several years), this computational time is negligible, and our method is suitable for the application in practice.

Table 2 Objective values and the computing time obtained by the proposed algorithm

Problem	Best	Mean	Worst	Standard deviation	Average time (s)
O7	89.25	89.25	89.25	0	33.13
O8	185.00	185.30	186.00	0.46	36.93
O9	185.00	185.45	186.5	0.57	41.80
SC30	3441.57	3663.21	3792.71	103.50	180.27
SC35	3347.94	3423.70	3555.89	69.06	225.94

Table 3 Objective values, dimensions of the facilities, space utilizations resulting from the best solutions found by GRAPH [21] and our proposed algorithm

Problem	GRAPH[21]			The proposed algorithm			Diff. in TTD (%)
	TTD	Layout dimension ($W \times H$)	Space utilization (%)	TTD	Layout dimension ($W \times H$)	Space utilization (%)	
O7	115.93		96.00	89.25	12×13.5	68.52	23.01
O8	239.00		96.00	185.00	12×16.5	74.24	22.59
O9	227.10		96.00	185.00	15×16.5	63.03	18.54
SC30	3601.20	15×12	90.56	3441.57	12.75×21.59	59.21	4.43
SC35	3351.12	16×15	80.00	3347.94	15.14×24.7	51.34	0.09

We also compare our proposed solution methodology with other existing algorithms in the literature and use them as benchmarks. We first compare the solution qualities obtained by our approach and the GRAPH algorithm [21], which attains the best known solutions for the computational instances. As shown in Table 3, our proposed outperforms GRAPH [21] in terms of TTD for all computational instances, but the space utilizations resulting from our algorithm are less than those from GRAPH. In our UA-FLPs, departments are arranged within an open-field floor

(as shown in Fig. 4 and Fig. 5) and the loss of consideration on space utilization in the objective function of the proposed model mainly causes the poor performance on space utilization. Therefore, a multi-objective optimization considering minimization of TTD and maximization of space utilization may be our future research topic. For reference, the best layouts solutions obtained by our proposed approach are shown in Appendix 1. The proposed algorithm also has an advantage of a shorter computing time due to its capability of searching in the restricted solution spaced. We conduct another study to examine the efficiency of our proposed methodology. In addition to GRAPH [21], we also include SEQUENCE [20] for the comparison of computational efficiencies of the different approaches. Computational results in Table 4 suggest that our algorithm takes a significantly shorter time to solve the UA-FLP. More importantly, the computing time appears to grow almost linearly as the number of departments increases. Compared with the approaches that appear to have an exponential computing time in the number of departments, our approach is apparently more suitable for problems of a larger size. The linear increase of computing time may be attributed to the proposed zone-LP algorithm which can construct the layout solution effectively. In each iteration of SA, the number of zones produced for putting departments by MP method is $O(N)$ according to the zone algorithm. The proposed LP model is just used to determine the dimensions of departments. In general, the computing time is approximately proportional to the number of generated zones in each SA iteration and thus grows almost linearly as the number of departments increases. This further demonstrates the value of our proposed approach in advancing the computational performance for solving UA-FLP.

Table 4 Average computing times of the SEQUENCE [20], GRAPH [21], and our proposed algorithm

Instance	Computing time (s)		
	SEQUENCE [20]	GRAPH [21]	Our proposed algorithm
O7	1644.0	228.0	33.13
O8	3056.0	390.0	36.93
O9	3879.0	222.0	41.80
SC30	7282.8	2442.0	180.27
SC35	9590.4	4728.0	225.94

5. Conclusion and future research

This paper deals with UA-FLP and proposes a novel approach, which we call a combined zone-LP and SA algorithm, for solving large-sized UA-FLP. The zone-LP algorithm is a new layout construction method and consists of two phases. In the first phase, the shapes of departments are fixed to a rectangle with an allowable aspect ratio. Those departments of fixed shapes are then placed sequentially by using an improved zone algorithm, where an MP method is proposed to locating departments. By using relative positions of the departments obtained in this first phase, an LP is formulated to determine the exact locations and dimensions of departments. We also implement an SA algorithm to search the sequences of locating the departments within the facility and to prevent the search procedure from getting trapped at a local optimum. Computational experiments indicate that our proposed combined zone-LP and SA has a reasonably good performance, compared with existing algorithms in the literature on widely tested computational instances.

We also note that there can be future directions extended from this research. In reality, departments of irregular shapes are commonplace; a future study may consider the facility layout problem with the consideration of departments with irregular shapes. Moreover, the results of computing experiments suggest that our proposed solution methodology may generate solutions leading to a low space utilization. For this case, we may adopt a multi-objective optimization approach whose goals are to minimize the material handling cost and to maximize space utilization. The trade-off between these two performance measures would also be an interesting research topic for investigation in the future.
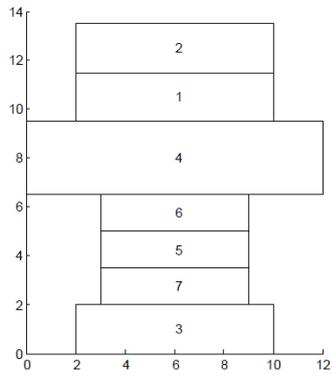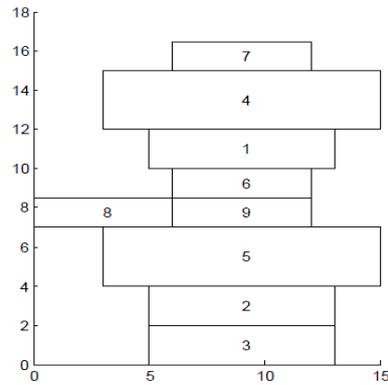
Acknowledgement

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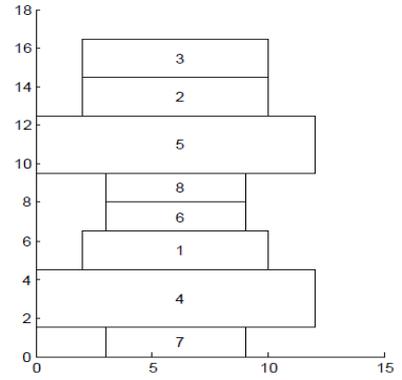
Appendix 1

The best layouts obtained by our proposed algorithm for the experimental instances from Meller *et al.* [10] and Liu and Meller [19].
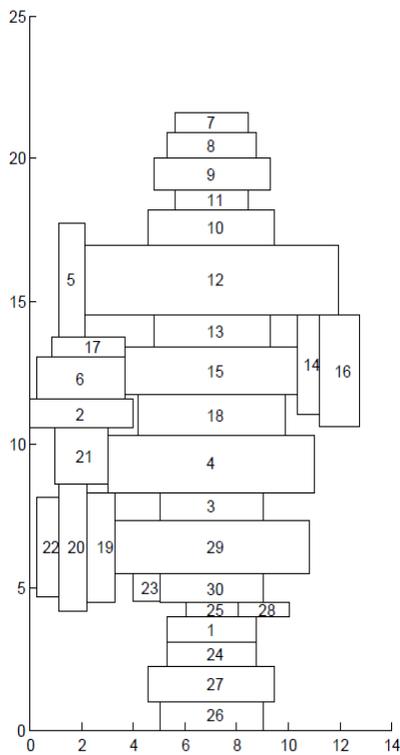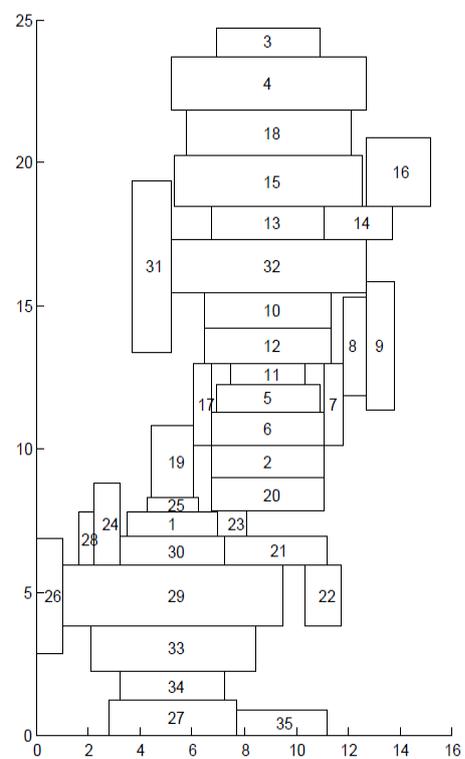

Layout with seven departments

Layout with eight departments

Layout with nine departments

Layout with 30 departments

Layout with 35 departments