

Classification of thin regular map representations of hypermaps*

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Abstract

There are two well-known map representations of hypermaps, namely the Walsh and the Vince map representations, that are dual to each other. They correspond to normal subgroups of index two of a free product $\Gamma = (C_2 \times C_2) * C_2$ which decompose as “elementary” free product $C_2 * C_2 * C_2$. However, Γ has three normal subgroups that decompose as “elementary” free product $C_2 * C_2 * C_2$, the third of these subgroups giving the less known Petrie-path map representation. By relaxing the “elementary” free product condition to free product of rank 3, and under the extra condition “words of smaller length” on the generators, we prove that the number of map representations of hypermaps increases to 15 (up to a restrictedly dual), all of which are described in this paper.

Keywords: Map representation, hypermaps, maps, regularity, restricted regularity, orientably regular.

Math. Subj. Class. (2020): 05C10, 05C25, 05C65, 05E18, 20F65

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Klasifikacija tankih regularnih predstavitev hiperzempljevidov z zemljevidi*

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Povzetek

Obstajata dve dobro znani predstavitvi hiperzempljevidov z zemljevidi, in sicer Walshova in Vinceova; ti dve predstavitvi sta druga drugi dualni. Ustrezata dvema podgrupama edinkama indeksa dve prostega produkta $\Gamma = (C_2 \times C_2) * C_2$, ki razpade na "elementarni" prosti produkt $C_2 * C_2 * C_2$. Vendar pa ima Γ tri podgrupe edinke, ki razpadejo na "elementarni" prosti produkt $C_2 * C_2 * C_2$, pri čemer tretja od teh podgrup edink ustreza manj znani zemljevidni predstavitvi z uporabo Petriejevih poti. Če umaknemo pogoj, da mora biti prosti produkt "elementaren" oz. oblike $C_2 * C_2 * C_2$, in zahtevamo le, da ima rang 3, ter ob dodatnem pogoju, da so "besede iz generatorjev krajše", dokažemo, da število predstavitev hiperzempljevidov z zemljevidi naraste na 15 (do omejenega duala natančno); in prav vse takšne predstavitve so prikazane v tem članku.

Ključne besede: Predstavitve z zemljevidi, hiperzempljevidi, zemljevidi, regularnost, omejena regularnost, orientabilno regularen.

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