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Modeliranje naključnih termomehanskih napetostno-deformacijskih stanj

Thermo-Mechanical Modelling of Stochastic Stress-Strain States

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Deformacijski postopek je najbolj razširjena metoda napovedovanja dobe trajanja dinamično obremenjenih delov na področju malociklične trdnosti. Uporablja se tako pri nizkih, srednjih kakor tudi visokih temperaturah, če je temperatura med obratovanjem nespremenljiva. Deformacijski postopek je računsko izjemno hiter in narekuje le uporabo analiz elastičnosti s končnimi elementi. Zaradi številnih prednosti, razširjenosti in potencialnih možnosti je bil prilagojen tako, da je uporaben tudi za spremenljive temperature. Napetostno-deformacijsko stanje je popisano s Prandtlovimi operatorji. Postopek temelji na stabilnih histereznih zankah, pri čemer lezenje ni upoštevano. Analiziran je tudi vpliv filtriranja konic sunkov. Narejena je primerjava izsledkov raziskav z rezultati meritev in Skeltonovim modelom.

(Ključne besede: utrujanje termomehansko, enačbe konstitutivne, stanja napetostno-deformacijska, modeliranje)

The isothermal strain-life approach is the most commonly used approach for determining fatigue damage, particularly in low-cycle fatigue. It is used for low, medium and high temperatures if the temperature remains constant during the test. Computationally, it is extremely fast and generally requires elastic finite-element analyses only. For this reason it has been adapted for variable temperatures. The local temperature-stress-strain behaviour is modelled with an operator of the Prandtl type. The hysteresis loops are supposed to be stabilized and no creep is considered. The consequences of reversal point filtering are analysed. Finally, the approach is compared to several thermo-mechanical fatigue tests and the Skelton model.

(Keywords: thermomechanical fatigue, constitutive equations, stress strain states, modelling)

0 UVOD

Številni izdelki, npr. motorji z notranjim zgorevanjem, izpušni in hladilni sistemi, turbine, jedrski reaktorji, so v času uporabe obremenjeni z naključnimi termomehanskimi obremenitvami (TMU), ki lahko privedejo do utrujenostnih razpok [1]. Z razvojnega vidika je zato pomembna napoved dobe trajanja izdelka v razvojnih fazah pred preskušanjem prototipov oziroma izdelkov. Doba trajanja je odvisna predvsem od obremenitev, uporabljenih materialov, oblike izdelka in okoliščin. Za njeno dokazovanje se uporabljajo naslednji preizkusi ([2] in [3]):

- deformacijsko nadzorovani malociklični preizkus (MCUP - LCF) pri nespremenljivi temperaturi,
- termomehanski preizkus delov in izdelkov in
- toplotni sunek.

0 INTRODUCTION

Many products, like internal combustion engines, exhaust and cooling systems, turbines, nuclear reactors, are subjected to stochastic thermo-mechanical fatigue (TMF) loading that can cause fatigue failures during their usage [1]. From the development perspective, fatigue-life prediction in the development phases before prototype or product testing is important. Fatigue life depends primarily on loads, applied materials, product geometry and environmental effects. Its approval is generally based on the following tests ([2] and [3]):

- isothermal strain-controlled low-cycle fatigue (LCF) tests,
- TMF tests on specimens and components,
- thermal shock tests.

MCUP se izvajajo prednostno, ker so v primerjavi z drugimi preprostejši [4]. Poleg tega se opravljajo že več desetletij, postopek testiranja in zapisovanja rezultatov testiranj pa je standardiziran [2]. Obstajajo tudi baze podatkov o materialnih lastnostih, potrebnih za napoved dobe trajanja malociklično obremenjenih delov. Končni cilj je razviti metodologijo napovedovanja dobe trajanja, ki bo omogočala zanesljive napovedi brez termomehanskih testiranj in testiranj s toplotnim sunkom. Področje je poglobljeno predstavljeno v [3] do [9].

V prispevku je obravnavan problem napovedovanja napetostno-deformacijskih stanj ob upoštevanju spremenljive temperature obratovanja. Predpostavimo, da je vpliv lezenja in prehodnih pojavov, npr. utrjevanja ali mehčanja, neznaten. Napovedovanje napetostno-deformacijskih stanj torej temelji na stabilnih histereznih zankah in konstitutivnih enačbah elastoplastičnosti. Izmed razpoložljivih reoloških modelov ([9] do [11]) je bil izbran model vzporedno vezanih vzmeti in drsnikov [11], izražen s Prandtlovim modelom s temperaturno odvisnimi gostotami in temperaturno neodvisnimi drsnimi površinami. Model omogoča razširitev deformacijskega postopka na področje spremenljivih temperatur, pri čemer se velika hitrost računanja ne zmanjša.

Napetostno nadzorovan in temperaturno preoblikovan reološki model je obravnavan v [12] in [13]. V primeru deformacijskega nadzora potrebujemo enakovreden, a drugačen model.

V drugem poglavju so obravnavane temperaturno odvisne napetostno-deformacijske krivulje in ocenjevanja parametrov. V tretjem poglavju je prikazan razvoj temperaturno preoblikovanega in deformacijsko nadzorovanega modela vzmet – drsnik. V četrtem poglavju so analizirane posledice izločanja konic sunkov. V petem poglavju je opisano ujemanje novega modela z rezultati preizkusov in s Skeltonovim modelom, v zadnjem poglavju pa so podani pomembnejši sklepi.

1 TEMPERATURNO ODVISNE NAPETOSTNO-DEFORMACIJSKE KRIVULJE

Ker večina materialov preide po nekem manjšem številu obremenitvenih ciklov v stabilno področje, se za napoved dobe trajanja navadno uporabijo materialne lastnosti, ki ustrezajo polovični dobi trajanja preskušanca. Temperaturni časovni poteki obremenitev so v splošnem naključni. V literaturi je mogoče najti napetostno-deformacijske krivulje za le nekaj različnih

In terms of complexity, LCF tests are favoured because of their simplicity [4]. In addition, because they have been carried out for decades, the testing procedures and data recording is standardized [2]. There also exist databases with material properties required for the fatigue-life prediction of low-cycle loaded parts. The final goal is, therefore, to develop a methodology of fatigue-life prediction that will enable reliable estimations by avoiding expensive TMF and thermal shock tests. The methodology is discussed comprehensively in [3] to [9].

The present work is concerned with a cyclic stress-strain response prediction suitable for variable temperatures. Assuming that creep and transient effects, such as cyclic hardening and cyclic softening and their effect on damage accumulation are negligible, the hysteresis loops are supposed to be stabilized and the constitutive equations for elastoplasticity are applicable for stress-strain behaviour modelling. From among the available models ([9] to [11]), the strain-controlled spring-slider model [11] with temperature-dependent Prandtl densities and temperature-independent yield surfaces has been chosen. It enables the extension of the isothermal strain-life approach to non-isothermal problems by preserving a high computational speed.

The stress-controlled and temperature-modified spring-slider model is explained in [12] and [13]. However, if the strain is controlled, an equivalent but distinct spring-slider is required.

In Section 2 the temperature-dependent cyclic stress-strain curves and parameter assessment is discussed. In Section 3 the temperature-modified strain-controlled spring-slider model is developed. In the 4th section the consequences of reversal point filtering are analysed. In Section 5 the spring-slider model is compared to several TMF tests, and the final section lists the conclusions.

1 TEMPERATURE-DEPENDENT CYCLIC STRESS-STRAIN CURVES

Most metals approach a cyclically stable state after a certain number of cycles. Cyclically stable or half-life material properties are usually used in fatigue analyses. Generally speaking, the temperature–time history is stochastic, but cyclic stress–strain curves for only a few distinct temperatures can be found in the literature. To temperatur. Napoved napetostno-deformacijskih stanj pri temperaturah, pri katerih napetostno-deformacijske krivulje ne obstajajo, je mogoča z uporabo interpolacije, aproksimacije ali neparametričnih metod. Predvsem slednje so se v zadnjem času izkazale za najboljše. Za popis elastoplastičnih lastnosti gradiv se najpogosteje uporablja Ramberg-Osgoodova enačba [14]:

obtain the material parameters that have not been measured, linear parameter interpolation, approximation or non-parametric methods can be used. The latter turned out to be the most appropriate recently. Elasto-plastic hardening solids are most frequently modelled by the Ramberg–Osgood relation [14]:

$$\varepsilon = g(\sigma,T) = \frac{\sigma}{E(T)} + \left(\frac{\sigma}{K'(T)}\right)^{1/n'(T)} \qquad (1),$$

kjer je $E(T)$ modul elastičnosti, $K'(T)$ in $n'(T)$ pa sta utrjevalni koeficient in utrjevalni eksponent. Interpolacija je bila obravnavana v [12].

where $E(T)$ is Young's modulus and $K'(T)$ and $n'(T)$ are the cyclic hardening coefficient and the cyclic hardening exponent respectively. Parameter interpolation has been dealt with in [12].

2 MODELIRANJE NAPETOSTNO-DEFORMACIJSKIH STANJ

V [12] je bilo dokazano, da Masingov model in spominski modeli niso veljavni, če se temperatura med obremenitvenim potekom spreminja. Masing je izsledke svojih raziskav utemeljil z reološkim modelom vzmet – drsnik in predpostavil, da so parametri modela neodvisni od časa [11]. Da bi odpravili omejitve omenjenih modelov, je bil model vzmet – drsnik temperaturno spremenjen ([12] in [13]). Model v sedanji obliki je primeren za modeliranje elastoplastičnih lastnosti gradiv in nelinearnega kinematičnega utrjevanja pri sočasnem nadzoru napetosti. Če je deformacija nadzorna spremenljivka, je na sliki 1 prikazan primernejši model za modeliranje napetostno-deformacijskih stanj.

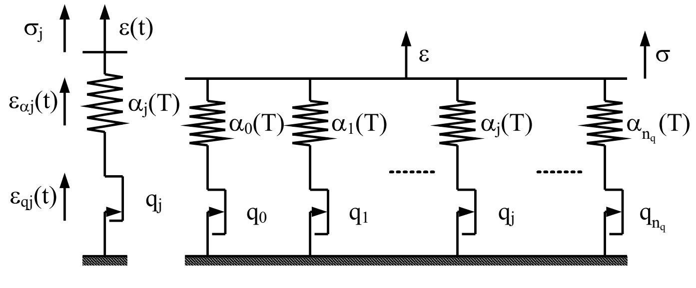
Iz ravnotežja na segmentu vzmet – drsnik je mogoče izraziti celotno deformacijo ε: $\varepsilon = \varepsilon_{qj} + \varepsilon_{\alpha j}$. Deformacija drsnika $|\varepsilon_{qj}|$ ne more nikoli preseči navidezne drsne deformacije q_j. Deformacija vzmeti

2 STRESS-STRAIN-TEMPERATURE BEHAVIOUR

It has been shown in [12] that the Masing and Memory models are not valid if the temperature varies during the cycle. Masing based his finding on the rheological spring-slider model and assumed that the model parameters are time independent [11]. Therefore, the spring-slider model has been adapted for variable temperatures ([12] and [13]). The model developed so far is capable of modelling elasto-plastic hardening solids and non-linear kinematic hardening under stress control. If the strain is controlled, the rheological spring-slider model depicted in Fig. 1 proves to be more convenient.

From the equilibrium in a single spring-slider segment, the total strain ε is obtained $\varepsilon = \varepsilon_{qj} + \varepsilon_{\alpha j}$, where the slider strain $|\varepsilon_{qj}|$ can never exceed the fictive yield strain q_j. The spring strain can now be expressed as:

$$\varepsilon_{\alpha j} = \varepsilon - \operatorname{sign}(\varepsilon - \varepsilon_{\alpha j}) \min\{q_j, |\varepsilon - \varepsilon_{\alpha j}|\} \qquad (2)$$



Sl. 1. *Reološki model vzmet – drsnik*
Fig. 1. *Rheological spring-slider model*

ustreza operatorju ohlapa s splošno začetno vrednostjo ([15] do [17]):

and corresponds exactly to the play operator with the general initial value ([15] to [17]):

$$\varepsilon_{\alpha j}(t_i) = \max\{\varepsilon(t_i) - q_j, \min\{\varepsilon(t_i) + q_j, \varepsilon_{\alpha j}(t_{i-1})\}\} \tag{3}$$

za $0 < t_1 < t_2 < \cdots < t_n$. Trenutna deformacija $\varepsilon_{\alpha j}(t_i)$ je odvisna od deformacije v predhodnem trenutku $\varepsilon_{\alpha j}(t_{i-1})$, imenovanem spominska točka. Predpostavimo, da zaostalih deformacij ni. Odtod izhaja $\varepsilon_{\alpha j}(0) = 0$ in $\sigma_j(0) = 0$. Napetost v odseku vzmet – drsnik je tedaj:

for $0 < t_1 < t_2 < \cdots < t_n$. Thus, the current strain state $\varepsilon_{\alpha j}(t_i)$ depends on the previous state $\varepsilon_{\alpha j}(t_{i-1})$ called the memory point. Presumably, there is no residual strain initially, so $\varepsilon_{\alpha j}(0) = 0$ and $\sigma_j(0) = 0$. The stress in the spring-slider segment is then:

$$\sigma_j(t_i) = E_j(T_i)\varepsilon_{\alpha j}(t_i) = \alpha_j(T_i)\varepsilon_{\alpha j}(t_i) \tag{4}$$

$T_i = T(t_i)$ in $\alpha_j(T_i)$ je Prandtlova gostota. S seštevanjem napetosti po posameznih odsekih je mogoče izraziti celotno napetost s Prandtlovimi operatorji ([15] do [17]):

where $T_i = T(t_i)$ and $\alpha_j(T_i)$ is the Prandtl density. Adding the spring-slider stresses results in the total stress in the form known as the operator of the Prandtl type ([15] to [17]):

$$\sigma(t_i) = \sum_{j=0}^{n_q} \alpha_j(T_i)\varepsilon_{\alpha j}(t_i) \tag{5}$$

s temperaturno odvisnimi gostotami. Ker je operator rege, definiran v en. (3), neodvisen od časa in temperature, ga je treba spremeniti tako, da bo zagotavljal ravnotežje v odseku vzmet – drsnik tudi pri spremenljivih temperaturah.

Če je v trenutku t_{i-1} j-ti odsek v ravnotežju, pomnožimo en. (3) z $\alpha_j(T_{i-1})$ (sl. 2). Odtod izhaja:

with temperature-dependent Prandtl densities. The play operator given in Eq. (3) is independent of time and temperature. Therefore, it is modified to assure equilibrium in the spring-slider.

Let us presume that at t_{i-1} the spring-slider segment j is in equilibrium and multiply Eq. (3) by $\alpha_j(T_{i-1})$ (Fig. 2). This yields:

$$\sigma_j(t_{i-1}) = \max\{(\varepsilon(t_{i-1}) - q_j)\alpha_j(T_{i-1}), \min\{(\varepsilon(t_{i-1}) + q_j)\alpha_j(T_{i-1}), \sigma_j(t_{i-2})\}\} \tag*{(6),}$$

kjer velja $\sigma_j(t_{i-2}) = \sigma_j(t_{i-1})$. Če se v koraku $[t_{i-1}, t_i]$ spremenita temperatura ali deformacija, se meje operatorja ohlapa premaknejo in:

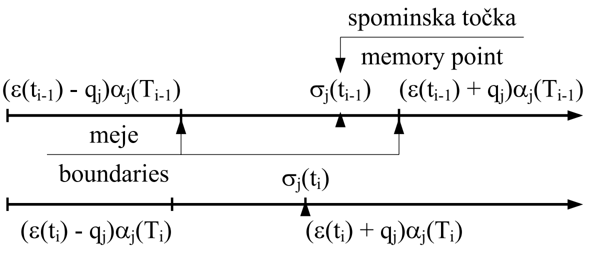
where the memory point $\sigma_j(t_{i-2}) = \sigma_j(t_{i-1})$. If the temperature or strain change in the interval $[t_{i-1}, t_i]$, then the boundaries of the play operator move and:

$$\sigma_j(t_i) = \max\{(\varepsilon(t_i) - q_j)\alpha_j(T_i), \min\{(\varepsilon(t_i) + q_j)\alpha_j(T_i), \sigma_j(t_{i-1})\}\}$$

Če nato enačbo delimo z $\alpha_j(T_i)$, dobimo:

If the equation is divided by $\alpha_j(T_i)$, then:

$$\varepsilon_{\alpha j}(t_i) = \max\left\{\varepsilon(t_i) - q_j, \min\left\{\varepsilon(t_i) + q_j, \frac{\sigma_j(t_{i-1})}{\alpha_j(T_i)}\right\}\right\} \tag{7}$$



Sl. 2. *Ravnotežje v modelu vzmet – drsnik*
Fig.2. *Equilibrium in the spring-slider model*

in nazadnje:

and finally:

$$\varepsilon_{\alpha j}(t_i) = \max\left\{\varepsilon(t_i) - q_j, \min\left\{\varepsilon(t_i) + q_j, \frac{\alpha_j(T_{i-1})}{\alpha_j(T_i)}\varepsilon_{\alpha j}(t_{i-1})\right\}\right\} \quad (8),$$

kjer je $\sigma_j(t_{i-1}) = \alpha_j(T_{i-1})\varepsilon(t_{i-1})$. Z enačbama (8) in (5) je mogoče modelirati napetostno-deformacijska stanja tudi v primeru naključno spreminjajočih se temperatur, saj temperaturno spremenjen operator ohlapa vedno zagotavlja ravnotežje v vseh odsekih vzmet – drsnik.

Ker so ponovitvene napetostno-deformacijske krivulje znane, je Prandtlove gostote mogoče izračunati vnaprej. Naj bo $n_\mathrm{q}+1$ navideznih drsnih deformacij enakomerno razporejenih med ničelno in največjo pričakovano amplitudo deformacije (sl. 3). Za vsako navidezno drsno deformacijo q_j v območju $j = 0,\ldots,n_\mathrm{q}$ in vsako temperaturo T_k v območju $k = 0,\ldots,n_\mathrm{T}$ je z en. (1) mogoče izračunati deformacije $\varepsilon_j(T_k)$ in napetosti $\sigma_j(T_k)$. Prandtlove gostote izračunamo tako, da v en. (5) vstavimo napetosti:

as $\sigma_j(t_{i-1}) = \alpha_j(T_{i-1})\varepsilon(t_{i-1})$. When Eq. (8) is inserted into Eq. (5), the temperature-dependent stress-strain behaviour can be modelled, since the temperature-modified play operator guarantees the equilibrium in the spring-slider segments at any time and temperature.

As the temperature-dependent cyclic stress-strain curves are known, the Prandtl densities can be precalculated. Let us disperse $n_\mathrm{q}+1$ fictive yield strains equidistantly between the zero strain and the maximum expected strain amplitude (Fig. 3). Thus, for each fictive yield strain q_j in the range $j = 0,\ldots,n_\mathrm{q}$ and each temperature T_k in the range $k = 0,\ldots,n_\mathrm{T}$, the strain $\varepsilon_j(T_k)$ and the stress $\sigma_j(T_k)$ can be determined from Eq. (1). By inserting q_j and $\sigma_j(T_k)$ into Eq. (5) the Prandtl densities are obtained:

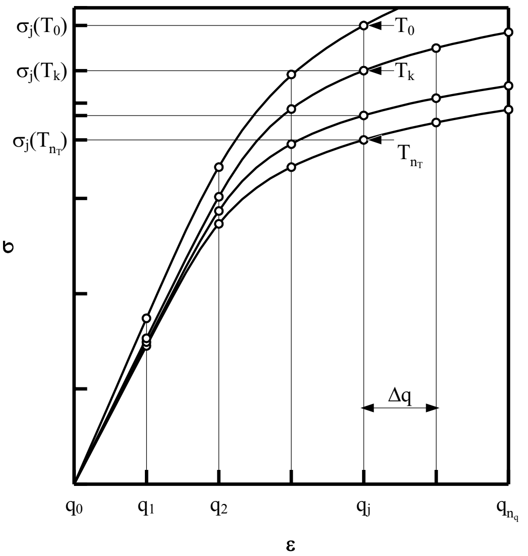
$$\alpha_j(T_k) = \frac{1}{\Delta q}(\sigma_{j+1}(T_k) - 2\sigma_j(T_k) + \sigma_{j-1}(T_k)) \quad j = 0,\ldots,n_\mathrm{q} \quad k = 0,\ldots,n_\mathrm{T} \quad (9),$$

kjer je $\sigma_{-1}(T_k) = \sigma_0(T_k) = 0$, širina razreda navidezne drsne deformacije pa je Δq.

Prandtlove gostote je pametno izračunati vnaprej, jih shraniti v preglednico rešitev ter šele nato modelirati napetostno-deformacijska stanja z enačbama (8) in (5). Tako ohranimo veliko hitrost računanja.

where $\sigma_{-1}(T_k) = \sigma_0(T_k) = 0$ and the fictive yield strain class width is Δq.

The Prandtl densities can be precalculated and stored in a table before the stress-strain-temperature trajectory modelling using Eqs. (8) and (5) starts. In this way a high computational speed is preserved.



Sl. 3. *Temperaturno odvisne napetostno-deformacijske krivulje*
Fig. 3. *Temperature dependent cyclic stress-strain curves*

3 POSLEDICE IZLOČANJA KONIC SUNKOV

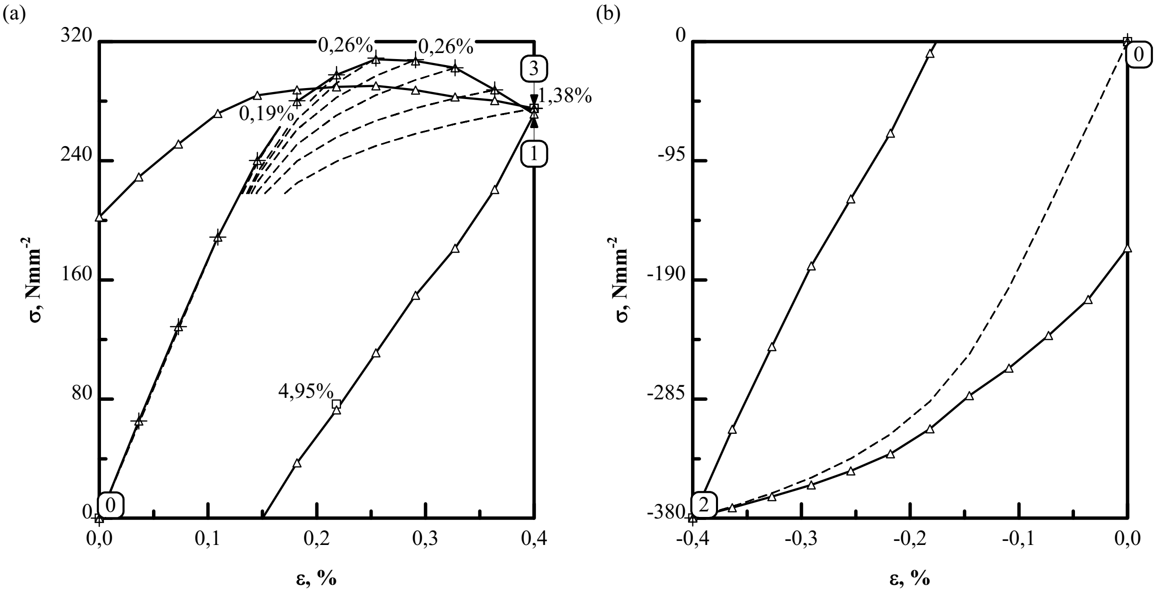
Če so materialne lastnosti časovno neodvisne, temperature pa nespremenljive, lahko poteke obremenitev pred začetkom modeliranja napetostno-deformacijskih stanj zgostimo in tako izločimo le zaporedje konic sunkov brez časovne komponente. Poteke obremenitev pregledujemo sproti in izločamo vse točke, ki niso lokalni ekstremi ter obremenitvene cikle, ki ne prispevajo k zbiranju poškodb [17]. Ker se z zgoščevanjem poteki močno skrajšajo, je deformacijski postopek numerično bistveno hitrejši od zahtevnejših postopkov in se zato tudi pogosto uporablja.

Če se temperatura med posameznimi konicami sunkov spreminja, lega naslednje konice ni več odvisna le od lege predhodne konice, ampak tudi od prehoda med njima. Slika 4 prikazuje vpliv spreminjanja temperature na krivuljo σ-ε za zlitino 9Cr2Mo, ki se začne v točki 0 ($T_0 = 270$ °C), gre skozi točki 1 ($T_1 = 570$ °C) in 2 ($T_2 = 270$ °C) ter se konča v točki 3 ($T_3 = 570$ °C). Predpostavimo, da sta poteka deformacij in temperatur nezgoščeni in izračunajmo krivuljo napetost-deformacija (glej debelo polno črto) po enačbah iz prejšnjega poglavja. En. (8) zagotavlja ravnotežje v odsekih vzmet – drsnik v vseh točkah, označenih s trikotniki. Izkaže se, da se izračunane napetosti ne

3 CONSEQUENCES OF REVERSAL POINT FILTERING

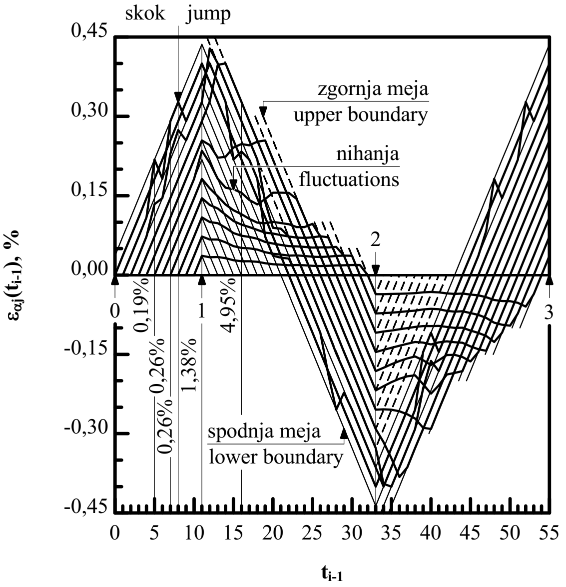
For rate-independent material behaviour and constant temperature, only the reversal points in the load histories are needed for stress-strain trajectory modelling and cycle counting. Load histories can thus be compressed and the time component can be eliminated. The load histories are scanned online and any point that is not the reversal point is discharged from the history. Similarly, all reversal points corresponding to a change in amplitude smaller than a certain threshold are filtered [17]. As history compression results in a considerable reduction of history lengths the strain-life approach is much faster than competing ones. For this reason it has been used frequently.

However, if the temperature changes between reversal points, the position of the next point does not depend only on the current point, but also on the transition between the points. Fig. 4 shows the influence of temperature variations on the σ-ε trajectory for a 9Cr2Mo alloy. It starts at point 0 ($T_0 = 270$ °C), goes through points 1 ($T_1 = 570$ °C) and 2 ($T_2 = 270$ °C) and ends at point 3 ($T_3 = 570$ °C). Let us suppose that the strain and temperature histories are not filtered and calculate the stress-strain-temperature trajectory as explained in the preceding section (the thick solid line). Eq. (8) ensures equilibrium in the spring-slider model at any point indicated by triangle markers. It has been observed that the stresses do not necessarily coincide

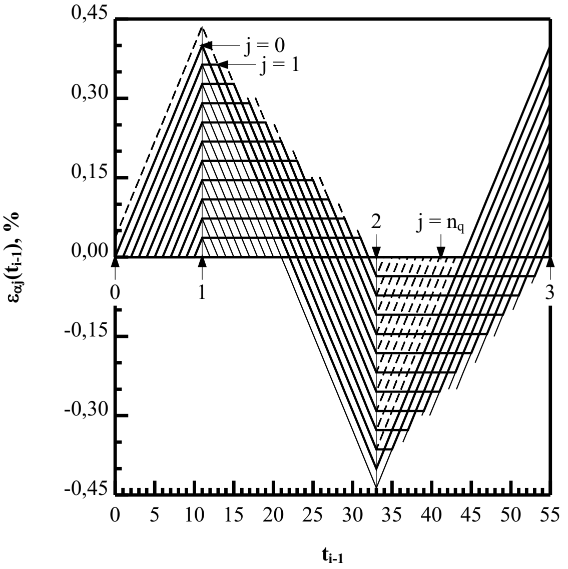


Sl. 4. *Vpliv temperature na spremembe σ-ε krivulj*

Fig. 4. *Influence of temperature variations on σ-ε trajectory*



Sl. 5. *Spominske točke pri spremenljivi temperaturi*
Fig. 5. *Memory point history at variable temperature*



Sl. 6. *Spominske točke pri nespremenljivi temperaturi*
Fig. 6. *Memory point history at constant temperature*

ujemajo vedno z napetostmi na izotermnih napetostno-deformacijskih krivuljah (črtkane črte s simboli v obliki križcev). S tem je dokazan vpliv prehodov med konicami sunkov na krivuljo σ-ε. Podobna odstopanja se pojavijo v primeru zgoščenih potekov obremenitev, kjer se lega točk v ravnini σ-ε računa le za konice sunkov (kvadratni simboli). Odstopanja so odvisna od spominskih točk in temperaturnih sprememb med konicami sunkov.

Na slikah 5 in 6 so prikazani poteki spominskih točk za $j = 0,\ldots,n_q$ operatorjev ohlapa (debele polne črte), omejenih z zgornjimi (tanke črtkane črte) in spodnjimi (tanke polne črte) mejami. Točke 0 do 3 ustrezajo konicam sunkov. Če je temperatura $T_0 = \cdots = T_3 = 570\,°\mathrm{C}$, se spominske točke vedno ujemajo z eno od meja oziroma ostajajo nespremenjene (sl. 6). Če pa se temperatura spreminja (sl. 5), lahko spominske točke preskočijo z ene meje na drugo ali pa med njimi nihajo. To je torej razlog za odstopanja napetosti.

S slike 5 je razvidno, da so odstopanja neodvisna od dolžine krivulje σ-ε. Ko ε spremeni smer in se začno spominske točke približevati spodnji ali zgornji meji, se začno odstopanja zmanjševati. Ker ostajajo odstopanja po vrednosti vedno omejena, je izločanje konic sunkov sprejemljivo tudi v primeru spremenljivih temperatur.

with those on the isothermal cyclic stress-strain curves (the dashed lines with cross markers). This way the influence of transitions between reversal points on the σ-ε curve is proved. Similar deviations are observed in the case of the compressed load histories, where the point positions in the σ-ε plane are calculated at load reversals only (square markers). The deviations depend on the memory points and the temperature variations between the reversal points.

In Figs. 5 and 6 the memory-point histories for the $j = 0,\ldots,n_q$ play operators (thick solid lines) bounded by their upper (thin dashed line) and lower (thin solid line) boundaries are shown. Points 0 to 3 correspond to the reversal points. If the temperature $T_0 = \cdots = T_3 = 570\,°\mathrm{C}$, the memory points coincide with one of the boundaries or stay constant, as depicted in Fig. 6. However, if the temperature changes (see Fig. 5), the memory points can jump from one boundary to another or fluctuate between the boundaries. This is actually why stress deviations occur.

From Fig. 5 it can be observed that the deviations do not depend on the length of the σ-ε trajectory. Whenever ε changes its direction and memory points start to approach either the lower or upper boundaries, the deviations start to decrease. As deviation are always limited, only reversal points should be processed under variable temperatures, too.

4 PREVERJANJE

Deformacijsko nadzorovan in temperaturno spremenjen model, sestavljen iz vzporedno vezanih odsekov vzmet – drsnik, smo primerjali z več preizkusi TMU, ki jih je opravil Skelton [4] pri 0,6 % celotne deformacije zlitine 9Cr2Mo. Temperaturno odvisni parametri, ki se pojavljajo v Ramberg-Osgoodovi enačbi, podatki o materialu in eksperimentalnih tehnikah so pojasnjeni v [2], [4] in [18].

Pri prikazu poteka deformacij in temperatur običajno nanašamo deformacije na ordinatno, temperature pa na abscisno os [4]. V prispevku sta obravnavani pot PKRM in zahtevna ponovitev (črtkana črta), prikazani na sliki 7. Medtem ko predstavlja pot PXRM 45° deltoidni cikel, izveden proti smeri urnega kazalca, je pot PMRK enaka ciklu, izvedenem v smeri urnega kazalca. Podobno velja tudi za 135° deltoidni cikel, omejen s točkami PKRZ. Cikla pravokotne oblike PXRZ in PKRM sta poimenovana 45° in 135° pravokotni cikel. Poti GEHF in GFHE tvorita dvojni termični cikel ter cikel zahtevne oblike je prikazan s črtkano črto.

Izmerjene histerezne zanke so omejene s križci na slikah 8 in 9. Trikotniki označujejo napetostno-deformacijske krivulje, izračunane s Skeltonovim [4] algoritmom, debele polne črte pa krivulje, ocenjene po našem algoritmu.

S slike 8 je razvidno, da je napoved obremenitvenih ciklov, izvedenih v smeri proti urnemu kazalcu, boljša od napovedi ciklov, izvedenih

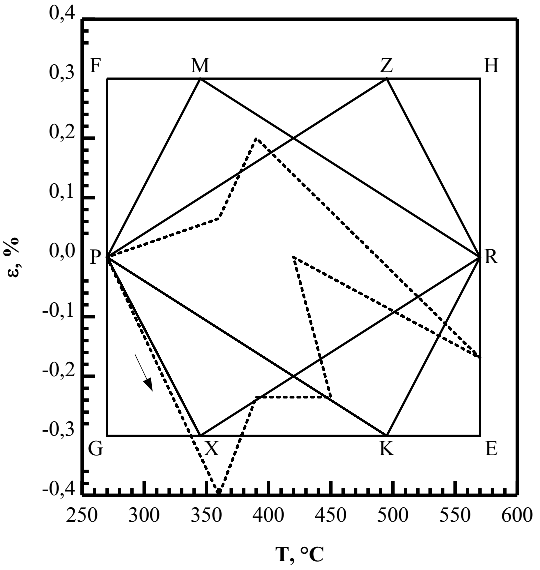
4 VERIFICATION

The strain-controlled and temperature-modified spring-slider model is compared to several TMF tests conducted by Skelton [4] at a total strain range of 0.6% on the 9Cr2Mo alloy. The temperature-dependent parameters associated with the Ramberg-Osgood relation as well as the material data and experimental methods are given in [2], [4] and [18].

In deciding the temperature-strain path, the convention is to plot strain on the vertical axis and temperature on the horizontal axis [4]. The paper concerns the path PKRM and the complex cycle (dashed line) given in Fig. 7. The path PXRM represents a 45° kite cycle performed in an anticlockwise way while the corresponding clockwise cycle is taken in the PMRX order. A similar scheme applies in the 135° kite cycle PKRZ. The parallelogram-shaped PXRZ and PKRM are labelled as the 45° zero strain and the 135° zero strain respectively. Finally, the anticlockwise GEHF and the clockwise GFHE bi-thermal cycles as well as the complex cycle given in the dashed line are considered.

The observed hysteresis loops from the tests are plotted as crosses in Figs. 8 and 9. The triangle markers and the thick solid lines denote the stress-strain trajectories modelled by the Skelton [4] and our algorithm respectively.

It is clear that in Fig. 8 the shapes of the anticlockwise loops are predicted well, whereas for



Sl. 7. *Vrste TMU ciklov*
Fig. 7. *TMF cycle types*

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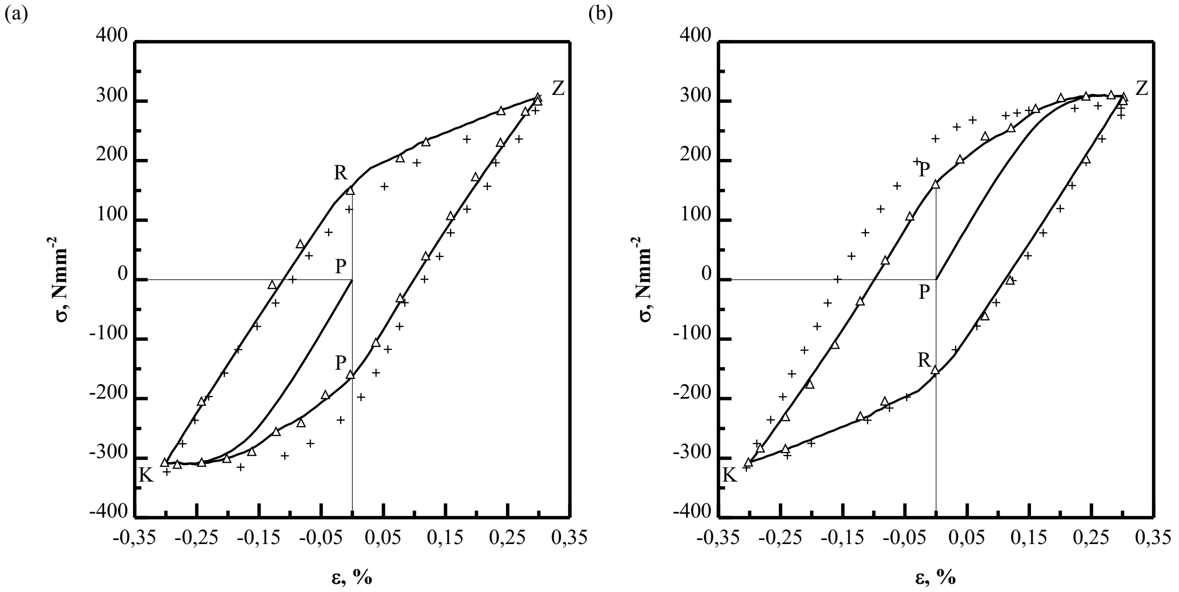
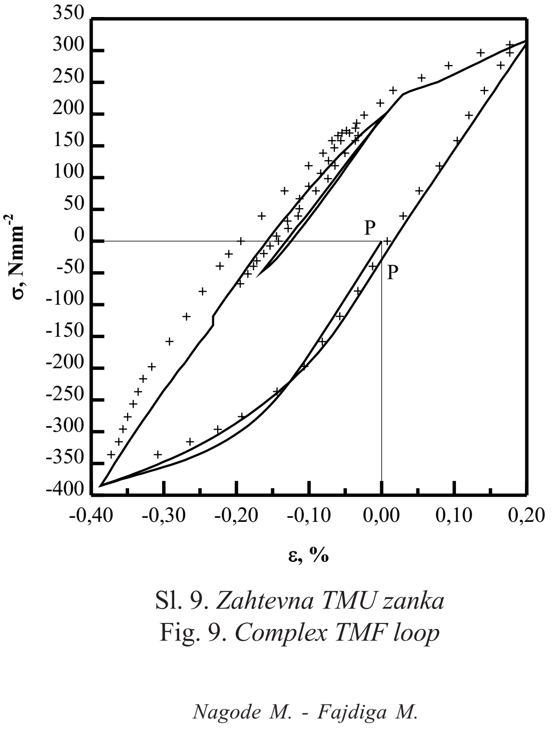


Sl. 8. *TMU 135° zanka (a) proti smeri urnega kazalca (b) v smeri urnega kazalca*
Fig. 8. *TMF loop 135° kite (a) anticlockwise, (b) clockwise*

v smeri urnega kazalca. Pri slednjih so testne histerezne zanke širše od napovedanih. Širše histerezne zanke niso le posledica nelinearnega ohlajanja, pač pa predvsem nekaterih parametrov, ki doslej še niso bili prepoznani. Ujemanje med Skeltonovimi in našimi rezultati je skoraj popolno. Postopek je bil nazadnje uspešno preverjen na ciklu zahtevne oblike, prikazani na sliki 9.

the clockwise style tests the observed loops are wider. The wider loops do not appear just because of non-linear cooling. Consequently, some parameters undefined so far seem to be more influential. The agreement between the Skelton [4] and our algorithm is almost perfect. The method was finally applied to the complex cycle of Fig. 9 successfully.



Sl. 9. *Zahtevna TMU zanka*
Fig. 9. *Complex TMF loop*

5 SKLEP

Deformacijski postopek je alternativa napetostnemu postopku napovedovanja utrujenostne poškodbe, ki se uporablja predvsem, če se pojavi tečenje. V prispevku je predstavljen in z več TMU preizkusi preverjen temperaturno spremenjen deformacijski postopek napovedovanja napetostno-deformacijskih stanj. Razvit in preverjen je bil deformacijsko nadzorovani model, sestavljen iz vzporedno vezanih odsekov vzmet – drsnik, ki omogoča modeliranje lokalnih temperaturno-napetostno-deformacijskih stanj. Za primer napetostnega nadzora je bil v [12] razvit enakovreden model. Izločanje konic sunkov je za elastoplastična gradiva sprejemljivo. Velika hitrost računanja običajnega deformacijskega postopka se ohrani. Razviti model je formalno dobro definiran z modelom Prandtlovega tipa in pomeni posplošenje Skeltonovih [4] izsledkov. Čeprav predlagani model ne omogoča napovedovanja lezenja in nekaterih drugih pojavov, lahko dosežemo izboljšano napoved temperaturno-napetostno-deformacijskih stanj na podlagi standardnih MCV preizkusov.

5 CONCLUSIONS

The strain-life approach is an alternative to the stress-life approach for determining fatigue damage, particularly when yielding occurs. An online temperature-modified strain-life approach has been developed and verified through several TMF tests. The spring-slider model has been developed and verified to enable local temperature-stress-strain trajectory modelling. The model is applicable if the strain is controlled or else the model given in [12] can be used. Reversal point filtering turned out to be acceptable for elasto-plastic hardening solids. The high computational speed of the classical strain-life approach can thus be preserved. The developed model is a generalization of the Skelton [4] findings and is formally well defined through an operator of the Prandtl type. Although neither creep damage nor some other effects can be assessed by the proposed approach, an improved temperature-stress-strain state prediction based on standardized LCF tests is possible.

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