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Online Formative Assessment in Mathematics Education: Prospective Primary Teachers' Understanding of Rational Numbers

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Abstract/Izvleček This study sheds light on prospective primary teachers' understanding of rational numbers. This is an experimental study that aims to evaluate prospective primary teachers' mathematical and didactic knowledge of rational numbers through online formative assessment. The participants were 38 prospective primary teachers from a primary teacher education study program at a public university in Pekanbaru, Riau, Indonesia. This study indicates that the prospective teachers have insufficient knowledge of rational numbers, and they possess better mathematical than didactic knowledge. This study also reveals a significant increase in prospective teachers' mathematical and didactic scores from the first test to the second test.

Keywords:
mathematical and didactic knowledge; online formative assessment; rational numbers; teacher knowledge

Spletno formativno ocenjevanje pri pouku matematike: Kako bodoči osnovnošolski učitelji razumejo racionalna števila?

V članku se osredinjamo na vprašanje, kako bodoči osnovnošolski učitelji dojemajo racionalna števila. Predstavljena je eksperimentalna raziskava, katere osnovni namen je oceniti matematično in didaktično znanje bodočih učiteljev na področju racionalnih števil pri spletnem formativnem ocenjevanju. Udeleženci v raziskavi so bodoči osnovnošolski učitelji (n = 38) na študijskem programu izobraževanja učiteljev na javni univerzi v Pekanbaruju, Riau, Indonezija, in sicer na programu matematika za višje razrede osnovne šole. Rezultati raziskave kažejo, da izkazujejo bodoči osnovnošolski učitelji pomanjkljivo znanje o racionalnih številih ter da njihovo didaktično znanje presega njihove matematične zmožnosti. Obenem raziskava kaže pomembne razlike v matematičnem in didaktičnem znanju pred testom in po njem.

Ključne besede:
matematično in didaktično znanje; spletno formativno ocenjevanje; racionalna števila; znanje učiteljev

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Introduction

Digital technology has a significant role in learning and teaching in the 21st century, especially during the Coronavirus disease (COVID-19) outbreak. The role ranges from learning management (Hoyos, 2012) to implementing online assessments or tests (Drijvers et al., 2014). Many studies have shown that teachers and students have a positive attitude toward using digital technology in education (Afify, 2019; Drijvers et al., 2014; Jupri, Drijvers, and van den Heuvel-Panhuizen, 2015; Putra, Witri, and Yulita, 2019). The exploitation of technology for online assessment, when used effectively, can be particularly valuable in the assessment of 21st-century learning (Boitshwarelo et al., 2017). Online assessment in mathematics is a challenge because mathematics learning involves not only texts but also mathematical symbols. A study conducted by Drijvers et al. (2014) has confirmed that students who studied and did a test using an online algebra environment scored slightly below students benefiting from offline learning. One of the main factors involved was the schools' experience with digital technology in learning and teaching. Nevertheless, online assessment is highly efficient, fast and reliable, and beneficial for many students (Gipps, 2005) because digital technology can record student answers and give feedback directly. Teacher feedback is essential for student learning, as students can obtain rapid, specific feedback on their performance (Ho et al., 2018). In addition, students will likely be encouraged to employ online peer and collaborative evaluations (Gipps, 2005). In this study, we are interested in investigating how online assessment is applied in mathematics education, especially in primary teacher education institutions. This study focuses on prospective primary teachers' knowledge of rational numbers because this topic is complex for students to learn and for teachers to understand and teach (Siegler and Lortie-Forgues, 2017).

Online assessment in mathematics education

In 2003, Graff (2003) conducted a study on using online learning and assessment methods. He defined online assessment as a method of using computers to deliver and analyse tests or exams, and it has been used since the 1970s. The advantage of online assessment is that it is possible to give students immediate feedback on their understanding, especially in large classes (Pezzino, 2018).

Online assessment has also been used in mathematics education (Brouwer et al., 2009; Drijvers et al., 2014; Pezzino, 2018).

Brouwer et al. (2009) conducted a study focusing on using frequent online assessments powered by a mathematical engine named Maple TA. They found that students in the bachelor courses in mathematics responded positively to online assessment, and the use of online assessment tools was straightforward. Furthermore, the interactive online assessment programme in mathematical exercises also provided rapid feedback (Brouwer et al., 2009). Similar results were also found in a study by Pezzino (2018): the students were very impressed with online assessment because they received feedback and support. Moreover, online Maple TA-based assessment could have been the only factor that helped students considerably enhance their performance level in class (Pezzino, 2018). In general, there are two types of assessment (Graff, 2003), and these also apply to online assessment in mathematics education. The first of these, formative assessment, is defined as an evaluation conducted during the course presented as a means of monitoring student learning. Formative assessment has some benefits, such as monitoring the learning experience and intervening early, learning from errors, offering improvement suggestions, and meeting individual needs (Dopper and Sjoer, 2004; Pastor, 2011). The second type, summative assessment, aims to determine examination outcomes, and it is mainly given to students at the end of a period of study. Summative assessment tests whether the predetermined learning results align with the programmed objectives (Mohamadi, 2018). The present study focuses on the online formative assessment of rational numbers given to prospective primary teachers during their studies because this tool has much to offer in terms of improving student learning experience when some factors are considered, such as feedback on each item and a guided student learning process (Dopper and Sjoer, 2004). The online formative assessment in this study aims to monitor prospective primary teachers' understanding of rational numbers. This topic is part of a whole course of mathematics education for primary schools in upper grades. With the advancement of digital technology in education, students can receive feedback from teachers to improve their learning process and develop a better understanding of knowledge.

Teachers' knowledge of rational numbers

Teachers' knowledge of rational numbers has been the main focus of many studies in mathematics education (Browning, Edson, Kimani, and Aslan-Tutak, 2014; Depaepe et al., 2015; Ma, 1999; Newton, 2008; Putra, 2019a, 2019b; van Steenbrugge, Lesage, Valcke, and Desoete, 2014).

Most studies have shown that teachers have difficulty with and misconceptions about rational numbers and their operations. For example, a seminal work by Ma (1999) revealed that teachers in the USA have struggled to explain the meaning of the division of fractions. Ma (1999) identified teachers' difficulties in both subject-matter knowledge of dividing fractions and knowledge of how to teach this topic. Similarly, Güler and Çelik (2019) revealed that prospective teachers performed well in evaluating what misconceptions students may have shared in the scenario. However, most of them did not perform well in delivering content in terms of educational strategies. These findings also apply to the Indonesian perspective and in-service teachers' knowledge of rational numbers (Putra, 2018, 2019b).

Two main issues underlie students' and teachers' inadequate understanding of rational number arithmetic: inherent sources and culturally contingent issues (Siegler and Lortie-Forgues, 2017). Concerning the first issue, inherent sources of difficulty relate to students' understanding of individual rational numbers, the relationship between rational and whole number arithmetic, and the relationships among rational number operations. For instance, to add two rational numbers, $\frac{1}{2}+\frac{1}{4}$, a student cannot directly apply a procedure of adding both numbers based on their position, but he/she needs to change them into the same denominator. Siegler and Lortie-Forgues (2017) pointed out that those are independent of the educational system and the community in which learners live. On the other hand, culturally contingent sources of difficulty differ depending on the particulars of student lives. Those cultural contingent sources include teachers' knowledge, textbooks, and language. For instance, Indonesian school textbooks provide only about 10% context-based tasks (Wijaya et al., 2015), so explaining the meaning of partitive reasoning of fraction division using contextual situations could be a challenge for many teachers in Indonesia because they are not as familiar with what appears in the textbook.

Concerning inherent difficulties, many teachers tend to teach students based on a single meaning of rational numbers, namely a part-whole relationship (Putra, 2018).

Other meanings of rational numbers, such as ratio, operator, quotient, and measurement (Charalambous and Pitta-Pantazi, 2007), are rarely given attention by teachers in teaching fractions. The meaning of rational numbers as quotient, for instance, will lead students to understand that a/b is a single number, so they can use this understanding to solve arithmetic operations of rational numbers. Students may understand why adding two fractions cannot be done based on their positions instead of changing fractions into fractions with a common denominator and adding numerators.

Understanding individual rational numbers leads students to know the relations between rational and whole number arithmetic and the relations among rational numbers.

The second issue of students' and teachers' understanding of rational numbers is culturally contingent. Teacher knowledge is the central aspect of this difficulty. Many teachers build their knowledge based on their learning experiences in primary schools and instruction developed in pedagogical institutions (Putra, 2019b, 2019a). For example, some studies in the literature show that many prospective teachers struggled to explain the meaning of multiplication and division of fractions when they graduated from teacher education programs (e.g. Putra, 2018, 2019b). Another example is to figure out the density of rational numbers, many prospective teachers provided different answers to find how many numbers are between 2/5 and 4/5 and how many numbers are between 0.4 and 0.8. Most prospective primary teachers did not realise that the two tasks were the same (Putra, 2019b). Mathematics textbooks constitute another cultural contingent because these provides fewer opportunity-to-learn, context-based tasks for students (Wijaya et al., 2015) and pay more attention to procedural tasks and techniques. In addition, the language used in learning fractions in primary school in Indonesia also becomes culturally contingent. For example, a fraction is always interpreted as a part of a whole, such as a fraction of 1/3 in Indonesian, 'one-third' translating into one out of three parts.

Teachers' difficulties and challenges provide open questions for many researchers to investigate. In the present study, we focus on prospective teachers' understanding of rational numbers and their operations. This study aims to reveal the extent to which the educational system and the culture within which prospective teachers live affect their understanding of rational numbers. Specifically, we formulate the research question for this study as "To what extent is prospective teachers' understanding of rational numbers assessed through online formative assessment?"

Research Methodology

Participants

Participants were second-year prospective primary teachers (2 men and 36 women) from a primary teacher education study program at a public university in Pekanbaru, Riau, Indonesia. All participants were in the even semester of the 2019/2020 academic year, and they were taking a course on mathematics education for the upper grades of primary school to teach mathematics from grades 4 to 6.

For half a semester, the mathematics instruction focused on fractions and measurements. It subsequently continued with didactic projects related to designing media and tools in mathematics, task design, lesson plans, and teaching scenarios.

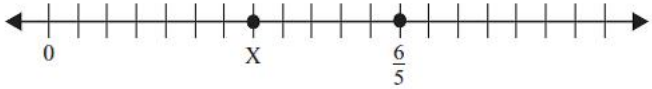
Those participating in this study had taken two foundational courses: mathematics and mathematics education. The mathematics content of these courses included numbers, algebra, geometry, measurement, and data. Meanwhile, the content of the course on mathematics education was to develop learning instruction on teaching mathematics for lower primary school grades.

Task

The tasks comprise two types of problems: mathematics and didactics. The mathematical tasks focus on assessing prospective mathematical knowledge of rational numbers. The didactic tasks aim to evaluate the prospective construction of contextual problems posed by rational numbers. There are eight mathematical tasks and two didactic tasks. The mathematical tasks cover two mathematics domains, namely the structure and operations of rational numbers. Each task requires prospective teachers to complete the task and to provide their mathematical reasoning behind it.

The mathematical tasks about the structure of rational numbers consist of four tasks: positioning a fraction on a number line, structuring the set of rational numbers, and comparing fractions and equivalent fractions. The tasks of positioning a fraction on a number line and equivalent fractions are presented using figures, and the task of comparing fractions is presented using a real-life context. Figure 1 illustrates the mathematical task of positioning a fraction on a number line (Figure 1).

Write an appropriate fraction for X.

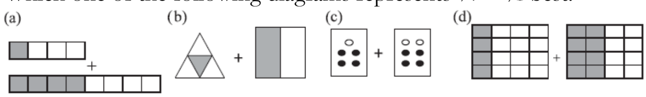


Give reason for the answers.

Figure 1. A mathematical task of positioning a fraction on a number line

The mathematical tasks about operations of rational numbers also consist of four tasks: fraction addition, two fraction division tasks, and a mixed fraction operation. Three tasks are given within real-life contents, and the other task about fraction addition is presented using diagram representation. Figure 2 presents the task of fraction addition.

Which one of the following diagrams represents ¼ + ½ best?



Give reason for the answers.

Figure 2. A mathematical task of fraction addition (Li, 2014)

The didactic tasks ask prospective teachers to pose a real-life/contextual situation for a given mathematical operation. For example, the first problem-posing task is fraction multiplication, while the other is about fraction division. We illustrate the task of fraction division in Figure 3.

Pose a real-life /contextual situation to the following fraction division

$$\frac{1}{3} \div \frac{1}{2}$$

Figure 3. A didactic task of fraction division

The tasks presented in this study are limited, especially the didactic problems, but they are sufficient to accommodate what is in the primary school curriculum for the upper classes (Kemdikbud, 2020).

For instance, student ability to solve problems related to fraction operations becomes one of the core competences for sixth-grade students. They are expected to achieve this goal by the end of the grade.

Procedures and Process

All tasks were presented in a learning management system called Schoology. The participants completed the first test before they had online learning instruction about rational numbers. Then, the participants retook the test a month later to evaluate their understanding of consistency about rational numbers. Between these two tests, the participants attended four meetings, consisting of investigation of rational number concepts from the academic journal (e.g., Castro-Rodríguez et al., 2016; Vamvakoussi and Vosniadou, 2004), designing learning instruction for teaching fractions, and discussion in small groups, as well as online classroom discussion.

Finally, the tasks were assessed by prospective teachers randomly. These tasks aim to avoid cheating on the tests, and the time to complete the tests was 75 minutes.

The three sets of online learning instruction about fractions provided prospective teachers with several learning activities and materials. Before attending the synchronous lectures, prospective teachers were asked to study a module and some scientific articles about fractions. For instance, the main topic for the first meeting was about sub constructs of fractions; therefore, the participants were asked to read two scientific articles about this topic, and one of them was an article written by Charalambous and Pitta-Pantazi (2007) about drawing on a theoretical model to study students' understanding of fractions. Then, the synchronised lectures were conducted using Skype, during which the lecturer and the participants discussed the concept of sub constructs of fractions. In the second meeting, the prospective teachers worked in small groups (3 to 4 participants) to design a learning scenario of fractions for primary school students. The design was then uploaded to Schoology and reviewed by the lecturer. In the next meeting of the synchronised lecture, some groups presented their design learning scenarios and received feedback from the lecturer and other participants.

Prospective teachers' answers for each of the ten tasks were evaluated for correctness. Each answer to mathematical tasks was scored 0, 1, or 2: score 0 for an incorrect answer to both the mathematical problem and its reason, score 1 for a partially correct answer such as a correct answer of the mathematical problem but no reason or incorrect reason, and score 2 for getting both answer and reason correct. Each answer on the didactic tasks was also scored 0, 1 or 2: score 0 for an incorrect answer, score 1 for a partly correct answer such as faulty units or meaningless context, and scoring 2 for an entirely correct answer. The first authors coded all prospective teachers' answers. Coding reliability was checked through additional coding by the third author, who coded a random range of 60% of the answers. The inter-rate reliability of this coding resulted in a value of .82. These findings suggest that the coding was reliable.

Results

The results for prospective teachers' understanding of rational numbers are presented in Figure 4. The prospective teachers have better mathematical than didactic knowledge. Their average score on the first test was less than half the answers correct for mathematical and didactic tasks.

The average score increased on the second test, but it was still less than 75% correct answers. We noticed that prospective teachers had more difficulty with the didactic tasks than with the mathematical tasks. A paired-sample *t*-test revealed that the differences were statistically significantly different between prospective teachers' mathematical and didactic knowledge on the two tests [first test, $t\ (1,37) = 2.305$; $p < .05$; and second test, $t\ (1,37) = 3.293$; $p < .05$]. Their performance improved significantly on both mathematical [$t\ (1,37) = 7.134$; $p < .001$] and didactic tasks [$t\ (1,37) = 3.939$; $p < .001$]. Their cumulative performance increased from test 1 ($M = .91$; $SD = .47$) to test 2 ($M = 1.30$; $SD = .52$), and it was statistically significant [$t\ (1,37) = 5.938$; $p < .001$].

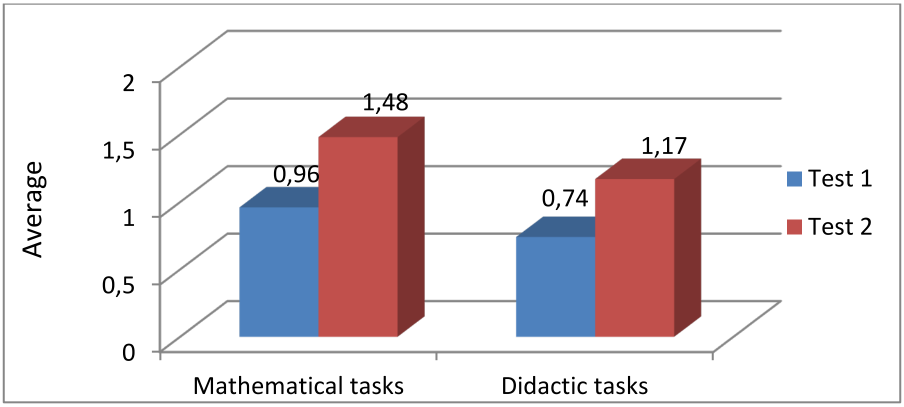


Figure 4. Prospective teachers' average scores on mathematical and didactic tasks

Prospective teachers' performance was moderate on the mathematical tasks concerning the structure of rational numbers (Figure 5). The average score increased by .30 from test 1($M = 1.22$; $SD = .53$) to test 2 ($M = 1.52$; $SD = .56$). A paired-sample *t*-test revealed that the difference was statistically significantly different between test 1 and test 2 ($t(1,37) = 4.071$; $p < .001$). We noticed that prospective teachers had more difficulty with the mathematical task of comparing fractions than with the others. However, their performance increased significantly on the mathematical task of positioning a fraction on a number line after participating in four meetings regarding understanding fractions and constructing a learning instruction for teaching fractions to students.

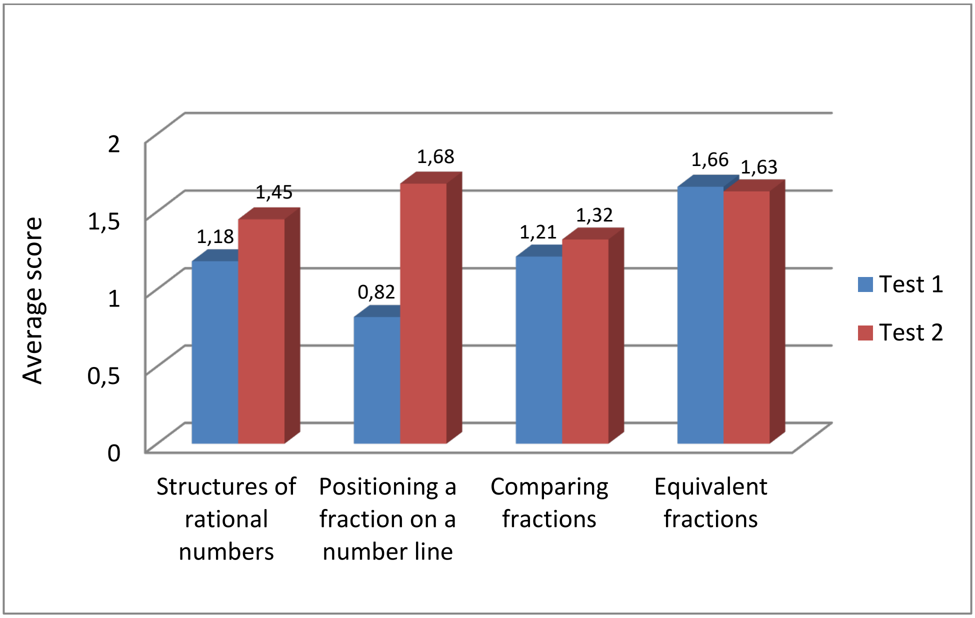


Figure 5. Prospective teachers' average scores on the structure of rational numbers

Among the four mathematical tasks about the structure of rational numbers, positioning a fraction on a number line (Figure 1) was the most challenging task in the first test, where the average score for this task was below 1 point (Figure 5). This score means that many prospective teachers could not find a fraction shown by X on a number line. For example, one of the prospective teachers, coded as PST23, answered that X was equal to 1/5 because the position of X was on the left side of 6/5, so the numerator was getting smaller. She counted the numerator backwards from 6 to 1, from 6/5 to X. She did not become aware that the distance between 0 and X should be longer than that between X and 6/5. However, she completed the same task correctly on test 2. She wrote, "6/5 = 1.2. When we look at the line, 6/5 is in the 12th line, so 12/10 = 1.2. We can also change it to 120% when X is in the 7th line, which equals 7/10=0.7. So, X equals 7/10 or 0.7 or 70%" (Figure 6). Her answer on test 2 was correct, and she could also give several representations for the same rational number

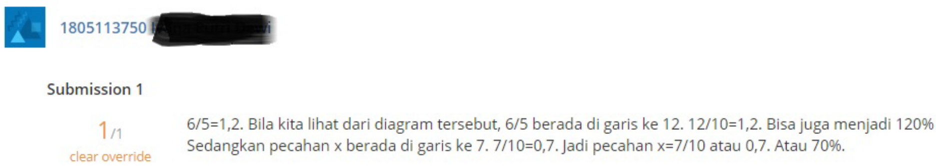


Figure 6. PST23's answer on the second test

Prospective teachers' performance was poor on the operation of fractions on test 1 (M = .70; SD = .49) but significantly increased on test 2 (M = 1.44; SD = .68) (Figure 7). A paired-sample *t*-test revealed that the differences were statistically and significantly different between test 1 and test 2 ($t(1,37) = 7.945$; $p < .001$). We noticed that the prospective teachers had more difficulty with the mathematical task of mixed fraction operation, and their performance significantly increased on fraction addition.

Surprisingly, the prospective teachers performed much better on fraction division than on the other two tasks. To show why this could be the case, we present an example of prospective teachers' answers to the task of fraction addition (Figure 2). To the first test, PST10 answered the correct representation for ¼ + ½ is answer b. She argued, "the shaded area on the triangle shows ¼, which is one shaded part from 4 triangles, and the shaded area of the rectangle shows 1/2, which is one shaded part from 2 rectangles". She was not aware that the two diagrams were in different shapes. However, she could give a correct answer on the second test. She answered that the correct representation for ¼ + ½ was D because ¼ is half of ½, and the two fractions were represented in the same shapes. So, diagram D represents a fraction of 4/16 equals ¼ and a fraction of 8/6 equals ½. This answer indicated that she was aware of the need for the same diagram to represent adding two fractions.

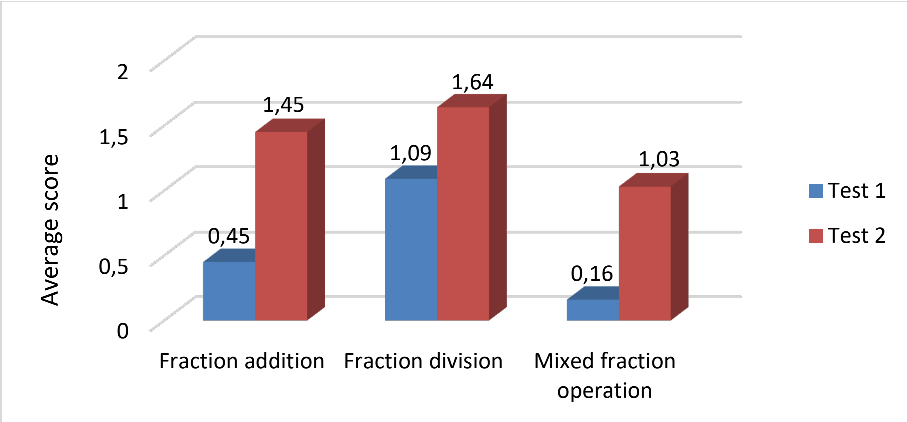


Figure 7. Prospective teachers' average scores on the operation of rational numbers

The prospective teachers were found to perform poorly on didactic tasks (Figure 8). The average score increased by 34 from test 1(M = .74; SD = .73) to test 2 (M = 1.17; SD = .74). A paired-samples T-test revealed that the differences were statistically significantly different between test 1 and test 2 (t(1,37) = 3.939; p < .001). We noticed that the prospective teachers had more difficulty with the didactic task of problem posing on fraction division than on fraction multiplication.

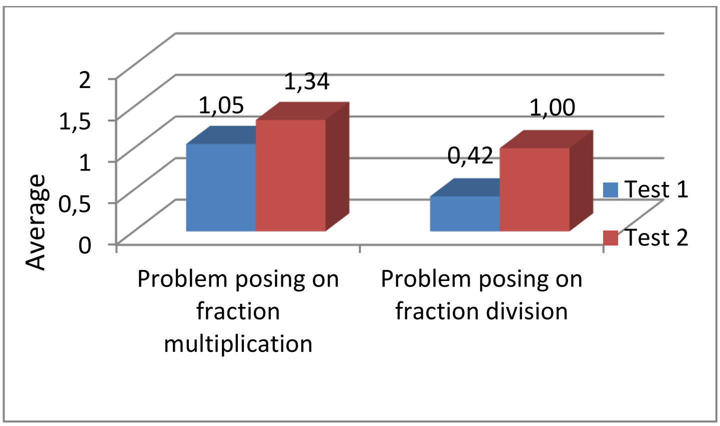


Figure 8. Prospective teachers' average scores on problem-posing tasks

To illustrate the prospective teachers' didactic knowledge of rational numbers, we present an example of one prospective teacher's response to posing a contextual problem on fraction division. PST20 provided an incorrect answer on the first test. She wrote, "Kiki has 1/3 of a brownie. ½ of the brownie will be shared with her sister. How many brownies will her sister get?" (Figure 9). This contextual task represented a task of fraction multiplication instead of fraction division. In the second test, PST20 could give the correct answer. She represented the task using partitive division. She wrote "Dodo has 1/3 kg of rice in his kitchen. Then, the rice is put in a plastic container measuring ½ kg of rice. How much of the container is filled with rice?" (Figure 10). We also found that some prospective teachers, given the correct answers on the second test, represented the division task based on a ratio. For instance, PST18 wrote, "A mother has 1/3 kg of sugar to make a cake. Suppose the cake recipe requires ½ kg of sugar. How many cakes can she make?" (Figure 11).

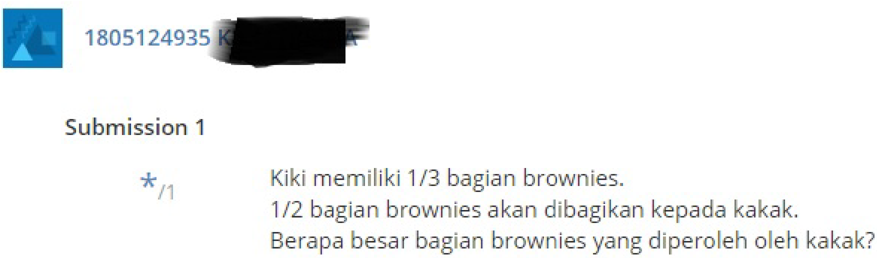


Figure 9. PST20's answer on the first test of the fraction division task

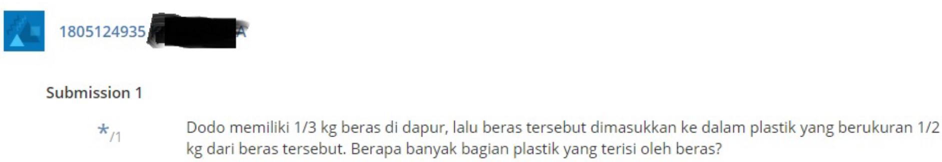


Figure 10. PST20's answer on the second test of the fraction division task

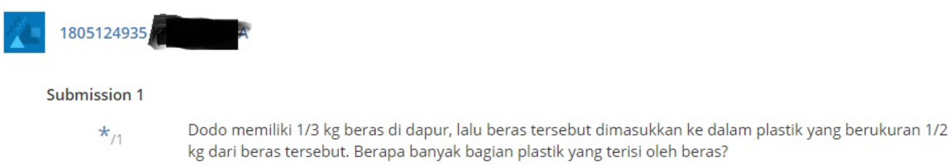


Figure 11. PST18's answer on the second test of the fraction division task

Discussion

This study sought to investigate prospective primary teachers' understanding of rational numbers through a formative online assessment. The study focuses on their mathematical and didactic knowledge of rational numbers. The results indicate that prospective teachers improved their mathematical and didactic knowledge of rational numbers during the study.

Concerning mathematical knowledge, the mean score was only half the answers correct on the first test and three-quarters of the answers correct on the second test, even after they had received instruction on the structure and operation of rational numbers. This finding is in line with the study conducted by Depaepe et al. (2015). They found that prospective teachers have limited mathematical knowledge even though they have received instruction on rational numbers during their training. Therefore, it is suggested that the mathematical content of the course should emphasize conceptual and procedural knowledge of rational numbers. Meanwhile, prospective teachers' difficulty with rational numbers could be caused by their understanding of the structure and the operation of rational numbers (Siegler and Lortie-Forgues, 2017). Some prospective teachers know only a single meaning of rational numbers, a part-whole relationship (Putra, 2018). However, this understanding is insufficient to deal with different task types involving rational numbers. For instance, less than half the prospective teachers could place a fraction on a correct number line in the first test because many of them understood a fraction only as part of a whole.

Prospective teachers' didactic knowledge of rational numbers was much more limited than their mathematical knowledge. Although they did improve their performance on the second test, the mean score was just slightly above half the answers correct. This finding is similar to previous studies on teachers' didactic knowledge of rational numbers (Depaepe et al., 2015; Izsák, 2008; Lo and Luo, 2012; Putra, 2019b). Lo and Luo (2012) also found that prospective primary teachers, who are highly proficient at primary and middle school mathematics, have similar challenges with representing fraction division using word problems or pictorial diagrams, which is also the case for this study. Only half the prospective teachers in the second test could construct an appropriate contextual or actual word situation for the division task. Prospective teachers' limited mathematical knowledge could be caused by their limited classroom experience (Depaepe et al., 2015).

As we know, this study's participants were second-year students who lacked classroom experience, especially in teaching rational numbers. However, a previous study conducted by Putra (2018) revealed that final-year prospective teachers also had insufficient mathematical and didactic knowledge of rational numbers. They had received some classroom experience, but this did not guarantee that they had acquired better didactic knowledge. Therefore, training in teaching institutions should undoubtedly address the topic of transforming a mathematical notation or model into a contextual or word problem, and vice versa. The pedagogical quality of teachers' work and student progress is determined by their mathematical and didactic knowledge (Hill et al., 2005). The gaps in prospective teachers' mathematical and didactic knowledge of rational numbers become a challenge for improving teaching quality in primary school. As revealed by this study, many prospective teachers struggled with didactic tasks more than with mathematical tasks. The difference between the two knowledge constructs is statistically significant. Although this study did not examine the relationship between prospective teachers' mathematical and didactic knowledge, some previous studies have revealed that the construction of teachers' didactic knowledge has a significantly positive relationship with their mathematical knowledge (Depaepe et al., 2015; Tirosh, 2000). Therefore, prospective teachers should have sufficient mathematical knowledge for them to develop their didactic knowledge of rational numbers (Depaepe et al., 2015). However, didactic knowledge is much more complex than just mathematical knowledge because teachers need more than just sufficient mathematical knowledge, but also other knowledge, such as pedagogical and technological knowledge, as well as knowledge of students and contexts.

Conclusion

The prospective teachers have insufficient knowledge of rational numbers. Although their knowledge is much better on mathematical tasks than on didactic ones, they still could not achieve a mean score above three-quarters of answers correct on the second test; this is the minimum score for professional teachers in Indonesia (Kemdikbudristek, 2021). These prospective teachers' mathematical knowledge significantly increased in positioning a fraction on a number line and in fraction addition.

These two tasks were not the most difficult, but a lack of conceptual understanding of rational numbers led prospective teachers to incorrect answers and reasoning.
The prospective teachers clearly struggled with didactic tasks, especially in constructing a contextual or real-life situation involving fraction division. Only half the participants could deal with that task. Their difficulty was interpreting the symbolic meaning of division in a real-world situation.
The online formative assessment used in this study revealed a change in prospective teachers' performance on the tests. Their mathematical and didactic knowledge significantly increased on the second test because, between the two tests, they received online intervention on understanding rational number concepts, constructing instructional learning, and online discussion. Familiarity with the use of digital technology during the course and the intervention helped them in adjusting to the process of online formative assessment conducted in this course, especially as an effect of the COVID-19 pandemic. However, we recognise some limitations of this study. First, the problems on the two tests are the same, and that could lie behind the participant' precise performance on the second test. During the intervention, they could discuss the tasks with their peers. We suggest giving different mathematical and didactic tasks for the first and second tests for further study. Second, there were more mathematical tasks than didactic tasks. The limited number of didactic tasks could affect the results for prospective teachers' didactic knowledge. It could be different if the didactic tasks covered both the structure and operation of rational numbers. Lastly, this study was conducted in a class at a teaching institution. A further study needs to consider a control class for assessing the effectiveness of the intervention and the online formative assessment.

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