

Redundantly globally rigid braced triangulations*

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Abstract

By mapping the vertices of a graph G to points in $\mathbb{R}^3$, and its edges to the corresponding line segments, we obtain a three-dimensional realization of G . A realization of G is said to be globally rigid if its edge lengths uniquely determine the realization, up to congruence. The graph G is called globally rigid if every generic three-dimensional realization of G is globally rigid.

We consider global rigidity properties of braced triangulations, which are graphs obtained from maximal planar graphs by adding extra edges, called bracing edges. We show that for every even integer $n \geq 8$ there exist braced triangulations with $3n - 4$ edges which remain globally rigid if an arbitrary edge is deleted from the graph. The bound is best possible. This result gives an affirmative answer to a recent conjecture. We also discuss the connections between our results and a related more general conjecture, due to S. Tanigawa and the third author.

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Keywords: Triangulation, globally rigid graph, braced triangulation, rigidity.

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Redundantno globalno toge oporne triangulacije*

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Povzetek

Če preslikamo vozlišča grafa G v točke prostora $\mathbb{R}^3$, povezave pa v ustrezne daljice, dobimo tridimenzionalno predstavitev grafa G . Predstavitev grafa G se imenuje globalno toga, če njene povezave enolično določajo predstavitev, do skladnosti natančno. Graf G se imenuje globalno tog, če je vsaka njegova generična tridimenzionalna predstavitev globalno toga.

Obravnavamo globalne togostne lastnosti opornih triangulacij; to so grafi, ki jih dobimo iz maksimalnih ravninskih grafov, če jim dodamo dodatne povezave, t.i. oporne povezave. Pokažemo, da za vsako sodo število $n \geq 8$ obstajajo oporne triangulacije s $3n - 4$ povezavami, ki ostajajo globalno toge tudi še potem, ko iz grafa izvzamemo poljubno povezavo. Ta meja je najboljša možna. Ta rezultat daje pritrdilen odgovor na nedavno postavljeno domnevo. Razpravljamo tudi o povezavi med našimi rezultati in sorodno splošnejšo domnevo S. Tanigawe in tretjega avtorja.

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Ključne besede: Triangulacija, globalno tog graf, oporna triangulacija, togost.

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