BLED WORKSHOPS
IN PHYSICS
VOL. 17, NO. 1
p. 33

Proceedings of the Mini-Workshop
Quarks, Hadrons, Matter
Bled, Slovenia, July 3 - 10, 2016

Soliton model analysis of SU(3) symmetry breaking for baryons with a heavy quark*

H. Weigel, J. P. Blanckenberg

Physics Department, Stellenbosch University, Matieland 7602, South Africa

Abstract. In these proceedings we review the construction of the collective coordinate Hamiltonian that describes the spectrum of baryons with a single heavy quark and *up*, *down* or *strange* degrees of freedom in the context of chiral soliton models.

1 Introduction

This presentation is based on Ref. [1] that describes the numerical results of this soliton model analysis in detail. The derivation of the Hamilton for the (light) flavor degrees of freedom (*up*,*down*, *strange*) and, in particular, the origin of the constraint that projects onto certain flavor $SU(3)$ representations are discussed only by the way in Ref. [1]. We therefore provide more details of the derivation here.

2 Collective rotations in flavor symmetric SU(3)

The approach builds up from a chiral soliton generated from light flavors and heavy meson fields that are bound to the soliton. Both acquire *strangeness* components by collectively rotating in flavor $SU(3)$. Without symmetry breaking this corresponds to approximating time dependent configurations by large zero–mode fluctuations.

2.1 Chiral soliton

The major building block for the chiral soliton is the non–linear representation of the pseudoscalar mesons in form of the chiral field U but also vector mesons ρ and ω may be included. In a first step we construct the stable static soliton (with winding number one). Subsequently we approximate time dependent solutions and introduce collective coordinates for the flavor orientation $A(t) \in SU(3)$. Generically we write this as

$$U(\mathbf{r}, t) = A(t) U_0(\mathbf{r}) A^\dagger(t) \,, \tag{1}$$

* Talk delivered by H. Weigel

where $U_0(\mathbf{r})$ represents the classical (static) soliton. The time dependence is most conveniently parameterized via eight angular velocities Ω_a

$$\frac{i}{2}\sum_{a=1}^{8}\Omega_a\lambda_a = A^\dagger(t)\,\frac{dA(t)}{dt}\,. \tag{2}$$

The resulting collective coordinate Lagrange function has the structure

$$L_l(\Omega_a) = -E_{cl} + \frac{1}{2}\alpha^2\sum_{i=1}^{3}\Omega_i^2 + \frac{1}{2}\beta^2\sum_{\alpha=4}^{7}\Omega_\alpha^2 - \frac{N_c}{2\sqrt{3}}\Omega_8\,. \tag{3}$$

The term linear in the time derivative originates from the Wess–Zumino–Witten action [3] and therefore carries an explicit factor N_c (number of colors). The coefficients α^2 and β^2 are radial integrals of the profile functions and represent moments of inertia for rotations in isospace and the strangeness subspace of flavor $SU(3)$, respectively. The form of Eq. (3) is generic. The particular numerical values for the classical energy and the moments of inertia are, of course, subject to the particular model. They are reviewed in Ref. [2].

2.2 Heavy meson bound states

In the heavy flavor limit the pseudoscalar and vector meson components become degenerate [4]. In contrast to the light sector it is hence inevitable to include both components. Since the soliton configuration itself has non–zero orbital angular momentum the most strongest coupling to the solution dwells in the P–wave channel [5] (P and Q_μ are $SU(3)$ flavor spinors):

$$\begin{aligned}
P &= \frac{e^{i\omega t}}{\sqrt{4\pi}}\Phi(r)\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\tau}}\chi\,,\\
Q_0 &= \frac{e^{i\omega t}}{\sqrt{4\pi}}\Psi_0(r)\chi\,,\\
Q_i &= \frac{e^{i\omega t}}{\sqrt{4\pi}}\left[i\Psi_1(r)\hat{r}_i + \tfrac{1}{2}\Psi_2(r)\epsilon_{ijk}\hat{r}_j\tau_k\right]\chi\,,
\end{aligned} \tag{4}$$

where $\chi = \chi(\omega)$ is a three component spinor that is constant in space but should be viewed as the Fourier amplitude of the heavy meson wave–function. Since the coupling to the light mesons occurs via a soliton in the isospin subspace, only the first two components of χ are non–zero. The parameterization that emerges by left multiplication with $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\tau}}$ has different profile functions and leads to the S–wave bound states.

The field equations for heavy mesons turn into coupled linear differential equations for the profile functions in eq. (4) with the soliton generating a binding potential. This (so–called bound state) approach assumes the soliton as infinitely heavy and corresponds to (formally) assuming the large N_C limit. Normalizable solutions to these differential equations only exist for certain frequencies ω below the heavy meson mass.

To quantize the Fourier amplitudes, χ as harmonic oscillators it is necessary to properly normalize the bound state profiles. The normalization condition is that occupying the bound produces one unit of heavy charge (charm or bottom). This heavy charge arises from the Noether current associated with an infinitesimal phase transformation of the heavy field. We write the Lagrange function for the heavy meson as

$$L_H(\omega) = \int d^3r \left[\frac{\omega^2}{2}\varphi^t\hat{M}\varphi + \omega\varphi^t\hat{\Lambda}\varphi + \varphi^t\hat{H}\varphi\right]\chi^\dagger(\omega)\chi(\omega)\,, \tag{5}$$

where $\varphi^t = (\Phi, \Psi_0, \Psi_1, \Psi_2)$ contains the bound state profiles while the soliton determines the matrices $\hat{M}$, $\hat{\Lambda}$ and $\hat{H}$ are matrices that also contain differential operators. Since the phase transformation in Eq. (4) can be modeled as $\omega \to \omega + \delta\omega$, the normalization condition reads

$$\left|\int d^3r \left[\omega\varphi^t\hat{M}\varphi + \varphi^t\hat{\Lambda}\varphi + \varphi^t\hat{H}\varphi\right]\right| \stackrel{!}{=} 1\,. \tag{6}$$

We require absolute values because bound states with $\omega < 0$ have opposite heavy charge and eventually describe heavy pentaquarks with a heavy anti–quark. The heavy meson fields are spinors in $SU(3)$ flavor space and thus subject to the collective flavor rotation from Eq. (1),

$$P \longrightarrow A(t)P \qquad \text{and} \qquad Q_\mu \longrightarrow A(t)Q_\mu\,, \tag{7}$$

where the right hand sides contain the bound state profile functions. It is then very instructive to compute the time derivative

$$\begin{aligned}\dot{P} &= A(t)\left[i\omega + A^\dagger(t)\dot{A}(t)\right]\frac{e^{i\omega t}}{\sqrt{4\pi}}\Phi(r)\begin{pmatrix}\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\tau}}\chi\\ 0\end{pmatrix}\\ &= iA(t)\left[\omega + \frac{1}{2\sqrt{3}}\Omega_8 + \frac{1}{2}\sum_{a=1}^{7}\Omega_a\lambda_a\right]\frac{e^{i\omega t}}{\sqrt{4\pi}}\Phi(r)\begin{pmatrix}\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\tau}}\chi\\ 0\end{pmatrix}\end{aligned}$$

because it shows that $\frac{\partial L_H(\omega)}{\partial\Omega_8} = \frac{1}{2\sqrt{3}}\frac{\partial L_H(\omega)}{\partial\omega}$. Then the normalization condition enforces

$$-\frac{\partial L}{\partial\Omega_8} = \frac{1}{2\sqrt{3}}\left(N_c - \text{sign}(\omega)\chi^\dagger\chi\right) = \frac{1}{2\sqrt{3}}\left(N_c - N\right)\,, \tag{8}$$

where we have also identified the charge of the heavy quark. Finally, the collective rotation of the bound state yields the hyperfine coupling [6]

$$L_{hf} = \rho\chi^\dagger\left(\boldsymbol{\Omega}\cdot\frac{\boldsymbol{\tau}}{2}\right)\chi\,, \tag{9}$$

where ρ is an integral involving all profile functions, including those of the classical soliton. The bound state also contributes to the moments of inertia, α^2 and β^2, but numerically that contribution is negligible since the bound state is localized at the center of the soliton.

3 Symmetry breaking and mass formula

Though it is appropriate to work with $m_u = m_d$, the deviation $m_s \gg m_u$ is substantial and requires to add terms like

$$\mathcal{L}_{sb} \sim \frac{f_\pi^2 m_\pi^2}{4} \text{Tr}\left[\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & x\end{pmatrix}\left(U + U^\dagger - 2\right)\right] + \ldots \qquad \text{where} \qquad x \,\widehat{=}\, \frac{2m_s}{m_u + m_d} \gg 1\,. \quad (10)$$

to the effective chiral Lagrangian that describe different masses and decay constants of strange and non–strange mesons[1]. These symmetry breaking terms yield an explicit A dependence of the collective coordinate Lagrange function

$$L_{sb} = -\frac{x}{2}\widetilde{\gamma}\left[1 - D_{88}(A)\right] \quad \text{with} \quad D_{ab} = \tfrac{1}{2}\text{Tr}\left[\lambda_a A \lambda_b A^\dagger\right]\,. \qquad (11)$$

Again, $\widetilde{\gamma}$ is a radial integral[2] over all profile functions. Collecting Eqs. (3,9) and (11) and Legendre transforming to the right $SU(3)$ generators $R_a = \frac{\partial L}{\partial \Omega_a}$ yields the Hamilton operator whose eigenvalues are the baryon masses that are expressed in the mass formula

$$E = E_{cl} + \left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)\frac{r(r+1)}{2} + \frac{\epsilon(x)}{2\beta^2} - \frac{1}{24\beta^2}\left(N_c - N\right)^2 + |\omega| N + \frac{\rho}{2\alpha^2}\left[j(j+1) - r(r+1)\right] N\,. \qquad (12)$$

The moments of inertia are the same for the light and heavy degrees of freedom as they result from a single local Lagrangian. In Eq. (12) $\epsilon(x)$ is the eigenvalue of $O_{sb} = \sum_{a=1}^{8} R_a^2 + x\beta^2\widetilde{\gamma}\left[1 - D_{88}(A)\right]$ according subject to the constraint $R_8 = (N_c - N)/2\sqrt{3}$. For odd N_c and $N = 1$ this constraint requires diquark $SU(3)$ representations. The total spin is j and $r(r+1)$ is the eigenvalue of $\sum_{i=1}^{3} R_i^2$. It is zero and one for the anti–symmetric and the symmetric diquark wave–functions, respectively.

Obtaining the eigenvalues $\epsilon(x)$ of the operator O_{sb} amounts to a non–perturbative treatment of light flavor symmetry breaking. Yet, the approach can be illuminated in the language of perturbation theory as it corresponds to linearly combining states that belong to different $SU(3)$ representations, but otherwise have identical quantum numbers. Possible representations are subject to the constraint on R_8: For the physical value $N_c = 3$ representations with the lowest eigenvalue of the quadratic Casimir operator, $\sum_{a=1}^{8} R_a^2$ are the anti–triplet and the sextet with $r = 0$ and $r = 1$, respectively. That is, these are the quark model representations. With symmetry breaking added an anti–fifteen–plet and a 24 dimensional representation follow suit [7]. Increasing to the next odd value, $N_c = 5$, an anti–sextet, a mixed– and a fully symmetric fifteen–plet are allowed by the constraint. The latter has $r = 2$ and does not have a counterpart for $N_c = 3$. Hence the N_c counting effects (heavy) baryon masses via modified eigenvalues of O_{sb}.

---

[1] Symmetry breaking for the heavy mesons, proportional to *e.g.* $M_{B_s}^2 - M_B^2$, is also included.

[2] The notation is chosen to distinguish it from $\gamma = x\widetilde{\gamma}$ in the literature [2].

4 Summary

In these short proceedings we have explained the origin of the collective coordinate Hamiltonian from treating baryons with heavy quark as a heavy meson bound to a chiral soliton. The resulting spectrum and its comparison with empirical data has been discussed at length elsewhere [1]. We stress that light baryons are simultaneously described in this approach by setting $N = 0$ in Eq. (12) and in the constraint on R_8. In particular, we have unique moments of inertia and symmetry breaking coefficients regardless of the value for N. This is in contrast to the approach of Ref. [8] that employs different Lagrangians in the light and heavy sectors.

Acknowledgments

One of us (HW) is very grateful to the organizers for providing this worthwhile workshop. This work supported in parts by the NRF under grant 77454.

References

1. J. P. Blanckenberg, H. Weigel, *Phys. Lett.* **B750**, 230 (2015).
2. H. Weigel, *Lect.Notes Phys.* **743**, 1 (2008).
3. E. Witten, *Nucl. Phys.* **B223**, 422, 433 (1983).
4. M. Neubert, *Phys.Rept.* **245**, 259 (1994).
5. J. Schechter, A. Subbaraman, S. Vaidya, H. Weigel, *Nucl.Phys.* **A590**, 655 (1995).
6. M. Harada, A. Qamar, F. Sannino, J. Schechter, H. Weigel, *Nucl.Phys.* **A625**, 789 (1997).
7. A. Momen, J. Schechter, A. Subbaraman, *Phys.Rev.* **D49**, 5970 (1994).
8. G. S. Yang, H. C. Kim, M. V. Polyakov, M. Praszałowicz, arXiv:1607.07089 [hep-ph].