

Intersecting families of graphs of functions over a finite field*

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Abstract

Let U be a set of polynomials of degree at most k over $\mathbb{F}_q$, the finite field of q elements. Assume that U is an intersecting family, that is, the graphs of any two of the polynomials in U share a common point. Adriaensen proved that the size of U is at most q^k with equality if and only if U is the set of all polynomials of degree at most k passing through a common point. In this manuscript, using a different, polynomial approach, we prove a stability version of this result, that is, the same conclusion holds if $|U| > q^k - q^{k-1}$. We prove a stronger result when $k = 2$.

For our purposes, we also prove the following results. If the set of directions determined by the graph of f is contained in an additive subgroup of $\mathbb{F}_q$, then the graph of f is a line. If the set of directions determined by at least $q - \sqrt{q}/2$ affine points is contained in the set of squares/non-squares plus the common point of either the vertical or the horizontal lines, then up to an affinity the point set is contained in the graph of some polynomial of the form αx^{p^k} .

Keywords: Direction problem, Erdős-Ko-Rado, finite field, polynomial.

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Sekajoče se družine grafov funkcij nad končnim obsegom*

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Povzetek

Naj bo U množica polinomov stopnje največ k nad $\mathbb{F}_q$, končnim obsegom s q elementi. Predpostavimo, da je U sekajoča se družina, kar pomeni, da imata grafa poljubnih dveh polinomov iz U skupno točko. Adriaensen je dokazal, da je moč množice U največ q^k , kjer velja enakost natanko takrat, ko je U množica vseh polinomov stopnje največ k , ki gredo skozi skupno točko. V tem prispevku z uporabo drugačnega, polinomskega, pristopa dokažemo stabilnostno različico tega rezultata, kar pomeni, da isti sklep velja, če je $|U| > q^k - q^{k-1}$. Dokažemo močnejši rezultat za primer, ko je $k = 2$.

Za naše namene dokažemo tudi naslednje rezultate. Če je množica smeri, določena z grafom f , vsebovana v aditivni podgrupi $\mathbb{F}_q$, potem je graf f premica. Če je množica smeri, ki jo določa vsaj $q - \sqrt{q}/2$ afinih točk, vsebovana v uniji množice kvadratov/nekvadratov in skupne točke navpičnih ali vodoravnih premic, potem je, do afinitete natančno, množica točk vsebovana v grafu nekega polinoma oblike αx^{p^k} .

Ključne besede: Problem smeri, Erdős-Ko-Rado, končni obseg, polinom.

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