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Line graphs and geodesic transitivity

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Abstract

For a graph Γ, a positive integer s and a subgroup $G \leq \mathrm{Aut}(\Gamma)$, we prove that G is transitive on the set of s-arcs of Γ if and only if Γ has girth at least $2(s-1)$ and G is transitive on the set of $(s-1)$-geodesics of its line graph. As applications, we first classify 2-geodesic transitive graphs of valency 4 and girth 3, and determine which of them are geodesic transitive. Secondly we prove that the only non-complete locally cyclic 2-geodesic transitive graphs are the octahedron and the icosahedron.

Keywords: Line graphs, s-geodesic transitive graphs, s-arc transitive graphs.

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Linijski grafi in geodetska tranzitivnost

Povzetek

Za dani graf Γ, pozitivno celo število s in podgrupo $G \leq \mathrm{Aut}(\Gamma)$ pokažemo, da je G tranzitivna na množici s-lokov grafa Γ natanko tedaj, ko ima graf Γ ožino vsaj $2(s-1)$ in je G tranzitivna na množici $(s-1)$-geodetk njegovega linijskega grafa. Rezultat uporabimo najprej za klasifikacijo 2-geodetskih tranzitivnih grafov stopnje 4 in obsega 3 ter ugotovitev, kateri od njih so geodetsko tranzitivni, potem pa še za dokaz, da sta oktaeder in ikozaeder edina nepolna lokalno ciklična 2-geodetsko tranzitivna grafa.

Ključne besede: Linijski grafi, s-geodetsko tranzitivni grafi, s-ločno tranzitivni grafi.