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On p-gonal fields of definition*

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Abstract

Let S be a closed Riemann surface of genus $g \geq 2$ and φ be a conformal automorphism of S of prime order p such that $S/\langle\varphi\rangle$ has genus zero. Let $\mathbb{K} \leq \mathbb{C}$ be a field of definition of S. We prove the existence of a field extension $\mathbb{F}$ of $\mathbb{K}$, of degree at most $2(p-1)$, for which S is definable by a curve of the form $y^p = F(x) \in \mathbb{F}[x]$, in which case φ corresponds to $(x, y) \mapsto (x, e^{2\pi i/p}y)$. If, moreover, φ is also definable over $\mathbb{K}$, then $\mathbb{F}$ can be chosen to be at most a quadratic extension of $\mathbb{K}$. For $p = 2$, that is when S is hyperelliptic and φ is its hyperelliptic involution, this fact is due to Mestre (for even genus) and Huggins and Lercier-Ritzenthaler-Sijslingit in the case that $\mathrm{Aut}(S)/\langle\varphi\rangle$ is non-trivial.

Keywords: Riemann surfaces, p-gonal curves, automorphisms.

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O p-gonalnih definicijskih obsegih*

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Povzetek

Naj bo S sklenjena Riemannova ploskev roda $g \geq 2$ in naj bo φ tak konformni avtomorfizem ploskve S praštevilskega reda p, da ima $S/\langle\varphi\rangle$ rod nič. Naj bo $\mathbb{K} \leq \mathbb{C}$ definicijski obseg ploskve S. Dokažemo obstoj razširitve obsega $\mathbb{F}$ obsega $\mathbb{K}$, stopnje največ $2(p-1)$, za katero je S definiran s krivuljo oblike $y^p = F(x) \in \mathbb{F}[x]$, kjer φ ustreza $(x, y) \mapsto (x, e^{2\pi i/p}y)$. Če je, poleg tega, φ definiran tudi nad $\mathbb{K}$, potem se da $\mathbb{F}$ izbrati tako, da je kvečjemu kvadratna razširitev definicijskega obsega $\mathbb{K}$. Za $p = 2$, ko je ploskev S hipereliptična in je φ njena hipereliptična involucija, je za ta rezultat zaslužen Mestre (za sodi rod), ter Huggins in Lercier-Ritzenthaler-Sijslingit, ki so to pokazali za primer, ko je $\mathrm{Aut}(S)/\langle\varphi\rangle$ netrivialen.

Ključne besede: Riemannove ploskve, p-gonske krivulje, avtomorfizmi.

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