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The CEPS Journal is an international peer-reviewed journal with an international board. It publishes original empirical and theoretical studies from a wide variety of academic disciplines related to the field of Teacher Education and Educational Sciences; in particular, it will support comparative studies in the field. Regional context is stressed but the journal remains open to researchers and contributors across all European countries and worldwide. There are four issues per year. Issues are focused on specific areas but there is also space for non-focused articles and book reviews.

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In 2001, the Centre for Educational Policy Studies (CEPS; see http://ceps.pef.uni-lj.si) was established within the Faculty of Education to build upon experience acquired in the broad reform of the national educational system during the period of social transition in the 1990s, to upgrade expertise and to strengthen international cooperation. CEPS has established a number of fruitful contacts, both in the region – particularly with similar institutions in the countries of the Western Balkans – and with interested partners in EU member states and worldwide.

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Revija je namenjena obravnavanju naslednjih področij: poučevanje, učenje, vzgoja in izobraževanje, socialna pedagogika, specialna in rehabilitacijska pedagogika, predšolska pedagogika, edukacijske politike, supervizija, poučevanje slovenskega jezika in književnosti, poučevanje matematike, računalništva, naravoslovja in tehnike, poučevanje družboslovja in humanistike, poučevanje na področju umetnosti, visokošolsko izobraževanje in izobraževanje odraslih. Poseben poudarek bo namenjen izobraževanju učiteljev in spodbujanju njihovega profesionalnega razvoja.

V reviji so objavljeni znanstveni prispevki, in sicer teoretični prispevki in prispevki, v katerih so predstavljeni rezultati kvantitavnih in kvalitativnih empiričnih raziskav. Še posebej poudarjen je pomen komparativnih raziskav.

Revija izide štirikrat letno. Številke so tematsko opredeljene, v njih pa je prostor tudi za netematske prispevke in predstavitve ter recenzije novih publikacij.

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Editorial

Problem Solving and Problem Posing: From Conceptualisation to Implementation in the Mathematics Classroom

Problem solving and problem posing are leading mathematical activities that stimulate mathematical thinking. From the theoretical point of view, these activities are very complex, partly due to the various issues that describe/define problem solving and problem posing and their role in the process of teaching and learning mathematics. Problem solving and problem posing are interrelated activities; we could say that they are in an interdependent relationship: we solve the problems we pose, we pose the problems in a way that we can solve them. However, the two processes are not equally present in every situation.

Research into problem solving focuses mainly on the following areas: the basic characteristics of a mathematical problem; the nature (conceptual, procedural) and role of representation (interplay between internal and external) of a mathematical problem; mental schemas for problem solving; heuristics as principles, methods and (cognitive) tools for solving problems; types of generalisations and reasoning (abductive, narrative, naïve, arithmetic, algebraic); problem solving as a challenging activity for mathematically gifted students; and the role of the teacher in guiding problem solving as a way of implementing student problem solving in the classroom. Regarding problem posing, there are also some critical questions: How can the existing definitions of problem posing be categorised? How is problem posing conceived by the research community in relation to other mathematical constructs? What are the possible ways of implementing problem posing in research and teaching settings? Regarding problem solving, problem posing is formulated/used in research findings for generating (formulating, finding, creating) new problems; reformulating existing problems; creating and/or reformulating problems; raising questions and viewing old questions from a new angle; and an act of modelling.

Research has demonstrated and frequently confirmed that (mathematical) problem posing and solving possess great potential for learners, but the reality in terms of teaching practice, external examinations, teaching material, and mathematics curricula seems out of alignment with the research findings. In this focus issue, we have considered two aspects of problem posing and problem solving: conceptualisation and implementation in the mathematics classroom.

This issue contains five articles that address the issues of problem posing and problem-solving. The authors come from different backgrounds (Greece, Croatia, Hungary, Germany), which means that diverse perspectives and research findings in this field are confronted. Furthermore, each school system responds to the implementation of problem solving in different ways, and the research presented is related to this. Problem solving in mathematics education as a conceptual premise is not new; it has its roots in 1945, in the book *How to Solve It* (Polya), yet we are constantly faced with new challenges in implementing the findings of researchers on problem solving in mathematics education, such as how to create an environment for problem solving in mathematics education, how to place problem solving in certain mathematical content, how to establish an appropriate teaching role for the teacher (we are beyond believing that the student will become a good problem solver or problem poser on his/her own, without the teacher's intervention), how to assess problem solving, where to get problems that are appropriate for different age groups of students and their abilities, pre-knowledge and similar. Unfortunately, the latter (where to get mathematical problems) has a vital connection with the authors of the textbook materials used by teachers, which regulate the proportion of problem solving in mathematics lessons. It is true that problem solving in the sense presented here cannot be the main topic of instruction (problem solving cannot replace the learning of fundamental mathematical concepts and content), but the inclusion of selected problems and related strategies and heuristics in the classroom certainly makes sense from at least two points of view: the deepening and application of mathematical knowledge, and the acquisition of the generic problem-solving skills expected of us in an ever-changing society.

The introductory article in this focus issue, entitled Multiple Approaches to Problem Posing: Theoretical Considerations Regarding its Definition, Conceptualisation, and Implementation, by Ioannis Papadopoulos, Nafsika Patsiala, Lukas Baumanns, and Benjamin Rott answers the question of the conceptualisation of problem-setting.

In their theoretical research paper, they attempt to capture different meanings and aspects of problem posing by approaching it from three different levels: (1) by comparing definitions, (2) by relating it to other constructs, and (3) by referring to research and teaching settings. Their analysis of the documents and research findings considering the first level shows no consensus regarding the conceptualisations of problem posing, which certainly causes much uncertainty in terms of implementation in the classroom and in research. In the second level, they examine how problem posing is conceived by the research community compared to other mathematical constructs, such as problem solving, mathematical creativity, or modelling. In empirical research on the connection of problem posing to these constructs, it is noticeable that the focus is mainly on the products, meaning the problems posed. Their discussion emphasises that there is a lack of research to evaluate the process of problem posing when investigating connections to problem solving or creative mathematical thinking.

Furthermore, they argue that the products can only reflect one component of the activity of problem posing. In practice, the descriptions in this article can be helpful in understanding the enormous spectrum of conceptualisations of problem posing. This may enable a targeted selection and assessment of appropriate problem-posing activities for educational purposes to be achieved. The third level (research and teacher settings) summarises possible ways of implementing problem posing in research and teaching settings as depicted in the relevant literature. The authors do not offer definitive answers; their intentions are to stimulate discussion on this far-reaching and complex topic; a future systematic literature review may provide insights of greater validity into definitions, conceptualisations, and implementations of problem posing in research and practice.

In the second paper, Reading Mathematical Texts as a Problem-Solving Activity: The Case of the Principle of Mathematical Induction by Ioannis Papadopoulos and Paraskevi Kyriakopoulou, we are faced with the possibility of implementing problem solving in mathematics lessons in conjunction with the reading of mathematical texts. Reading complex mathematical texts is closely related to the effort of the reader to understand its content; therefore, it is reasonable to consider such reading as a problem-solving activity. In this paper, the principle of mathematical induction was introduced to secondary education students through mathematical text; their efforts to comprehend the text were examined to identify whether significant elements of problem solving are involved. The findings show that while negotiating the content of the text, the students went through Polya's four phases of problem solving. Moreover, this approach of reading the principle of mathematical induction in the sense of a problem that must be solved seems a promising idea for the conceptual understanding of the notion of mathematical induction. The article opens the door to a new understanding of problem solving by showing that reading mathematical texts also might have characteristics of problem solving in terms of the process experienced by the problem solver in this activity.

There is less research on geometry problems than on arithmetic problems, which is understandable (given that less attention is given to school geometry in comparison to other topics), as solving geometric problems requires complex geometric knowledge, which, due to the nature of concepts at higher levels of schooling, becomes much more abstract because it is based on a good understanding of definitions and the hierarchy between concepts. The paper entitled Factors Affecting Success in Solving a Stand-Alone Geometrical Problem by Students aged 14 to 15, by Branka Antunović-Piton and Nives Baranović, investigates and considers factors that affect success in solving a stand-alone geometrical problem by students of the 7th and 8th grades of elementary school. The starting point for consideration is a geometrical task from the National Secondary School Leaving Exam in Croatia (State Matura), utilising elementary-level geometry concepts. The task was presented as a textual problem with an appropriate drawing and a task within a given mathematical context. After data processing, the key factors affecting the process of problem solving were singled out: visualisation skills, detection and connection of concepts, symbolic notations, and problem-solving culture. The obtained results are the basis of suggestions for changes in the geometry teaching-learning process. They conclude that the selected sample of students lacked fully developed problem-solving skills, understanding certain geometrical concepts, and the skill to identify and connect conceptual properties, resulting in students' inability to find a systematic way to the required solution. The underdeveloped visualisation skills were observed as a particular issue, as fully-developed visualisation skills are required for the problem-solving process of geometrical tasks. All the aforementioned difficulties experienced throughout the problem-solving process indicate that the learning and teaching of geometry should emphasise the visualisation skills (drawing, interpretation, formation of connections among different notations, etc.) and systematic notetaking. The authors conclude that this skill set can be learned and developed by solving geometry problems of different cognitive requirements, and the role of the teacher should not be underestimated, as we mentioned in the introduction of this editorial.

It has been repeatedly shown that involving readers in research can make a difference to a teacher's teaching. Of course, the question remains to what extent these changes remain in the teaching after the project or research activity is over. In an optimistic scenario, at least 'traces' of the research or changes in teachers' teaching remain, as well as the possibility for a qualitative upgrading of professional-didactic knowledge in the area of integrating problem solving into mathematics teaching. The paper Management of Problem Solving in a Classroom Context by Eszter Kónya and Zoltán Kovács addresses the role of the professional development of teachers in implementing problem solving in the mathematics classroom. The authors discuss the results of a professional development programme involving four Hungarian teachers of mathematics. The programme aims to support teachers in integrating problem solving into their classes. The basic principle of the programme, as well as its novelty (at least compared to Hungarian practice), is that the development takes place in the teacher's classroom, adjusted to the teacher's curriculum and in close cooperation between the teacher and researchers. The teachers included in the programme were supported by the researchers with lesson plans, practical teaching advice and lesson analyses. The progression of the teachers was assessed after the one-year programme based on a self-designed trial lesson, focusing particularly on how the teachers plan and implement problem-solving activities in lessons, as well as on their behaviour in the classroom during problem-solving activities. The findings of this qualitative research are based on video recordings of the lessons and on the teachers' reflections. The authors conclude that the lesson plans and the self-reflection habits of the teachers contribute to the successful management of problem-solving activities.

The last paper in this focus issue, titled MERIA – Conflict Lines: Experience with Two Innovative Teaching Materials by Željka Milin Šipuš, Matija Bašić, Michiel Doorman, Eva Špalj and Sanja Antoliš, presents and evaluates the implementation of two tasks as part of didactic scenarios for inquiry-based mathematics teaching, examining teachers' classroom orchestration supported by these scenarios. The context of the study is the Erasmus+ project MERIA – Mathematics Education: Relevant, Interesting and Applicable, which aims to encourage learning activities that are meaningful and inspiring for students by promoting the reinvention of target mathematical concepts. As innovative teaching materials for mathematics education in secondary schools, MERIA scenarios cover specific curriculum topics and were created based on two well-founded theories in mathematics education: realistic mathematics education and the theory of didactical situations. With the common name Conflict Lines (Conflict Lines – Introduction and Conflict Set – Parabola), the scenarios aim to support students' inquiry about sets in the plane that are equidistant from given geometrical figures: a perpendicular bisector as a line equidistant from two points, and a parabola as a curve equidistant from a point and a line. They examine the results from field trials in the classroom regarding students' formulation and validation of the new knowledge and describe the rich situations teachers may face that encourage them to proceed by building on students' work. This is a crucial and creative moment for the teacher, creating opportunities and moving between students' discoveries and the intended target knowledge. These situations indicate the numerous creative moments when teachers make decisions, create opportunities, and challenge the situation to support a productive exchange of mathematical ideas, which could be of interest to any practising teacher. Such studies are expected to contribute to a better understanding of how to support teachers in the crucial and creative moments when they try to recognise and use opportunities for moving between students' discoveries and intended target knowledge. Once again, the teacher is in the role of using his/her professional-didactic knowledge to identify the potential of the problem, to understand the student's solution and to guide him/

her towards the achievement of the goal, if this is the purpose of the problem situation in mathematics education. It is certainly possible to design scenarios for teaching problems, but the classroom situation is alive, always new and, to a certain extent, unpredictable and requires a skilled teacher who improves his or her teaching by constantly questioning his or her performance in the classroom.

This issue also includes a varia section that covers different topics: the history of foreign language values in Sweden from the seventeenth century to the present, relationships between statistics anxiety (SA), trait anxiety, attitudes towards mathematics and statistics, and academic achievement among university students, a survey on outdoor lessons of the subject Science and Technology in education, effects of parental supervision and school climate on the relationship between exosystem variables (time spent with media and perceived neighbourhood dangerousness) and peer aggression problems and effect of altruism in the relationship between empathic tendencies, the nature relatedness and environmental consciousness.

Another valuable contribution to this issue is the reviews of the two books: *Inclusion in Education: Reconsidering Limits, Identifying Possibilities* (editor Pavel Zgaga) by Melina Tinnacher and *Visible Learning for Mathematics: Grades K-12* (authors John Hattie, Douglas Fisher, Nancy Frey) by Monika Zupančič.

Here we present only a few highlights from the reviews, inviting the reader to read books that address contemporary issues, stimulate deep reflection and open up space for finding solutions to raise the quality and breadth of teaching. Inclusion is a major issue in every place and time, and the teaching of mathematics has never been subject to more 'innovation' than it is today.

'What distinguishes this book from other volumes on inclusion is the broader context in which it situates its discussion, namely society in general rather than the more restricted frame of reference of the school system. It also provides an unprecedented insight into the inclusion debate in Slovenia, comparing it with other countries.' (from the review of the book Inclusion in Education)

'One theme that we believe makes an essential contribution to the value of this book is the importance of using mathematical language in the process of teaching and learning mathematics. The content of this book, through teachers' reflection, represents a great opportunity to strengthen teachers' didactic knowledge and contribute to the quality of their teaching.' (from the review of the book Visible Learning for Mathematics)

Tatjana Hodnik and Vida Manfreda Kolar

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Multiple Approaches to Problem Posing: Theoretical Considerations Regarding its Definition, Conceptualisation, and Implementation

Ioannis Papadopoulos*[1], Nafsika Patsiala[2], Lukas Baumanns[3] and Benjamin Rott[3]

The importance of mathematical problem posing has been acknowledged by many researchers. In this theoretical paper, we want to capture different meanings and aspects of problem posing by approaching it from three different levels: (1) by comparing definitions, (2) by relating it to other constructs, and (3) by referring to research and teaching settings. The first level is an attempt to organise existing definitions of problem posing. The result of this analysis are five categories, which shows that there is no consensus regarding the conceptualisations of problem posing. In the second level, we examine how problem posing is conceived by the research community compared to other mathematical constructs, such as problem solving, mathematical creativity, or modelling. Finally, in the third level, we summarise possible ways of implementing problem posing in research and teaching settings as they are depicted in the relevant literature. Given this broad variance regarding the conceptualisations of problem posing, we attempt to provide some arguments as to whether there is a need for consensus on a commonly accepted concept of problem posing.

Keywords: problem posing, definition, conceptualisation, implementation

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Večdimenzionalni pristop k zastavljanju problemov: teoretični premisleki glede njegove opredelitve, konceptualizacije in izvedbe

Ioannis Papadopoulos, Nafsika Patsiala, Lukas Baumanns in Benjamin Rott

Pomen zastavljanja matematičnih problemov je obravnavalo že več raziskovalcev. V tem teoretičnem prispevku želimo zajeti različne opredelitve in vidike zastavljanja problemov, h katerim smo pristopili na treh različnih ravneh: 1) primerjava opredelitev; 2) povezave z drugimi konstrukti; 3) izvedene raziskave in poučevalne prakse. Na prvi ravni smo želeli organizirati različne obstoječe opredelitve zastavljanja problemov. Rezultate te analize smo uvrstili v pet kategorij, s čimer pokažemo, da ni konsenza glede opredelitve zastavljanja problemov. Na drugi ravni povzamemo, kako je zastavljanje problemov sprejeto v skupnosti raziskovalcev in kako se ta povezuje z drugimi konstrukti, kot na primer z reševanjem problemov, s kreativnostjo, z modeliranjem. Nazadnje, na tretji ravni, iz relevantnih raziskav povzamemo mogoče načine raziskovanja zastavljanja problemov in implementiranja v poučevanje. Upoštevajoč precejšnja odstopanja glede konceptualizacije zastavljanja problemov, poskušamo navesti nekaj argumentov za potrebo po soglasju o splošno sprejetem konceptu zastavljanja problemov.

Ključne besede: zastavljanje problemov, opredelitev, konceptualizacija, implementacija

Introduction

Problem posing is considered an important topic that, from time to time, attracts the attention of the research community (Cai & Hwang, 2020; Stanic & Kilpatrick, 1988). For example, Einstein and Infeld (1938) emphasise the importance of problem posing by claiming that 'the formulation of a problem is often more essential than its solution' (p. 95).

The potential of problem posing to enhance students' learning in mathematics has been acknowledged by many researchers, thus confirming its importance (English, 1998; Silver, 1994). They attribute this potential to the fact that problem-posing activities are cognitively demanding tasks (Cai & Hwang, 2002), which require students to expand their thinking beyond already known procedures to improve their understanding by reflecting on the structure of the given problem. In this sense, problem-posing activities are considered an ingredient in doing high-quality mathematics (Hadamard, 1954). In the 'Principles and Standards for School Mathematics' by the National Council of Teachers of Mathematics (NCTM, 2000), it is considered important for students to 'formulate interesting problems based on a wide variety of situations, both within and outside mathematics' (p. 258), and it was recommended that students make and investigate mathematical conjectures in order to learn how to generalise and extend problems by posing follow-up questions. These standards explicitly emphasise that problem posing (in combination with problem solving) leads to a more in-depth understanding of the mathematical contents, as well as the process of problem solving and, thus, to a better grasp of doing mathematics itself.

Moreover, problem posing is important for teachers who regularly formulate and pose worthwhile problems for their students, no matter whether they are selecting and modifying standard textbook problems or developing self-generated problems (NCTM, 1991).

As a reasonable consequence of the research interest on this topic, there are many publications on problem posing. However, these are not homogeneous; instead, a wide range of different approaches on the meaning of problem posing and of its relations with other mathematical constructs can be identified in the relevant literature. On the one hand, there are papers in which the need for 'a clear distinction between problem posing and the general practice in raising questions in mathematics' (Mamona-Downs & Downs, 2005, p. 392) is emphasised. On the other hand, there are papers in which the importance of training students and teachers in problem posing, the use of problem posing as a measure of creativity, and the role of technology in problem-posing activities are examined (Cai et al., 2015). There are also papers in which the respective authors discuss methodological issues (among others) about problem posing and speculate about the future directions of the relevant research (Cai & Hwang, 2020).

Against this background, this paper is an attempt to organise the different aspects of problem posing. We do not provide a full systematic literature review but rather want to present and discuss a broad spectrum of literature on problem posing. We aim to stimulate reflection and initiate discussion rather than to propose irrefutable answers. This narrative synthesis is a summary of the current state of knowledge in relation to the following three research questions:

1. In which way can existing definitions of problem posing be categorised?
2. How is problem posing conceived by the research community in relation to other mathematical constructs?
3. What are the possible ways of implementing problem posing in research and teaching settings?

Theoretical Considerations on Problem Posing: Definitions, Conceptualisation, and Implementation

This section constitutes an attempt to examine the wide diversity of problem-posing aspects. We aim to sort the broad spectrum of literature thematically. Through an extensive examination of representative literature compiled by two research groups working on problem posing, three focal points were identified. First, the paper navigates the most common definitions found in the research literature and their differences (subsection 2.1). The second aspect concerns the connection of problem posing to further constructs at the research level (subsection 2.2). Finally, the third aspect deals with the implementation of problem posing in research and teaching settings (subsection 2.3). The theoretical considerations within each subsection are the product of an inductive category development in a qualitative content analysis (Mayring, 2014). Various definitions of several papers were analysed with regard to their content-related key aspects.

Problem posing: Examining the existing definitions

The analysis of the definitions found in the relevant literature resulted in five categories: Problem posing as (1) only the generation of new problems, (2) only the re-formulation of already existing or given problems, (3) both the generation and/or re-formulation of problems, (4) raising questions, and (5) an act of modelling. Please note that these five categories are not necessarily disjunctive; there might be definitions that fit into more than one. It is not our goal to provide distinct categories but to initiate a discussion.

This effort was initiated by the acknowledgement that there are many different perspectives on and definitions of problem posing (Silver & Cai, 1996). Of course, there have been other efforts to organise problem-posing definitions. For example, Olson and Knott (2013) organised the existing definitions into two groups according to whether they focus on students or teachers. The main difference between these two categories is the aiming goal. Students (mainly college students) pose problems to exhibit their conceptual understanding. Teachers pose problems to cultivate the mathematical thinking of their students. Similarly, Cai and Hwang (2020) specify problem posing separately for students and teachers.

In the same paper, looking across different perspectives, Cai and Hwang (2020) propose the following:

> By problem posing in mathematics education, we refer to several related types of activity that entail or support teachers and students formulating (or reformulating) and expressing a problem or task based on a particular context (which we refer to as the problem context or problem situation). (p. 2)

We attempt to deepen this effort and elaborate the categorisation of Cai and Hwang (2020) by using the categories (1) to (5) mentioned above.

Please note that in this paper, the focus is only on definitions of *mathematical* problem posing. There are definitions from other domains, such as Freire's (1970), who sees problem posing as a way to make students 'critical thinkers' (p. 83), extending the concept of problem posing to various domains of knowledge. However, such definitions are beyond the scope of this paper.

Problem posing as generating new problems

As Kilpatrick (1987) mentions, in real life outside of school, many problems, if not most, must be created or discovered by the solver, who gives them initial formulations. He also adds that in some cases, problems emerge from the exploration of ill-defined problems with a given mathematical input. This is in accordance with Lakatos (1976), who said that a problem never comes out of the blue; it is always related to our background knowledge. Kilpatrick (1987) provides the following example: Let's say that one is looking at the divisors of various numbers. It is easy to notice that the number of divisors varies; therefore, considering numbers with very few divisors might be interesting. It is reasonable to look at extreme cases and think that numbers with 0 or 1 divisors are likely non-existent and therefore uninteresting. A further observation might result in the hypothesis that numbers with 3 divisors always seem to be squares of numbers with 2 divisors and, therefore numbers with 2 divisors are of special interest. Then it is possible to examine additional examples of primes and factorisations of numbers into primes and to ask whether the relationship between a number and its divisors is a function. Then, the new problem is generated: Any integer greater than 1 can be expressed as a product of primes in essentially only one way (Fundamental Theorem of Arithmetic).

In the classroom context, the generation of new problems might result from proposing a problem-posing situation. In Kwek's (2015) study, students are initially presented with the following situation: 'A gardener is planting a new orchard. The young trees are arranged in the rectangular plot, which has its longer side measuring 100 m' (p. 279). Then, they are asked to use the information above to pose a mathematical problem.

In these examples and, therefore, in this category, problem posing could exclusively be seen as the creation of new problems. Stoyanova and Ellerton (1996) describe it as a 'process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems' (p. 518). The same idea of problem posing as the generation of problems based on given situations or mathematical expressions or diagrams can be found in the work of Cai et al. (2020). This definition holds for both teachers and students. This line of thinking is also present in the context of teacher education, where problem posing is seen as a task designed by teachers asking students to generate word problems (Kwek, 2015).

Problem posing as reformulating already existing or given problems

Definitions in this category consider problem posing to be re-formulations of problems that already exist – out of interest after finding interesting problems or because problems were given by someone (e.g., a teacher), with the request to find new problems. In this sense, this could immediately be related to Pólya's (1957, p. xvi-xvii) suggestions of *Devising a Plan*: 'Could you restate the problem?', 'Could you restate it still differently?', and 'Try to solve first some related problem … more accessible, … more general, … more special, … analogous.'

Kilpatrick (1987) provides an example of what might be considered a re-formulation of an existing problem. Students are given a practical problem: a cloth-drying rack for the backyard must be made, for which there are two options. The clotheslines are string between two parallel supports (Figure 1, left) or between crossbars (Figure 1, right). The students are asked to find how many feet of clothesline are needed for each case, given the length of the outer side and the separation between adjacent lines.

Figure 1
The cloth-drying rack task

These two models presented above seem realistic, but they can be questioned by the solvers. They do not include the clothesline necessary for tying the ropes to the supports. Indeed, the original mathematical model is quite simplified and, therefore, this is an opportunity for students to attempt a second model that takes account of the extra clothesline. This new model constitutes a reformulation of the initial problem (cf. Verschaffel et al., 1994).

The re-formulation of an existing problem is very often connected to the sense of ownership of the new problem (Kilpatrick, 1987). Moreover, this re-formulation can be a series of transformations of the original problem. In this case, each re-formulation indicates progress towards a solution and provides possibilities for further expanding the scope of the original problem (Cifarelli & Sevim, 2015).

Problem posing as both generating new and/or reformulating given problems

This category refers to definitions that include both the generation of new problems and/or the re-formulation of existing ones, thus combining the previous two categories. Very early, Duncker (1945) used such a definition. However, perhaps the most frequently used among the definitions is that of Silver (1994), who defines mathematical problem posing as 'both the generation of new problems and the re-formulation of given problems' (p. 19) and, as a consequence, posing can occur before, during, or after the solution of a problem. A similar approach is that of Singer and Voica (2015), who suggest that 'problem posing refers to generating something new or to revealing something new from a set of data' (p. 142).

Cai and Hwang (2020) proposed a new one-for-all definition:

> By problem posing in mathematics education, we refer to several related types of activity that entail or support teachers and students formulating (or reformulating) and expressing a problem or task based on a particular context (which we refer to as the problem context or problem situation). (p. 2)

In this definition, they explicitly suggest both formulation and re-formulation of a problem made by either students or teachers, thus bringing forward the issue of teachers' education. A slightly modified version of this definition can also be found in Osana and Pelczer's (2015) working definition that considers problem posing as 'the act of formulating a new task or situation, or modifying an existing one, with a specific mathematical learning objective and a targeted pedagogical purpose in mind' (p. 485).

Problem posing as raising questions and viewing old questions from a new angle

Ellerton and Clarkson (1996) adopt an approach for problem posing that is inspired by Einstein's and Infeld's (1938) perspective of raising new questions and as possibilities to regard old questions from a new angle. One can object that seeing already existing questions from a different perspective is similar to reformulating a problem. However, the focus now is on the questions asked in a problem rather than on its set of data. So, in the cloth-drying rack problem, the focus is on the data, which is on the accuracy of the numbers given for the clothesline length. However, when raising questions, the focus is on the questions themselves, as explained in the example of Gonzales (1996) below.

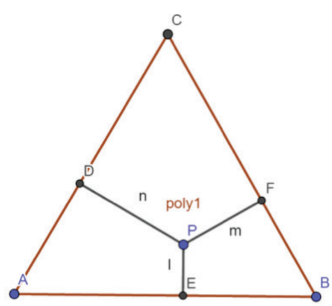
In the same spirit, Marquardt and Waddil (2004) say: 'Problem posing involves making a taken-for-granted situation problematic and raising questions about its validity' (p. 190). The same is written by O'Neil and Marsick (1994): 'Problem posing involves raising questions that open up new dimensions of thinking about the situation' (p. 22). 'The Principles and Standards for School Mathematics' (NCTM, 2000) are aligned with this spirit, emphasising that 'problem posing, that is generating new questions in a problem context, is a mathematical disposition that teachers should nurture and develop' (p. 117).

Estrada and Santos (1999) studied the concept of variation within a course in a group of Grade 11 students. The students received information showing the prices of a product, the behaviour of a school of fish, and the fluctuation of a currency (the peso). They were asked to examine the given data and formulate corresponding questions. The information provided to the students was given in formats that included tables, paragraph forms, or actual newspaper texts.

Some other researchers connect this to a specific situation where, for example, a picture is given without any explanation to students who are then asked to generate questions relevant to the situation. Gonzales (1996), for example, presented to the students a mathematical situation found in a newspaper in the form of a statistical graph that contains data but no built-in question. The students were expected to investigate the given situations and to pose several questions that could be answered by referring to the information provided in the graphs.

The issue of raising questions can also be connected to the application of the 'what-if-not' technique of Brown and Walter (1983). In this approach, the main elements of the task are identified, and then the solver starts negating them, asking what would happen if these elements were different. Mamona-Downs and Papadopoulos (2017) exemplify this using the task in Figure 2.

Figure 2
Task used for applying the 'what-if-not' technique.



Consider a point *P* internal to an equilateral triangle. Its distances from the sides of the triangle are 3, 4, and 5 cm.

Find the length of the altitude of the triangle.

The main elements of the task are that (1) the shape is plane, (2) it is a triangle, (3) the triangle is equilateral, (4) the point *P* is internal to the triangle, and (5) the distance from each side is considered. Negating each element, the solver can raise interesting questions: 'What if point *P* is not internal to the triangle?' (not 4), "What if the triangle is not an equilateral one?" (not 3), "What if we consider the distance from its edges instead of its sides?" (not 5), are some examples of interesting questions that can be generated using the 'what-if-not' technique.

However, as mentioned earlier, Mamona-Downs and Downs (2005) contend there is a need for a distinction between 'problem posing and the general practice in raising questions in mathematics' (p. 392) to avoid distorting effects in the relevant research literature.

Problem posing as an act of modelling

Finally, there is a limited number of papers that consider problem posing as an act of modelling. Referring to the definition of problem posing by Stoyanova and Ellerton (1996), Bonotto (2010a) says in one of her papers:

> I consider mathematical problem posing as the process by which students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. This process is similar to situations to be mathematised, which students have encountered or will encounter outside school. (p. 402)

She also adds that problem posing becomes an opportunity for interpretation and analysis of reality, and this takes place through activities that are quite absent from today's school context and are typical of the modelling process (Bonotto, 2010b). This definition of problem posing lies at the heart of modelling. Bonotto (2010a, 2010b) presents a relevant example. Children were given various menus (products on offer, prices, ingredients, cover charges, etc.). They were asked to compile an order according to their experience outside school, following the structural features of a blank receipt (description of goods, quantity, cost, etc.). In the end, they had to calculate the total amount they would have to pay.

Greer (1992), in an implicit way, sees mathematical problem posing as a task of 'translating from the natural language representation of a problem to the mathematical-language representation of the model' (p. 285). More explicitly, Stillman (2015) says that problem posing in a real-world situation occurs when a problem is formulated in such a way that is amenable to mathematical analysis. There are many situations in the world around us that can be transformed into a problem that can be solved.

How problem posing is conceived by the research community

This subsection examines how problem posing is conceived by the research community compared to other mathematical constructs. The analysis of several research papers reveals that problem posing is seen as (1) an autonomous mathematical construct viewing it as the (implicit or explicit) aim of tasks, and/or (2) interwoven with other mathematical constructs such as problem solving, creativity (which might also include giftedness), and modelling.

Problem posing as an autonomous concept

Despite problem posing being a counterpart of problem solving, for many years, the latter has attracted much more attention from mathematics education researchers than the former. This realisation initiated the emergence of studies examining a variety of aspects of problem posing. In these studies, students and teachers are engaged in problem-posing activities, and the potential effects of this engagement are examined (Koichu, 2020). Therefore, problem posing is viewed as a goal of an activity.

Being the goal of an activity might entail a variety of approaches. Some studies attempt to determine what kinds of problems are posed by the solvers (Lavy & Bershadsky, 2003; Silver et al., 1996), what the influence of different task formats on problem posing is (Leung & Silver, 1997), who poses problems, for whom and how problems are posed around particular situations (Singer et al., 2011), and what the role of computerised environments in problem posing is (Abramovich & Cho, 2006).

Others attempt to develop a better understanding of the ways pre- and in-service teachers use problem posing: What do they focus on when they pose mathematical problems (Stickles, 2011)? How do the pre-service teachers pose problems to the students? How do their practices change, and what factors contribute to the change (Crespo, 2003)? How do they develop their students' actual problem-posing abilities by explicitly teaching them about what are considered to be key elements of mathematical problem posing (English, 1998)? Moreover, how do they examine the extent to which the problems students pose are mathematical and solvable (Silver & Cai, 2005)?

Problem posing considered as interwoven with other mathematical concepts

Even though many research studies focus on problem posing per se, some studies focus on mathematical constructs connected to problem posing. Three common links that are examined are (1) between problem posing and problem solving, which is the most common, (2) between problem posing and creative mathematical thinking, and (3) between problem posing and modelling.

(1) Several researchers have conducted empirical studies examining potential connections between problem posing and problem solving. On the one hand, there are studies considering both as merely different sides of the

same coin. As Kilpatrick (1987) says, 'problem formulation is an important companion to problem solving'. Newell and Simon (1972) characterised problem posing as a process that is embedded within, and which is difficult to separate from, problem solving. Silver (1994) stated that problem posing and problem solving are interwoven activities in the means that problem posing can occur *prior*, *during*, and *after* a problem-solving process. Furthermore, as an extension, problem posing can be considered as a problem-solving process in which the solution is ill-defined since there are many problems that could be posed (Silver, 1995). Gonzales (1998), as well as Wilson et al. (1993), consider problem posing as the fifth step in Pólya's steps of problem solving. For Singer and Moscovici (2008), problem posing is an extension and application of problem solving, both included in a learning cycle in constructivist instruction.

In contrast, there are studies examining the various effects of problem posing on problem-solving skills and competencies. Silver and Cai (1996) identified a high correlation between problem-solving and problem-posing performances. More precisely, good problem solvers generated more and more complex mathematical problems than their less successful classmates did. Silver (1994), reviewing several studies relevant to problem posing, found that they give evidence about the positive influence of problem posing on students' ability to solve word problems. Moreover, there is a relationship between the students' use of abstract problem-solving strategies and their ability to pose extension problems, meaning problems that go beyond the given information (Cai & Hwang, 2003).

(2) Another construct closely connected to problem posing is creative mathematical thinking (CMT) (cf. Joklitschke et al., 2019). Again, there are two approaches here. On the one hand, problem posing is considered as a distinct and creative act (Dillon, 1982) equal to or more valuable than finding a solution or as a form of creative activity that can operate within rich-situated tasks (Bonotto & Dal Santo, 2015; Freudenthal, 1991). Leung (1997), examining the relationship between CMT and problem posing, claims that creativity is in the nature of problem posing and that, in essence, creating a problem is a creative activity.

On the other hand, many researchers use problem posing to promote, facilitate, and evaluate CMT. Jay and Perkins (1997, p. 257) identify problem posing as a key aspect of creative thinking and creative performance, and not only in mathematics. Silver (1997) claims that CMT lies in the interplay between problem solving and problem posing and that is 'in this interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity' (p. 76). Kontorovich et al. (2011) use Guilford's (1967) categories of fluency, flexibility, and originality as indicators of creativity in students' problem posing. It seems that students who pose coherent and original problems through changes made in their formulation have creative skills (Singer et al., 2011). Moreover, when students pose mathematical problems, they gradually develop fluency skills, and they generate problems of high-quality elaboration and originality (Van Harpen & Sriraman, 2013).

(3) Despite some research papers in which problem posing is defined as an act of modelling, other researchers view both as two closely interlinked concepts. For example, English, Fox, and Watters (2005) state that in mathematical modelling as a rich problem situation, the generation of problems and questions, as well as solving those, occur naturally. When students attempt to make sense of incomplete, ambiguous, or undefined information as in a modelling situation, numerous questions naturally arise for the children as they try to make sense of this information, elicit and work with the embedded mathematical ideas, and modify and refine their model. This perspective on modelling mirrors problem posing. Barbosa (2003) claims that modelling is strongly linked to problem posing and modelling activities to the act of creating questions/problems. As Barbosa exemplifies:

> Imagine that the teacher proposes that students study the impact of the social contribution tax. This is a tax deducted from people's salaries by the Brazilian Government for the maintenance of social welfare. The students certainly will have to formulate questions, search for information, organise it, draw up strategies, apply mathematics, evaluate the results, etc. (p. 230)

Christou et al. (2005) suggest that problem posing constitutes an integral part of modelling cycles, which require the mathematical idealisation of real-world phenomenon; and Stickles (2011) confirms that mathematical modelling starts with the posing of a problem.

Problem-posing activities in research and teaching settings

Finally, this third subsection summarises possible ways of implementing problem posing in research and teaching settings.

For the former, Kilpatrick (1987, p. 123) mentions the shift from viewing problem posing not only as a goal of instruction but also to use it as a means of instruction. Koichu (2020) adds to that, highlighting that problem-posing activities serve genuine mathematical or pedagogical needs. For the latter, the main way of implementation is as a diagnostic tool aiming to deepen our understanding of the difficulties students face in the learning of mathematics.

Below, we present three main ways of its implementation: (1) problem posing as a tool for teacher training leading to the enhancement of their subject didactic competence, (2) problem posing as a pedagogical/educational tool, and (3) problem posing as a diagnostic/assessment tool. Presenting only these three ways does not mean that they are the only ones. These were chosen as the most frequent ways represented in the research literature (Hošpesová & Tichá, 2015).

Problem posing as a tool for teacher training leading to the enhancement of their subject didactic competence.

Teachers' training seems to be a promising area for problem posing. Ellerton (2013) highlights the importance of integrating problem posing in teacher training, providing examples on how problem posing can be an integral part of mathematics teacher education programs through the active learning framework, thus contributing to two areas.

The first area is the improvement of the teachers' strategies. An example of this is shown by Crespo (2015), who reported significant improvements in a group of elementary pre-service teachers' problem-posing strategies during a semester-long engagement in problem-posing tasks. The participants were required to pose problems collaboratively for pupils or when open-ended exploration of a mathematical situation precedes problem posing (Crespo & Sinclair, 2008). In a similar spirit, Grundmeier (2015) explored how problem-posing activities can benefit prospective elementary and middle school teachers. After the course provided to them, the participants developed their problem-posing abilities regarding their techniques.

The second area is the enhancement of teachers' subject didactic competence. Tichá and Hošpesová (2013) used problem posing to produce that. The findings from their study indicate that pre-service teachers gained a deeper understanding of concepts realising the need to use various representations. Similar results can be found in the work of Malaspina et al. (2015).

Problem posing as a pedagogical/educational tool

Pólya (1981), in his book 'Mathematical Discovery', emphasises that letting students formulate problems not only motivates them to work harder but it teaches them a desirable attitude of mind, thus highlighting the pedagogical value of problem posing. Examples of studies that focus on this aspect include the work of Silver and Cai (2005) about teachers posing problems related to a specific situation in the class, or the work of English (1998) about teachers using problem posing to enhance students' learning of mathematics.

Downton and Sullivan (2017) used problem posing with Grade 3 students, aiming to prompt them to use more sophisticated strategies. The results of this study reveal that the tasks prompted the use of more sophisticated thinking. Thus, problem posing could be a useful educational tool.

Finally, in the context of teacher training, the case of educators offering examples of posed problems and pointing out flaws, mistakes, and misconceptions (Tichá & Hošpesová, 2013) is also an option for the educational use of problem posing.

Problem posing used as a diagnostic/assessment tool, which helps teachers/researchers to uncover deficits and obstacles in students' knowledge.

Problem posing has the potential to be used as a diagnostic/assessment tool. Following Tichá and Hošpesová (2009), according to the posed problems, it is possible to investigate both the level of understanding and the difficulties of a specific mathematical concept. They used problem posing to reveal pre-service primary school teachers' shortcomings in their conceptual understanding of fractions (Tichá & Hošpesová, 2013). Silver and Cai (1996) used problem posing as a tool for evaluating students' performance and revealing the connection between their problem-posing and problem-solving abilities based on a written 'story-problem'. They also suggest problem posing as a useful assessment tool for students' learning. Measuring the mathematical and linguistic complexity of the generated problems, teachers could evaluate students' conceptual understanding (Silver & Cai, 2005). Teachers can design and use problem-posing tasks to understand students' mathematical learning and assess students' understanding, as well as the obstacles to understanding and misconceptions (Caniglia, 2016). As an example, Verschaffel et al. (2009) use problem posing as a diagnostic tool for students' understanding of division-with-remainder problems. Problem posing around certain mathematical concepts can be a useful tool for assessing students' understanding. This provides important information to teachers who then optimise their quality of instruction (Lin, 2004). Cai et al. (2013) also used problem posing to measure the curricular effect of learning on middle-school students in order to compare the implementation of two different curricula.

Concluding Remarks

The contents of this paper aim to initiate a discussion on questions such as: Does this broad range of definitions constitute an advantage or an obstacle for researchers who work on problem posing and especially for those who start working in this area? Is there a need for consensus on a commonly accepted definition of problem posing? Does the diversity of the existing definitions enhance or reduce the robustness of the research findings?

The broad range of definitions, conceptualisations, and implementations of problem posing in research on mathematics education demonstrates that 'the field of problem posing is still very diverse and lacks definition and structure' (Singer et al., 2013, p. 4). While a differentiated range of definitions offers the opportunity to select suitable ones for respective research interests, underlying definitions and conceptualisations within studies should be made transparent, which sometimes fails to happen. Consequently, studies pursuing a similar research aim are hardly comparable due to different usages of similar terms. Furthermore, comparatively young research fields, such as the field of problem posing, suffer from the abundance of different understandings, which may have a detrimental effect on the actual teaching of problem posing. As Ruthven (2020) says:

> [A]n increasingly diverse range of concerns are finding a place under the banner of problem posing. […] There is a danger, then, of usage of the term becoming so diffuse as to undermine its analytic power and reduce it to a nebulous slogan. (p. 1)

Therefore, researchers must be aware of the spectrum of understandings so that they are able to make a differentiated and reasoned choice from them in order to make their understanding of problem posing comprehensible to recipients of their research.

This paper also illustrates how problem posing is addressed in research: sometimes in isolation and sometimes in connection with other constructs, such as problem solving, Creative Mathematical Thinking (CMT), and modelling. In empirical research on the connection of problem posing to these constructs, it is noticeable that the focus is mainly on the *products*, meaning the problems posed. There is a lack of research to evaluate the *process* of problem posing when investigating connections to problem solving or CMT. We argue that the products can only reflect one component of the activity of problem posing. Looking at CMT, for example, a comparatively insignificant problem may have been posed as a result of a highly creative process for the particular student. A mere evaluation of the problem would not do justice to this.

In practice, the descriptions in this article can be helpful in understanding the enormous spectrum of conceptualisations of *problem posing*. This may enable a targeted selection and assessment of appropriate problem-posing activities for educational purposes to be achieved.

As with this paper, we only intend to stimulate discussion on this far-reaching and complex topic; a future systematic literature review may provide more valid insights into definitions, conceptualisations, and implementations of problem posing in research and practice.

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Reading Mathematical Texts as a Problem-Solving Activity: The Case of the Principle of Mathematical Induction

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Reading mathematical texts is closely related to the effort of the reader to understand its content; therefore, it is reasonable to consider such reading as a problem-solving activity. In this paper, the Principle of Mathematical Induction was given to secondary education students, and their effort to comprehend the text was examined in order to identify whether significant elements of problem solving are involved. The findings give evidence that while negotiating the content of the text, the students went through Polya's four phases of problem solving. Moreover, this approach of reading the Principle of Mathematical Induction in the sense of a problem that must be solved seems a promising idea for the conceptual understanding of the notion of mathematical induction

Keywords: mathematical induction, reading mathematical text, problem solving, secondary education students

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Branje matematičnih besedil kot dejavnost reševanja problemov: primer principa matematične indukcije

Ioannis Papadopoulos in Paraskevi Kyriakopoulou

Branje matematičnih besedil je tesno povezano s prizadevanjem bralca, da razume njegovo vsebino, zato je smiselno takšno branje obravnavati kot dejavnost reševanja problemov. V tem prispevku je obravnavan primer principa matematične indukcije, ki je bil posredovan v branje dijakom srednješolskega izobraževanja. S preučevanjem njihovega prizadevanja po razumevanju besedila smo želeli ugotoviti, ali ta dejavnost vsebuje ključne elemente reševanja problemov. Ugotovitve pokažejo, da so dijaki med razpravljanjem o vsebini besedila prešli skozi štiri faze reševanja problemov po Polyi. Poleg tega menimo, da je pristop branja korakov matematične indukcije na način reševanja problema obetaven za pridobitev konceptualnega razumevanja matematične indukcije.

Ključne besede: matematična indukcija, branje matematičnega besedila, reševanje problemov, dijaki

Introduction

Mamona-Downs and Downs (2005), considering the 'identity' of problem solving, raised a series of issues including, among others, the reading of mathematical texts and considered whether this could involve significant elements of problem solving. More precisely, they acknowledge that 'reading mathematical text often needs an effort from the reader to understand and assimilate its content' (p. 386), and this prompted the question of whether it is reasonable to consider such reading as a problem-solving activity.

However, not all kinds of mathematical text can be appropriate. It seems that the kind that is the most suitable is proof. As Mamona-Downs and Downs (2005) explain, there are three aspects in examining a proof relevant to the issue. The first concerns 'the locating and examining of the implications occurring in the argument' (p. 397). The students can check whether the implications are logically sound and ensure that the necessary conditions are properly accounted for. The second concerns the understanding of how the overall reasoning is structured. Finally, the third concerns the extracting of meaning from the exposition in the sense that 'the reader creates concept images in order to relate the material to intuitively understood schemas' (p. 397).

Moreover, reading a proof seems difficult simply because of the style of exposition. The technical complexity in the use of notation, the leaping from one statement to another without justifying the leap, thus assuming too much, obscures much of the real argument that takes place in the proof. The readers must independently 'retrieve' their own version of how to interpret, evaluate and assimilate the material in front of them. A typical proof cannot be too detailed and excessively broken up into little lemmas, because one can become lost in the details. Thus, reading proofs is not easy if the student attempts to read them like a novel, in a comfortable way with little concentration.

This perspective opens a new area in the research agenda of problem solving; we are not aware of a study that examines this perspective. The few studies that exist in the relevant literature approach this issue from the authors' point of view, focusing on how mathematical texts should be written to make their text as comprehensible as possible (Konior, 1993; Morgan, 1996).

As mentioned above, reading mathematical text has been seen mainly from the authors' point of view. Morgan (1996) acknowledges that the language used in a textbook does not transparently transmit the authors' intentions; different readers may construct different meanings from the same text. This is in alignment with Freudenthal's (1983) view that the way each one of us is thinking is not always possible to be transferred satisfactorily to other people, especially if we differ in background and experiences.

As Konior (1993) mentions, reading such mathematical texts demands certain techniques that can be taught but are not given to the students together with the alphabet. He emphasises the importance of the authors structuring their text in a certain way to determine the process of its reading. This is related to the fact that mathematical texts are mainly conceived as 'highly compact, precise, complex, and containing technical vocabulary' (Österholm & Bergqvist, 2013, p. 751), and it is not clear how, or even if, mathematical texts, in general, can be described in common linguistic terms. Mathematical reading involves both linguistic comprehension skills and knowledge of the 'language of mathematics' (Adams, 2003). Moreover, Österholm (2006) found that comprehending a mathematical text (concerning basic concepts of group theory) becomes even more difficult when the text includes symbols. In particular, he found that if the texts include symbols, they require a special type of skill for reading comprehension, while if the texts are written in natural language, they then merely need a more general reading ability.

If, however, we are restricted to the mathematical texts of proofs, it seems that when students read and reflect on them, they tend to focus on the superficial features of the proofs' arguments, and their ability to determine whether these arguments are proven is very limited (Selden & Selden, 2003). A prooftext can be read in two different ways. One is to validate the proof, meaning to determine whether or not it is valid (Selden & Selden, 1995). The other is reading for comprehension. In the latter case, the validity of the proof is assumed by virtue of its author or source, and the goal of the reader is to understand the proof, not to check its validity (Mejía-Ramos & Inglis, 2009). Furthermore, the students' skills in reading comprehension, in general, are closely linked to their reading and understanding of mathematical texts. Vilenius-Tuohimaa, Aunola, and Nurmi (2008), working with Grade 4 students (9–10 years old), found that their reading comprehension was strongly related to their performance in mathematical word problems.

For securing the validity of proof, the step-by-step presentation of the mathematical proof moving from hypothesis to conclusion is considered suitable. In the case, however, of comprehension and therefore mathematical communication, this linear way does not work. Instead, Leron (1983) proposes the 'structural method' whose basic idea is to divide the proof into levels proceeding from the top down. These levels can be considered short autonomous 'modules', each embodying one major idea of the proof. In a very general (but precise) manner, the top level provides the main line of the proof. The next level proceeds to elaborate on these generalities of the top level. Proofs for unsubstantiated statements, more details for general descriptions, and similarly, are provided at this level. If a sub-procedure is somehow complicated, then a 'top level' description is given in this second level, and details are pushed further down to lower levels. The hierarchy continues similarly. Leron (1983) claims that this method could increase the comprehensibility of these ideas retaining at the same time their rigour. This process is often supported by what Raman (2003) calls a 'heuristic idea', which is an idea based on informal understanding, which gives a sense of understanding but not a conviction. It is more like a sense that something ought to be true. Another method, suggested by Grugnetti and Jaque (2005), is to ask students to look for a mistake in arguments or to examine the validity of their peers' evidence (see also Selden & Selden, 2003).

Mamona-Downs and Downs (2005) claim that reading a mathematical text can be a real problem-solving activity and that understanding a mathematical text can be just as challenging as developing a strategy for solving it. In this sense, this endeavour is connected to the theory of problem-solving (Polya, 1957; Schoenfeld, 1985, 2013). A recent study in this spirit is by Papadopoulos and Iatridou (2010), who presented the Pick's theorem in the form of an open problem to Grade 11 students and recorded the different ways the students approached this problem. The main feature of this approach is that the responsibility of understanding the mathematical text is transferred to the students themselves and not to the teachers or the textbook authors, who by default are considered responsible for making mathematical reading as clear and easy as possible.

This process of comprehending a mathematical text involves aspects of executive control, but, as Schoenfeld (1985a) indicates, the students' metacognitive skills, in general, are remarkably poor. The explanation is that standard instruction focuses on the mastery of facts and procedures and does not deal with metacognition. This is why Schoenfeld (1985) suggests some approaches that could be adapted to support students' needs while reading a mathematical text (Mamona-Downs & Downs, 2005). Yang (2012), working with Grade 9 students, found that good comprehenders tend to use more metacognitive reading strategies for planning and monitoring comprehension compared with moderate and poor comprehenders. In contrast, Weber et al. (2008), working with advanced undergraduate students, found that they use sophisticated comprehension-fostering and monitoring strategies in comprehending texts in the 'definition-theorem-proof' format.

In our study, the focus is on the students' responsibility to be engaged in comprehending the given mathematical text. The text chosen for the purpose of the study is negotiating the notion of Mathematical Induction, taken from a mathematics textbook that is no longer used in regular mathematics teaching and, therefore, is completely unknown for the students.

Mathematical induction and students' difficulties with mathematical induction

The Principle of Mathematical Induction (PMI) is usually (on the grounds of simplicity) expressed in the terms of the properties of natural numbers and with two options about the first number (since 0 and 1 are commonly used). Some authors, however, use a non-negative integer a_o.

The PMI can be stated as: *If 1 has a property P, and if any n having the property P implies that n+1 also has the property P, then every n has the property P.* In a more formal way, this can be expressed as: *If P(1) and if for all n, P(n) implies P(n+1), then for all n, P(n).* A typical proof by induction, therefore, must follow the steps below (Ernest, 1984):

Theorem: $\forall n \in N$, P(n).
Proof: By mathematical induction.
Basis: Prove that P(1) is true.
Inductive hypothesis: Assume P(n) is true.
Induction step: Prove that P(n+1) is true from the inductive hypothesis.
End of proof: Hence, from the PMI, P(n) is true for all natural numbers n.

The research shows that mathematical induction is a very difficult concept for secondary education students to learn, as well as for undergraduate students (Dubinsky, 1989). Induction presents specific cognitive obstacles (some of them will be presented below); therefore, students still fail with this given that the teaching methods do not pay attention to these difficulties. Bell (1920) very early expressed concerns about mathematical induction in secondary education: '[…] mathematical induction has no place in elementary teaching" (p. 413). Baker (1996) examined thoroughly the difficulties students faced when dealing with mathematical induction and identified nine such difficulties. Among others, he found that (i) students have difficulty with the mathematical content of induction, especially with being unable to operate with symbols, (ii) they rely exclusively on procedures lacking thus conceptual understanding, (iii) they are mainly reliant on examples to recognise that something is proven, and (iv) they exhibit poor metacognitive control abilities. To a certain extent, these can be attributed to the way the proof, in general, is negotiated in classrooms. In most textbooks, the relevant proof activities (not only for mathematical induction) begin with phrases such as: 'show that ...' or 'prove that...'. The theorem is given, and the students have to accept its truth and to prove it. This approach leaves behind how the theorem emerged (Avital & Hansen, 1976; Papadopoulos & Iatridou, 2010). Ernest (1984) relates these students' difficulties to the fact that the solvers assume what they have to prove, and then they prove it. He also finds it reasonable that the students 'wonder why this rather complex and seemingly arbitrary principle is adopted' (p. 183). The consequence is that the students are able to deal with mathematical induction tasks and feel comfortable with them, but it is questionable whether they really learned induction and are able to provide a coherent explanation of the induction steps (Allen, 2001).

Despite all these difficulties that the students face with PMI, there are also some findings showing that in some cases students are able to identify the critical properties that justify why mathematical induction works (Palla et al., 2012).

The fact is, however, that no matter the students' difficulties, the significance of mathematical induction cannot be ignored. NCTM (2000), in the section about reasoning and proof in Grades 9 through 12, acknowledges that 'students should learn that certain types of results are proved using the technique of mathematical induction' (p. 345).

Therefore, in this context, our research questions are shaped as follows:

1. Is it plausible to consider the reading of a mathematical text as a problem-solving activity? To what extent can elements of problem-solving be identified in the students' work while they try to comprehend a mathematical text (in our case, the Theorem of the Principle of Mathematical Induction)?
2. Does this approach contribute and/or facilitate its conceptual understanding?

Method

Sample of participants

The participants were 15 students from Grades 10, 11, and 12 (15 to 17 years old) in a public school in northern Greece who participated voluntarily. The students worked in pairs or small groups. For the purposes of this work, we follow one group of five students of Grade 11 since their work provided us with a representative complete set of instances found across the whole sample. The students' performance in mathematics was average and, according to the curriculum, they should have developed some of the mathematics skills required for learning mathematical induction, such as being (i) familiar with the reasoning and proof process, (ii) able to eliminate parentheses and reduce similar terms through the distributive property, (iii) able to use basic identities, and (iv) able to factorise polynomials.

Even though mathematical induction is included in the students' textbooks, the Number Theory chapter that negotiates this topic has been excluded from teaching for the last five years. Therefore, the participants had fluency in the above-mentioned algebraic acts, but they were not familiar with proof by mathematical induction.

Presentation of the task

In the context of the present study, the mathematical text given to students was the Principle of Mathematical Induction. It was an extract of an older textbook (Ntzioras, 1979) that was addressed to high school students (Figure 1). Therefore, despite the content being completely unknown to them, it was written for students of their age and, therefore, it was considered proper to use it in our study.

Figure 1

The PMI in the form given to the students

§ 19. **Θεώρημα** *(πρώτη μορφή τῆς τέλειας ἐπαγωγῆς)*.—**Ἄν p(ν) εἶναι ἕνας προτασιακός τύπος μέ σύνολο ἀναφορᾶς τό σύνολο N τῶν φυσικῶν ἀριθμῶν, τέτοιος ὥστε :**

α) νά εἶναι ἀληθής ἡ πρόταση p(1), καί

β) νά εἶναι ἀληθής ἡ πρόταση : $\forall k \in N,\ p(k) \Rightarrow p(k+1)$,

τότε (δηλ. ὅταν συμβαίνουν τά α) καί β)) **ὁ προτασιακός τύπος p(ν) εἶναι ἀληθής (ἰσχύει) γιά κάθε** $\nu \in N$.

Note. Adapted from Ntzioras, 1979.

Its translation is as follows:

If P(n) is a mathematical statement in the set N of natural numbers, and: (a) P(1) is true, and (b) ∀k∈N, P(k) => P(k+1), is also true, Then the P(n) statement is also true ∀n∈N

At the same time, the students were given an application of the theorem taken from the same textbook (Figure 2)

Figure 2

The application of the PMI given to the students

ΕΦΑΡΜΟΓΕΣ

1η: **Νά ἀποδείξετε ὅτι γιά κάθε φυσικό ἀριθμό ν ἰσχύει ὁ τύπος:**

$$p(\nu): \quad 1+2+3+\cdots+\nu = \frac{\nu(\nu+1)}{2} \qquad (1)$$

Ἀπόδειξη α) Γιά $\nu = 1$ ἡ (1) γίνεται: $1 = \frac{1(1+1)}{2}$, ἀληθής.

β) Ἄς δεχθοῦμε ὅτι ἡ (1) ἰσχύει γιά $\nu = k$ ($k \in \mathbf{N}$), δηλ. ὅτι:

$$1+2+3+\cdots+k = \frac{k(k+1)}{2} \qquad (2)$$

Θά ἀποδείξουμε τώρα ὅτι ἡ (1) ἰσχύει καί γιά $\nu = k+1$, δηλ. ὅτι:

$$1+2+3+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2} \qquad (3)$$

Πράγματι, ἄν προσθέσουμε καί στά δύο μέλη τῆς (2) τό $(k+1)$ ἔχουμε:

$$(1+2+3+\cdots+k)+(k+1) = \frac{k(k+1)}{2}+(k+1) = \frac{(k+1)(k+2)}{2},$$ δηλαδή ἡ (3) ἰσχύει.

γ) **Συμπέρασμα**: Στό α) ἀποδείξαμε ὅτι $p(1)$ εἶναι ἀληθής. Στό β) ἀποδείξαμε ὅτι: ἄν $p(k)$ εἶναι ἀληθής, τότε καί ἡ πρόταση $p(k+1)$ εἶναι ἀληθής *. Συνεπῶς, σύμφωνα μέ τό θεώρημα τῆς τέλειας ἐπαγωγῆς, ἡ (1) ἰσχύει γιά κάθε $\nu \in \mathbf{N}$.

Note. Adapted from Ntzioras, 1979.

Its translation is as follows:

Statement: Prove that

$$\mathbf{1+2+3+\ldots+n = \frac{n(n+1)}{2}, \ \forall n \in N}. \qquad (1)$$

Proof: (a) if n = 1, then $\mathbf{1} = \frac{\mathbf{1(1+1)}}{\mathbf{2}}$, true.

(b) Let us assume that (1) is true for n = k (k∈N), that is

$$\mathbf{1+2+3+\ldots+k = \frac{k(k+1)}{2}}, \qquad (2)$$

We will prove that (1) is true for n = k + 1, e.g.,

$$\mathbf{1+2+3+\ldots+k+(k+1) = \frac{(k+1)(k+2)}{2}}, \qquad (3)$$

Indeed, if we add (k + 1) at both sides of (2) we take

$$\mathbf{(1+2+3+\ldots+k)+(k+1) = \frac{k(k+1)}{2}+(k+1) = \frac{(k+1)(k+2)}{2}},$$

which means that (3) is true.

(c) Conclusion: we proved P(1) is true (step a). We proved that if P(k) is true, then P(k+1) is also true (step b). Therefore, according to the PMI, (1) is true $\forall \mathbf{n} \in \mathbf{N}$.

The participants were asked to read the two texts (Figures 1 and 2) and then answer the questions:

- *Do you understand the Theorem of Mathematical Induction?*
- *Can you identify the series of steps in a proof by Mathematical Induction?*

- *Are you able to explain why this series of steps constitute proof?*
- *How do these steps convince you about the validity of the statement to be proved?*

The specific choice ensured that the students would be involved in a completely unknown mathematical text but, given that it was addressed to high school students, the suitability of its wording and use of symbolism is also ensured.

The students had almost one and a half hours to complete the task. They were asked to vocalise and discuss their thoughts with each other. The only interventions aimed to answer questions related to terminology and symbols in the text.

Collecting and analysing data

The session was audio-recorded, and the students were constantly asked to 'think aloud'. The students' worksheets collected at the end of the session, combined with the transcribed protocols, constituted the data of this study. These data were examined and analysed in the context of qualitative content analysis (Mayring, 2014) at two different levels: first, detecting instances related to Polya's problem-solving steps (*Getting familiar with the problem, Devise a plan, Carrying out the plan, Looking back*); second, seeking evidence of the students' conceptual understanding of mathematical induction. In the context of this study, by the conceptual understanding of mathematical induction, we mean (i) the students' convincing explanation of why the steps of induction constitute evidence for the general truth of the statement, (ii) how these steps relate to each other, (iii) why the case of $n = 1$ is used as a first step, and (iv) why these series of steps constitute a proof.

The data were initially examined independently by the two authors. The coding results were compared, codes were clarified, and some data were recoded until agreement.

Results

In this section, the students' interaction while reading the task will be presented. The presentation will be based on the use of certain extracts from the transcribed protocols. All the excerpts will be accompanied by an alphanumeric string on the left indicating the group (e.g., B), the number of the task (e.g., 2), and the lines of the protocol (e.g., 22–24). Students' discussion and actions will be commented on.

The students initially spend some time individually to think about the task before starting any discussion. After that, their first effort was to connect this task with concepts already known to them, such as the concept of sequence:

[B2.22–24]: *The application is about a sequence that increases by 1 each time. Is this theorem true for every sequence that increases by 1?*

[B2.25–26]: *I thought it was an arithmetic progression.*

They verify the statement for different values of n:

[B2.29–33]: *If we put n = 2…*

[B2.34]: *The statement is true for n = 3.*

They understand that according to the theorem, a statement is true if the two conditions (a) and (b) of the theorem are satisfied (see above), and they describe the problem in their own words:

[B2.41–43]: *Any statement referring to natural numbers and satisfies these two conditions (a) and (b) is true for every natural number.*

They match without difficulty the first step, P(1), of the theorem with the first step of its implementation, and they observe that the proof includes the generalisation element:

[B2.67–68]: *It starts with n = 1, which is a natural number to show that the statement is true for natural numbers in order to proceed then to the case of the statement being true for n = k. Actually, the aim is to generalise.*

They understand the algorithmic part of the inductive step, and they suspect that the inductive step is somehow related to the generalisation of the statement:

[B2. 71–74]: *...Then it adds k + 1 on both sides… to show that we can take any natural number.*

They understand that the second step of the proof serves the generality of the statement:

[B2.104]: *I think that the 2nd step just generalises.*

They wonder whether the information given in the statement of the task is unnecessary:

[B2.107–109]: *The third step seems useless.*

This part of their negotiation brings them quite close to the question of why the given content is a proof:

> [B2.113–114]: *Because k + 1 is again a natural number. That is, from the moment we found that the statement is true for n=k that belongs to N, it means it will be true also for n = k + 1.*

They start to notice that the idea of successive numbers might be a key element in the proof by mathematical induction:

> [B2.115–116]: *Let us say... If we put k = 2… we could also put k = 3.*
>
> B2.117–118]: I*t is not actually two random numbers. n = k is a random number, and n = k + 1 is a bigger random number. Why was k + 1 chosen?*

They regularly come back to the wording of the task and analyse more closely words and data:

> [B2.121]: *The step for n = k is not proof. It says, 'assuming that the statement is true for n = k'.*

This part of the induction makes them feel confused, and they spent time moving back and forth to understand the situation. In particular, they focus on the issue of successive numbers and on assuming what they have to prove before proving it.

> [B2.185–186]: *2 is a random number, n and k+1 are random too.*
>
> [B2.200–201]: *It assumes that the statement is true for n = k... How is it possible to assume that? It is supposed that we want to prove it and we did not it yet.*

They gradually realise that k and k+1 are not specific numbers. They can be any natural numbers, but they always remain successive.

> [B2.211–213]: *We cannot determine the size of this sequence. The numbers can be until 100, until 1000, and so on…*
>
> [B2. 230–231]: *Also, they are random… k is random. And k + 1 is the next one...*

However, they remain unable to describe fully why these steps are proof and re-match the theorem's steps with the implementation steps. They mistakenly believe that the steps of the theorem are themselves the proof.

> [B2.254–255]: *If we can show that it is possible to go from P(k) to P(k + 1) then I think this is enough to convince us.*

Therefore, at this point they think they have solved the problem:
[B2.269]: *We cannot find any arithmetical mistakes in the process. Therefore, the question has been answered.*

They were satisfied with their thinking that if the statement has been verified for a known specific natural number and assuming it is true for a random k, then it is not necessary to go further.

The setting changes from the moment one of the members observed that both 1 and k are values that are assigned to n.

[B2.350]: *Actually, when we say n = 1 or n = k, it is the same action. We have the variable n, and we substitute n with 1 or k.*

Finally, they approach the essence of the theorem.

[B2.353–355, 359]: *Oh, I think I understood. In the beginning, we show it is true for n = 1. Then we assume that it is true for n = k, and then we show that this will be true* ***for every next number****. So, it is true if we put 1 in the place of k. Then, it is true for its next number 2 and the same for the next 3 and so on…*

They end with an implicit reference to the general validity of the theorem for all the natural numbers.

[B2.370]: *We assumed it is true for a random number, and we proved it for its successor. So, the statement applies to all the natural numbers.*

[B2.371–372]: *Because it applies for n = 1 and ... it also applies to n = 1 + 1 = 2. In the same way, it applies for n = 3 ...*

Discussion

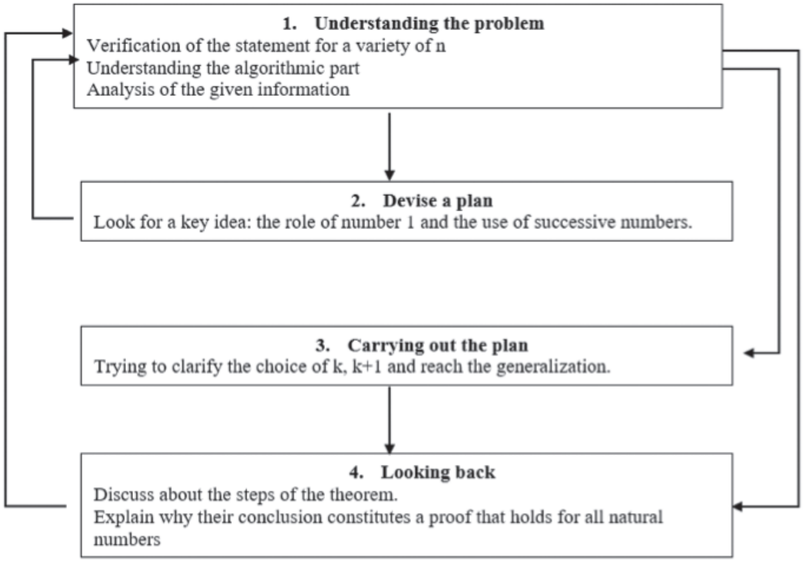
Before discussing the two research questions, it might be said that we do not intend to overgeneralise our findings. This study might better be considered as a case study that gives evidence for thinking on our research questions and support our better understanding of the situation. The next subsections try to shed light on these questions building upon the research findings presented above.

Reading a mathematical text as a problem-solving activity.

The analysis of the findings gave evidence on whether it is plausible to consider the reading of mathematical texts as a problem-solving activity (first research question) since Polya's problem-solving steps can be easily detected in the students' work, as presented in Figure 3.

Figure 3

Schematic representation of the problem-solving process of the group while reading the mathematical text



In 'Getting familiar with the problem', the students took the initiative to ensure they understand the problem correctly. Thus, they verified the statement for different values of n (B2.29–33), they invested time to analyse the given (B2.67–68, 71–74, 75–78, 104) and examine whether the included information was sufficient, insufficient, or redundant (B2.107–109).

Then they started wondering whether they had seen something familiar before, which is in accordance with Polya's 2^{nd} step of 'Devising a plan'. They recalled sequences and arithmetic progression (B2.22–26). Polya, in this step, also invites the solvers to pose to themselves the question 'Could you restate the problem', an action taken by the students at B2.41–43.

They turned towards looking for a key idea, which was used later to solve the problem ('Carrying out the plan'). This idea was evolved around the questions: Why do we start with number 1? What is the role of the use of the consecutive numbers k and $k + 1$? (B2. 117–118). This leads to the solution (B2.353–355, 359).

Finally, actions that could be linked to Polya's fourth step of problem-solving (Looking back) are the efforts made by the students to explain the generality of their arguments for all the natural numbers, starting from the smallest one and its successor (numbers 1 and 2) and then expanding this to the numbers 2 and 3, and so on (B2.370–372).

Therefore, it can be said that the students' effort to deal with this task can be seen as a problem-solving process, and the students exhibited instances of all of Polya's four steps, although in a non-linear way. There was a continuous interplay between the 'Getting familiar' and 'Devising a plan' phases. However, it was 'Getting familiar with the problem' that dominated the students' problem-solving process. The group came back several times to this step after devising or carrying out a plan (see relevant arrows in Fig. 2). This was the reaction of the participants every time they did not know how to move on or when a plan did not seem promising.

Reading mathematical texts and conceptual understanding.

The second research question concerned whether this approach of reading the particular mathematical text facilitated or contributed to the conceptual understanding of mathematical induction in the sense of whether the students have come to substantiate why the sequence of steps in the theorem of mathematical induction is a proof.

It can be said that there was a certain path leading this group towards conceptual understanding. They made choices; these choices were negotiated and revised, were abandoned or improved, and the participants gradually started exhibiting bits of understanding about the cognitively demanding mathematical induction. Based on the analysis of our collected data, we can distinguish five steps in the process of conceptualisation followed by the participants in this study.

<u>Step One</u>: The first step is connected with the design principle of the study to transfer responsibility from authors and/or teachers to students. This takes place through the experience of reading a mathematical text as a potential problem that has to be solved. In our case, the 'problem' we gave was an original mathematical text, the Principle of Mathematical Induction. Its solution is multistep. The students had to identify the series of necessary steps and then explain why these steps constitute proof.

<u>Step Two</u>: The students considered the algorithmic part as the actual proof (B2.254–255). Baker (1996) characterises this as a 'difficulty with proof by mathematical induction predicted in conceptual understanding'.

<u>Step Three</u>: The students gradually started being aware that if the statement is true for a random number then it is also true for its successor. This is a unique property of natural numbers. The set of all-natural numbers forms a (well-) ordered sequence. So, if the initial number (one) has a property and if

it is passed along the ordered sequence from any natural number to its successor, then the property will hold for all natural numbers since they all occur in the sequence (all of them can be generated from a single initial number, e.g., number one) (Ernest, 1984).

Step Four: The students realised that the concept of successive numbers was of critical importance. As Movshovitz-Hadar (1993) explains, this step, if completed successfully, makes it possible to deduce the truth of "For all n N, P(n)" and presents it as an infinite chain of applications of the basic law of inference.

Step Five: The students became finally able to make the connection between the steps (B2.353-372): Given the awareness of Step-3, if one starts with the smallest natural number 1 then the statement is true for its successor 2, and then for the next successor 3, and so on. This substantiates the validity of the statement for the whole set of natural numbers.

It seems, therefore, that this series of steps worked as a scaffolding that initially facilitated students' understanding of the process of proof by mathematical induction. At the same time, there was evidence that the students finally became able to appreciate why this process warrants the truth of the given statement.

Conclusions

This paper has attempted to show that reading a mathematical text could be considered a problem-solving activity. Students are engaged in reading mathematical texts with content completely unknown to them. This signals a shift. Traditionally, the teacher was responsible for the transmission of knowledge and the clarification or explanation of concepts. Now, this responsibility is transferred to the students. The students' effort to comprehend the mathematical meaning of the text became a problem that required a solution. In the students' effort to solve this 'problem', it was feasible to identify all the four problem-solving steps of Polya with the step of 'Getting familiar with the task' to dominate the students' work which confirms that reading of mathematical text might be considered as a problem-solving activity.

Moreover, it seems that this approach contributed to the conceptual understanding of mathematical induction. According to the literature, students do not conceptually understand mathematical induction regardless of whether they are able to apply it and prove statements (Allen, 2001; Baker, 1996). This is why alternative methods of teaching proof beyond the traditional model have been suggested. In our study, however, we introduce the students to the proof by mathematical induction as something that is progressively revealed to them. Therefore, our approach provides them with the mathematical text, and they are required to interpret it. We believe that the findings of this study gave evidence that this process leads to conceptual understanding. The participants were able to grasp the reason the steps of the theorem of mathematical induction constitute proof.

The difficulty to linguistically comprehend the text (Adams, 2003), especially when it includes symbols (Österholm, 2006), became obvious in the students' effort. In the end, however, we can say that this endeavour was successful, and this success seems to has its origin in the combination of three elements: (i) responsibility for understanding the mathematical content was transferred to students, (ii) proof had been selected as the most suitable kind of text, and (iii) the collaborative nature of the problem-solving process.

Given that this study might better be considered a case study, it is obvious that we cannot overgeneralise its results. However, they are promising for planning a future study since some questions arise. Is it possible to obtain similar results when the selected text is not relevant to proof? What is the role of the metacognitive skills the participants already possess? What is the role of the teacher? What aspects of social metacognitive control emerge while students attempt to cope with the task?

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Factors Affecting Success in Solving a Stand-Alone Geometrical Problem by Students aged 14 to 15

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This paper investigates and considers factors that affect success in solving a stand-alone geometrical problem by 182 students of the 7th and 8th grades of elementary school. The starting point for consideration is a geometrical task from the National Secondary School Leaving Exam in Croatia (State Matura), utilising elementary-level geometry concepts. The task was presented as a textual problem with an appropriate drawing and a task within a given context. After data processing, the key factors affecting the process of problem solving were singled out: visualisation skills, detection and connection of concepts, symbolic notations, and problem-solving culture. The obtained results are the basis of suggestions for changes in the geometry teaching-learning process.

Keywords: geometrical problem, mathematic language, problem solving, visualisation

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Dejavniki vplivanja na uspešnost reševanja strogo geometrijskega problema pri učencih med 14. in 15. letom starosti

Branka Antunović-Piton in Nives Baranović

Prispevek obravnava in razišče dejavnike, ki vplivajo na uspešnost reševanja geometrijskega problema 182 učencev 7. in 8. razreda osnovne šole. Izhodišče za razmislek je geometrijska naloga z nacionalnega srednješolskega zaključnega ocenjevanja znanja na Hrvaškem (državna matura), ki zahteva uporabo osnovnih geometrijskih pojmov. Naloga je bila predstavljena kot besedilni problem z ustrezno grafično reprezentacijo in kot naloga v danem kontekstu. Po obdelavi podatkov so bili izbrani ključni dejavniki, ki vplivajo na postopek reševanja problemov pri učencih: spretnost vizualizacije, odkrivanje in povezovanje pojmov, simbolni zapisi in kultura reševanja problemov. Pridobljeni rezultati so osnova za predloge sprememb v procesu poučevanja in učenja geometrije.

Ključne besede: geometrijski problem, matematični jezik, reševanje problemov, vizualizacija

Introduction

Throughout all stages of mathematical education, students are involved in solving various types of mathematical tasks. Such tasks are used as motivational and thematic introductions, learning foundations for novel procedures, algorithms and formulas, and tools for the revision and application of obtained knowledge in new situations and contexts, as well as a discernment of new contents (Kurnik, 2000).

By solving different types of tasks, including tasks of different complexity, students have the opportunity to develop and improve various procedural skills, acquire adequate conceptual knowledge and experience the application of such knowledge and skills in more complex and new mathematics environments (Hsu, 2013). The acquisition and development of problem-solving skills allow students to acquire and apply various mathematical concepts and processes, as well as to develop and cherish adequate mathematical competencies.

In the teaching process, the teacher is a designer and executant of all teaching activities (Cavanagh, 2008; Odluka, 2019). Hence, teachers are tasked with choosing from among various types of mathematical tasks while ensuring an adequate learning environment characterised by equal opportunities to acquire and develop the knowledge, abilities and skills.

The current methods of teaching mathematics are often implemented using several topics/subjects focused on a particular subject area (e.g., triangles, quadrilaterals, linear function, vectors, etc.) (MZOS, 2006). Additionally, tasks are chosen to accommodate the learning, practice, and application of area-specific concepts and processes. Upon completing a specific topic, an assessment of students' skills and competences is carried out using relevant tasks. In such a learning context, students often complete the course without difficulties before moving to the context of the next subject area. However, when students are assessed outside of area-specific contexts, for example, during the PISA international benchmarking tests, TIMSS, and similar, and national level tests (State Matura in Croatia), they often underperform (PISA, 2012; Priručnik DM, 2017; Priručnik TIMSS, 2017).

The most significant underperformance occurs in geometrical tasks during external assessment. Numerous studies (e.g., Baranović, 2019; de Villiers, 2009; Fujita & Jones, 2007) indicate that students have difficulties working with geometrical concepts and processes (visualisation, classification, proving, etc.). Specifically, students perform poorly when given a mathematical task, for example, a mid-level task that can be solved using different paths and strategies outside of the context of the subject area. Thus, this task becomes an isolated problem (defined as a stand-alone (SA) problem). To solve an SA task, one has to implement a network of mathematical knowledge and skills. If this is not the case, meaning if knowledge and skills are insufficiently developed or have remained at the level of disjoined subject areas, students are unable to solve the given problem successfully.

This paper examines the possible factors affecting the students' success in solving stand-alone geometrical task.

Problem Tasks

In this paper, the concept of a 'problem' or 'problem task' implies any task for which a solution or solving method cannot immediately be found. Such a task can indeed represent a challenge for some students, whereas it is a matter of routine for others, highlighting individuals' knowledge and experience in solving problem tasks. These differences in perspective to the problem task can present themselves as an obstacle to learning and teaching, as well as in evaluating students' knowledge; a teacher who knows a certain subject matter well and visualises it skilfully tends to underestimate the difficulties faced by students who encounter the subject matter for the first time (van Hiele, 1986, p. 17).

Current research indicates that solving problem tasks provides multiple benefits for students, such as developing mathematical thinking (Foong, 2002; Leikin & Lev, 2007), stimulating and developing creativity (Klavir & Gorodetsky, 2011), ensuring the engagement of the majority of students during classes (Klavir & Herskovitz, 2014) and at appropriate learning levels (Sullivan, 2009), enabling the identification of mathematical giftedness (Leikin & Lev, 2007), developing communication and a positive atmosphere in math classes (Schukajlow et al., 2012) and similar. Also, the approach that uses problem tasks can transform the teacher's role 'from a transmitter of knowledge to a facilitator of learning' (Cavanagh, 2008, p. 123). Utilising problem tasks, the teacher has the opportunity to guide the flow of students' thoughts through discussion, encourage conclusion-making and creativity, and link the knowledge from disjoined subject areas into one functional unit.

The evaluation of the task-solving process is highly complex, as tasks can be solved in various ways, yet very advantageous as it provides teachers with a better insight into the students' thinking process (Bingolbali, 2011). Furthermore, the inclusion of students into the process of evaluation by their teachers provides students with the possibility of visualising and solving challenges while finding various approaches to the same problem. Discussing possible problem-solving approaches, students have the opportunity to develop and appreciate different solving strategies, such as analysis, synthesis, specialisation, generalisation, visualisation, analogy, among others (Kurnik, 2000). More precisely, solving problem tasks and the analysis of their respective solving processes enable students to develop their learning culture and various problem-solving strategies. Consequently, a practical intertwinement of mathematical concepts allows for a continuity of knowledge, a higher level of thinking, and a deeper understanding of mathematical concepts and processes.

This paper analyses a stand-alone problem (hereinafter abbreviated as 'SA problem') as a distinct type of problem task. The SA problem is defined as a 'problem to find', a problem task that requires greater cognitive effort to discover and link basic mathematical ideas and concepts, facilitating the acquisition of conceptual understanding. As such, the SA problem requires the application of prior knowledge using different approaches and problem-solving strategies, yet simultaneously, it serves as a learning platform for the discovery of new knowledge and concepts. The SA problem can be presented visually, symbolically, and contextually when it is necessary to establish connections between different representations. Furthermore, the SA problem is given outside area-specific confinements, allowing for various didactic purposes. To solve an SA problem, one must become familiar with the basic problem-solving principles of 'problem to find', as proposed by Polya.

The 'problem to find' consists of three main parts: given data, unknown elements, and the conditions linking them (Polya, 1966, p. 92). Therefore, to solve the 'problem to find' means recognising all possible mathematical objects that meet the conditions and their relationship to the given data. As problem tasks require greater cognitive effort, Polya (1966, p. 5) proposes four phases for an effective problem-solving process and finding the solution: understanding the problem, devising a plan, carrying out the plan, and looking back.

In the understanding phase, to develop an intuition about the possible solutions, it is necessary to observe and connect the main parts of the task by combining various task representation methods (text, visual presentation, symbolic notation). To understand the problem, the mathematical language and the students' reading literacy are essential (Gal & Linchevski, 2010; Polya, 1966; Yang & Li, 2016). According to Polya, a partial understanding of the task, which occurs either because of insufficient concentration or interest to solve the task independently, is the most common drawback in the task-solving process (Polya, 1966, p. 58).

In the devising-a-plan phase, it is very important to visualise the path towards the solution by gradually analysing the given situation, remembering a similar task, varying the task, examining the sketch or given figure, setting a particular expression, equation, formula, etc. During sketching or observation of the figure, the visualisation skill and spatial and geometrical thinking become prominent (Duval, 1999; Boonen et al., 2014). Polya emphasises that many students often become lost along the way to determining the solution as they start computing without a plan or idea, which results in a lack of control over their process (Polya, 1966, p. 58).

Once the problem has been well understood and the solving plan created, the carrying out of the plan phase is simple since it only remains to carry out the chosen procedures and computations. However, this phase requires patience and control over the process, as a mistake can easily pass unnoticed, and the goal can be missed. Suppose one becomes stuck or a mistake is noticed. In that case, one can always go back and try again because a clear plan facilitates the control of the solving process (Polya, 1966).

The looking-back phase has numerous advantages: it allows for the additional development and consolidation of knowledge by verifying whether a solution is in line with the given elements and contextual conditions. Teaching praxis has shown that students often pass over the opportunity to check their solutions as they are satisfied by reaching any, especially if their solution coincides with one of multiple choices. The review of the problem-solving process, which includes a discussion and a comparison of different problem-solving approaches to a single task, creates preconditions for the detection and development of adequate solving strategies while allowing for new knowledge acquisition. It is far more useful to solve a particular task in many different ways and then to compare chosen strategies mutually than to solve similar tasks using the same method (Yanhui, 2018).

In line with the phases given by Polya, the task-solving process often occurs as a nonlinear, phase-interchangeable process that constantly evolves by looking back at understanding, revising the plan, and repeating the process until a clear path to the required solution is achieved (Hodnik Čadež & Manfreda Kolar, 2018). Many current mathematics curricula emphasise the importance of developing mathematical processes, which are especially pronounced when solving problem tasks (Odluka, 2019).

Visualisation in Mathematics Teaching

Each person has a biological tendency to visualise his/her thoughts and conclusions. Although the tendency for visual thinking varies among individuals, educational research confirms that the frequency of these tendencies in the general population follows a normal distribution. Hence, some people will never turn to visual representation, whereas others always will, regardless of the opportunity to choose otherwise. However, in a proper context, the majority of people will turn to visual solutions (Presmeg, 2014, p. 152).

The status and the role of visualisation in mathematics learning and teaching is changing. Visualisation has been present and influential since the very beginning of mathematics, especially in geometry. However, the development of mathematical formalism downgraded geometry, and visualisation became secondary. While strict formalism was prevailing, the mathematics community thought of visualisation as a second-rate activity, resulting in its poor application throughout learning, teaching and knowledge evaluation processes. Although most mathematicians use visualisation in their work as an efficient help and support for learning various mathematical topics, they commonly describe visualisation as only an aid along the path to 'true' mathematics. In such an environment, students also develop an attitude that those utilising visualisation are not successful enough, resulting in the neglect of visual explanations and arguments (Dreyfus, 1991).

The development of modern technologies has increased the value of geometry and visualisation. The previous two decades of educational research, specifically research focused on the role of visualisation in learning and teaching mathematics, has uncovered anew the potential of visual reasoning in discovering, describing, debating, and evaluating mathematical results (Duval, 2014; Presmeg, 2006). Therefore, a new idea has gradually developed, suggesting that an overemphasis of abstract and analytical thoughts can have an adverse effect on mathematics teaching and indicating the importance of developing students' visualisation skills (Gal & Linchevski, 2010; Rellensmann et al., 2017; Sinclair et al., 2018).

In literature about learning and teaching mathematics, visualisation, as a notion, is described in different ways, usually implying both the product (visual representation) and process, and often followed by a definition that integrates various other definitions, by Arcavi (2003):

> Visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (p. 217)

As such, visualisation is crucial for the learning and teaching of geometrical concepts and for solving geometrical problems. However, to achieve a deeper understanding of geometrical concepts, a flexible transition between the spoken language, visual representation, and symbolic notations within the problem is required. This problem-solving process that utilises multiple representations is neither linear nor simple but can be mastered by learning and teaching (Duval, 1999). Nevertheless, the visual representation of complex conceptual structures requires high cognitive effort to observe and establish connections between adequate elements of these structures. As such, this process is not a routine, nor is there a procedure for students to rely upon as there is when working with formal symbolic notations (for instance, linear equations solving), which is also one of the reasons why students easily give up visualisation (Dreyfus, 1991), meaning that they have difficulties in geometry (too).

Moreover, as Duval (2014) states:

> For a mathematician and a teacher, there is no real difference between visual representations and visualisation. But for students, there is a considerable gap that most are not always able to overcome even throughout their mathematics education. They do not see what the teacher sees or believes that they will see. (p. 160)

Aims and Research Problem

This research aimed to examine the approaches by which students solved a chosen geometrical task, specifically, to analyse the entire solving process, not only the final solution. We investigated how students established connections among the task text, the visual representation of the described situation, and the symbolic notation. Namely, we asked to what extent students developed problem-solving skills with respect to Polya's phases. The chosen task encompassed several basic geometry concepts of elementary school mathematics, mainly focusing on the concept of the inscribed angle. The task was given to 7^{th}- and 8^{th}-grade students, since the inscribed angle topic is taught and applied in the 7^{th} and the 8^{th} grade, respectively, using various geometrical problems. To examine whether the posing of the problem impacted the problem-solving success, three variations of the same task were offered.

Our research questions were as follows:

1. What factors are contributors to the SA problem-solving process, whether as an asset or an obstacle?
2. How does the posing of the problem affect the success of its solving?

Method

To answer the research questions, students' assignments were processed using the descriptive method with qualitative analysis of the problem-solving process regarding the four phases by Polya.

Participants

The research involved 182 randomly selected 14- to 15-year-old students attending the 7^{th} and 8^{th} grades in different Croatian urban elementary schools. Personal information about the participants was not requested. The participation was voluntary and anonymous; each student was assigned a unique ID code in accordance with ethical research practice (Cohen et al., 2007, p. 61).

Instrument

For research purposes, a geometrical task from the National Secondary School Leaving Exam in Mathematics (DM, 2012) was selected. This task was a multiple-choice textual task with one correct out of the four offered answers (Figure 1). The compiling criteria of the State Matura tests classified the task as an intermediate-level task (40–59% of correct resolutions) for the assessment of the student's ability to apply the properties of inscribed angles intercepting the same circular arc or chord (Priručnik DM, 2017, p.20). The State Matura results showed the task was an advanced level task (20–39% of correct resolutions) and mostly solved by the guessing method. The selected task was offered to research participants in three versions.

The first version (Task 1) of the task was identical to the State Matura task (Figure 1). The second version (Task 2) contained the text of the original task without multiple-choice answers, but with the addition of incomplete drawing (Figure 2); all the given elements were pointed out in the figure, but not the unknown angle.

Figure 1

Task from the State Matura exam – Task 1

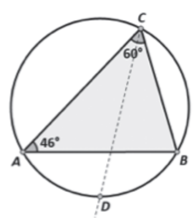
Consider triangle ABC. The measure of angle at vertex A is 46° and the measure of angle at vertex C is 60°. Angle bisector at vertex C intersects the circumscribed circle of the triangle at points C and D. What is the measure of angle ∠CBD?

Answers: (a) 104° (b) 120° (c) 134° (d) 150°

Figure 2

Task with incomplete drawing – Task 2

Consider triangle ABC. The measure of angle at vertex A is 46° and the measure of angle at vertex C is 60°. Angle bisector at vertex C intersects the circumscribed circle of the triangle at points C and D. What is the measure of angle ∠CBD?

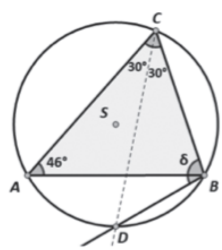


In the third version (Task 3), the text of the selected task was put in the context of an astronomy problem, without multiple-choice answers, and the complete drawing was added to the context (Figure 3). Also, the new terminology was used in the text: the descriptive term 'measure of the angle' was replaced with 'at what angle...is seen'. This version represented a 'dressed up' problem, a problem in which the focus of interest was not on the modelling but on the manner of establishing connections between the text, drawing and computations (Schukajlow et al., 2012).

Figure 3

Context-based task with drawing – Task 3

Young astronomer and mountain climber Sanjin (marked as point S) is observing 4 mountain peaks (marked as A, B, C and D), located at the edge of the horizon, as shown in the drawing.



After climbing the mountain peak A, with the help of astronomical tools, he measured that from the point A, the imaginary line segment $\overline{BC}$ is seen at an angle of 46°.

He also climbed the mountain peak C. He measured that from the point C the imaginary line segment $\overline{AB}$ is seen at an angle of 60° and the imaginary line segment $\overline{BD}$ at an angle of 30°.

Sanjin did not make any more measurements but instead he drew a drawing (as shown) and he applied his knowledge of mathematics and determined at which angle the line segment $\overline{CD}$ is seen from point B.

Explain how Sanjin determined at what angle the line segment $\overline{CD}$ is seen from point B; i.e. what is the measure of the angle δ in the drawing?

All versions of the task included an instruction for the participants to illustrate their work (drawings, computations) and to explain how they reached the result, enabling the qualitative analysis of participants' work. Participants were also allowed to use various geometric tools for their drawings, except the protractor.

Data Collection

To fulfil all the preconditions of the SA problem, the research was conducted at the end of the school year, following the completion of the curriculum. At the beginning of the class session, the participants were given the task and a 15-minute time-frame in which to solve it. The participants were not additionally prepared to solve the task. With a random selection, the participants were divided into three groups. Each group received one version of the task.

Data Analysis

Following data collection and the review of participants' work, the assessment criteria were defined, and three areas were agreed upon:

(1) The creation of the drawing in Task 1 and supplement of the drawing in Task 2,
(2) The correctness of the final answer, as well as the presented solution,
(3) The alignment of the task text, drawing and symbolic notation.

The drawing in Task 1 was assessed according to the emphasis of all the given and required elements, whereas the drawing in Task 2 was assessed according to the emphasis of the required elements. The codes of the given elements are: Type of triangle: Scalene (ST), Equilateral (ET), and Isosceles (IT); Given measure of Angles (GA), Circumscribed Circle (CC), Bisector of angle (BA), Point of Intersect (PI). The codes of the required elements are: Ray or segment BD (R) and Corresponding arc of angle CBD (A).

Regarding the correctness of the final answer, each work was sorted into the following categories: T – if the answer is correct, F – if the answer is incorrect, and O – if the answer is not given. Additionally, the task-solving process was assessed (textual description and symbolic notation) using another five categories: without any explanation (1), procedure completely incorrect (2), procedure partially correct (3), procedure fully correct, but unfinished (4) and procedure fully correct (5). Thus, each work was appointed one marking T1–T5, F1–F4 or O1–O4. For example, T4 indicated that the participant's answer was correct and with a fully correct procedure, but unfinished. There were no F5 and O5 markings as each fully correct procedure resulted in the right solution.

Students' assignments were also assessed for the alignment of the drawing with the task text (highlighting of given and required elements), the highlighting of additional elements and markings in the drawing and the quality of the connection between the drawing and the symbolic notation (Table 1). Thus, each Task 1 assignment was appointed a marking D11–D15, each Task 2 assignment was categorised as D21–D24, and each Task 3 assignment was categorised as D31–D34. Although Task 2 and Task 3 contained pre-set drawings, several students sketched their drawings, resulting in two parallel categories (D2 and D2* for Task 2; D3 and D3* for Task 3).

Table 1

Alignment code for the assessment of the task text, drawing and symbolic notation

Code	Description	Explanation
		Coding for Task 1
D11	without anything	There was no drawing and the computation has not been done.
D12	misaligned	The drawing was misaligned with the task text and the computation, i.e. certain highlighted elements were mismatched with the task conditions, and the calculation was incorrect accordingly.
D13	partially aligned	The drawing was correctly, but partially aligned with the text (not all the given elements were highlighted and there were no required elements), the drawing was partially supplemented with other elements and markings, and it was partially aligned to the conducted computation.
D14	aligned but unfinished	The drawing was correctly, but partially aligned with the text (all the given elements were highlighted, but not all the required ones), the drawing was partially supplemented with other elements and markings, and the computation was conducted accordingly.
D15	completely aligned	The drawing was fully aligned with the task text (all the given and required elements were highlighted), it was supplemented with additional elements and markings, and the computation was conducted accordingly.
		Coding for Task 2
D21	without anything	Nothing was highlighted or marked in the drawing and the computation was not conducted.
D22	misaligned	Nothing was highlighted or marked in the drawing, but the computation was conducted.
D23	partially aligned	The drawing was partially supplemented as per conditions of the task text, including additional elements and markings, and the computation was done accordingly.
D24	completely aligned	The drawing was fully supplemented as per conditions of the task text, including additional elements and markings, and the computation was done accordingly.
		Coding for Task 3
D31	without anything	Nothing was highlighted or marked in the drawing and there is no computation.
D32	misaligned	Nothing was highlighted or marked in the drawing, but the computation was conducted.
D33	partially aligned	The drawing was partially supplemented with additional elements and markings, and the computation was done accordingly.
D34	completely aligned	The drawing was supplemented with additional elements and markings, and the computation was done accordingly.

Results and Discussion

Out of 182 randomly selected participants, 66 (36.26%), 63 (34.62%), and 53 (29.12%) solved Task1, Task 2 and Task 3, respectively. The analysis and discussion of the observed factors contributing to the (lack of) problem-solving success were performed with respect to the features of Polya's four phases. This approach emphasised participants' strategies towards finding the solution. Three variations of the task (Task1-3) also allowed for the analysis of the problem-posing as a possible factor and contributor to the (lack of) problem-solving success.

According to Duval, a true understanding and a success of problem-solving in geometry are achievable when a student is capable of establishing connections among the spoken language (task text), visual representation (drawing, whether sketching a drawing or recognising elements in the given drawing), and the symbolic notation (computation) (Duval, 1999, p. 25), where the visualisation skills are essential. Therefore, to answer the research questions, we also examined the quality of the established connections among the task text, the visual representation, and the symbolic notation and computation. The analytical focus was placed on the visualisation skills as one's sketching or reading of the given drawing, not as a matter of choice but a requirement of the SA problem-solving process that provides insight and understanding of the described situation.

The Phase of Understanding the Problem

According to Polya, students understand a problem if they are able to identify all the given and required elements and at least anticipate the connection that will ensure their path to the solution (Polya, 1966, p. 6). The chosen SA problem emphasised (the lack of) this ability in sketching, identifying the given and required elements, as well as establishing the connections among them.

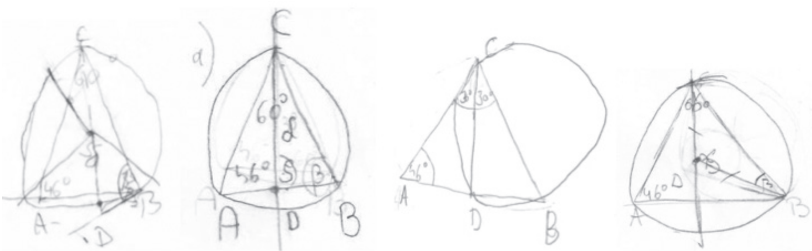
To solve Task 1, the participants had to draw a scalene triangle ΔABC, highlight or mark the given measure of angles at vertices A and C and draw a circumscribed circle. Also, they had to draw the angle bisector at vertex C and highlight point D in the intersection with the circle. As such, all given elements were visually represented (Figure 2). Finally, the participants had to highlight the leg (ray or segment) of BD and then mark the angle at vertex B, namely the angle ∠CBD, to visualise the required element (Figure 3).

Besides sketching, it is equally important to mark the drawing adequately. These markings contribute to the understanding of the problem as 'the student is forced to observe the objects to mark' (Polya, p. 6). Additionally, these markings are used as the symbolic notation for the observed concepts and their relations, enabling the computation.

In solving Task 1, many participants experienced immediate difficulties in drawing (Figure 4), making numerous mistakes in the process. Several participants drew the triangle ΔABC as isosceles or equilateral (for instance, Figure 4 (a)), and, as a result, equalised the angle bisector with the bisector of line segment AB and highlighted the centre of the circumscribed circle S on the angle bisector (for instance, Figure 4 (a)), where point D was on the AB side (for instance, Figure 4 (b)). This approach distinctively changed the situation of the given problem, where the required angle became the angle of the triangle ΔABC at vertex B. In this context, some participants drew the circumscribed circle to triangle ΔBCD instead of triangle ΔABC (for instance, Figure 4 (c)). Additionally, 29% (17 of 66) of participants mismatched the point of the circle and angle bisector intersection with the centre of the circumscribed circle (for instance, Figure 4 (d)).

Figure 4

Incorrect participants' drawings of an SA problem



(a) (b) (c) (d)

While solving Task 1 and Task 2, participants experienced significant difficulties when asked to highlight the required angle, which was crucial for the phase of understanding the problem (Table 2). Only 36.43% of participants (47 of 129) highlighted the required angle on the drawing. Among them, 48.94% (23 of 47) of participants answered correctly, 29.79% (14 of 47) answered incorrectly, and the remaining 21.28% (10 of 47) provided no answer.

In comparison, among the remaining 63.57% (82 of 129) of participants who did not highlight the required angle in the drawing, 24.39% of participants (20 of 82) answered correctly, 30.49% (25 of 82) answered incorrectly, and 45.12% (37 of 82) provided no answer. However, the majority of participants who failed to highlight the angle answered correctly by using the estimation based upon the drawing or by relying on the offered answer.

The poor results in Task 1 could be explained by untidy or incorrect drawings that led to an incorrect identification of the required angle. However, the results of Task 2, a task in which the drawing was given, indicate other underlying causes, such as a lack of understanding of the concept of angle and the symbolic notation '∠CBD' for the required angle (Linchevski & Gal, 2010). Additionally, these results also indicate the lack of ability to establish the relevant connections in the drawing, which is a pre-requisite of the visualisation process (Duval, 2000).

Table 2

The correlation between the required angle and the correct answer

Angle∠CBD	Task 1		Task 2		ALL	
Designed	N	%	N	%	N	%
T	15	22.73	8	12.70	23	17.83
F	7	10.61	7	11.11	14	10.85
O	1	1.52	9	14.29	10	7.75
S1	23	34.85	24	38.10	47	36.43
Not designed	N	%	N	%	N	%
T	18	27.27	2	3.17	20	15.50
F	13	19.70	12	19.05	25	19.38
O	12	18.18	25	39.68	37	28.68
S2	43	65.15	39	61.90	82	63.57
S1 + S2	66	100	63	100	129	100.00

To solve Task 2 and Task 3, 22.22% (14 of 63) and 13.21% (7 of 53) of participants sketched their drawings, respectively, in addition to the given drawing. In comparison to the drawings for Task 1, these drawings were neater, containing more correct elements, yet still reflected the aforementioned drawing-related difficulties. A possible explanation of the need for additional drawing could be the predominance of the triangle ΔABC on the given drawing and its effect on those with weaker visual skills, who usually rely on 'the thing they see first'. Another possible explanation could be that the participants found it easier to understand the connections between the given and required elements by sketching their drawing.

All these results suggest that many participants have underdeveloped visual skills (drawing/reading of the drawing as per the task text and highlighting of the required element), resulting in difficulties during the phase of understanding of the problem. These visual skills are crucial for the process of SA problem solving; lacking the necessary skills, the participants were unable to find the path to the solution. These results are comparable with other researchers' findings, which also emphasise that visualisation skills are not innate but have to be learned and developed (Duval, 1999; Rellensmann et al., 2017; Sinclair et al., 2018). Although many use the saying 'Geometry is the art of reasoning well from badly drawn figures', this ability to reason well cannot be expected from students lacking visualisation skills and the confidence in their knowledge. Not only are the process of drawing and the further use of the drawing not simple, but the reading and understanding of the representations made by others are quite complex processes (Duval, 2000, p. 59).

Although a necessary part of the understanding phase, the recognition of given and required elements alone is insufficient to find a solution, which indicates other factors affecting the (lack of) success in the problem-solving process.

The Devising a Plan Phase and the Carrying-out Phase

After reading the text, making the drawing, or identifying the elements on the drawing, one needs to develop an idea that would be the basis for the creation of the realisation plan, and 'the path from understanding the task to implementing the plan can be quite long and curvy' (Polya, 1966, p. 7). After the implementation of the plan, 'the execution of the plan is much easier' (Polya, 1966, p. 11).

The process that consists of creating/reading the drawing, observing the necessary configuration in the drawing, creating an adequate symbolic notation in line with the drawing and computing, also occurs as a nonlinear, interchangeable process. Such a process constantly evolves via looking back to the drawing, (re)creating notations and (re)computing, which requires a significant amount of knowledge, skills and experience (Duval, 1999; Polya, 1966).

Although encompassing several basic geometry concepts, the conceptual design of the chosen SA task revolved around three features: the equal inscribed angles at the same arc, the angle bisector (dividing into two equal parts), and the sum of interior angles of triangles. If utilising only these features, participants were able to determine the measure of the required angle. One possible approach (a good idea) was to notice that the required angle was one of the angles in ΔBCD triangle, meaning that the measure of the required angle could be determined using known measures of the other two angles of the triangle. Another good idea was to notice that the required angle consisted of two angles with the B vertex, meaning that the measure of unknown angle could be determined from the measures of these angles with the B vertex.

In general, our research sample consisted of participants who had a good idea that resulted in an elegant way to the solution (Figure 5), but also of those who got lost along the path to the solution, despite having a good idea (Figure 6). Additionally, some participants started with a good idea but failed to find their path to the solution (Figure 7), whereas several participants had no idea and came to the (correct) solution by guessing and estimation or incorrect computation (Figure 8).

Figure 5
Solution to Task 3 with the code D34 and T5

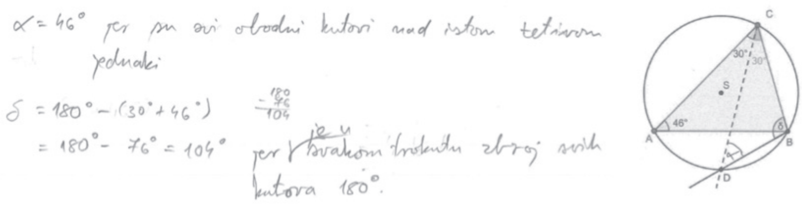


Figure 6
Solution to Task 2 with the code D23 and F3

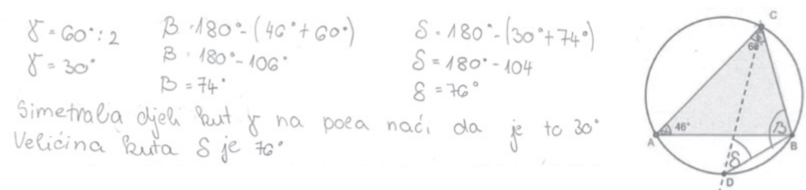


Figure 7
Solution to Task 2 with the code D24 and O4

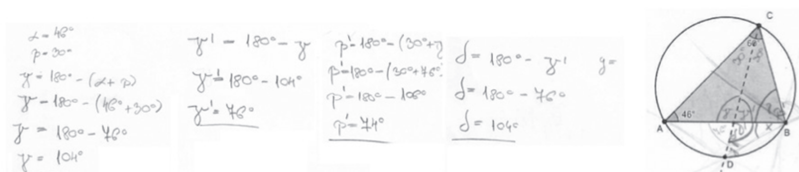
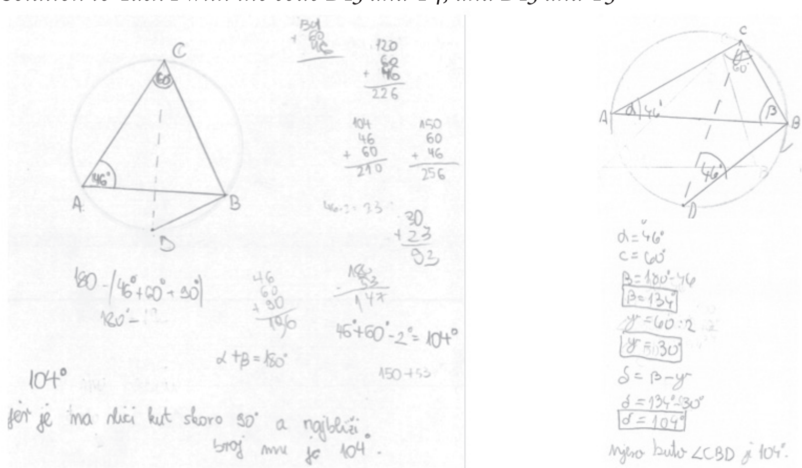


Figure 8
Solution to Task 1 with the code D13 and T4, and D13 and T3



In the problem-solving process of the chosen SA problem, participants utilised various configurations but experienced tremendous difficulties when observing the configuration with the required element and utilising this particular configuration to determine the measure of the required angle. Hence, participants experienced difficulties while discovering the path plan (e.g., Figure 8). The inability to determine the necessary configuration is a reflection of underdeveloped visual skills (Duval, 1999), whereas the inability to find one's way inside the chosen configuration reflects a poor knowledge of adequate concepts and insufficient experience in working with these concepts (Polya, 1966, p. 7).

Many participants also had difficulties in making symbolic notations of the observed elements in the drawing, their connections, and the applicable rules (e.g., Figure 6), all of which were necessary preconditions for the successful realisation of the plan. As they lack ideas and the path plan towards the solution, participants tend to note and compute everything they observe and to perform computations irrespective of the drawing and the highlighted markings (Figure 8). Consequently, many participants used the same marking for different angles or a marking that was not highlighted for the computation. Also, some participants utilised one marking in the drawing, and another in the computation, although both markings represented the same object.

The observed difficulties indicate the existence of important factors that affect the (lack of) success in determining the problem solution: the visualisation skill while reading complex drawings, the knowledge and experience when working with certain concepts, the skill to apply symbolic notations to the observed elements in the drawing and the conduction of computations based upon applicable rules.

The analysis results of the problem-solving process and its respective connections between the task text, the drawing and the symbolic notations, as a basis of the computation, are given in Table 3.

Table 3

Establishing connections between the text, drawing, notation and computation

	Task 1						Task 2						Task 3						All together	
matching text, image, notation and computation		T	F	O	S1	%		T	F	O	S2	%		T	F	O	S3	%	S	%
completely aligned	D15	14	5	1	20	30.30	D24	8	7	9	24	38.10	D34	14	5	1	20	31.75	64	35.16
aligned but unfinished	D14	6	3	2	11	16.67													11	6.04
partially aligned	D13	1	2	1	4	6.06	D23	0	5	8	13	20.63	D33	0	10	6	16	25.40	33	18.13
not aligned	D12	11	10	8	29	43.94	D22	2	7	15	24	38.10	D32	4	5	2	11	17.46	64	35.16
without anything	D11	1	0	1	2	3.03	D21	0	0	2	2	3.17	D31	0	0	6	6	9.52	10	5.49
SUM		33	20	13	**66**	100.00		10	19	34	**63**	100.00		18	20	15	**53**	100.00	**182**	100.00

In solving Task 1, 30.30% of participants (20 of 66) completely and correctly highlighted all the given elements and the required element in their drawings, connecting drawings to the computations. Only two participants (3.03%) did not have a drawing, and the remaining 66.67% of the participants (44 of 66) had difficulties highlighting the required angle, or they misrepresented the required elements, misaligning the required angle with the task's conditions. As a result, the misrepresentation of the drawing's conditions had a significant impact on the relevant notations and computations.

Although the drawing was given as a part of Tasks 2 and 3, participants failed to make full use of it. In solving Task 2, 38.10% of the participants (24 of 63) filled the given drawing completely and correctly, resulting in the aligned computation. In solving Task 3, this was accomplished by 37.74% of participants (20 of 53).

Since participants did not take care to use systematic, precise and correct symbolic notation, nor for matching a symbolic notation with the drawing, and were often impatient in the problem-solving process, we conclude that the finality of the answer was of greater importance to them than the process itself. Without a plan, participants turned to the guessing method utilising what they 'see' on the drawing, especially when the answers were offered, or they simply 'tuned' their computation accordingly. As members of the true *click generation:* students gravitated towards the 'first instance' solution, meaning the solution 'at the first click'.

A general strategy applied by our sample participants was to calculate whatever was possible, whatever came first to mind and then attempt to find a connection with what was required while hoping to be lucky enough to be successful.

Insufficient patience and perseverance in the problem-solving process are important factors affecting the (lack of) success. The offered solutions have a disastrous impact on the problem-solving process, especially when students lack an idea of how to do so; instead, they align their answers with the offered solution.

All the aforementioned prevents participants from mastering the problem-solving process and finding the path to the final solution. It also hinders those who assess participants' work to clearly and fully recognise the flow of thoughts and the degree of participants' understanding in this process.

The Looking Back Phase

The problem-solving process does not end with the solution; the looking back phase requires checking whether the obtained solution makes sense, whether the conducted procedure is correct, and whether there is another, more economical path to the solution.

Because the participants mostly failed to work and develop a plan, computing despite their partial understanding of the context, it is possible that their problem-solving process ended with the solution. Their notations showed writing, then erasing, then writing again, but more as an on-the-fly process, which stopped after reaching the solution.

To summarise, the third phase of the problem-solving process was predominantly seen in participants' assignments, along with participants' intention to reach the final solution as quickly as possible. Failing to check the meaningfulness of the obtained solution and to monitor their process, participants passed over the possibility of finding possible mistakes, as well as to utilise the important and instructive working phase for strengthening one's knowledge and task-solving skills fully (Polya, 1966, p. 12).

Problem-posing and Solving Success

Finally, it is important to single out participants who fully connected the task text (all the given and required elements), the visual notation (whether they drew or used the given drawing) and symbolic notation (code D11, D21, D21*, D31 or D31*), for which the computation was correct (code T5). There were 19.70% (13 of 66), 11.11% (7 of 63), and 22.64% (12 of 53) of such participants who solved Task 1, Task 2 and Task 3, respectively.

These results suggest that the problem-posing impacted solving success. Participants who solved Task 3 were the most successful (22.64%), providing the largest number of the most elegant solutions and more than one way of solving the task. Therefore, we conclude that the realistic context of Task 3 greatly motivated participants to complete the solving process. Also, the completeness of the given drawing possibly contributed to the process, guiding participants' attention in the right direction. Surprisingly, participants who solved Task 2 were the least successful (only 11.11%). Task 2 offered no answers to lead participants or an interesting context to motivate them. The drawing of Task 2 was less helpful, as the required angle was not highlighted.

Moreover, Task 2 had the highest percentage of assignments without an answer (53.97%, 34 of 63). To solve Task 1, participants were required to draw, which contributed to the poor results, as those participants who were insecure and possessed underdeveloped visual skills could easily draw using the offered solutions 'as a last resort'. Hence, Task 1 had the least number of assignments without an answer (19.70%, 13 of 66), which clearly indicates that the offered answers directed participants toward guessing or aligning the solution.

To conclude, numerous factors improve or hinder a successful path to the required solution. The most prominent factors are the visualisation process (making or reading of the drawing, observing the relevant elements and connections), the understanding of the concepts and the connections necessary to solve the problem, the correct symbolic notation aligned with the drawing and text, the gradualism and patience in conducting of the procedure (through the four phases), control over one's process, testing the meaningfulness of the obtained solution, and the posing of the problem.

Conclusion

The obtained results allow us to conclude that the selected sample of students lacked fully developed problem-solving skills, the understanding of certain geometrical concepts, and the skill to identify and connect conceptual properties, resulting in students' inability to find a systematic way to the required solution (which is in agreement with the previous State Matura results, as well as the PISA 2015 results). The underdeveloped visualisation skills were observed as a particular issue, as fully-developed visualisation skills are required for the problem-solving process of geometrical tasks. The poor connections among the task text, the visual representation, symbolical notations and computation were noted as another issue; without these connections, it is impossible to find the path to the solution (Duval, 1999). In the task-solving process, a student often conducted only the third phase (the notation that is usually not connected to the drawing and computation), and lacking a plan, resolved to use of 'calculate whatever you can' strategy and, if possible, 'at first click'.

All aforementioned difficulties that students experienced throughout the problem-solving process indicate that the learning and teaching of geometry should emphasise the visualisation skills (drawing, interpretation, formation of connections among different notations, etc.) (Duval, 2014; Sinclair et al., 2018) and systematic notetaking. This skillset can be learned and developed by solving geometry problems of different cognitive requirements. Additional emphasis should be on the contribution of the visualisation towards the development of geometric thinking, imagination and creativity, not only in mathematics but also in other areas (Duval, 2014). Hence, it is of utmost importance to increase the awareness among teachers and pre-service teachers about the role of the visualisation in the teaching process.

In addition, the assessment of the problem-solving process, especially of the SA problem-solving process, is a highly complex and demanding endeavour. However, this assessment provides remarkably useful information about the flow of thoughts and the background processes relevant to students' understanding. Teachers can utilise these cognitions to effectively change their teaching and the implementation of problem tasks. By involving students in the assessment process, teachers can create an environment where each student will have the chance to explore, discover and relate various mathematical concepts and thus improve their mathematical literacy, mathematical communication skills, and problem-solving culture (Odluka, 2019, p. 94). A discovery of horizontal and vertical connections among the mathematical knowledge can consequently strengthen students' interest in mathematics (Yanhui, 2018).

For further insights, it is desirable to conduct more research, for instance, with secondary school students or with pre-service teachers, and to examine the extent to which the aforementioned factors affect the success in problem-solving. It would be important to study how teachers approach geometric problems, how they help students solve the aforementioned problems, how they utilise the visualisation in teaching, and similar issues. Moreover, it would be useful to enrich professional teacher training with workshops for the SA problem design and further examine whether such problem tasks incite students' interest or improve their success in solving problem tasks.

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Management of Problem Solving in a Classroom Context

Eszter Kónya*[1] and Zoltán Kovács[2]

We report on the results of a professional development programme involving four Hungarian teachers of mathematics. The programme aims to support teachers in integrating problem solving into their classes. The basic principle of the programme, as well as its novelty (at least compared to Hungarian practice), is that the development takes place in the teacher's classroom, adjusted to the teacher's curriculum and in close cooperation between the teacher and researchers. The teachers included in the programme were supported by the researchers with lesson plans, practical teaching advice and lesson analyses. The progression of the teachers was assessed after the one-year programme based on a self-designed trial lesson, focusing particularly on how the teachers plan and implement problem-solving activities in lessons, as well as on their behaviour in the classroom during problem-solving activities. The findings of this qualitative research are based on video recordings of the lessons and on the teachers' own reflections. We claim that the worked-out lesson plans and the self-reflection habits of the teachers contribute to the successful management of problem-solving activities.

Keywords: classroom discussion, problem solving, professional development, reflective thinking

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Vodenje reševanja problemov pri pouku

Eszter Kónya in Zoltán Kovács

V prispevku so predstavljeni rezultati programa strokovnega izpopolnjevanja, v katerem so sodelovali štirje madžarski učitelji matematike. Cilj programa je bil podpreti učitelje pri vključevanju reševanja problemov v njihovo poučevalno prakso. Osnovna načela programa pa tudi njegova novost (vsaj v primerjavi s poučevalno prakso na Madžarskem) so, da razvijanje poučevanja poteka v praksi, da je prilagojeno učiteljevemu časovnemu načrtu poučevanja ter da poteka v tesnem sodelovanju med učiteljem in raziskovalci. Učitelje, vključene v program, so raziskovalci podprli pri načrtovanju učnih ur pa tudi s praktičnimi nasveti in z analizami izvedenih učnih ur. Napredek učiteljev je bil ocenjen po enoletnem programu. Temeljil je na učiteljevi samostojno zasnovani učni uri, njihovem načrtovanju in na izvajanju dejavnosti reševanja problemov pri pouku pa tudi na njihovem odzivanju med dejavnostmi reševanja problemov. Ugotovitve kvalitativne raziskave temeljijo na videoposnetkih učnih ur in učiteljevih refleksijah opravljenega dela. Trdimo, da premišljeno načrtovanje poučevanja in učiteljeva samorefleksija prispevata k uspešnemu vodenju dejavnosti reševanja problemov pri pouku.

Ključne besede: razredna diskusija, reševanje problemov, profesionalni razvoj, refleksija

Introduction

Teaching problem solving in mathematics classrooms is not an unknown element of the tradition of teaching mathematics in Hungary and is closely related to Polya's principle of active learning (Pólya, 1981). Along with arithmetic fluency, problem-solving activities became more important in the Complex Mathematics Teaching Experiment led by Tamás Varga (1919–1987) in Hungary in the 1960s and 1970s (Varga, 1988). Nevertheless, our experience is that these activities are becoming less common in Hungarian teaching practice today: teachers often ignore problem solving as a means of achieving a better understanding of mathematical concepts.

In order to support teachers in incorporating problem solving in their classes, we elaborated on a research-based professional development (PD) programme. One of the core principles of the programme is that we only want to make incremental changes in teachers' practice, focusing on the problem-oriented approach to learning mathematics. This principle also appears in the research of Niss, who claims that "instead of making more radical changes in curricula, in teaching and learning materials, and in assessment, corresponding to the changes in the audiences, authorities have attempted to preserve the goals and the ethos of mathematics education of the past, at least in spirit, while making series of piecewise adjustments so as to avoid too drastic discontinuities in the transition from the past to the present" (Niss, 2018, p. 79).

The origin of our framework was the Japanese lesson study model (Fernandez & Yoshida, 2004), thus the PD programme is linked to the everyday teaching practices of the participants. The lesson study process includes the following three steps: (1) collaborative lesson planning; (2) one participant teaches while the others observe his/her work; (3) discussing the study lesson, reflections and suggestions. After revising the lesson, an updated version of the lesson plan is prepared. The collaboration in our model means that the teachers and researchers work together in planning the study lessons. We retain the feature of the model that the participating teachers visited and reflected upon each other's lessons. The lesson analysis was done in two steps: after the lessons and after the semester.

In line with Walsh (cited by Rott (2019)), we aimed at improving the quality of teaching within the existing curriculum, focusing on learning in the form of problems. Pehkonen et al. (2013) go one step further and emphasise the role of open problems, which also contribute to a better understanding of key principles and concepts. The open approach to teaching mathematics "leads almost automatically to problem-centred teaching and increases communication in class, thus approaching instruction that is more open and pupil-centred" (Pehkonen et al., 2013, p. 12).

In our view, the problem-centred or problem-oriented approach to learning mathematics is characterised by three properties (Kovács & Kónya, 2019):

(1) students analyse a mathematical problem situation;
(2) students critically adapt to their own and their classmates' thinking;
(3) students learn to explain and justify their thinking.

This approach is closely related to problem-solving strategies. In this respect, we rely on our preliminary research on problem solving conducted in various student age groups from grades 4–12. We found that observing and following a pattern is a well-known and often used problem-solving strategy in the age group of 11–12 years, which is why patterning has become a feature of our programme. We also found that the further phases of the inductive thinking process, such as the formulation, testing and explaining of conjecture, are barely observable. The students usually did not feel the need for such an explanation. In our programme, item (3) is therefore given particular emphasis and is closely related to (2), as reasoning is often done in pairs or as teamwork. Consequently, the nature of students' mathematical thinking should be taken into account in the design and implementation of lessons, emphasising inductive thinking and patterning (Kónya & Kovács, 2018).

The successful implementation of problem solving in mathematics classrooms is strongly dependent on teachers' behaviour during the phases of the problem-solving process. Rott (2019) classifies teachers' behaviour as:

- closely managed, i.e., preferring only one approach, to which the students are led;
- emphasising strategies, i.e., encouraging students to pursue their ideas, aiming for strategic diversity;
- neutral, i.e., neither narrowing down students' approaches nor emphasising strategic diversity.

In order to better investigate the behavioural type of a teacher, Rott divided the problem-solving process into the four phases of Polya: (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and (4) looking back. In additional, the teacher's intervention regarding their students' problem-solving process was interpreted for each of these phases. The teachers' behaviour during these problem-solving phases was coded following Rott's grid.

The third core element of our PD programme is lesson analysis. We agree with Lee (2005) that self-monitoring and a reflective attitude help teachers to become more successful. According to Šarić and Šteh (2017), critical reflection should mean looking for new solutions and paths in teaching in order to introduce the changes that contribute to the transformation of the community for better learning. Therefore, we emphasise the importance of the teachers developing reflective thinking: why they employ certain instructional strategies, how they can improve their teaching, and how they can evaluate their work from different points of view.

Our PD programme has a narrow goal: the teacher should progress in implementing problem-oriented learning in his or her classes, i.e., s/he should be able to process the curriculum in a problem-oriented way and realise classroom discussion. Taking into consideration all of the research-based principles described above, we consider whether the PD programme we have developed is effective or not. Its effectiveness is scrutinised through the following two questions focusing on the participating teachers' progression:

Q1 Is the teacher completing the PD programme able to incorporate a problem into the lesson plan that reflects a problem-oriented processing of the curriculum?

Q2 Is the teacher able to organise classroom discussion so that the students explain and justify their ideas, while also critically adapting to their own thinking and that of their classmates?

Method

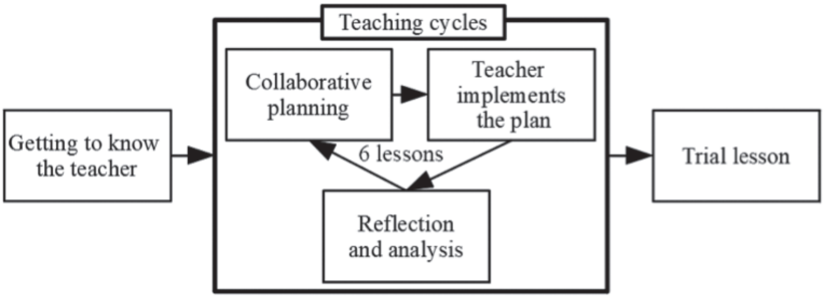
After a pilot study (Kovács & Kónya, 2019), we established the structure of our PD programme. As a first step, the researchers took part in the classes of the teachers and characterised their teaching style.

After selecting the study lessons from the teachers' agenda, the researchers developed detailed lesson plans. The teachers gave their opinions and suggestions, and determined the final lesson plan. One of the participants taught the lesson, which was video recorded, while another teacher from the PD programme took part in the class as an observer.

We organised six teaching cycles and concluded the year-long programme with one trial lesson (Figure 1).

Figure 1

The structure of the PD programme



Four teachers and two researchers (the authors of the present article) participated in the programme. The teachers had 15–20 years of experience in teaching mathematics and were employees of the same school in a Hungarian town. Having chosen one of their classes, the teachers took part in the programme together with the head of the school. Although the students of the selected classes were motivated to learn mathematics, they did not show any special interest in this school subject.

As a starting point, we visited some lessons in the experimental classes and discussed the teachers' professional views and the goal of the developmental programme with the teachers. We found that they all preferred a closely managed way of teaching: they explained the new material, asked the students direct questions and did not feel any need to initiate open classroom discussion.

After completing the last teaching cycle, the participating teachers were asked to plan a trial lesson individually in the same spirit as the previous six lessons planned by us. The lesson themes were chosen freely by the teachers from their course calendar. They had an opportunity to discuss the plan with us before its implementation. The lessons were held in the same class as the previous study lessons. After the lessons, we analysed the video recordings and a few weeks later organised a closing discussion with the teachers.

In the present paper, we focus on four trial lessons (Table 1) that were held in April 2019.

Table 1
Information about the trial lessons

	Teacher A	Teacher B	Teacher C	Teacher D
Grade	5	8	9	10
Age of students (years)	11–12	14–15	15–16	16–17
Class size	23	11	12	14
Lesson theme	Coordinate plane	Arithmetic sequence	Line reflection	Trig. functions in geometry
Lesson objective	Introduction	Introduction	Systematisation	Application
Didactical function of problem solving	Deepening new knowledge	Deepening new knowledge	Applying updated knowledge	Practising, modelling

The first research question is answered by analysing the problem situation incorporated in the lesson plan according to the following aspects: whether the problem is in line with the theme and objective of the lesson, and whether it provides an opportunity for a problem-oriented approach as outlined in the PD programme. Concerning the second research question, we use a content analysis of the transcripts made from the video recordings and evaluate the teacher's behaviour according to Rott's grid.

Results

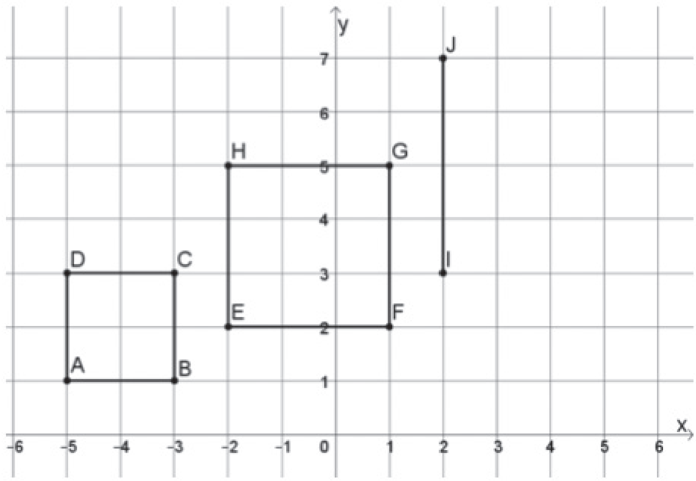
In line with our two research questions, we present our results by first providing a brief description of the problem addressed in the trial lesson (Q1) and then highlighting some typical and informative moments of the classroom discussion during the problem-solving activity (Q2).

Problem A

a) *Write the coordinates of the points marked on the drawing. (Figure 2)*
b) *Continue the sequence.*

Figure 2

Diagram for Problem A

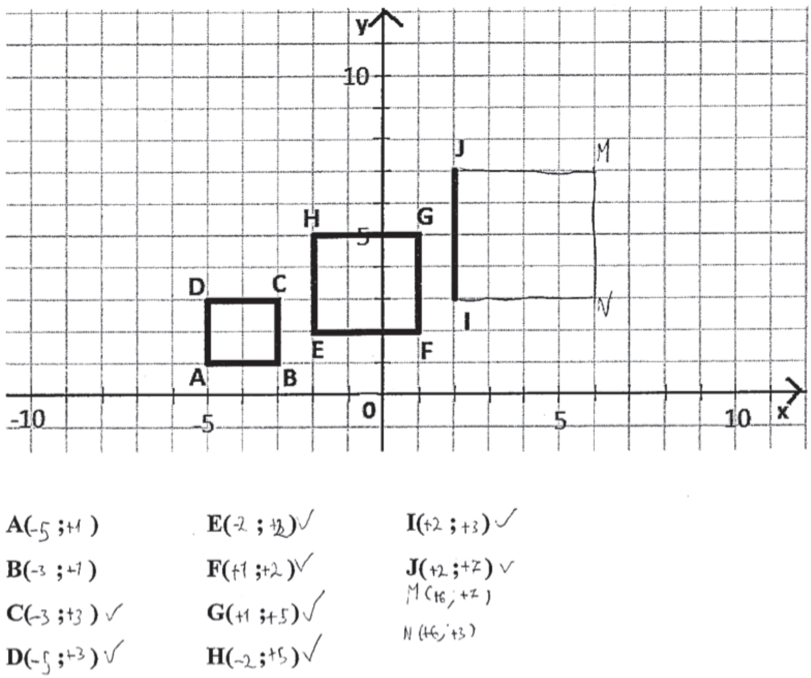


Ad Q1. The theme of the lesson in Grade 5 was to introduce the Cartesian coordinate system. The purpose of setting this complex problem was to deepen the new knowledge and practise the patterning strategy. In order to draw a new square, the students have to recognise the geometric regularity and orient themselves in the coordinate system, after which reading the coordinates of points was a straightforward application of the new material.

Ad Q2. Teacher A set this problem at the end of the lesson. The first part (writing the coordinates of the marked points) was presented to the whole class. Only those who answered the first question quickly had time to deal with the second part, i.e., the patterning problem.

From a total of 23 students, 8 answered the patterning problem and 4 solved it correctly (Figure 3).

Figure 3
A correct solution to Problem A



There was no classroom discussion on the second part of the problem.

Problem B

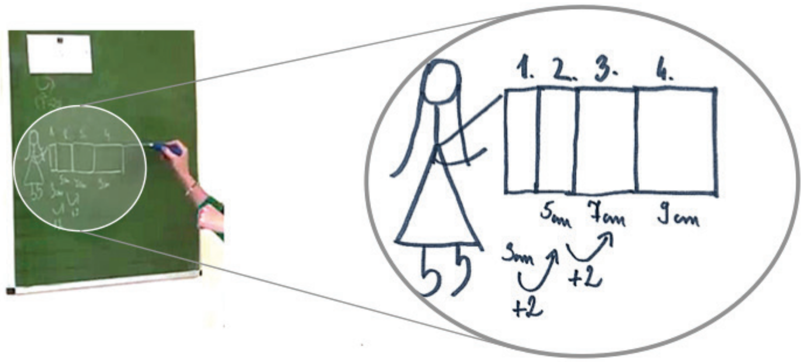
a) *Zsuzsi decided to knit a scarf […] the first day she produces a 3 cm long section, […] she makes two centimetres more each day than the previous day.*

b) *How many centimetres will Zsuzsi knit on the 2nd, 3rd, 9th, 20th and nth day?*

Ad Q1. Teacher B used the problem to introduce new material (arithmetic sequences) at the beginning of the Grade 8 lesson. The task design reflected the patterning phases (determine close members of the series, then a distant member and finally a general member). The problem in the lesson led to the formula of the general element of the sequence, knowing the first element and the difference. The rule itself was created by the students during a classroom discussion.

Ad Q2. After giving the students a few minutes to think about the solution individually, Teacher B drew and wrote on the board the students' answer regarding the 2nd, 3rd, 4th and 5th day (Figure 4).

Figure 4
Teacher B writes and interprets the students' answers on the board



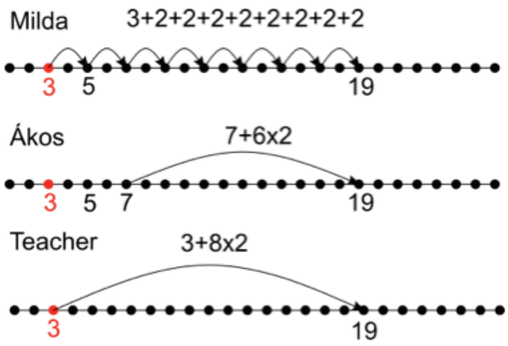
After that, the following classroom discussion was initiated.

01 Teacher B: How many centimetres will Zsuzsi knit on the 9^{th} day?
02 Milda: 19.
03 Teacher B: The result is 19. How did you calculate it?
04 Milda: I continued the sequence.
05 Teacher B: You continued the sequence. On the 6^{th} day, she knitted 13 cm, on the 7^{th} day 15 cm, on the 8^{th} day 17 cm, …, very good!
06 Ákos: (interrupts the teacher) I multiplied the difference by 6 because the 3^{rd} member is already there, and there are six more members, and I added 12 to the 3^{rd} member.
07 Teacher B: Ákos says he did not count one by one, but he calculated how much to add to the 5^{th} day … to get the 9^{th} day … 6^{th}, 7^{th} (she uses her fingers to count, and then she becomes uncertain) … From what did you start?
08 Ákos: From the 3^{rd} day…
09 Teacher B: Yes, from the 3^{rd} day. So, he calculated that from the 3^{rd} day, he added …
10 Ákos: six …
11 Teacher B: Six times two, right and that gives the result.
12 Teacher B: Why did you start from the 3^{rd} day? You could have started … (waiting for somebody to continue her sentence)
13 Zalán: … from the very first number (continues the teacher's thought)
14 Teacher B: … from the very first. Usually, we start from the very first, but Ákos had a very good idea … Usually, we start from the first element. After all, if we were, for example, asking for the 100^{th} member, it would

take quite a while to determine all 99 members before it …

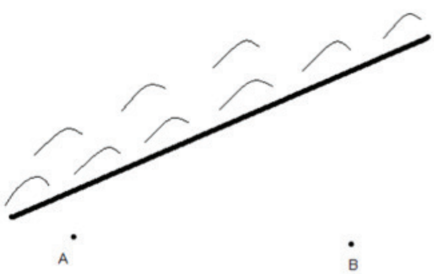
When calculating the 9th element, there was one student who enumerated all of the elements (Line 04), but another student started with the 3rd member: he already considered the previous stage and added six times the difference directly (Line 06). Teacher B did not ignore this idea but used it to steer the discussion in the direction she had imagined: to determine the elements of the sequence from the first member and the difference (Lines 12–14). We summarise the suggested solution strategies in Figure 5.

Figure 5
Different ways of calculating the 9th element of the sequence



Problem C
The man who lives in house "A" should take water from the river to the house "B" every morning. How can he make the shortest route? (Figure 6)

Figure 6
The "water from the river" problem



Ad Q1. Problem C is an indirect application of line reflection strongly related to the theme of the lesson. It provides an opportunity for ninth-grade students to come up with strategies to find the shortest path, as well as to discuss individual ideas. However, the problem appeared in an isolated way in the middle of the lesson, i.e., neither the previous nor the subsequent items referred to the problem.

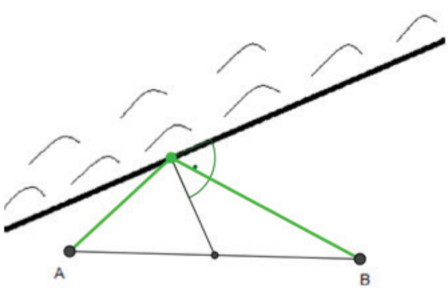
Ad Q2. After reading the text and working on it in pairs, the classroom discussion started with the following brainstorming.

01 Teacher C: Who has some idea of how to find this shortest route?

02 Panni: He will take the water from the river at that point (she shows it with her arms) …we connect points A and B and draw a line from the midpoint perpendicular to the line [river]. (see Figure 7) … so the distances will be the same.

Figure 7

Panni's idea



03 Teacher C: Why?

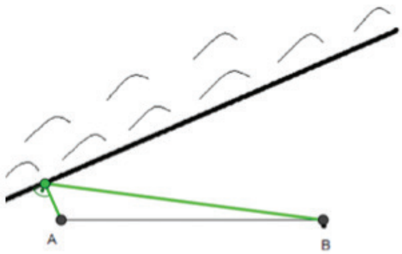
04 Panni: If he goes directly from A to the river, the route will be longer.

05 Teacher C: Please measure the distance then compare your result with those of your pair.

06 Teacher C: Are there any other ideas?

07 Tibi: We draw a line from point A to the river … (hand gestures) perpendicularly … and draw it further to point B. (see Figure 8)

Figure 8
Tibi's idea



08 Teacher C: Hmm … I'll try to give you some hints. Listen. What is the shortest way between two points?
09 The class together: A straight line.

Two students shared their ideas with the class during the brainstorming phase (Line 02 and 07), but Teacher C realised that the conversation was coming to a dead-end and gave a direct hint to move forward (Line 08). However, this hint did not guide students toward the right solution.

Problem D
What is the angle of the slope in degrees? (Figure 9)

Figure 9
Traffic sign

Ad Q1. The purpose of setting this problem was an application of trigonometric functions in an everyday situation, as well as making a mathematical model for a well-known traffic sign.
Problem D is an open problem because its starting situation is open. In the phase of planning, we decided together with Teacher D to open up the original problem by skipping the first two sentences from the text: *Before a steep street, 12% is written on a traffic sign. It means that the rise of the slope is 12% of the horizontal road.* The original form of the text was well-known by the teacher, but new (in both forms) for tenth-grade

students. The open aspect of the problem provided an opportunity to discuss the relative nature of the concept of percentages, as well as the role of the definition in both everyday life and mathematics. Problem D was well-integrated into the lesson structure along with other tasks.

Ad Q2. The classroom conversation began immediately after the students had read the text.

01 Teacher D: What does this mean? What do you think?
02 Anna: It means that the degree is the same as the percentage.
03 Teacher D: Is it that simple, the same in degrees? Hmm … it's clear to me that if it's rising by, then …
04 Juli: the road is higher by , … by …
05 Teacher D: If I run … How?
06 András: horizontally …
07 Teacher D: Originally horizontally, compared to this … (she shows the vertical direction with her arm)
08 Áron: The road is rising by of the horizontal road?
09 Teacher D: Good. Do you understand? For example, if I run metres … (waiting for Áron to complete her sentence)
10 Áron: … then it rises by metres.
11 Teacher D: I will be higher by metres. What if I want to generalize? Let's see, I run x metres?
12 Áron: then …
13 Teacher D: Very good. We helped you too much. I'll still show you what Áron said. If I run a certain number of metres, then I should calculate of that, which is the value of the rise. (She shows the triangle with the necessary data on the board.) Now tell me what the angle is in degrees? Anna said degrees.

The first answer was a typical, expected wrong answer (Line 02); the students simply tried to guess the meaning of the given data (Line 04, 08). The discussion was strongly directed by Teacher D, who closed it with a detailed solution plan.

Discussion

In this section, we discuss the findings obtained from the analysis of the presented episodes and the teachers' reflections on the lessons.

We establish that the teachers selected appropriate problems for the chosen lesson theme and lesson objective (see Table 1). Moreover, the didactical function of the problem solving in the lessons was appropriate to the aim of teaching mathematics. The purpose of setting problems, such as deepening new knowledge or applying updated knowledge as well as modelling, is highly relevant in mathematics education today. Although methodologically not all four problems were appropriate (Problems A and C were too complex for the investigated classes), we can conclude that the participants understood the essence of the problem-oriented teaching style.

When evaluating the teachers' behaviour, we need to separate Teacher A from the other three, as she did not realise the planned classroom discussion. During the individual work of the students, Teacher A's behaviour was neutral at all stages of problem solving. She did not interpret the problem, did not provide strategic assistance, and did not check the solutions during the lesson. This corresponds to the use of the task for differentiation: only the students who had done the basic task dealt with the problem. Although the content of the problem was appropriate to the lesson theme, we agree with the reflection of Teacher A that the problem was not suitable for an introductory lesson: it was too complex for fifth graders at this time.

Many elements taught in the PD programme, such as patterning, appeared in the trial lesson of Teacher B. This teacher had previously experienced problems with her reactions in unexpected situations. On this occasion, however, she built on the students' idea. Her typical former behaviour was closely managed, while in the trial lesson she made a conscious effort to change. In the "devising the plan" phase she was neutral: her students shared their ideas freely, but the evaluation of these ideas was made by the teacher instead of the classmates. The teacher always felt a need to interpret the responses for the whole class. Nevertheless, the presented episode highlights Teacher B's professional development well.

The content of Problem C was wholly connected to the lesson theme. Teacher C's behaviour at the first two phases of the problem-solving process was appropriate, i.e., neutral or emphasising strategies, but after she realised that her students were unable to find the right solution, she suddenly switched to a closely managed style and gave them a direct hint without referring to the ideas they had previously communicated. The hint (see Line 08) was therefore useless: the students stopped thinking about the problem itself and only tried to follow the teacher's thought, giving short answers instead. The implementation of Problem C in this form was inappropriate for these ninth-grade students, as they had no chance whatsoever of finding the right solution. Teacher C's colleagues reflected that it would have been better to use the "water from the river" problem as a worked example, and after understanding the key steps to set similar problems based on the ideas of the minimal route and line reflection.

Teacher D, too, was a closely managed type teacher before and after the PD programme. Her students are usually successful in the final exam of high school, which she regards as justification of her closely managed behaviour. Although the problem was planned as an open problem, it was implemented as a closed problem. The class did not discuss the different possible interpretations of the traffic sign and did not search for its correct meaning. Consequently, the teacher missed the opportunity to deal with the role of the definition in everyday life, as well as in mathematics. Teacher D explained the solution method while the students listened carefully. We can conclude that the teacher's behaviour hindered the independent problem-solving activity of the students.

Table 2 summarises the behaviour of the teachers in the phases of the problem-solving activities they managed in the classroom context. Teacher A did not realise classroom discussion regarding Problem A.

Table 2

Teachers' behaviour during the classroom discussion

	Teacher A	Teacher B	Teacher C	Teacher D
Understanding the problem	–	closely managed	neutral	closely managed
Devising a plan	–	neutral	1. emphasising strategies 2. closely managed	closely managed
Carrying out the plan	–	1. neutral 2. emphasising strategies	closely managed	neutral
Looking back	–	missing	missing	missing

Referring to the work of Rott (2019), where several behavioural types of teachers are listed, we can establish that Teacher D exactly fits the statements (a) "Some teachers make sure that their students understand the problem before they start working on it. This way, they take responsibility for the Understand phase, which could lead to their students not learning to analyse tasks on their own" and (f) "There are teachers that give content-related aid that directly leads to a solution very early without trying motivational or strategical aids beforehand" (Rott, 2019, p. 905).

The behaviour of Teachers B and C shows a more diverse picture. We can detect the effect of the PD programme, especially in the 2nd and 3rd phases. The teachers tried to change their former closely managed teaching style, although this change could be considered productive only in the case of Teacher B.

A common characteristic of the lessons is the lack of the "Looking back" phase. However, the analysis of this phenomenon goes beyond the scope of the present paper.

At the end of the school year, we organised a closing discussion about the findings and their possible explanations with the four participating teachers. In line with our research questions, we highlight two topics and the teachers' brief reflections on them.

What was the novelty for you in the problem-oriented approach?

Teacher A: My students are allowed to speak in the classroom and express their opinion now.

Teacher B: I became more aware than I was before.

Teacher C: I realised that we could connect different mathematical topics by problems.

Teacher D: Now I often look for useful tasks from everyday life.

The answers strengthen our findings related to the first research question (Q1); namely, that the programme was successful from the point of view that all of the participants understood the essence of problem-oriented learning design and found appropriate problems for their lesson themes, at least regarding the content. In their reflections, the teachers pointed out some of the main characteristics of the method.

The method of implementation of the chosen problem showed a diverse picture. We found that management is strongly dependent on teachers' behaviour. We could, however, detect some indications of changes; for example, instead of a closely managed style, Teachers B and C tried to be neutral or to emphasise strategies, but after an unexpected situation they reverted into the previously used closely managed style.

One of the typical phenomena was that *often the teacher reacts to students' ideas instead of their classmates.* When asked why, they responded as follows:

Teacher B: Teachers like to correct the answer immediately; students expect it, too.

Teacher D: My students learn everything I want but they don't deal with the "Why?" questions.

Teacher A: They don't understand what the other child says. I repeat the student's idea in a simpler form.

Teacher C: The older the student, the less active he or she is. Those who don't like and know mathematics, don't like and care about classroom discussion, either.

The teachers' reflections show that they are aware of their typical "repetitive" behaviour, but in their opinion, the students expect this behaviour from them. Teachers C and D, who teach ninth- and tenth-grade classes, respectively, reported a lack of motivation of students to take part in mathematical discussions. The reason for this well-known phenomenon may be the teacher-centred teaching-learning practice to which the older students are already accustomed.

Conclusions

This paper discusses the experiences of a PD programme for teachers of mathematics elaborated by the authors. The clear goals of the programme were as follows: the teachers should (1) process the curriculum in a problem-oriented way and (2) realise classroom discussion. Two research questions were formulated:

Q1 Is the teacher completing the PD programme able to incorporate a problem into the lesson plan that reflects the problem-oriented processing of the curriculum?

Q2 Is the teacher able to organise the classroom discussion so that the students explain and justify their ideas, while also critically adapting to their own thinking and that of their classmates?

On analysing trial lessons (the outputs of the programme) and the self-reflection of the teachers related to their work, we established that the teachers selected the appropriate problems for the chosen lesson theme and lesson objective (see Table 1). Moreover, the didactical function of the problem-solving in the lessons – according to the aim of teaching mathematics – was adequate. We can therefore conclude that the participants understood the essence of a problem-oriented teaching style.

The classroom implementation of the problem-solving activities can be considered partly successful. At the beginning of our experiment, we recognised the rather rigid professional habits of all of the participating teachers and their engagement with teacher-centred instruction. After investigating their work and reflections, we can report that the teachers tried to change their habits and behaviour during the lessons as a result of the PD programme, but they did not always succeed, often returning to their old, well-established teaching habits, especially when an unexpected situation arose.

The answers to the research questions show that the applied form of the PD programme – the essential elements of which are teacher-research cooperation, direct connection to the everyday practice of the teachers, and the reflective activity – all seem to be productive. What to expect from a programme of six teaching cycles and what realistic opportunities for improvement were available in a relatively short period became clear during the programme. In our experience, it is feasible for a teacher to develop in lesson planning and to initiate a behavioural change. Among the problems not solved by this PD programme, we can highlight the development of the ability to respond to unexpected situations. The programme needs to be adjusted to this in the future.

This phenomenon reflects the dual-process model of cognition. According to Feldon (2007), information processing occurs simultaneously on parallel pathways: controlled (high cognitive load) and automatic (low cognitive load). Controlled and automatic processes operate independently but intersect at certain points to produce human performance. When teachers process high levels of cognitive load, they are less able to dedicate working memory resources to other mental processes. Some dual-process cognitions, such as evaluation of an unexpected situation, may therefore rely almost entirely on their automatic components and operate without conscious monitoring. Studying teachers' cognitive load could therefore be a possible direction for our further research.

Another meaningful extension of our recent research to refine the presented PD programme could be the conscious development of teachers' reflective thinking. According to Lee, "… an awareness of the need for reflective thinking might be the first condition for its improvement. This should be followed by continual practice of reflection in various formats and on multiple specific issues" (Lee, 2005, p. 711).

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MERIA – Conflict Lines: Experience with Two Innovative Teaching Materials

Željka Milin Šipuš*[1], Matija Bašić[2], Michiel Doorman[3], Eva Špalj[4], and Sanja Antoliš[4]

The design of inquiry-based tasks and problem situations for daily mathematics teaching is still a challenge. In this article, we study the implementation of two tasks as part of didactic scenarios for inquiry-based mathematics teaching, examining teachers' classroom orchestration supported by these scenarios. The context of the study is the Erasmus+ project MERIA – Mathematics Education: Relevant, Interesting and Applicable, which aims to encourage learning activities that are meaningful and inspiring for students by promoting the reinvention of target mathematical concepts. As innovative teaching materials for mathematics education in secondary schools, MERIA scenarios cover specific curriculum topics and were created based on two well-founded theories in mathematics education: realistic mathematics education and the theory of didactical situations. With the common name *Conflict Lines* (Conflict Lines – Introduction and Conflict Set – Parabola), the scenarios aim to support students' inquiry about sets in the plane that are *equidistant* from given geometrical figures: a perpendicular bisector as a line equidistant from two points, and a parabola as a curve equidistant from a point and a line. We examine the results from field trials in the classroom regarding students' formulation and validation of the new knowledge, and we describe the rich situations teachers may face that encourage them to proceed by building on students' work. This is a crucial and creative moment for the teacher, creating opportunities and moving between students' discoveries and the intended target knowledge.

Keywords: inquiry-based mathematics teaching, realistic mathematics education, teaching scenarios, theory of didactic situations

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MERIA – Razmejitvene črte: izkušnje z dvema inovativnima učnima gradivoma

Željka Milin Šipuš, Matija Bašić, Michiel Doorman, Eva Špalj in Sanja Antoliš

Oblikovanje preiskovalnih nalog in problemskih situacij pri dnevnem poučevanju matematike je še vedno izziv. V tem prispevku prikazujemo vključevanje dveh nalog v pouk matematike kot primera didaktičnih scenarijev preiskovalnega načina poučevanja matematike in analizo učiteljevih implementacij teh scenarijev v svojih razredih. Prispevek je nastal pod okriljem Erasmus+, projekta MERIA – *matematično izobraževanje: relevantno, zanimivo in uporabno*, katerega cilj je spodbujati učne situacije, ki imajo za učence pomen, so zanje motivirani, saj omogočajo samostojno odkrivanje izbranih matematičnih pojmov. Scenariji MERIA kot inovativno učno gradivo za matematično izobraževanje v srednjih šolah zajemajo izbrane vsebine učnega načrta in temeljijo na dveh dobro utemeljenih teorijah matematičnega izobraževanja: realistična matematika in teorija didaktičnih situacij. S splošnim imenom Razmejitvene črte (razmejitvene črte – uvod in razmejitvena množica – parabola) je namen scenarijev podpreti preučevanje študentov o množicah točk v ravnini, ki so enako oddaljeni od danih geometrijskih objektov: pravokotnica kot premica, ki je enako oddaljena od dveh točk, ter parabola kot krivulja, ki je enako oddaljena od točke in premice. V prispevku prikazujemo proces učenčevega oblikovanja znanja in učne situacije, ki za učitelja predstavljajo dobro izhodišče za nadgrajevanje učenčevega znanja. Prepoznavanje takih situacij je ključno, saj ustvarjajo prostor za nove priložnosti oz. premikanje od učenčevih odkritij do ciljnega znanja.

Ključne besede: raziskovalno poučevanje matematike, realistično poučevanje matematike, scenariji poučevanja, teorija didaktičnih situacij

Introduction

In many countries worldwide, today's curricula promote student-centred teaching approaches and, in the case of mathematics, students' reinvention of mathematics is at stake. Although these ideas are not new in mathematics teaching, their large-scale expansion coincides with the immense expansion of human knowledge in all fields of activity. Society's demands for skill development, including inquiry and problem-solving skills, critical thinking, reasoning and creativity, are certainly reasonable in a knowledge-based society, and are seen as the responsibility of education. Mathematics, especially, is perceived as the educational domain in which these so-called twenty-first century skills could be addressed. The Erasmus+ project MERIA[5] – Mathematics Education: Relevant, Interesting and Applicable, which is one of many educational projects worldwide, thus aims to promote inquiry-based mathematics teaching (IBMT) as one of the teaching approaches that provides opportunities to address these demands. IBMT supports students' own inquiry of unstructured problem situations in which they work similarly to researchers by posing questions, experimenting and hypothesising, validating and evaluating. These ideas emerged even earlier in science education (Artigue & Blomhøj, 2013), and many projects have been launched that support the development and implementation of inquiry-based science education (IBSE), such as projects promoting relevant school science education at the secondary level (e.g., Holbrook & Rannikmäe, 2014). Similar ideas were in fact suggested in the 1940s by the educational researcher John Dewey, who saw teaching as related to students' experiences and promoted focusing on activities in which students "learn by doing" (Winsløw, 2017). Nowadays, IBMT attracts the attention of education researchers worldwide, in particular by considering its relation to the already well-established problem-solving tradition and other theoretical frameworks in mathematics education programmes (Artigue & Blomhøj, 2013). The design of appropriate tasks or problem situations that have the potential to engage students in inquiry activities is a very important element in this process, since, as the evidence shows, such tasks with teachers' instructions are often missing (Bruder & Prescott, 2013).

Theoretical framework

In order to support the implementation of IBMT in secondary schools, the Erasmus+ project MERIA offered specially designed teaching scenarios as an innovative teaching materials project for IBMT, with tasks and problem

5 See more on the project https://meria-project.eu/.

situations that were selected or developed for the purpose. The design was supported by two well-founded theories in mathematics education: realistic mathematics education (RME) and the theory of didactic situations (TDS). Realistic mathematics education is based on the idea that students' experiences can be considered as a starting point for their own inquiry in mathematics (Freudenthal, 1991). It promotes the learning of mathematics as meaningful and related to different kinds of human activity, including pure mathematics itself, by using rich contexts that are "familiar for students and provide relevant and challenging elements that need to be organized or schematized mathematically so as to have the potential to evoke their (informal) knowledge" (Kieran et al., 2013, p. 53). The theory of didactic situations (Brousseau, 1997) assumes that students can construct new knowledge in an appropriately constructed *didactical milieu*, as a specially designed teaching environment that contains a mathematical problem and with which students interact autonomously, without the teacher's guidance. To this end, a milieu should have adidactical potential that enables students to work alone, and that incorporates feedback potential that provides students with possibilities to validate their work (Hersant & Perrin-Glorian, 2005). TDS serves as a tool both to organise and analyse teaching, as well as to hypothesise didactical situations that support students' learning. It highlights five phases of teaching – devolution, action, formulation, validation and institutionalisation – and their character, which is either didactical (performed by a teacher who is teaching directly) or adidactical (without the teacher's interference). The latter usually exists in students' work during the action, formulation and validation phases. These are the phases for students' autonomous work, in which they try to solve the problem, formulate and test hypothesis, and finally presumably formulate the new requested knowledge. Validation, if achieved by students, is beneficial if it is also recognised as correct by the other students involved, but this is the phase in which the teacher can intervene, as well. In the (didactical) devolution phase, the teacher announces the problem to the students and presents the milieu, while in the phase of institutionalisation, which is also carried out by the teacher, s/he sums up the students' solutions (new knowledge) and connects it to the official (institutional) answer to the posed problem.

The teacher's role in IBMT requires further attention. Teachers are called upon to orchestrate teaching during the inquiry process, which means they need to withdraw, to "keep their hands in the pockets", to avoid the temptation to explain to the students what to do, but also to balance the process in order to prevent students becoming frustrated when stuck. Moreover, teachers are called upon to formalise new knowledge by building on the students' ideas. The last issue is especially recognised by Sherin (2002) as a pedagogical tension that requires the teacher to balance seemingly opposing demands between various students' ideas and productive mathematical discussion. The author notes that this demand is not easy to resolve.

The MERIA scenarios aim to provide support for teachers to orchestrate inquiry-based teaching, which they can adapt to their purposes and conditions. The scenarios are structured by the successive TDS phases, with timings that describe the lesson, especially by considering the didactical or adidactical character of the phases. In addition, the scenarios describe the assumed roles of the students and the teacher in each phase. The expected students' pre-knowledge and the assumed target knowledge are explicated, and the students' role is described by their possible strategies and learning issues for the action phase, as well as the possible realisations of the target knowledge in the formulation and validation phases as phases of the production of new knowledge. The description of the teacher's role includes an illustration of how the teacher devolves a certain phase and supports the students' activity. Some of these elements in a designed scenario can be treated as so-called didactical variables: they are the choices made by the teacher that influence learning but can be changed during a particular lesson, e.g., the number of students in a group, the anticipated time of a phase, the technology at the teacher's disposal, the size and shape of the geometrical objects in a milieu, etc. The designed scenarios do not (and cannot) anticipate all of the students' actions, but they build a hypothesised reference that allows a comparison with the classroom implementation.

When working with a task, the students' personal knowledge often develops in a way that is directly related to the context of the problem. Although it can be further developed and formalised when shared and discussed with other students, it is still different from the official (institutional) knowledge. The teacher's role in the phase of institutionalisation is didactical: s/he reformulates the students' personal knowledge relying as much as possible on their own work.

> "It is essential that the teacher challenges his students' personal knowledge by posing new problems which require knowledge they have not yet fully developed. In this way, personal knowledge is being validated. It can be validated either by the teacher, by the problem situation itself, or compared to other students, e.g. to their strategies when solving a problem. In this way, personal knowledge is transformed and becomes more formalized. This means that the knowledge becomes closer to what can be regarded as institutional knowledge." (Winsløw, 2017, p. 32).

In the process of inquiry, the teacher faces diverse student work that needs to be verified as (un)productive, situations that may have unexplored potential, overlooked student actions, or other unexpected situations. This requires the teacher to have a range of teaching skills and proficient knowledge, both mathematical and pedagogical, in order to respond to these challenges and to create opportunities for a transition between students' personal knowledge and institutional knowledge. Nonetheless, the phase of formulation provides an opportunity both for the teacher to observe students' reasoning, and for the students to practise their communication skills. Moreover, while presenting their solution, students make their stream of thought explicit, which then enables other students to engage in the discussion and provide feedback. Hence, it is not only the milieu that can be used to validate the solution, but the whole class becomes an environment that supports meaningful learning.

Bearing in mind what has been said thus far, we may state that our focus in the present article is twofold and formulate our research questions as follows:

1. How do the designed scenarios provide students with opportunities to build mathematical knowledge during classroom implementation?
2. Which elements of the scenarios related to the interaction of the teacher and the students guide the lesson to its goal?

Method

In the Erasmus+ project MERIA, several teaching scenarios were developed by teachers and researchers in mathematics education and mathematics from partner countries. They were implemented in the classroom in field trials during the school years in the period 2017–19 in schools in Croatia, Slovenia, Denmark and the Netherlands. The aim of the field trials was to test the theoretical assumptions of the design in practice in terms of their feasibility. In particular, the field trials aimed to identify the didactical actions of teachers and learning opportunities for students in order to refine the scenarios for future use. The first selection of scenarios was based on the requirement that the prescribed target knowledge formed part of curricula in all of the partner countries. The implementations of the scenarios selected for the field trials were observed by at least one teacher, usually from the same school, and the actions of the students and the teacher were documented in a report. All of the students' productions were collected. After the field trials, the next selection of proposed scenarios was made and the selected scenarios were published on the project's website. In order to discuss situations that are challenging for teachers and rich in teaching potential, we present experiences with two subsequent scenarios, Conflict Lines – Introduction[6] and Conflict Set – Parabola[7]. Both of these scenarios are published on the project's website: the first scenario is among the selected exemplary MERIA scenarios, while the second is published in the repository. Since it has target knowledge that does not cover a curriculum topic in all of the partner countries, the second scenario was not chosen in the first group.

In Croatia, some 500 students were included in testing of the first scenario, and approximately 200 students for the second. Among all of the classroom implementations, the students' productions presented in this article were selected by teachers who participated in the field trials as the most informative. The classroom implementation of both scenarios took place in a high school with an emphasis on mathematics, science and computer science, and was performed by a (single) experienced mathematics teacher. It was implemented in the second grade in a class of 25 students. Student pre-knowledge included knowledge of a perpendicular bisector as a line perpendicular to a given segment and passing through its midpoint, as well as knowledge of the parabola as the graph of a quadratic function. During the two meetings at which the scenarios were implemented, the students worked in groups. In the formulation phase, they wrote down their hypothesis and arguments on posters to facilitate classroom discussion. All of the students' productions were collected.

As mentioned above, we used the theoretical frameworks of RME and TDS in the design of the tasks and scenarios, making theoretical assumptions on their didactical potential. We also hypothesised the teacher's role in classroom implementation. Our focus is now on exploring the implementation, on what is observed, what may happen in practice or what is missing. We focus mostly on the adidactical and feedback potential of the created learning environments hypothesised by the scenarios, when students work alone in their own inquiry and formulate their new personal knowledge. Based on the available observations, we also discuss the teacher's actions based on the MERIA scenarios in terms of which elements of the scenarios may facilitate the teacher's decisions and what is possibly missing. We describe the rich situations that the teacher may face regarding the students' formulation and validation as a basis for the teacher's institutionalisation. During the field trials, we identified the moments that were not described in detail in the scenarios but need to be considered in further institutionalisation. The implementation of the scenarios was also analysed with respect to the realisation of the TDS phases from the lesson plans. Our conclusions are based on the teachers' reflection and reports,

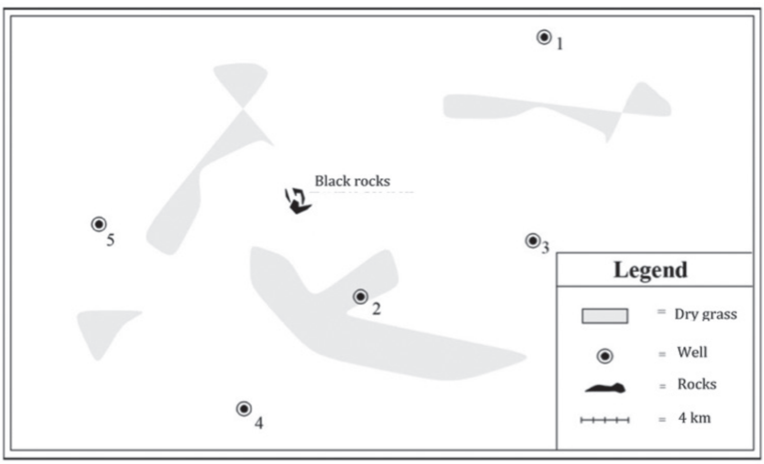
6 See more at https://meria-project.eu/activities-results/meria-teaching-scenarios.
7 See more at https://meria-project.eu/repository.

the students' work and the explicit formulation of the elements of the specific didactical situations that were hypothesised by the design (and now refined) and that could be transferred to other didactical situations.

Mathematical context

In the first scenario, Conflict Lines – Introduction, the task is intended to involve students in what is, for them, a meaningful path to the target knowledge: perpendicular bisectors are lines whose points are equidistant from a pair of points, thus creating the partition of the plane into regions, the so-called Voronoi diagram. In the second scenario, Conflict Set – Parabola, the parabola is perceived as a locus of points in the plane equidistant from a given (horizontal) line and a point. Both scenarios fall under the umbrella of *conflict sets*, that is, sets that are *equidistant* from a given set of points: in the first scenario the set of (discrete) points in the plane, and in the second scenario a line and a point.

Task 1: There are some water wells in a desert, as shown on the map. A thirsty person naturally goes to the closest well. Which well should the thirsty person go to from different points in the desert? Make a partition of the desert into areas in such a way that for all points in an area, a certain well is the closest of all of the wells.



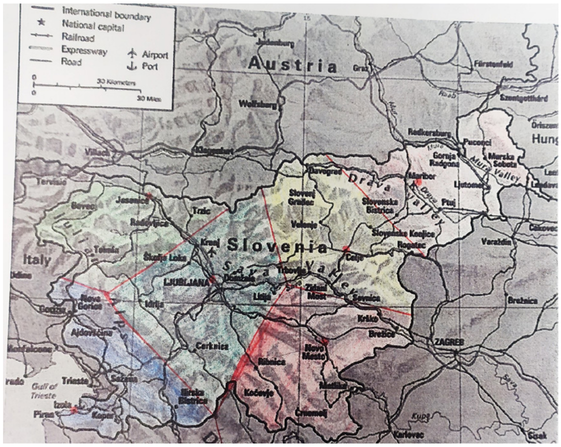
Task 2: In the coordinate plane, consider the line p given by the equation $y = 2$ and the point $A(5, 4)$. Show that points $B(1, 7)$ and $C(7, 4)$ are equally distant from the line p and point A. Find all of the points with the same property!

The first task, the water well problem, is a well-known task chosen for the present article partly because it has already been presented at a ProMath conference (Holzäpfel et al., 2016), where it was investigated as a task that offers a rich situation for students' own explorations. As stated in Holzäpfel et al. (2016), researchers' assumptions about possible strategies (heuristics) include guessing, reducing the complexity of the problem (e.g., working only with two water wells), or searching for familiar details and formulating an analogous (easier) situation (e.g., someone put one water well in another position to get a better idea of the problem). As argued by Doorman et al. (2020), the task and its possible sub-questions offer students the opportunity to work in geometry as postulated by RME, starting from their own experience and continuing to appreciate formal, precisely defined geometrical objects with their properties understood.

In the MERIA field trials, the students' pre-knowledge of perpendicular bisectors is already available from primary school. It is assumed by the scenario that in the devolution, the teacher would offer a reduction of the problem to two points. This could evoke the concept of a perpendicular bisector of a segment determined by the points, which then defines a border of the requested regions in this case. Further reduction to three points is not part of the devolution: it could evoke an additional concept, as given three points that can be considered as vertices of a triangle, there is a point that is equally distant from all of them, the centre of a circumscribed circle to a triangle. The rich context of this problem could allow further questioning about the situation of four points if there is – or when there is – a point that is equally distant from all of them. This could also be seen as a didactical variable, with the different positioning of the four points leading to a cyclic quadrilateral. Another didactical variable is the context of the task: it may be presented in a variety of other contexts, some of which could be more meaningful to the students. In the field trials during the MERIA project, one teacher from Slovenia changed the context of the task from wells in a desert to hospitals in a country that are reached by helicopters in the case of an emergency (Figure 1). This possibility was similarly suggested in Holzäpfel et al. (2016), where the context was changed to restaurants on a map.

Figure 1

Changed context of the water well task in the devolution phase



In the second task, a parabola is described as the curve (locus of points) on a plane satisfying a geometrical property, that is, as a set of points that are equidistant from a given point and a given line. The students' pre-knowledge of a parabola is that it is a graph of a quadratic function, and this knowledge was realised directly prior to the implementation of the lesson. This scenario therefore aims to support the students' linking of different viewpoints of a parabola: the geometrical viewpoint as a locus of points in a plane equidistant from a given horizontal line and a point (*parabola as a curve*), and the algebraic viewpoint as a graph of a quadratic function (*parabola as a function graph*). Both of these are curriculum topics in Croatia, although the establishment of these links is not required. The scenario also assumes a motivating activity for the students during the first devolution of the problem: one student stands in the class and the others have to form a navigation route for a robot to avoid the student and the closest wall in the class by maintaining an equal distance to them. The students' action phase is assumed to begin by checking whether two given points satisfy the condition of being equally distant from a line and a point. It is then assumed that the students will try to find more concrete points with the same property, initially probably by guessing. Since it is not a straightforward task, the teacher may suggest the use of a strategy such as considering the points lying on a certain line, considering certain symmetrical points, etc. By plotting the generated points in the coordinate system, the students may first notice that the requested equidistant set is not a line. If the shape is evoked as a parabola, the question arises as to how to prove it, that is, whether it is possible to find a quadratic relationship between the variables.

Results

Rich context with unexplored situations in Task 1

Four groups of students worked on this task and were engaged in the inquiry. They made a number of provisional drawings during the action phase. Two groups completely realised the assumed mathematical target knowledge and presented it as such in the formulation phase on a poster. The other two groups struggled with various strategies. The teacher assumed that the task, together with the described devolution based on a reduction of the problem to two points, offered enough feedback potential for the students to verify their assumptions or initial ideas. If the students failed to achieve this alone, the teacher decided to leave the case of three perpendicular bisectors for the classroom discussion as a rich unexplored situation. Here we present the final formulations of the two groups that did not get involved in exploring the case of three bisectors.

Figure 2.1
Group 1

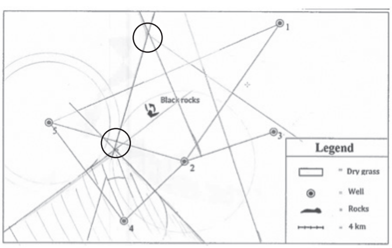
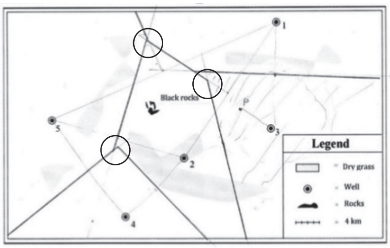


Figure 2.2
Group 2



Group 1:

The students drew circles around the wells in order to represent the idea of "being equally distant from a point" (Figure 2.1). However, they considered this strategy as unproductive and did not persist with it. They simply connected pairs of points to obtain segments and drew the perpendicular bisectors without referring further to the circles. The regions that belong to a certain well were difficult to distinguish. This group did not analyse the special case of what happens where bisectors (seem to) meet (marked by red circles).

Group 2:

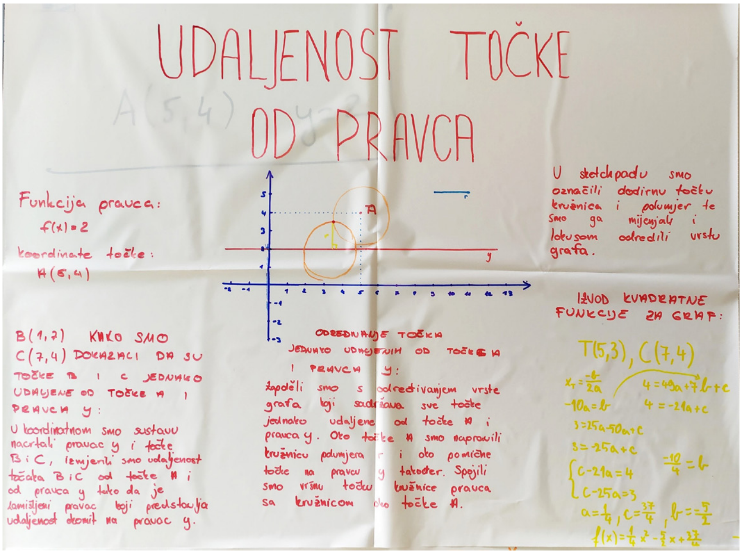
The students started by connecting points and drawing perpendicular bisectors in order to determine the regions. Due to their rather imprecise work, the triples of bisectors did not meet (Figure 2.2, marked by red circles). The question arises as to what to do in the resulting area, that is, in the triangle, but the students left this situation unexplored.

Rich context with unexpected formulations in Task 2

In connection to Task 2, we address the students' diverse and rich solution formulations, which present a challenging task for the teacher to be evaluated during a lesson. The teacher needs to recognise productive formulations or to question misleading ones in the new personal knowledge of the students. However, the teacher usually also questions the students' formulations based on his/her own expectations regarding how far the students can or should reach, how formal the students' mathematics language or reasoning can or should be, how often s/he should interfere and/or provide a feedback, and similar. In the classroom implementation of Task 2, the students worked in four groups with the same teacher. All of the students were engaged in the inquiry. As intended by the scenario of Task 2, after the action phase, the students were asked to formulate their hypotheses and possible validations. In the validation phase, they presented their group work using posters and shared the explanations and results with their colleagues. The students were required to evaluate their own ideas and those of others. The formulations on the posters of all of the groups are presented below.

Group 1:

Figure 3
Formulation of Group 1



The first group worked with technology (Dynamic Geometry Software, DGS) from the beginning of their action phase. The students verified that points *B* and *C* were equidistant from the given point and the given straight line (Figure 3). Using technology, they tried to obtain all of the points that satisfy this property. They drew circles with the same radius around point *A* and around an arbitrary point on the line. The measurements showed that the intersection of these circles did not satisfy the property. The students therefore concluded that the point they called the "peak point" of the circle with its centre on the line should be observed (Figure 4.1). However, this "peak point" for an arbitrary point on the line (as the centre of a circle) did not even belong to the fixed circle with the centre at point *A*. The students then moved the arbitrary point along the line and thus adjusted the "peak point" to the circle with its centre at point *A* (Figure 4.2). In this way, they obtained one point of the required curve. The construction of all of the points by DGS required teacher support. After concluding that the required curve had the form of a parabola, the students used the vertex (5, 3) and point (7, 4) to set the system of equations. The solutions were the coefficients of the quadratic function $f(x) = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{37}{4}$.

This group had difficulty constructing a new point equidistant to the given line and the point. They used the fact that all points on a circle are equidistant to its centre (the given point *A*), but did not know how to connect it to being equidistant to a line. They argued that it may be related to perpendicularity, which is why they invented the point called the "peak point" (Figure 4.1), that is, the intersection of the circle constructed around an arbitrary point on line $y = 2$ and a perpendicular line passing through the arbitrary point. The students experimented to obtain the requested new point (Figure 4.2).

Figure 4.1
The idea of the "peak point"

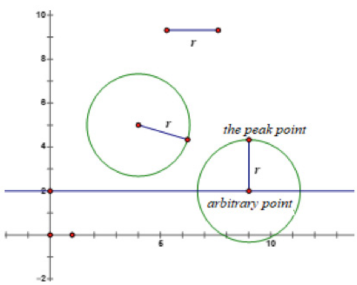
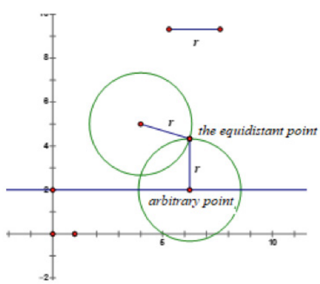
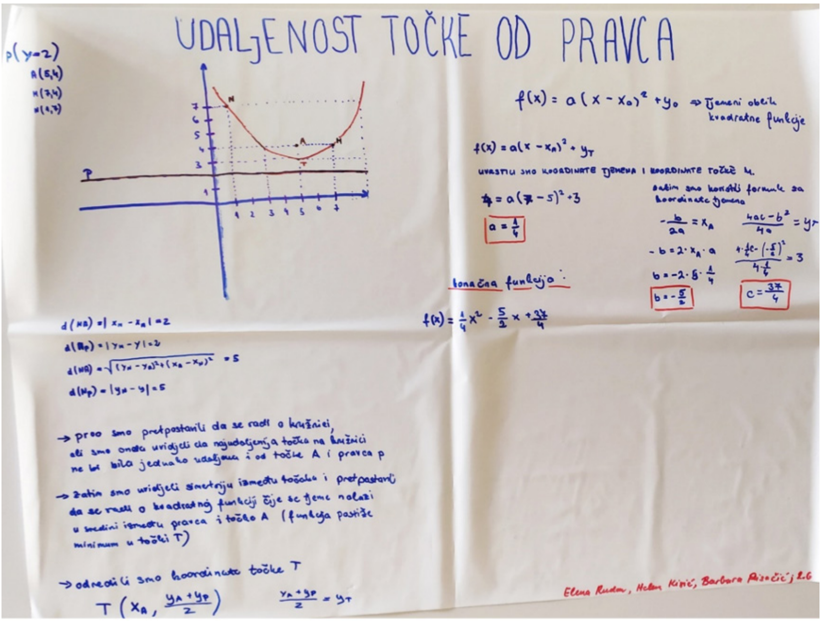


Figure 4.2
Construction of a new point



Group 2:

Figure 5
Formulation of Group 2

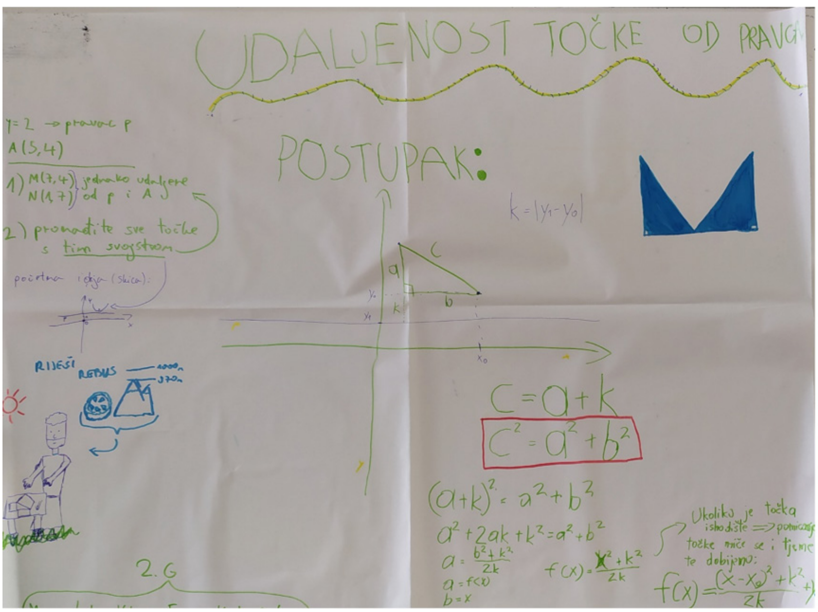


Rather than using technology, the students in this group used the distance formula to verify that points *B* and *C* satisfy the property (Figure 5). Based on the shape they formed in the first devolution (the motivating activity), they assumed it was a circle. After discussion, they rejected this assumption with the argument: *We saw that the farthest point on the circle would not be equidistant from point A and line p*. They then found several points that were equally distant from the given point and the line: the ones symmetrical with respect to the line passing through the given point and perpendicular to the given straight line. Thinking of a curve that is symmetrical, they assumed that it was a graph of a quadratic function whose vertex was halfway between the line and point *A*, and concluded that the function reached a minimum at point *T*. The students found the coordinates of point $T(x_A, \frac{y_A + y_P}{2})$ and used the vertex form of the quadratic function: $f(x) = a(x - x_A)^2 + y_T$. Substituting the coordinates of point *M* resulted in the coefficient $a = \frac{1}{4}$. Coefficients *b* and *c* were obtained by solving the system of two equations with two unknowns. They this obtained the formula of a quadratic function $f(x) = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{37}{4}$.

After the teacher's instruction to prepare their arguments and conclusions for presentation, the students commenced a group discussion on how to be sure about the solution. They tried to prove that all points with coordinates $(x, ax^2 + bx + c)$ are equidistant from point $A(5, 4)$ and the straight line $y = 2$, but they failed in their attempt. In dialogue with the teacher, they concluded that they should work with points $(x, \frac{1}{4}x^2 - \frac{5}{2}x + \frac{37}{4})$. They presented this idea in the class and did the proof as homework.

Group 3:

Figure 6
Formulation of Group 3

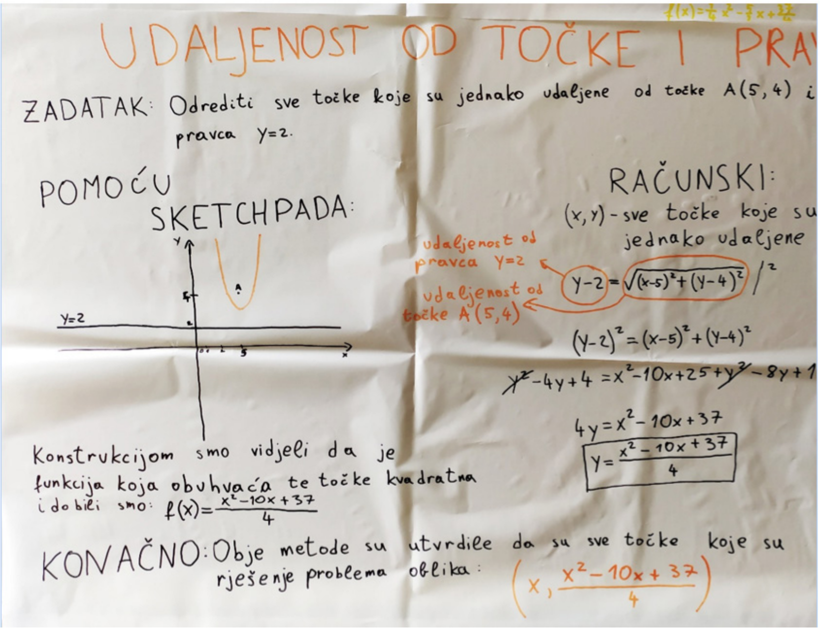


The students in this group did not use technology. They started from a point that satisfied the condition and obtained a quadratic relation of the coordinates (Figure 6). In so doing, they found the relation generally, using the translation of the coordinate system. They began with a given point as the origin and a line parallel to the x axis and k units distant from it. They denoted the point that satisfied the required condition by (b, a). The distance from that point to the origin is denoted by c. Then $c^2 = a^2 + b^2$, and according to the task $c = a + k$. From there, with a mistake in signs, they got $a = \frac{b^2 + k^2}{2k}$ (correct would be $a = \frac{b^2 - k^2}{2k}$). By substituting a = $f(x)$, $b = x$, the function becomes

$f(x) = \frac{b^2 + k^2}{2k}$ for the translated coordinate system. Finally, $f(x) = \frac{(x - x_0)^2 - k^2}{2k} + y_0$. The students stopped at this point. The correct general solution is (the difference being due to the mistake the students made). After substituting $(x_0, y_0) = (5, 4)$ and $k = 2$ into the last equation, the solution turns out to be $f(x) = \frac{x^2 - 10x + 37}{4}$, as required.

Group 4:

Figure 7
Formulation of Group 4



The students used DGS technology. They independently planned the steps of the research and independently performed the constructions (Figure 7). Using technology, they obtained points that satisfy the property and noticed that the curve has the shape of a parabola. They thus determined the rule $f(x) = \frac{x^2 - 10x + 37}{4}$. After this, they did some computation. They denoted by (x, y) all of the equidistant points and using the distance formula $y - 2 = \sqrt{(x - 5)^2 + y - 4)^2}$ resulted in $f(x) = \frac{x^2 - 10x + 37}{4}$. Finally, they concluded that both methods gave the same answer, so all of the points that satisfy the given condition have the coordinates $(x, \frac{x^2 - 10x + 37}{4})$.

Discussion

In a teaching approach such as inquiry-based teaching, according to the theoretical assumptions behind the design of the lesson, each scenario carries a certain adidactical potential that enables students to work autonomously and possibly verify their hypothesis by themselves. If students are stuck when solving a task, the teacher decides on the extent to which s/he can rely on the feedback potential offered by the task, or whether to scaffold the students in their work. In both of the classroom implementations investigated, we observed that the students were actively involved in the inquiry and found the tasks motivating and challenging to explore. We also noted that the adidactical potential of the tasks was realised, at least in creating the initial students' strategies, whereas in some groups, almost all of the students realised the expected main target knowledge, especially in the second task. The feedback potential of the first task, assumed as the possibility to directly test the properties of the points on the map, was not extensively used by the students. The two groups whose productions are presented here mostly tried to avoid such reasoning and to directly implement the conclusion from the first devolution that the perpendicular bisector provides points that are equally distant from the two points given. In the second task, the students relied much more on the feedback potential of the task and, even using technology, explored different points that enabled them to realise the pattern required.

Finally, regarding the teacher's role, the classroom implementation revealed numerous details regarding the feasibility of the scenarios. The main idea behind the scenarios is to facilitate teaching in an inquiry environment, especially for inexperienced teachers, who often express that it is not enough just to get a task. The scenarios offer the teacher some (assumed) insight into the students' thinking, which may or may not be subsequently observed in the classroom. Even more important, however, is to realise whether some crucial moments in the possible students' thinking are missing, in order to refine the scenarios. The teachers who implemented these two scenarios largely agree that the scenarios provided a solid support for their teaching. However, even in the present study, we have identified questions of a very generic nature for which the teacher needs to be prepared, and which are not emphasised in the scenarios.

In the first scenario, Conflict Lines – Introduction, a question arose concerning a particular (unexplored) rich subtask (the intersection of three perpendicular bisectors obtained by three wells). Although this question is briefly mentioned in the scenario, it could easily be overlooked by a reader of the scenario's rather long text. If not prepared in advance, it is not easy for the teacher to recognise such rich situations during the lesson. This situation provides an opportunity for the teacher to scaffold the students and to establish links with their prior knowledge.

In the second scenario, Conflict Set – Parabola, in the first group, the teacher was challenged to grasp the informal student communication that emerged with the students' creative productive strategy. The teacher asked the students to explain the meaning and deepen the idea of the *peak point.* Although the students failed to express themselves using more formal mathematical language, the teacher encouraged them to persist. It is up to the teacher whether to question the students and to determine the level of mathematical formality that s/he is satisfied with.

In the second group, the students initially assumed that the required set of points was a circle, and the teacher did not intervene. It is very common for a teacher to have an initial urge to correct the student's reasoning; however, the teacher decided to remain withdrawn. It was up to the students to realise that their hypothesis was incorrect. The teacher's preparation allowed her to consider that the milieu ensured enough potential for the students to test and validate their hypothesis. After the students had rejected the idea of a circle, they assumed that the solution was a parabola and used the data to determine the equation. Finally, they proved that all points on the parabola, that is, on a graph of the quadratic function $f(x) = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{37}{4}$, are equidistant from the given point and line.

In the third group, the teacher noticed that there was a mistake in the calculation that would not influence the idea behind the solution. The strategy itself was very innovative and unexpected, as the group translated the coordinate system in order to simplify calculations, although they did not express their idea in this way. At first, the students' sketch was also misleading, giving the impression that they had used similar triangles with no clear rationale. The students' reasoning was very informal, and the calculation was incorrect and incompletely provided. This strategy was not anticipated in the scenario, so the teacher needed to recognise that the students' idea was different than the official solution due to the mistake they had made. The teacher had an opportunity to discuss the innovative idea, understanding that the mistake did not have a major impact.

In the fourth group, from the perspective of teacher, the solution was the most complete. After plotting some points that satisfied the required property, the students devised the hypothesis that the solution was a parabola and calculated its coefficients. Seeing it was still a hypothesis, the students autonomously decided that they needed some kind of validation. They therefore verified the solution algebraically starting from the property of equal distance and ended up with the same equation. The students called this general argument "the computational approach". Still, the teacher did not interfere with the work of this group, although the precise mathematical statement involved logical equivalence in formulation. Regarding the mathematical target knowledge, the students needed to discover that the point lies on the conflict set *if and only if* its coordinates satisfy a certain equation. Students usually find a way to show one direction or the other, but not both. In this case, Group 2 proved one direction and Group 4 the other. Thus, the formulation phase in this task provides an opportunity for the teacher to build on the students' work in the institutionalisation phase, and to discuss the logical equivalence in the statement as a subtle interplay between algebra and geometry: a set of points that is the *conflict set* of a point and a line is given by quadratic dependence (the parabola as a graph of a quadratic function) and, vice-versa, all points of a parabola as a graph of a quadratic function has the property of the *conflict set*. The teacher can encourage students to compare the proofs given by Group 2 and Group 4, and thus to notice equivalence.

At the end of a lesson designed in this way, after the different strategies are presented by the groups, the teacher needs to decide what can be validated and institutionalised for all of the students. The final discussion allows the student to confront her/his new personal knowledge with the knowledge of other students, to improve it, and finally, with the help of a teacher, to connect it with the official knowledge.

Conclusion

Teaching is a complex task in an open inquiry environment, requiring the teacher to have a range of proficiency skills. It is more challenging than addressing rote techniques by direct teaching. It seems that not only mathematical and pedagogical knowledge and skills, but also beliefs influence teachers' behaviour (Rott, 2020). Beliefs are especially important for IBMT because teachers may incorporate their previous experiences and belief systems concerning mathematics into their teaching, which can be a constraint when introducing IBMT into practice. However, pursuing the IBMT approach, we still assume that "the goal of teaching is for the students to acquire a certain established and culturally recognized knowledge, which they will be able subsequently to use without the teacher's help" (Hersant & Perrin-Glorian, 2005, p.113). Teaching scenarios designed for the purpose of IBMT within the Erasmus+ project MERIA offer a solid ground for teachers to orchestrate such teaching. The scenarios have been designed based on two theoretical frameworks, RME and

TDS, and the experience that the members of the project team brought from different institutions. The presented teaching sequence on *Conflict Sets* has not yet been thoroughly discussed within the scope of the project, and the analysis presented in this paper completes that work. In this article, we discussed several student productions as evidence of the students' newly constructed knowledge. These productions present the rich and inspiring situations that the teachers confronted and exploited to build new official knowledge. In addition to the didactical situations hypothesised by the teaching scenarios, we described the outcomes of their classroom implementation, which show that the theoretical ideas behind the design of the scenarios can work in practice, and that the scenarios helped the teacher to navigate the inquiry process. We observed the diverse student reactions, but also noted that it was feasible for the teacher to perceive a variety of strategies and proceed with the lesson by using them. However, we also identified situations that were not emphasised by the scenarios. These situations indicate the numerous creative moments when teachers make decisions, create opportunities and challenge the situation in order to support a productive exchange of mathematical ideas, and as such could be of interest to any practising teacher. In future investigations, we aim to consider whether we can refine our investigation into the characteristics of tasks for secondary school mathematics in addition to their adidactical potential, such as *linkage, deepening and research potential*, as assumed for the purpose of university mathematics education in Gravesen et al. (2017). Such studies are expected to contribute to a better understanding of how to support teachers in the crucial and creative moments when they try to recognise and use opportunities for moving between students' discoveries and intended target knowledge.

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The Dynamics of Foreign Language Values in Sweden: A Social History

Parvin Gheitasi*[1], Eva Lindgren[2] and Janet Enever[3]

This paper gives an account of the history of foreign language values in Sweden from the seventeenth century to the present. The paper is informed by sociocultural standpoints on language and language learning according to which language is a dynamic tool that is appropriated by individuals to achieve particular purposes, and that dialogically creates and renews our social world(s). Since the sixteenth century, three languages (German, French and English) have been taught in Sweden as foreign languages during particular eras. In this paper, we explore how language value can be understood as a system that evolves over time as a result of triggers such as power, trade and personal benefits. The impact of these variables on Swedish society's efforts to invest in learning a particular language during specific eras is critically examined from the perspectives of nested systems.

Keywords: dynamic system, foreign language learning, foreign language values

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Dinamika vrednot tujih jezikov na Švedskem: socialna zgodovina

Parvin Gheitasi, Eva Lindgren in Janet Enever

Članek predstavlja zgodovino vrednot glede tujih jezikov na Švedskem od sedemnajstega stoletja do danes, pri čemer izhaja iz sociokulturnih stališč o jeziku in učenju jezikov, skladno s katerimi je jezik dinamično orodje, ki si ga posamezniki prisvojijo za doseganje določenih namenov in ki dialoško ustvarja pa tudi obnavlja naš(e) družbeni(e) svet(ove). Od šestnajstega stoletja naprej so se na Švedskem v določenih obdobjih učili tri tuje jezike (nemškega, francoskega in angleškega). V članku preučujemo, kako je mogoče jezikovne vrednote razumeti kot sistem, ki se razvija skozi čas kot posledica sprožilcev, kot so: moč, trgovanje in osebne koristi. Z vidika ukoreninjenih sistemov kritično preučujemo vpliv navedenih spremenljivk na prizadevanja švedske družbe za vlaganje v učenje določenega jezika v specifičnih obdobjih.

Ključne besede: dinamični sistem, učenje tujih jezikov, vrednote tujih jezikov

Introduction

Since the start of the twenty-first century, the English language has become the most important global lingua franca. Trade, tourism and social contact use English for communication across countries and continents. Sweden is no exception. English is used by most people on a daily basis and Swedish 15-year-olds surpass their European peers in English language competencies (ESLC, SurveyLang, 2012). At the same time, their competence in foreign languages other than English is considerably lower than that of many other European 15-year-olds (special Eurobarometer 386, 2012). However, the remarkably strong position of English as a foreign language in Sweden is a recent phenomenon, and is preceded by a complex and dynamic history of German and French dominance.

When chaos theory was introduced in the 1960s it was a way of explaining how complex phenomena like weather are actually systems that interact over time and space (Oestreicher, 2007). Seemingly random events in one part of the world can severely affect other places. The explanation is that these phenomena are organised systems that can be pushed into chaos by diverse occurrences, but will always strive to reorganise themselves as systems. The idea of complex systems has since been applied to a number of other areas, including language, language use and language learning. Ellis (2008, p. 233) explains that "Language learning and language use are dynamic processes in which regularities and systems arise from the interaction of people, brains, selves, societies, and cultures using languages in the world".

In this paper we argue that the dominance of one or more foreign languages in a society is interrelated with the values that are connected with the language(s). Language is a sort of human capital (Breton, 2000) and the dynamics of the spread and decline of a language depend on its utility and communication potential to individual users (De Swaan, 2001), that is, the position of a language is determined by its communicative value (ibid.). Languages serve speakers' purposes, and the value of a language depends on the benefits it brings to the user. The value of a language is subjective and dynamic, changing according to time and place, and to the speakers' needs. Thus, the higher the value, the more likely it is that a language will be promoted, learnt and used. The value of a language can be related to the benefits derived from the potential of broader exchange and communication. In addition, according to Church and King (1993), the 'external network effect' can increase language value. Thus, the more people join the language network, the more valuable it becomes. As De Swaan (2001) proposes, "All human groups on the globe engage in relations of power, trade, migration and cultural exchange. These relations all involve verbal transactions; they are necessarily embedded in language" (p. 177). In the present paper we explore how language value can be understood as a system that evolves over time as a result of triggers such as power, trade and personal benefits. Our case is Sweden and the Swedish system of language values during four periods from the seventeenth century to the present. The dynamics of this system, its development and change, are discussed from the perspectives of nested systems, e.g., from the macro (state) to the micro (individual); timescales, e.g., an event at one point in time may trigger the language system at another point in time (Larsen-Freeman & Cameron, 2008); or space, e.g., an event in one geographic position may affect the language system in another place. Aspects of policy, trade, economy, culture, ideology, technology, social context and investment in both the domestic and international arenas are included as parts of the dynamics of the language value system in Sweden, in view of how they act as triggers for particular influence. In this paper we focus on foreign languages in Sweden. We acknowledge that the official national minority languages – Sámi languages, Meänkieli, Finnish, Romani and Yiddish – have been and still are present in Sweden. We also acknowledge the historical assimilation policies towards the Sámi people, which have resulted in severe language loss (see Hornberger & Outakoski, 2015).

Theoretical framework

In this study we understand foreign language (FL) value predominantly as a sociocultural phenomenon in which a range of contextual factors interact to trigger change. Foreign language is defined as any language other than Swedish that is officially used or promoted by the state or other official organisations. In our analysis we firstly draw on dynamic systems theory to acknowledge the complexity of FL values. We acknowledge the fact that at any given moment a number of languages will be used in a society such as Sweden. However, not all will carry the same value. Secondly, we discuss FL values in Swedish society from an investment theory perspective, identifying cultural capital as a key driver of FL values and consequently of language choices made by states and individuals. Our main assumption is that high FL values in a society are reflected in which foreign languages are promoted by the authorities of that society for official communication or education and/or used by a majority of the population.

Towards a dynamic systems rationale

Language use in society can be described as the means of communication between people in a society. When communicating, people use one or more 'named' languages, such as Swedish, English, Arabic, etc., in a mix that is suitable for the particular situation. As a result, different combinations of named languages will be used in different societies and these will fluctuate over time. Language in society is interrelated and can be viewed as nested within the languages of other societies, creating a complex system where change on one level may lead to change on other levels. In order to understand these dynamics, Larsen-Freeman and Cameron (2008, p. 202) suggest that "Instead of investigating single variables, we study modes of system change that include self-organization and emergence. Emergent properties or phenomena occur when change on one level of social grouping or on the timescale of a system leads to a new mode on another level or timescale".

Changes in language value and use can be attributed to changes in society. Such change may be brought about by shifts in the economy, trade, language contact or social prestige, for example. New social situations induce people to communicate in new ways and as a result their language changes (Crowley & Bowern, 2010). These processes have been described as complex and dynamic changes to systems of languages (Elman, 1995). Similarly, individual language users' learning and development of language has been described as a dynamic system. Ellis (2008) explains that processes of usage, change, perception and learning are integral parts of the dynamics of second language learning.

Looking at the dynamics of FL values in society across a particular time span – i.e., how foreign languages have been and are used as a result of societal, individual and other variables – inevitably becomes a complex process in which a number of factors interact. Indeed, as Larsen-Freeman and Cameron (2008, p. 203) suggest, "If we think in terms of reality as a web […] [t]he independence of any individual variable then becomes questionable, as does the idea of a single cause giving rise to a complex event. Rather, it is likely that there are multiple and interconnected causes underlying any shift or outcome".

Language as investment and cultural capital

In the social process of learning, we view an individual as situated within a specific context (with its own particular culture and history) engaging in activities that are culturally valued, that is, the individual is required to develop certain behaviours using cultural tools, and during this process the activities and the tools might also change. In this sense, language skills can be regarded as a form of capital that provides access to specific rewards (Norton & Toohey, 2011).

Bourdieu (1977, p. 488) introduces the notion of cultural capital, defining it as "instruments for the appropriation of symbolic wealth socially designated as worthy of being sought and possessed". Bourdieu (1991) refers to the knowledge, qualifications, modes of thought and language that differentiate various classes, stressing the often unequal relationships between interlocutors and the significance of power in structuring speech. He argues that cultural capital has different values in different social contexts. Accordingly, the notion of culture as a capital suggests that it can be saved, invested and used to obtain other resources (Darvin & Norton, 2015).

Drawing on Bourdieu's work on cultural capital and language and the role of identity, Norton (2000) introduced the construct of investment, with an emphasis on the socially and historically constructed relationship between a language learner and the target language. The investment theory of language learning positions language learners as *investing* in a second language at particular times and in particular settings, in the interest of gaining a broader scope of symbolic resources (such as language and education) and material resources (such as property and economic wealth), which will, in turn, strengthen the value of their cultural capital and social power. Extending this construct further, we suggest that it may be significant to investigate how a particular language becomes popular for investment in the sense of having highly sought-after symbolic value that leads to social prestige. According to Norton, the notion of investment recognises that language learners have complex social histories and variable desires. By using the target language, learners organise and reorganise their identities during a process of constant change across time and space. Thus, it can be concluded that the relationship between the learners and the target language is socially and historically constructed.

Grin (2002) argues that the economic imperative may also be an influential factor in the dynamics of language choice, both at individual and societal levels, with regard to the potential rewards and drawbacks that specific choices may offer. For Grin (2002), one of the influential factors in the analysis of language policy is economics, since it helps to look at different choices about language in terms of benefits and drawbacks, both at individual and societal levels. Other contributing factors could be social, cultural and political signals that indicate how the knowledge of a particular language provides credentials for social inclusion and exclusion. Some scholars also claim that the literacy rate of a society has a positive influence on learning a foreign language (Church & King, 1993; Ginsburgh et al., 2017). According to this argument, higher literacy rates should make learning easier by lowering the cost of learning and thereby promoting the learning of foreign languages.

These perspectives demand the contextualisation of FL values, FL use and FL learning within a specific socio-historical period. Thus, our aim in this paper is to present an account of FL values, language shift and change reflecting how and why communities select languages for specific purposes at particular moments in time. Acknowledging that language permeates every facet of life at the individual and social levels, we argue that knowledge of a foreign language, particularly one that is widely spoken in other communities (lingua franca), can make a difference to people's lives in many ways. It may grant access to other speech communities, it may empower both individuals and societies by serving as a resource to establish and transform social and personal identities and relationships, and it may provide access to important socioeconomic and political markets.

Analytical approach

In the following sections we critically examine the language value system of Sweden at four different points in time in order to explore the reasons behind the strong influence of English in current Swedish society. We adopt a qualitative approach arguing that language value in society is a dynamic performance in time in response to complex patterns of events. This country case study interrogates socio-historical evidence to tease out the complexities of language values in society over time. We understand the system of language value as a system that is nested with other systems in time as well as in space (Larsen-Freeman & Cameron, 2008). For example, we acknowledge that global events outside Sweden may trigger changes in the language value system in Sweden. Similarly, events that occur at one point in time may prompt changes in the system at another point in time. Other causes for the development of the Swedish language value system that we include in the analysis are policy, trade, the economy, culture, ideology, technology, social context and investment, both on macro (state) and micro (individual) levels.

The history of foreign language values in Sweden

As a small country, Sweden has always needed to pay particular attention to languages for the purposes of trading, both regionally and internationally. The establishment of a national basic education system (mid-nineteenth century) led to both German (standard variety) and French being introduced as foreign languages in schools, with English being introduced from the mid-twentieth century and Spanish in the early twenty-first century.

The construct of Sweden as a nation state can be dated back to the early sixteenth century, when Gustav Vasa was elected king by the Swedish council in Stockholm (Scott, 1988, p. 121). Prior to this, different regions were engaged in trade across northern Europe during the dominant period of the Hanseatic League (13^{th}–15^{th} century), often operating through the lingua franca of low German (Plattdeutsch/Niederdeutsch, spoken today mainly in the Saxony region).

In the following section we include an outline of the presence of languages in the region prior to the eighteenth century. This is followed by three further sections that explore the dynamic role of language in Swedish society across the eighteenth, nineteenth and twentieth centuries, respectively.

Formation of the nation state, 14th–17th century

From the fourteenth century, regional varieties of German (often described as *Low German)* were positioned as the most powerful European language(s), becoming a lingua franca across the Nordic region. From the fifteenth to eighteenth centuries, German was widely used among the nobility and the Swedish court (Phillipson, 2000). Germany was a destination for young Scandinavian noblemen for education, and hence they had to learn the German language. The invention of the printing press facilitated large-scale book production, contributing to greater literacy, not only in Latin but also other languages (Oakes, 2001). Alongside this, the Swedish vernacular became increasingly dominant as the country severed ties from Denmark and from the influences of Rome and thus the Latin language. In this period, the Swedish language became fairly standardised in its usage (Oakes, 2001).

During the sixteenth century, there was increasing social stratification in Sweden as the aristocracy sought to distinguish itself from the peasantry (Oredsson, 2000). Education was mainly accessible to the elite of Swedish society. Their higher literacy skills gave access to written literature, including Latin and German texts. This access resulted in features of Latin and High German appearing in the language of Swedish noblemen (Oredsson, 2000). In parallel, the translation of the Bible into Swedish in 1540–41 “had a great influence on the Swedish language, acting as a standard for Swedish grammar, vocabulary and orthography” (Oakes, 2001, p. 75).

In the early seventeenth century, Sweden adopted the German university model and German became the language of academic study (Cabau-Lampa, 2005). Subsequently, the first printed resources for teaching and learning German in Sweden were produced (Gluck, 2014). In 1686, the Church introduced a literacy campaign that emphasised religious and Sunday-life reading for everyone. Increasingly, the Swedish government became aware that trade and diplomacy demanded that both the education system and the administration of the country be developed (Oredsson, 2000). During this period, scientific developments in France attracted many young Swedish noblemen to study at French universities.

In summary, during the formation of the nation state and the following century, Sweden could be characterised as a stratified society in which the need to invest in foreign languages for individuals as well as for the state was both varied and subject to change in response to new circumstances. Technology, such as the invention of the printing press, and religious ideologies promoted literacy among all classes, while social context defined the level of individual investment in languages. Those involved in trade and members of elite groups in society were most inclined to view language as capital and as something in which to invest. The Swedish state, nested within an international community, was strongly influenced in terms of policy, culture and trade. Thus, geographical space had an influence on language in society, with Germany being the main international power and collaborator at the onset of the period, but with a gradual shift in international power towards France by the end of the period.

The eighteenth century

During the eighteenth century, France gained political and military influence, becoming the dominant continental economic power and a major European cultural centre (Wright, 2006). French was increasingly viewed as a 'refined' language (Olsson, 2005). French language and culture became influential among the higher social classes in Sweden, being used as a lingua franca at court and in diplomacy among the intellectual, aristocratic and upper middle-class groups (Cabau-Lampa, 1999). In parallel with many European courts, Sweden adopted the prestige lingua franca of French as the language of diplomacy, the court and scholarly writing.

For much of the eighteenth century, the influence of the Swedish nobility was central to the national political system, which was deeply affected by the period of the French Enlightenment (Wolff, 2005). Contact with France was not only a matter of economic and political influence, it also involved the introduction to new literature, ideas and philosophies (Wolff, 2005). Reportedly, one significant influence of French culture in Sweden during this period was the growth of a letter-writing culture (mainly in French) among the upper classes. According to Olsson (2005), the use of the French language signalled the higher status and education of the writer. Thus, during this period, French was perceived as valuable cultural capital by Swedish society, with the main investors being amongst the societal elite. Through investment in their children's education, this elite was able to secure the family's social status, leading to both economic benefits and to gaining prestige in society.

However, although the influences of this period of French Enlightenment were evident among the upper classes, no similar pattern can be traced through the lower social classes. Education in the French language was neither accessible nor a perceived advantage for the majority of Swedish society. Cabau-Lampa (1999) suggests that this may be one explanation for French education never becoming dominant in the Swedish school context. Cabau (2014) also suggests that French may have been seen as a difficult language to learn, while German, with its closer proximity to Swedish, may have been perceived as easier. Throughout this period, the poorer, rural classes maintained communication through vernacular Swedish (Oakes, 2001).

Through much of the eighteenth century, Sweden was characterised by rapid cultural development, partly as a result of its close ties with France. Towards the end of the century, however, Sweden experienced a period of great social and ideological change, in which international trade declined as a result of the Napoleonic Wars, the nobility became less exclusive and social mobility increased. Ordinary people obtained the right to purchase land and were admitted to high government posts previously held only by the elite (Olsson, 2005). According to Schroder (2018, p. 34), referring to the German context, the French Revolution (1789) "marked the beginning of the end of French as an international language". No doubt much the same could be said of the influence of French in Sweden as the country entered the nineteenth century.

To sum up, through the eighteenth century, Sweden became consolidated as a nation state, establishing a parliament (Riksdag) and council that introduced a new constitution placing power in the hands of parliament. Thus, national and regional policy was determined by an elected body rather than the monarchy. The international focus shifted away from the educational and technological advances of Germany, with the elite increasingly engaging in cultural activities related to the rising influences of France and French literature, culture and style. The provision of foreign languages in secondary schools was limited, with Latin still taught in the *trivialskolor* (lower secondary school) and *gymnasier* (upper secondary school), but not in apologist schools. No foreign languages were taught in these public schools, but the complex dynamics of social context were again operating as a key determinant in refocusing the interest of the elite, together with a rising middle class. With a move away from the appeal of German universities and technical expertise, the Swedish upper classes turned to the French language, attracted by the sense of culture, refinement and literature associated with both the language and a country positioned as a powerful nation in Europe during the eighteenth century. Thus, we can identify a shift in the balance of power, with the Swedish elite moving away from the influences of Germany, attracted by images of high culture and prestige symbolised by the French nation. At the end of the century, however, the ideological shift that had precipitated the French Revolution was again to have a major impact on the European economic, technological and educational space, precipitating further restratification throughout Swedish society.

The nineteenth century

The early nineteenth century was marked by the French Revolution and the Napoleonic Wars, which affected Sweden dramatically. Napoleon's treaty with Russia led to a war between Sweden and Russia (1808–1809), resulting in the loss of Finnish territory to the Russian Empire (becoming the autonomous Grand Duchy of Finland). From the late eighteenth century through to the mid-nineteenth century, the Romanticist movement swept through Europe, bringing an artistic, literary, musical, cultural and intellectual period that influenced Swedish language and identity. According to Oakes (2001, p. 77), "Swedish was transformed from the language of the state to the language of a nation", with Swedish becoming a regular school subject from 1807.

From the early 1850s, a campaign for popular education developed, emphasising that everyone should be able to read and write from new books and texts printed or written in modern type. This resulted in the introduction of four-year compulsory basic schooling for all, leading to a dramatic improvement in literacy rates. With shifting perceptions related to foreign languages, the 1859 Swedish school ordinance criticised French as a difficult language to learn, pointing also to the association of French with elitism. This was very much in line with a parallel shift in Swedish society towards some forms of Swedish linguistic purism, together with an increased interest in Swedish culture, handicrafts and folk dance, which had emerged by the end of the century (Oakes, 2001). The gradual impact of the 1859 ordinance was further consolidated as one outcome of Germany's victory over France in the Franco-Prussian war in 1872. From this time, according to Cabau (2014), Swedish scholars were more attracted to Germany, German scholarship and the German language.

It should be noted that prior to 1871, German could not be described as a national language, since Germany as a nation state did not exist. Before the early 1800s, a mix of regional low-status varieties were spoken and learning German did not seem to be a priority for other Europeans (Schröder, 2018, p. 28). After 1871, however, the unification of Germany and the rise of the German Empire as a dominant nation restructured the European balance of power. Technical schools provided Germany with an educated and skilled population who could contribute to the industrial growth and development that led to Germany's industrial boom (Henderson, 2013). New innovations and technology turned Germany into the largest economy in Europe.

Following the Prussian defeat of France and the subsequent unification of Germany under Bismarck, French was no longer perceived as a profitable tool in which to invest; instead, Swedes were more interested in learning German and establishing stronger links with Germany, both for economic purposes and for education.

The attractiveness of both Germany's political and economic strength during the nineteenth century is reflected by the view that Germany was considered as the "motherland of pedagogy" and Sweden as "Germany's pedagogical province" according to Cabau-Lampa (2005, p. 103). Germany increasingly became a destination for Swedish scholars for education and consequently the need to learn German increased. During this period, Germany was associated with modernisation and civilisation. Its success attracted many European countries in the German sphere of influence (including Sweden) to aim at strengthening their own nations by following Germany as their role model (Bottenburg, 2001). Moreover, the economic link was crucial for Sweden, since Germany served as one of the main destinations for its high-grade iron ore exports.

German was the first foreign language taught at public schools in Sweden, and by 1895 it had become a compulsory subject for all students in all secondary school classes. The Swedish elite in particular were inspired by German universities, conservatories and art centres. The German language, as a powerful European lingua franca, provided a valuable tool for Sweden with which the country could enhance its education system through modelling it on the German system and its economy through import and export. The value of the German language was acknowledged both at the individual and societal level. Social stratification in Sweden was decreasing during this period and more people were gaining the opportunity to climb the social ladder. However, it should be mentioned that despite the social change, education at secondary schools was still not accessible to everyone in society. Compulsory school (*folkskola*) was for children aged 7–13 years and those students who wished to continue to secondary school (*realskola* and *gymnasier*) had to pay for their education. Therefore, although German language could provide valuable social and cultural capital for Swedish society, language education was not easily accessible for everyone in society to invest in.

Sweden was a poor country during the late nineteenth century, with a large proportion of the population relying on agricultural work. Extensive famine during this period led to large-scale emigration from Sweden to the United States. From 1850 to 1930, more than 1.2 million people (around 25 percent of the total population) emigrated to the United States in the hope of opportunities for economic advancement. Linked to this demographic shift, during the late nineteenth century and the beginning of the twentieth century, the new cultural artefact of silent movies, followed by the talkies, was widely available from the United States. These cultural imports signalled the start of a period during which Sweden opened its arms to American cultural imports to a large extent (Oredsson, 2000).

In summary, during the nineteenth century, a combination of events both at the domestic and international levels influenced language choices in Swedish society, leading to a change from French to German as the most widely used foreign language. Trade, science, industry and power shifts at the international level were among the significant motivating factors for Sweden's preference for German as a foreign language in which to invest. The rate of literacy increased dramatically in Sweden due to the establishment of a four-year compulsory schooling programme. Increasing changes in the sociopolitical structure led to criticism of elitism. Consequently, French language lost its status as a sign of elitism. Meanwhile, power structures in Europe leant more towards Germany, not only due to its political power after victory over France but also because of its economic strength. Germany's great economic and political power in Europe was one of the motivating factors for Sweden to invest in German language education, particularly given Sweden's extensive economic interest in trade with Germany. The period 1880–1945 marked Germany's rapid industrial development, and Sweden maintained close contact with Germany in areas such as trade, the scientific community and student exchange. With its powerful economy and industry, Germany was the main customer for Sweden's high-grade iron ore. Knowledge of the customer's language (i.e., German) could provide a tool for enhanced participation in trade with Germany. Thus, the benefits of trade can be seen as a key motivator for investment in the German language, reflecting the profitable market value attached to the language.

The twentieth and early twenty-first century

From the early 1900s, concerns regarding emigration from Sweden rose. It was understood that the United States was a more developed country than Sweden and Sweden needed to reduce this development gap to be able to survive as a nation (Alm, 2003). Hence, the United States became a role model for expanding agriculture, industry and efficient construction of inexpensive houses. According to Alm (2003), the United States was viewed as a pioneer of technology and efficient working methods, which Sweden could aim to imitate.

At the same time, the first decades of the twentieth century witnessed increasing interest in the ideas of National Socialism in Germany, while Sweden experienced a major political shift as a result of the decline in the power of the elite. By the 1920s, a social democratic government was established in Sweden, upholding the values of the working class by promoting a vision of the party as community, safety, and welfare for all.

In Sweden, early industry – mainly based on mining, electrical power and timber – grew rapidly throughout the rural areas rather than in the cities (Esping-Andersson, 1985). This decentralised industrial revolution allowed the rural poor to find work. The industries were export-oriented and were in significant demand at an international level, particularly during wartime. The end of the Second World War (WWII) represented a paradigm shift in Swedish cultural and political life as a result of the defeat of Nazi Germany. During the post-war period, the social democratic party in Sweden experienced a golden age (Esping-Andersen, 1985).

The rise in the popularity of social democracy in Sweden during the 1930s and 1940s led to the establishment of a social democratic administration, which positioned Sweden as an officially neutral country with regard to foreign policy throughout the period of WWII. This position enabled Sweden to benefit from economic growth during a period when the economies of many other European countries were being devastated by war. In parallel with this, the establishment of a welfare state model of governance (known as *folkhemmet*) consolidated Sweden's image as a neutral, socially equal society (Milani, 2007), reflecting a substantial shift from the early twentieth-century influences of the Romantic Movement that had previously contributed to Sweden's sense of national identity. In this sense, the growth of social democracy in Sweden, together with the negative view of a defeated Germany and the rejection of Nazi ideologies, led to the discrediting of the constructs of nationalism and national identity in Sweden. Oakes (2005, p. 159) goes so far as to suggest that a "negative or inverted nationalism developed into a popular myth" during this period. This so-called myth contributed to enhancing Sweden's position in the international community: at least in the late-twentieth century, it was renowned for both its neutrality and its comprehensive welfare policy. Alongside this, Swedish policy towards the indigenous Sámi population of northern Sweden cannot not be ignored. As a minority group, "the Sámi have since the 17th century been in the role of political and economic underdog" (Hornberger & Outakoski, 2015, p. 6). Even today, "[i]n Sweden, school children only have the right to receive their education through the medium of Sámi up to school year 6" (Lindgren et al., 2016, p. 5).

With the end of WWII, Germany's defeat and the empowerment of Britain and the United States, Sweden's language policy changed again; this time, English replaced German (Oredsson, 2000). Due to various sociopolitical and economic factors, Sweden identified English as a commodity representing a 'profitable' investment.

A combination of different events – such as political changes in Sweden and the declining power of the Swedish elite, together with the loss of prestige that Germany experienced after WWII – led to a major shift in language planning. In 1952, English replaced German as a compulsory language subject for Swedish students. Here again, trade and the economy played an influential role in Sweden's language policy; however, we should also acknowledge the effect of Sweden's increasing familiarity with English language and culture as a result of Swedish immigration to the United States and access to American cultural artefacts. Following WWII, Sweden's economy and industry developed rapidly and its industrial resources were in high demand at an international level. As a nation economically reliant on export, Sweden realised the immense symbolic value of a lingua franca (English) and hence developed a rather progressive multilingual policy. Knowledge of English allowed contacts with two of the greatest economic forces, the United States and the United Kingdom, as well as with international partners across Europe and beyond, in trading relationships.

In contrast to earlier periods, when French and German were learnt mainly by the elite, English language education was favoured by an expanding middle class, who realised the economic returns associated with learning English for trade and industry (Cabau-Lampa, 2005). As mentioned above, Sweden's industry mainly developed in the rural areas (particularly through the growth of the timber industry). Consequently, the middle and working classes, who were engaged in the industry and subsequent export, 'invested' in learning English with the understanding that they would acquire vital knowledge for their social and professional promotion. Family connections with earlier emigrants also enabled Swedish residents to gain knowledge about both the language and culture through visits and contact with their relatives.

Extensive exposure to American cultural artefacts such as films and TV shows also played a significant role in introducing English language and culture to Sweden, particularly given the general practice in Sweden of providing subtitles rather than dubbing or voice-over. Danan (1991, p. 613) suggests that "Subtitling corresponds to a weaker system open to foreign influences. Dubbing results from a dominant nationalistic system in which a nationalistic film rhetoric and language policy are promoted equally. Suppressing or accepting the foreign nature of imported films is a key to understanding how a country perceives itself in relation to others, and how it views the importance of its own culture and language".

Ricento (2015) refers to the visible relationship between the economic, cultural and political influences of the United States and the growth in the popularity of English in many countries around the world. Relating this argument to Sweden, Oakes (2005) explains that English has longstanding links to the Swedish self-image as modern and international. Since the early 1950s, Sweden has envisioned English as part and parcel of a modernising project with historical ties both to the United Kingdom and the United States.

Alongside the historically negative positioning of the indigenous population, the symbolic function of Swedish at national levels was reduced as internationalism became a social democratic priority during the 1970s. English served as a symbol of internationalisation in Sweden during this period (Milani, 2007), offering Swedes a tool with which to engage more widely in international trade. Through this process of sociocultural transformation, Swedish national identity has been shaped by the principles of neutrality, social equality and the welfare state, moving away from traditional understandings of nation-building as language and nationhood (Milani, 2007). Linguistically, for Sweden, being modern and international during this period meant speaking English (Oakes, 2005).

The twentieth century brought dynamic social, political and economic change to Sweden. The post-war decades meant increasing development for Swedish industries and increasing export at the international level. The globalised market demanded proficiency in a lingua franca. The increasing economic and political growth of the United States and the United Kingdom contributed to the spread of English as a lingua franca. Alongside this, English developed as a highly influential language in Swedish society, although political and economic factors were of significant importance for language shift in Sweden.

To sum up, the language value system in twentieth-century Sweden can be characterised by the dominance of English. This was triggered by global events (the end of WWII) that shifted global power to the Anglo-American space; by events in time, with the great famine in 1860 leading to massive emigration to the USA, leaving relatives in Sweden with strong connections to American language and culture; by a shift in political ideology, including compulsory education and more equal opportunities for all; and by economics and trade, which allowed the country to gain economically and increase the living standards for everyone, thereby allowing for the consumption of culture as well as technology. More recently, the emergence of a view of globalisation that emphasises the important role of English as a lingua franca has served to consolidate the dominant global role of English (Jenkins, 2015; O'Regan, 2014; Seidlhofer, 2010). On both macro (state) and micro (individual) levels, there have been strong incitements to invest in the English language in order to develop the economy and maintain participation in the international community.

The influences of languages: Past, present and future

In this paper, we have illustrated how historical patterns of events in and around Sweden have contributed to a process of constant negotiation and renegotiation in the forms of communication, a process that is inevitably provisional and subject to change over time. During the period in focus, stretching from the fourteenth to the early twenty-first century, we have seen how the pre-eminence of forms of low German, high German and French, in addition to Swedish, performed a significant communicative role related to specific domains of use in Swedish society. With reference to each historical period, we have indicated how language values and preferences were selected as a response to a complex range of societal needs related to economic, political and cultural priorities at particular points in time.

The contemporary twenty-first century period, with the pre-eminence of English, can be identified as unique in the history of languages in Sweden. It is a period during which the everyday use of English has been normalised across almost all domains of use, within all speech communities. The current National Curriculum positioning of English as a core subject from the start of compulsory schooling can be seen as a statement of political will, a signal of Sweden's engagement in the so-called 'global marketplace', or simply as a societal acknowledgement of bilingualism as a necessary attribute for a relatively small, northern European population. This last point, however, ignores the fact that Sweden is, and always has been, a multilingual country, with five official minority languages and more than 200 languages brought to Sweden by various waves of immigration. The enshrinement of English in the National Curriculum as a *core subject* rather than a *foreign language* implies the emergence of new patterns of language use in Sweden, whereby English has gained the status of a second language and, in some instances, *almost* of being a joint first language alongside Swedish.

In addition to the prioritisation of English in the school curriculum, the popularity of Spanish as a foreign language (in preference to German and French) can be seen as a further illustration of how complexity "gives rise to unpredictable patterns of emergence" (Baicchi, 2015, p. 9). While, on the one hand, the recent popularity of Spanish might be accounted for as a response to the availability of low-cost European air travel in recent years, together with an increased 'cool' factor that has emerged in many parts of Europe in response to the popularity of Latino culture promoted via music, film and TV, the low exam results achieved by Swedish school students studying Spanish (ESLC, SurveyLang, 2012) suggest that a more complex combination of both internal and external factors may be operating here (Letica Krevelj & Medved Krajnovic, 2015, p. 192). In our attempts to understand the complexity of the language value system in Sweden we must acknowledge that "[b]ecause the systems are open, what arises may be in nonlinear relation to its cause" (Larsen-Freeman, 2012, p. 205).

Turning to the question of how the future language value system in Sweden might dynamically evolve, we assume that its complex nature will be creative, operating within a system that is itself nested within other complex systems. Walsh (2006) emphasises the significance of the relationship between language and the economy, arguing that the success and failure of language planning and policy activities is closely connected to economic outcomes: if language policy and planning activities lead to desirable economic outcomes, they will succeed. In the historical account presented here, however, we have shown how the rise and fall in the popularity of both German and French as foreign languages in Sweden occurred "through multiple routes (...) mediated in different ways" (Larsen-Freeman, 2012, p. 205). We therefore suggest that dynamic evolution in response to the interaction of elements within the system, operating within other complex systems, is a more likely future pathway.

Linguistic diversity in Sweden has substantially increased during the twenty-first century as a result of a policy to support immigration. Provision for the maintenance of home languages in schools has been widespread, albeit with some difficulties in ensuring adequate provision for the variety of different languages, which now exceeds 200 languages (Språkrådet, 2012). Still, as illustrated in our analysis, the current language value system in Sweden positions English as more or less alone as the non-Swedish language with a high value. We might hypothesise, however, that given the new multilingual context, the next Swedish generation will demand an increased acknowledgement of their multilingual expertise, requiring more extensive schooling provision, including formal recognition of proficiency through a final school examinations system. Alternatively, with the shifting balance of economic power towards Asian countries, we might anticipate an increased demand for Mandarin Chinese in schools of the future. If this shift were to occur it would reflect an economic bias in line with Walsh's (2006) claim. A third alternative might be that English will continue to play a significant global role for the foreseeable future. With the ever-increasing global interconnectivity made possible via digital technologies, Swedish society, with its expertise in English, is already in a strong position to adapt and fully participate in any spontaneous new system that may emerge. Clearly, the scenarios outlined here are little more than speculative. In the spirit of dynamic systems theory, we therefore close with this final speculation on the nature of language (Larsen-Freeman, 2012, p. 207): "We give a language a name as if its borders were defined and it existed as a separate entity".

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That Old Devil Called 'Statistics': Statistics Anxiety in University Students and Related Factors

Melita Puklek Levpušček*[1] and Maja Cukon[2]

The present study investigated relationships between statistics anxiety (SA), trait anxiety, attitudes towards mathematics and statistics, and academic achievement among university students who had at least one study course related to statistics in their study programme. Five hundred and twelve students from the University of Ljubljana completed the Statistics Anxiety Rating Scale (STARS), State-Trait Anxiety Inventory, and answered questions about their perceptions of mathematics and statistics. The results showed below-average mean scores on the STARS dimensions, except for the Test and Class Anxiety with the average score around the midpoint of the scale. Female students reported higher levels of SA than male students did. The highest levels of SA were reported by students who perceived mathematics and statistics as a threat. The subscales of the STARS correlated positively with students' trait anxiety. Students who reported less enjoyment in mathematics in high school perceived statistics to be a less worthy subject and had a lower computation self-concept. Students who had better mathematics performance in high school and higher average study grades also reported a higher computation self-concept. In the present study, we translated the STARS questionnaire into Slovenian and confirmed the six-factor structure of the questionnaire. The results provide a basis for further research on statistics anxiety and further validation of the STARS questionnaire. The results can also aid statistics teachers in better understanding students' worries, fears, and attitudes towards statistics and in learning about the factors that affect students' statistics anxiety and their work in the course.

Keywords: statistics anxiety, trait anxiety, attitudes towards mathematics and statistics, university students

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Statistična anksioznost pri študentih in povezani dejavniki

Melita Puklek Levpušček in Maja Cukon

V raziskavi smo preučevali odnos med statistično anksioznostjo (SA), anksioznostjo kot potezo, stališči do matematike in statistike ter študijsko uspešnostjo pri študentih, ki so imeli v svojem študijskem programu vsaj en statistični predmet. 512 študentov Univerze v Ljubljani je izpolnjevalo vprašalnik za merjenje statistične anksioznosti (STARS) in lestvico anksioznosti (STAI-X2) ter odgovarjalo na vprašanja o doživljanju matematike in statistike. Rezultati so pokazali, da je bilo pet dimenzij statistične anksioznosti izraženih pod povprečjem, dimenzija testna anksioznost in anksioznost v razredu pa je bila povprečno izražena. Študentke so poročale o več statistične anksioznosti kot študenti. O najvišji statistični anksioznosti so poročali študenti, ki so doživljali matematiko in statistiko kot grožnjo. Dimenzije STARS so bile pozitivno povezane z anksioznostjo kot potezo pri študentih. Študenti, ki so bili manj navdušeni nad matematiko v srednji šoli, so poročali o nižji vrednosti statistike in imeli nižjo samopodobo na področju računskih sposobnosti. Študenti z višjo zaključno oceno pri matematiki v srednji šoli in višjo povprečno študijsko oceno so imeli višjo samopodobo na področju računskih sposobnosti. V raziskavi smo prevedli vprašalnik STARS in potrdili 6-faktorsko strukturo vprašalnika. Rezultati predstavljajo izhodišče za nadaljnje raziskovanje statistične anksioznosti in nadaljnje validacije vprašalnika. Učiteljem statistike lahko pomagajo pri boljšem razumevanju strahov in skrbi študentov, njihovih stališč do statistike ter dejavnikov, ki vplivajo na doživljanje statistične anksioznosti in delo pri predmetu.

Ključne besede: statistična anksioznost, anksioznost kot poteza, stališča do matematike in statistike, študenti

Introduction

Statistics is a mathematical discipline that investigates how statistical data can be collected, summarised and presented. Statistical knowledge is useful in many scientific fields (e.g., medicine, computer science, mathematics, economics, finance, etc.). For example, modern medicine is based on statistics: randomised controlled trials have been described as 'one of the simplest, most powerful and revolutionary tools in research' (Hand, 2008, p. 2). Governments use statistical analysis of data to explain economic and social issues. Farmers, food technologists, and supermarkets implicitly use statistics to decide what to grow, process, package, and finally distribute (Hand, 2008). Statistics is undoubtedly the basis of all science and scientific research. It is also a tool that can help us make important discoveries. In the modern data-driven world, it would be impossible to make decisions without using statistical methods.

Statistical knowledge helps to promote critical and logical thinking (Lehman & Nisbett, 1990; VanderStoep & Shaunghessy, 1997) and is essential in many academic and professional fields (Maat & Rosli, 2016). Learning statistics in the social sciences equips students with specific research skills, such as the appropriate use of scientific methods and procedures, the ability to define research problems and hypotheses, plan research, conduct research, analyse, and interpret results, report on results, and similar. Since mathematics and statistics are crucial to many academic disciplines, it is important to study students' attitudes and feelings towards these subjects. Research suggests that students' positive attitudes towards mathematics and science are important for achieving greater interest in science-related professions. Attitudes can change over time; if they change towards more positive attitudes, students are likely to be better prepared and interested in science careers (Ing & Nylund-Gibson, 2017). Unfortunately, students have identified statistics as their most anxiety-inducing course (Chew & Dillon, 2014), and 80% of university students experience some form of statistics anxiety (Onwuegbuzie & Wilson, 2003). Statistics anxiety is negatively related to various academic outcomes, such as failing a statistics course, dropping out of a course, lower academic grades, difficulties with the research part of a thesis, and the perception of statistics as unimportant (Siew et al., 2019).

In this article, we focused on statistics anxiety as experienced by students of the University of Ljubljana. We also examined factors associated with statistics anxiety, such as gender, academic performance, attitudes and experiences with mathematics and statistics, and students' trait anxiety. The study data could aid statistics teachers in better understanding students' worries, fears, and attitudes related to statistics and in learning more about the factors related to students' statistics anxiety.

Statistics anxiety has been defined as 'feelings of anxiety encountered when taking a statistics course or doing statistical analyses' (Cruise et al., 1985, p. 92). It occurs when people are confronted with statistics in any form and at any level (Onwuegbuzie et al., 1997). For example, students may experience anxiety when confronted with statistical ideas, problems or questions, instructional situations, or evaluative assessments (Onwuegbuzie & Daley, 1999; Onwuegbuzie & Seaman, 1995; Zeidner, 1991). Statistics anxiety is a pervasive phenomenon that occurs mainly in the social sciences, such as psychology, education, and sociology (Onwuegbuzie, 2004; Onwuegbuzie & Wilson, 2003; Ruggeri et al., 2008). It can affect both exam performance and the quality of learning statistics, as it refers to the interference of task-relevant thoughts with task-irrelevant thoughts, such as worries and ruminations. As a result, the cognitive resources required for successful task completion are reduced (Eysenck et al., 2007). Statistics anxiety is, in some ways, similar to mathematics anxiety, but various studies have also confirmed its different nature (Chew & Dillon, 2014; Paechter et al., 2017). Understanding mathematical concepts and using mathematical symbols is an important part of statistics anxiety, but learning basic mathematical concepts is different from learning statistics (Aksentijević, 2015). Statistical tasks in the fields of education, psychology, or sociology are more related to verbal comprehension (Buck, 1987) and require cognitive operations that include thinking about probabilities, possible effects, and understanding social phenomena.

Many authors conceptualised statistics anxiety as a multidimensional construct (Cruise et al., 1985; Onwuegbuzie, 1997; Onwuegbuzie et al., 1997). Using factor analysis, Cruise et al. (1985) identified six components of statistics anxiety: Interpretation Anxiety, Test and Class Anxiety, Fear of Asking for Help, Computation Self-Concept, Fear of Statistics Teachers, and Worth of Statistics. *Interpretation anxiety* is related to the fear that students feel when interpreting statistics results. *Test and class anxiety* refers to anxiety when a person attends lectures on statistics or takes an exam. *Fear of asking for help* is a component related to the feeling of anxiety when asking for help in understanding statistical material. *Computation self-concept* refers to the perception of one's own ability to solve mathematical or computational tasks. *Fear of teachers of statistics* is related to the negative perception of statistics teachers by individuals. Finally, *Worth of statistics* refers to the students' perception of the importance and usefulness of statistics.

A higher level of statistics anxiety relates to negative attitudes towards statistics (Chiesi & Primi, 2010; Kesici et al., 2011). However, more recent findings support the idea that positive attitudes towards statistics can mitigate the negative effects of statistics anxiety on students' academic performance (Najmi et al., 2018). González et al. (2016) claim that students with a higher level of statistical self-image and the intrinsic value of statistics; specifically, those students who show confidence in their competence to learn statistics and who believe that statistical courses and content are valuable experience less anxiety during class, use more self-regulating and deep-learning strategies, and show more persistence in accomplishing difficult tasks, leading to better statistical performance. Baloğlu et al. (2011) found that students who believe that statistical skills are important for future career development show lower computational anxiety, lower fear of statistics teachers, lower fear of asking for help, and lower interpretation anxiety. Accordingly, they show a more positive attitude towards statistics.

Students' experiences and attitudes towards mathematics contribute to their attitude towards statistics, which in turn contributes to feelings of statistics anxiety (Marchis, 2011). Some students experience statistics anxiety due to a lack of mathematics knowledge, lower previous achievement, and negative previous experience in mathematics courses and fear of mathematics (Lalayants, 2012; McGrath, 2014). Students with less experience in mathematics reported higher levels of statistics anxiety and difficulties in following lectures in statistics and often found statistics to be a difficult and useless course (Baloğlu, 2001). Similarly, Trimarco (1997) found that people with a higher level of knowledge in scientific research and statistics reported a lower level of statistics anxiety.

Previous research reported associations of statistics anxiety with stable personality traits, such as trait anxiety. It refers to individuals' tendency to experience stressful situations as threatening, which in turn increases the level of anxiety (Ronchini Ferreira & Ribeiro Silva, 2016). Anxious people tend to have low self-esteem and have a higher fear of failure (Lamovec, 1988). Macher et al. (2012) reported a positive correlation between anxiety as a personality trait and statistics anxiety. Anxiety is also an important component of neuroticism, which determines how we experience and overcome stressful situations (Musek, 2010). It has been shown that neuroticism is related to the dimensions of statistics anxiety, such as the lower worth of statistics, the fear of asking for help and the fear of statistics teachers (Chew & Dillon, 2014).

The Present Study

In brief, the present study aimed to examine the level of statistics anxiety and its dimensions in a sample of Slovenian university students who took at least one statistics course during their studies. Furthermore, we were interested in personal factors that might be related to statistics anxiety. For this study, we translated the Statistics Anxiety Rating Scale (STARS, Cruise et al., 1985) from English into Slovenian and tested the factor structure of the questionnaire.

Based on the results of previous studies, we hypothesised that statistics anxiety would be expressed in above-average responses to the items. Students of different disciplines experience statistics as a course that causes most of their anxiety (Onwuegbuzie & Wilson, 2003; Zeidner, 1991).

Secondly, we hypothesised that female students would have a higher level of statistics anxiety than male students did. Research has shown that women have higher anxiety as a personality trait than men do (Benson, 1989; Demaria-Mitton, 1987; Macher et al., 2012; Zeidner, 1991). However, research on the relationship between statistics anxiety and gender has not yet produced consistent results. In some studies, female students reported higher levels of statistics anxiety than male students did (Onwuegbuzie, 1995; Stroup & Jordan, 1982; Zeidner, 1991), but some authors also reported a nonsignificant gender difference in statistics anxiety (Baloğlu, 2003; Cruise & Wilkins, 1980).

Thirdly, students who perceive mathematics and statistics as a threat will report the highest levels of statistics anxiety.

Finally, we hypothesised that students' enjoyment of mathematics, final grade in mathematics in the fourth year of secondary school, and average study grade would correlate negatively with their statistics anxiety, while students' trait anxiety would correlate positively with their statistics anxiety.

Method

Participants

The sample consisted of 512 students at the University of Ljubljana who had at least one study course related to statistics in their study program. There were 400 female students (78.1%) and 112 male students (21.9%); most of the sample were undergraduate students (386 students, 75.39%). The participants were students in the Faculty of Arts (161 students, 31.4%), the Faculty of Social Sciences (106 students, 20.7%), the School of Economics and Business (150 students, 29.3%) and the Faculty of Education (95 students, 18.6%). The average age of the students was 21.6 years (SD = 2.03).

Participants of the Faculty of Arts were students of psychology (n = 150), geography (n = 10), and one student of sociology. There were 122 undergraduate students of psychology (six statistics courses, 32 ECTS; e.g., Descriptive Statistics, Methodology of Psychological Research, Statistical Inference) and 28 graduate students (one statistics course (Applied Psychometrics), 7 ECTS). Students of geography were undergraduate students who take a compulsory statistics course (Methods for Geographers, 4 ECTS), while students of sociology (bachelor's level) take two statistics courses (Basics of Sociological Research I and II, 9 ECTS). Participants of the Faculty of Social Sciences were undergraduate students of the International Relations (n = 19, one statistics course (Statistics), 5 ECTS), Communication (n = 17, one statistics course (Statistics), 5 ECTS), Political Science (n = 6, one statistics course (Statistics), 5 ECTS), Cultural Studies (n = 10, one statistics course Statistics, 5 ECTS), Journalism (n = 23, one statistics course (Statistics), 5 ECTS), and Social Informatics (n = 34, 10 statistics courses, 50 ECTS). Seventy-five undergraduate students of the School of Economics and Business had one statistics course in their program (Basics of Statistics, 6 ECTS) and 75 graduate students who had one statistics course in their study programme (Research Methods and Techniques, 7 or 10 ECTS). The participants of the Faculty of Education were students of the Special and Rehabilitation Pedagogy (56 undergraduate students, one statistics course (Statistical Analysis of Data), 4 ECTS, and 8 graduate students, one statistics course (Statistical Multivariate Analysis of Data), 6 ECTS) and students of Speech Therapy and Surdopedagogy (21 undergraduate students, one statistics course (Statistical Analysis of Data), 4 ECTS and 10 graduate students, one statistics course (Statistical Multivariate Analysis of Data), 6 ECTS).

Instruments

The students first answered questions on gender, age, faculty, course of study, final grade in mathematics in the fourth year of high school and the average study grade in the current academic year. The following questions related to the students' experiences and their attitudes towards mathematics and statistics: *I enjoyed mathematics in my high school* (1 = I strongly disagree, 5 = I strongly agree); *How did you perceive mathematics in high school* ((a) as a threat, (b) as a challenge, (c) as something in between, and (d) none of these); *Did you expect statistics to be one of the courses in your study program?* (yes, no); *In my study program, the amount of statistics is …* (fair enough, too high, insufficient); *How do you perceive the statistics in your study program?* ((a) as a threat, (b) as a challenge, (c) something in between and (d) none of these). After these questions, the participants answered the questionnaires on statistics anxiety and trait anxiety.

Statistics Anxiety Rating Scale (STARS; Cruise et al., 1985)

The Statistics Anxiety Rating Scale (STARS; Cruise et al., 1985) is the most widely used measure for testing statistics anxiety, especially in the academic but also in the non-academic fields. The STARS contains 51 questions that describe six domains of statistics anxiety: (a) Worth of Statistics (example of an item: *statistics takes more time than it's worth*), (b) Interpretation Anxiety (example of an item: *interpreting the meaning of a probability value once I have found it*), (c) Test and Class Anxiety (example of an item: *studying for an examination in a statistics course*), (d) Computation Self-Concept (example of an item: *I have not done maths for a long time. I know I'll have problems getting through statistics*), (e) Fear of Asking for Help (example: *asking someone in the computer lab for help in understanding a printout*) and (f) Fear of Statistics Teachers (example: *statistics teachers speak a different language*).

The STARS consists of two parts. The first contains 23 items that describe different situations related to statistics. Participants rate the items on a 5-point scale according to their level of anxiety in each situation (1 – no anxiety; 5 – strong anxiety). The second part consists of 28 items that relate to the participants' attitude towards statistics. The participants circle the level of their agreement with the item (1 – strongly disagree; 5 – strongly agree)[3].

Cruise et al. (1985) reported that the items on the STARS load onto six factors, and they confirmed good internal reliabilities of the subscales: Interpretation Anxiety ($\alpha = .87$), Test and Class Anxiety ($\alpha = .68$), Fear of Asking for Help ($\alpha = .89$), Worth of Statistics ($\alpha = .94$), Computation Self-Concept ($\alpha = .88$), and Fear of Statistics Teachers ($\alpha = .80$). The construct

3 Higher scores on the subscales indicate higher levels of anxiety (i.e., interpretation anxiety, test and class anxiety, fear of asking for help) or higher levels of negative attitudes (i.e., fear of statistics teachers, less worth of statistics and lower computation self-concept).

validity of the STARS was examined by verifying the factor structure of the questionnaire and its relationship to other measures of statistics anxiety (Cruise et al., 1985; Hanna et al., 2008; Mji & Onwuegbuzie, 2004). Hanna et al. (2008) confirmed the six-factor structure of the questionnaire, first proposed and validated by Cruise et al. (1985).

The translation of items mostly followed the meaning of the original items, except for the three items that were slightly adapted to the Slovenian cultural and academic context.

We present the data on the construct validity and reliability of the STARS results obtained in a Slovenian sample of university students in the Results section.

State Trait Anxiety Inventory – STAI-X2 (Spielberger et al., 1983)

State Trait Anxiety Inventory – STAI-X2 (Spielberger et al., 1983; translated into Slovenian by Lamovec, 1988) measures anxiety as a personal characteristic. Participants assess how they feel in typical situations that everyone experiences daily and how they react to situations with different levels of anxiety. STAI-X2 consists of 20 items, which the participants answer on a 4-point scale (1 – almost never, 4 – almost always). Examples of the items include 'I worry too much over something that really doesn't matter', 'I am inclined to take things hard'. Kranjec, et al. (2016) found a high internal consistency of the scale ($\alpha = .90$), which was also confirmed in our study ($\alpha = .91$).

Procedure

We collected the data in May and June 2018. The participants were recruited through personal contact with professors of statistics at the four faculties of the University of Ljubljana. The professors who agreed to participate allowed us to apply the questionnaires to students during their lectures. The students were first informed about the purpose of the study, their voluntary participation, and their anonymity. Afterwards the participants answered the questionnaires in paper-pencil form. The whole procedure took about 15 minutes.

The data were analysed with Excel, SPSS 20.0 and R (laavan package).

Results

In a first step, we checked the results on students' general experiences and attitudes towards mathematics and statistics. The average level of students' enjoyment of mathematics in their high school was slightly above the scale's midpoint (M = 3.22, SD = 1.23). In addition, 11.3% of the students perceived mathematics at their high school as a threat, 35.7% as a challenge, 36.7% as something in between, and 15.8% of the students perceived mathematics neither as a threat nor as a challenge. The most frequent answer thus indicates that the perception of mathematics among students can be quite ambivalent (they perceive it both as a challenge and as a threat).

In contrast, we can also observe a positive attitude towards mathematics among more than one third of the respondents (who perceive it as a challenge). Compared to the attitude towards mathematics, there was a higher percentage of students who perceived statistics as a threat (22.3%), a similar percentage of students who perceived statistics as something between a challenge and a threat (36.3%), and a lower percentage of students who perceived it as a challenge (24.2%). In addition, 70.7% of students reported that they expected statistics to be one of the courses in their study program. Most students (61.5%) reported that the amount of statistics in their study program was fair enough, while almost a third of students (31.5%) thought that the amount of statistics was too high.

Since the STARS (Cruise et al., 1985) was translated into Slovenian and used for the first time in the Slovenian academic environment, we used confirmatory factor analysis (CFA) to validate the factor structure of the original questionnaire. We used the *weighted least squares with mean and variance adjusted* (WLSMV; Muthén, 1993) estimation. In assessing the fit of the model to the data, we followed the criteria for cut-off values recommended by Hu and Bentler (1999): CFI, TLI close to or > .90, $RMSEA$ < .08, and $SRMR$ < .08. The fit of the 6-factor model to the data in our study was marginally satisfactory: $\chi 2$ = 2077.304; df = 1209; p < .000; $RMSEA$ = .039, 90 % IZ [.037–.042]; CFI = .867; TLI = .860; $SRMR$ = .058. Table 1 shows the standardised loadings, reliabilities (α), means (M) and standard deviations (SD) for the scales of the STARS.

Table 1

The range of standardised factor loadings, Cronbach α coefficients, means and standard deviations for the STARS scales

STARS subscales	Factor loadings	*M*	*SD*	α
Interpretation	.47–.71	2.31	.71	.86
Test and class	.55–.76	3.13	.87	.87
Help	.65–.85	2.33	.93	.70
Computation SC	.37–.76	2.14	.81	.85
Teachers	.55–.73	2.21	.85	.79
Worth	.44–.82	2.41	.81	.94

Note. Interpretation = Interpretation Anxiety; Test and class = Test and Class Anxiety; Help = Fear of Asking for Help; Computation SC = Computation Self-Concept; Teachers = Fear of Statistics Teachers; Worth = Worth of Statistics. The subscale scores of the STARS were calculated as average score per item. The range of the response scale was 1 to 5. Higher scores on Computation Self-Concept and Worth of Statistics mean lower self-rates on the two constructs.

The standardised factor loadings presented in Table 1 indicate a good construct validity of the Slovenian version of the STARS. The average loadings for the six STARS subscales were: $M_{\text{Interpretation}} = .64$, $M_{\text{Test and class}} = .61$, $M_{\text{Help}} = .75$, $M_{\text{Computation SC}} = .67$, $M_{\text{Teachers}} = .65$ and $M_{\text{Worth}} = .70$. The six STARS subscales also showed good internal consistency with Cronbach α coefficients ranging from .70 to .94. Table 1 also shows the mean scale-scores with respect to the possible range of these scores, which correspond to the range of the response scale used (1 to 5). The scale scores close to value 3 indicate intermediate levels of a particular dimension of statistics anxiety, while the scores closer to the minimum and maximum values of the possible range indicate low and high levels of a particular construct. Participants scored below the scale's mid-point levels on the five STARS scales, while the average score of participants on the Test and Class Anxiety scale was around the midpoint of the scale.

Next, we performed a one-way ANOVA to find out whether students with different attitudes towards mathematics and statistics (which they each perceive as a threat, a challenge, something in between or none of these) differed in their statistics anxiety. The results showed that the four groups of students with different attitudes towards mathematics differed significantly in all dimensions of the STARS questionnaire: $F(3, 490) = 7.93$, $p < .001$, $d = .94$ (Interpretation Anxiety), $F(3, 492) = 9.66$, $p < .001$, $d = 1.20$ (Test and Class Anxiety), $F(3, 501) = 6.52$, $p < .001$, $d = .51$ (Fear of Asking for Help), $F(3, 494) = 11.19$, $p < .001$, $d = 1.04$ (Worth of Statistics), $F(3, 500) = 60.21$,

$p < .001$, $d = 2.07$ (Computation Self-Concept), $F(3, 499) = 4.19$, $p = .006$, $d = .65$ (Fear of Statistics Teachers). Similarly, significant differences in statistics anxiety scores were found among groups of students with different attitudes towards statistics: $F(3, 489) = 25.67$, $p < .001$, $d = .32$ (Interpretation Anxiety), $F(3, 491) = 59.38$, $p < .001$, $d = 1.91$ (Test and Class Anxiety), $F(3, 500) = 12.46$, $p < .001$, $d = 1.09$ (Fear of Asking for Help), $F(3, 493) = 43.11$, $p < .001$, $d = 1.72$ (Worth of Statistics), $F(3, 499) = 16{,}98$, $p < .001$, $d = 1.08$ (Computation Self-Concept), $F(3, 498) = 25.58$, $p < .001$, $d = 1.39$ (Fear of Statistics Teachers).

Tables 2 and 3 show descriptive statistics for the six subscales of the STARS in groups of students who perceived mathematics and statistics as a threat, a challenge, something in between, or none of these.

Table 2

Statistics anxiety (interpretation anxiety, test and class anxiety, fear of asking for help) according to students' perceptions of mathematics and statistics

		Interpretation	Test and class	Help
		M(SD)	*M(SD)*	*M(SD)*
Mathematics	threat	2.62 *(.70)*	3.55 *(.88)*	2.71 *(1.09)*
	something in between	2.35 *(.74)*	3.21 *(.83)*	2.39 *(.94)*
	challenge	2.29 *(.66)*	3.06 *(.83)*	2.28 *(.86)*
	none of these	2.03 *(.69)*	2.80 *(.92)*	2.04 *(.83)*
Statistics	threat	2.67 *(.75)*	3.87 *(.74)*	2.74 *(1.03)*
	something in between	2.40 *(.64)*	3.16 *(.71)*	2.32 *(.84)*
	challenge	1.95 *(.57)*	2.67 *(.76)*	2.06 *(.88)*
	none of these	2.16 *(.70)*	2.73 *(.81)*	2.19 *(.85)*

Note. Interpretation = Interpretation Anxiety; Test and class = Test and Class Anxiety; Help = Fear of Asking for Help. The results are shown as the average score per item.

The results in Table 2 show that students who perceived mathematics and statistics as a threat had the highest scores on Interpretation Anxiety, Test and Class Anxiety and Fear of Asking for Help subscales. Students who perceived mathematics neither as a threat nor as a challenge had the lowest scores in the three statistics anxiety dimensions, while students who perceived statistics as a challenge had the lowest anxiety in these dimensions.

Table 3

Statistics anxiety (worth of statistics, computation self-concept, fear of teachers) according to students' perceptions of mathematics and statistics

		Worth	Computation SC	Teachers
		M(SD)	*M(SD)*	*M(SD)*
Mathematics	threat	2.84 *(.86)*	3.00 (.86)	2.45 (.96)
	something in between	2.50 *(.82)*	2.34 *(.76)*	2.30 *(.85)*
	challenge	2.19 *(.73)*	1.70 *(.50)*	2.11 *(.79)*
	none of these	2.37 *(.78)*	2.05 *(.76)*	2.05 (.84)
Statistics	threat	3.00 *(.88)*	2.57 (.88)	2.74 (.99)
	something in between	2.39 *(.71)*	2.09 (.75)	2.19 (.73)
	challenge	1.92 (.58)	1.89 (.75)	1.87 (.68)
	none of these	2.38 *(.71)*	2.02 *(.66)*	2.06 *(.71)*

Note. Worth = Worth of Statistics; Computation SC = Computation Self-Concept, Teachers = Fear of Statistics Teachers. The results are shown as the average score per item. Higher scores on Computation Self-Concept and Worth of Statistics mean lower selfrates on the two constructs.

Table 3 shows that students who perceived mathematics and statistics as a threat had the lowest Worth of Statistics and Computation Self-Concept and the highest Fear of the Statistics Teachers. Students who experienced mathematics and statistics as a challenge reported the highest Worth of Statistics and Computation Self-Concept, while the group of students who perceived mathematics as neither a threat nor a challenge reported the lowest score on the Fear of Statistics Teachers.

Table 4

Gender differences in the STARS dimensions

	Gender	*M*	*SD*	*t*	*d*
Interpretation	male	2.06	.68	-4.29**	0.47
	female	2.38	.71		
Test and class	male	2.73	.88	-5.46**	0.59
	female	3.23	.84		
Help	male	2.32	.92	-0.17	0.02
	female	2.33	.93		
Worth	male	2.27	.81	-1.99*	0.21
	female	2.45	.81		

	Gender	*M*	*SD*	*t*	*d*
Computation SC	male	2.06	.78	-1.19	0.13
	female	2.16	.81		
Teachers	male	2.15	.87	-0.89	0.09
	female	2.23	.84		

Note. Interpretation = Interpretation Anxiety; Test and class = Test and Class Anxiety; Help = Fear of Asking for Help; Worth = Worth of Statistics; Computation SC = Computation Self-Concept, Teachers = Fear of Statistics Teachers. The subscale scores of the STARS were calculated as average score per item.
$^{*}p < .05$, $^{**}p < .01$.

As shown in Table 4, female students reported higher Interpretation Anxiety, Test and Class Anxiety, and lower Worth of Statistics than male students. The effect sizes (*d*) were of small to medium magnitude.

Table 5

Correlations between the subscales of the STARS and trait anxiety, enjoyment in mathematics in high school, final grade in mathematics in the last year of high school, and average study grade

STARS	Trait anxiety	Enjoyment in mathematics	Final grade in mathematics	Average study grade
Interpretation	.323**	-.047	.036	.061
Test and class	.366**	-.058	.122**	.029
Help	.364**	-.080	.053	-.059
Worth	.126**	-.209**	-.083	-.071
Computation SC	.312**	-.488**	-.307**	-.117**
Teachers	.194**	-.058	.053	.003

Note. Interpretation = Interpretation Anxiety; Test and class = Test and Class Anxiety; Help = Fear of Asking for Help; Worth = Worth of Statistics; Computation SC = Computation Self-Concept; Teachers = Fear of Statistics Teachers. Higher scores on Computation Self-Concept and Worth of Statistics mean lower self-rates on the two constructs.
$^{**}p < .01$.

The results in Table 5 show statistically significant and positive correlations between trait anxiety and all subscales of STARS. In addition, students who showed higher enjoyment of mathematics in high school reported higher Worth of Statistics and higher Computation Self-Concept. Students with a higher final grade in mathematics in high school reported a higher Test and Class Anxiety and higher Computation Self-Concept. in the end, students with a higher average study grade reported higher Computation Self-Concept.

Discussion

In this study, we investigated the occurrence of statistics anxiety in the sample of students of social sciences at the University of Ljubljana. We were also interested in personal factors related to the experience of statistics anxiety.

The findings indicate that our participants experienced average to below-average levels of statistics anxiety. We could speculate that students who have statistics courses in their study program have on average no significant difficulty interpreting statistics results and asking teachers or colleagues for help if they have problems understanding the course material. The students in our sample also showed relatively positive perceptions of their competencies to solve mathematical or computational tasks, and a positive perception of statistics as an important and useful course. The results of our research are, therefore, not consistent with other studies that showed substantial levels of statistics anxiety among university students (Chew & Dillon, 2014; Onwuegbuzie, 2004; Onwuegbuzie & Wilson, 2003; Ruggeri et al., 2008). It can be assumed that modern educational technology enables students to access various learning materials and use extensive information on statistics on the Internet. In this way, students can feel more comfortable studying statistics.

Nevertheless, teaching statistics in the social sciences means that more females than males are taught. Previous findings showed that women have a higher level of anxiety as a personality trait than men (Benson, 1989; Macher et al., 2012; Zeidner, 1991). Also, Stroup and Jordan (1982), Onwuegbuzie (1995), and Zeidner (1991) reported significant gender differences in the experience of statistics anxiety, with female students showing a higher level of statistics anxiety than male students did. In our study, gender differences occurred in certain dimensions of statistics anxiety. Female students reported a higher Interpretation Anxiety and Test and Class Anxiety, and a lower Worth of Statistics. Interpretation anxiety is a form of anxiety that students experience when interpreting statistical results. It may be that females question their statistics knowledge more than male students do, which in turn may affect their higher test and class anxiety. A lower Worth of Statistics may also indicate that statistics is less important for female students. For example, Štraus et al. (2013) found that Slovenian female adolescents do not like mathematics as much their male counterparts do, which may later be reflected in their more negative perception of statistics. At this point, it should be noted that effect sizes (*d*) in gender differences were small to medium, suggesting that gender is not so important in experiencing statistics anxiety.

Furthermore, we cannot generalise gender differences, since our sample included about 80% female students. Although the sample was unbalanced by gender, it is representative of the gender ratio of students in the faculties included in the study. However, further studies should investigate the gender differences in statistics anxiety using a more balanced sample.

In this study, we also examined the relationship between attitudes towards mathematics and statistics and the statistic anxiety. The results showed that students who perceived mathematics as a threat had the highest scores in all dimensions of statistics anxiety. In contrast, students with a neutral perception of mathematics (who perceived mathematics as neither a threat nor a challenge) scored lowest in most dimensions of statistics anxiety. The only exceptions were Worth of Statistics and Computation Self-Concept. Students who perceived mathematics as a challenge in high school reported the most positive beliefs about the value of statistics and the highest computational competencies, which may be because students who had a positive experience of mathematics in high school developed a similarly positive attitude towards statistics during their studies. The results are consistent with the study by Macher et al. (2012), which found that students with a more positive mathematical self-image reported lower levels of statistics anxiety. However, we found the lowest results in four dimensions of statistics anxiety among students with a neutral attitude towards mathematics. Such individuals probably did not develop a negative attitude towards mathematics in their earlier school years and were, therefore, less likely to develop statistics anxiety.

Similar results were obtained when we analysed the relationship between students' perception of statistics and their level of statistics anxiety. About one-fifth of the students in our sample perceived statistics as a threat, and they had the highest scores in all dimensions of statistics anxiety. Students who perceive statistics as a threat are likely to be more anxious about statistics because of their negative attitudes towards the subject, which may reinforce their anxiety. Also, individuals with higher statistics anxiety may be more likely to interpret their work in a statistics course as a threatening experience. Since anxious students need more structure in the classroom, it is important to provide clear and informative instructions and to ensure that the student workload and the complexity of statistical tasks increase gradually.

Regarding the experience of statistics as a challenge, our results show that students who experienced statistics as a challenge had the lowest level of anxiety in all statistics anxiety dimensions. Students who view statistics positively and see it as a challenge are likely to be less concerned about learning statistical material and performing statistical tasks and are not afraid of possible obstacles to understanding statistics. In this way, they can more easily meet the challenges of statistics. Again, the results are consistent with the study by Macher et al. (2012), in which the authors found that students who are more interested in statistical content are less anxious about statistics. The results also agree with Najmi et al. (2018), who showed that a positive attitude towards statistics can mitigate the negative effects of statistics anxiety on students' academic performance.

Our study confirmed that all dimensions of statistics anxiety are related to anxiety as a personality trait. The correlations were mostly low but statistically significant. A significant correlation between anxiety as a personality trait and statistics anxiety was also reported by Macher et al. (2012) and Walsh and Ugumba-Agwunobi (2002). Test and Class Anxiety and Interpretation Anxiety appear in situations such as passing exams in statistics, accepting or rejecting null hypotheses and interpreting statistics results (Cruise et al., 1985). In these situations, a highly anxious individual may perceive the possibility of making a mistake as a threat. Eysenck (1997) found that highly anxious individuals focus on potential threats while processing information from the environment, especially in situations that they perceive as ambiguous. Exams or lectures in statistics are examples of such situations, since there is always the possibility that the individual does not understand learning content or does not solve the tasks correctly.

The relationship between anxiety as a personality trait and the dimensions Fear of Asking for Help and Fear of Statistics Teachers can be explained by an individual's concern about possible social ignorance, which can be a stressful situation for anxious individuals. A similar explanation was provided by Onwuegbuzie and Daley (1999), when they explained the relationship between socially prescribed perfectionism and the fear of asking for help. Socially prescribed perfectionism refers to an individual's concern that society might perceive his or her efforts or work as inadequate or inferior. Such individuals are often very anxious while performing various tasks. Their performance is not motivated by the desire to succeed or be the best, but by the fear of failure or the desire to avoid shame, fear and guilt (Klibert, et al., 2005).

Furthermore, anxiety as a personality trait is related to students' self-concept about computing skills. From this, it could be concluded that people who are particularly concerned about their self-image are also more likely to be anxious when solving statistical tasks with mathematical operations.

Finally, the correlation between anxiety as a personality trait and the dimension Worth of Statistics was lowest but still statistically significant. Students who consistently show higher rates of anxiety tend to perceive statistics as an unimportant and useless course and perceive it as a waste of time. Enjoyment in mathematics was related only to the Worth of Statistics and the Computation Self-Concept. Based on these results, we can conclude that students who are intrinsically motivated to learn mathematics are more likely to value statistics and feel more competent in dealing with statistical challenges.

The students' final grade in mathematics correlated positively with the Test and Class Anxiety. The correlation was low but statistically significant. The result may reflect our sample unbalanced by gender. Females are generally more anxious about school, education, and passing exams, although they have a similar or even higher academic achievement than males. For example, Puklek Levpušček (2014) showed that 15-year-old girls were just as successful as boys in mathematics in the PISA 2012 study, but they were more concerned about poor grades in the subject.

Nonetheless, better high school mathematics performers reported a better self-image in computational skills. The students' average study grade related to the students' computation self-concept, while there was no relationship between students' academic success and other dimensions of statistics anxiety. It appears that academically more successful students do not necessarily experience lower statistics anxiety. Further studies should also include student grades in statistics courses, which could better illustrate the correlation between statistics anxiety and performance.

Conclusions

Our study showed an average to below-average levels of statistics anxiety among students of social sciences at the University of Ljubljana. It confirmed that statistics anxiety is related to anxiety as a personality trait and to the students' perceptions of mathematics and statistics. It also showed that previous achievements and motivation in mathematics play a role in experiencing statistics anxiety at university. Attitudes towards mathematics in primary and secondary school are extremely important, since a positive attitude towards mathematics can contribute to successful and effective problem solving (Marchis, 2011). At the entry point to upper secondary school, students generally have a positive attitude towards mathematics, but over the years, this attitude can change in a negative direction due to the difficulty of learning material in mathematics (Ma & Kishor, 1997). Therefore, it is important to motivate and help students throughout the educational process to develop a more positive learning self-concept and attitude towards mathematics and statistics. Also, it might be a good idea that teachers showed students all the benefits of studying and understanding statistics and used authentic research projects that link statistics knowledge to its application in the real world.

Interventions for students who experience higher levels of statistics anxiety should focus on their negative beliefs about their mathematical and computational skills that might undermine their motivation and willingness to learn statistics. Teachers of statistics should identify areas where students have concerns about the subject (e.g., low self-worth in computation, understanding the logic of statistical concepts, worries about the interpretation of statistical results, fear of exams) and identify possible misconceptions about the study of statistics. They should also think about how to reduce the distance to students and how to balance effective work with a warm classroom atmosphere. Experimental studies are also needed to determine which teaching methods could reduce statistics anxiety and lead to a better statistical knowledge of students. Also, future studies should examine other factors that might contribute to statistics anxiety, such as perfectionism, attitudes towards science, mathematics anxiety, and previous experience and achievement in statistics courses.

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The Mediating Role of Parents and School in Peer Aggression Problems

Tena Velki[1]

Starting from the ecological framework, the present study aimed to examine the mediating effects of parental supervision and school climate on the relationship between exosystem variables (time spent with media and perceived neighbourhood dangerousness) and peer aggression problems (peer aggression and victimisation). The participants were 880 primary school students. The data were analysed with multiple regression. The results show that both mediators (parental supervision and school climate) have statistically significant partial mediating effects on peer aggression and victimisation. If students experienced more parental supervision, there was a decrease in the relationship between a) time spent with media and peer aggression, and b) perceived neighbourhood dangerousness and peer aggression and victimisation. Identical findings were obtained for positive school climate. Thus, positive school climate and parental supervision served as protective factors against the negative influence of dangerous neighbourhoods and excessive use of media on peer aggression problems.

Keywords: parental supervision, school climate, peer aggression, peer victimisation, media, neighbourhood

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Posredniška vloga staršev in šole pri vrstniškem nasilju

Tena Velki

Izhajajoč iz ekološkega ogrodja, se je ta raziskava usmerila na preučevanje posredniških učinkov starševskega nadzora in šolske klime na razmerje med eksosistemskimi spremenljivkami (čas, namenjen medijem, in zaznava nevarnosti v bližnji okolici) in težavami, povezanimi z vrstniškim nasiljem (vrstniško nasilje in viktimizacija). Raziskava je bila izvedena na vzorcu 880 osnovnošolcev, pri čemer so bili podatki analizirani z multiplo regresijo. Rezultati kažejo, da imajo posredniki in starševski nadzor ter šolska klima statistično pomemben delni posredniški učinek na vrstniško nasilje in viktimizacijo. Če so bili učenci deležni več starševskega nadzora, se je zgodil upad v razmerju med a) časom, namenjenim medijem, in vrstniško agresijo; b) zaznavo nevarnosti v bližnji okolici ter vrstniškim nasiljem in viktimizacijo. Enake ugotovitve smo pridobili glede pozitivne šolske klime. Pozitivna šolska klima in starševski nadzor sta torej služila kot zaščitna dejavnika pred negativnim vplivom nevarnosti v bližnji okolici in pretirano uporabo medijev ob težavah, povezanih z vrstniškim nasiljem.

Ključne besede: starševski nadzor, šolska klima, vrstniško nasilje, vrstniška viktimizacija, mediji, bližnja okolica

Introduction

A significant amount of research deals with the issue of peer violence, but only a limited number addresses the complexity of this phenomenon and applies Bronfenbrenner's ecological approach for empirical testing (Ferrer et al., 2011; Khoury-Kassabri et al., 2004; Kim et al., 2011; Swearer et al., 2006; Velki, 2018; You et al., 2014; Yuhong, 2012). Starting from the definition of peer aggression as all behaviours intended to physically or psychologically hurt or harm another human being (Berkowitz, 1993; Hawley & Vaughn, 2003), the main goal of the aforementioned studies was to explore predictors of peer aggression or peer victimisation.

Influence of the microsystem on peer violence

According to Bronfenbrenner (1986), a microsystem is the system closest to the child with a direct influence on his/her behaviour. Family and school have been the most frequently investigated microsystems related to peer aggression problems.

Lack of parental supervision or parental monitoring has been confirmed not only as a good predictor of peer aggressive behaviour (Kim et al., 2011; Velki, 2018; Velki & Kuterovac Jagodić, 2015; You et al., 2014) but also good at predicting peer victimisation (Lereya et al., 2013). Nonetheless, a meta-analysis study found the protective effects of positive parenting, including parental supervision, to be generally small to moderate for victims of peer aggression (Lereya et al., 2013). In families in which parental control is poor, children are left alone, and there is no control over their activities or correction of problematic behaviour; consequently, they become more easily involved in peer aggression problems as victims or perpetrators.

Research has consistently shown that school climate is a good predictor of peer aggression (Barboza et al., 2009; Harel-Fisch et al., 2010; Lee, 2011; Petrie, 2014; Swearer et al., 2006) and victimisation at schools (Ferrer et al., 2001; Harel-Fisch et al., 2010; Khoury-Kasssabri et al., 2004). Low teacher support predicts both peer aggression and peer victimisation (You et al., 2014). In particular, deficits in the emotional aspect of the school climate are related to children's increasingly aggressive behaviour (Kasen et al., 2004). A child's negative emotional relationship with the teacher increases the likelihood of aggressive behaviour, especially among primary school children (Hanish et al., 2004). If teachers fail to provide social support to children and fail to participate in their life within the school, an increase in violent behaviour is found (Demaray & Malecki, 2003; Kasen et al., 2004).

Distal influence on peer violence

Although Bronfenbrenner's ecological theory (1986) hypothesises that variables from the distal system (i.e., the exosystem) exert only indirect influence on child behaviour, many researchers have investigated their direct influence; most of them were based on media and community characteristics (Barboza et al., 2009; Bowes et al., 2009; Gentile & Walsh, 2002; Kim et al., 2011; Kuntsche, 2004). For over half a century, media and community have constantly been proven to be important factors in the development of peer aggression (e.g., Aneshensel & Sucoff, 1996; Bandura, 1973; Huesmann & Eron, 1986); accordingly, their indirect influence should be empirically confirmed.

Excessive use of media

The negative effects of excessive use of media are well documented. Generally, exposure to violence through television has been confirmed as a risk factor in the development of violence among children (Barboza et al., 2009; Kim et al., 2011; Kuntsche, 2004; Zimmerman et al., 2005). Moreover, playing violent computer games increases the likelihood of students' engagement in physical confrontations with peers and arguments with their teachers (Barboza et al., 2009; Gentile & Walsh, 2002). Parental restrictions on playing violent games have a positive effect in reducing aggressive behaviour in children (Gentile et al., 2004). Spending large amounts of time browsing the Internet is a risk factor in the development of violent behaviour among children and also for victimisation (Dooley et al., 2009). Parental supervision of internet activities in adolescents resulted in less engagement in chat, social networking, video streaming, and multiplayer online games (Vaala & Bleakley, 2015).

Only a few studies have attempted to empirically investigate the indirect influence of excessive use of media as posited in ecological theory. The mediating role of parental monitoring results in lower exposure to violent media that, consequently, reduces aggressive behaviour in children (Gentile et al., 2014). Parental presence (i.e., their passive supervision) has moderating effects on the relationship between the time spent on three types of media use and peer aggression; negative relationships between the time spent watching TV, playing computer games and browsing the Internet and peer aggression decreased in the presence of parents (Velki & Kuterovac Jagodić, 2017). Moreover, a positive school climate had a mediating role in the association between watching TV and peer aggression, but not peer victimisation. In the case of students who often watch TV, a positive school climate significantly reduced aggressive behaviour (Barboza et al., 2009).

Neighbourhood dangerousness

Several studies conducted in the USA have shown that exposure to violence in the community is a risk factor in the development of peer aggression and victimisation at schools (Bowes et al., 2009; Bradshaw et al., 2009; Kim et al., 2011; Lambert et al., 2005; Tolan et al., 2003). Even low levels of community violence increase the likelihood of peer aggression problems at schools (Bradshaw et al., 2009). The most significant characteristics of neighbourhoods that adversely affect the development of aggression in children are exposure to violence and high crime rates (Leventhal & Brooks-Gunn, 2000; Trentacosta et al., 2009).

Alongside the direct influence of neighbourhood characteristics on peer aggression, neighbourhood characteristics have an indirect role through a moderating effect of parental supervision. For example, the association between neighbourhood safety and violence in children and adolescents is often indirect, moderated by parental control (Pettit et al., 1999). Furthermore, parental monitoring decreases the effects of exposure to community violence on peer aggression and victimisation by reducing adolescent involvement in deviant behaviours (Low & Espelage, 2014). School climate also has a moderating role: if the school climate is positive, the association between neighbourhood dangerousness and peer aggression (Velki, 2012) and externalising behaviours in adolescents (Gaias et al., 2019) decreases.

As reported above, most previous studies investigate only the direct influence of family and school on peer aggression problems (Bowes et al., 2009; Khoury-Kasssabri et al., 2004; Kim et al., 2011; Swearer et al., 2006; You et al., 2014). Although Bronfenbrenner's ecological model (1968) posits mediational influence of closer systems (i.e., microsystems) on the association between the distal system (i.e., exosystem) and the child's behaviour, only a few studies investigate interaction effects of these systems empirically (Gaias et al., 2019; Low & Espelage, 2014; Pettit et al., 1999; Velki, 2012; 2018; Velki & Kuterovac Jagodić, 2017), and the number of studies that examine mediation effects is even smaller (Barboza et al., 2009; Gentile et al., 2014).

The Current Study

Starting from Bronfenbrenner's ecological theory of peer aggression (Swearer & Doll, 2001; Swearer & Espelage, 2004), which states that exosystem variables influence students' behaviour only indirectly, that is, through microsystem variables, the goal of the current study was to examine mediation effects of two most frequently investigated microsystems (parents and school) on the relationship between two exosystem variables (media and neighbourhood) and peer aggression and victimisation.

Parental supervision and positive school climate are consistently shown to have a protective role against children's aggressive behaviour and to reduce levels of unwanted behaviour (Hanish et al., 2004; Kasen et al., 2004; Vaala & Bleakley, 2015; Velki, 2018). Some previous studies examined moderation effects (Velki & Kuterova Jagodić, 2017) and mediation effects (Gentile et al., 2014) of parental supervision on the relationship between media use and peer aggression but not peer victimisation. Furthermore, moderation effects (Low & Espelage, 2014; Pettit et al., 1999) of parental supervision on associations between neighbourhood violence and peer aggression and victimisation were tested, but the examination of mediation effects was missing. Only a few studies tested for moderation (but not mediation) effect of school climate on the association between neighbourhood dangerousness and peer aggression (Gaias et al., 2019; Velki, 2012), and none of them addressed peer victimisation. Also, mediating effect of school climate was found only in the association between watching TV and peer aggression (Barboza et al., 2009), but not in connection to peer victimisation or excessive use of other media.

In line with the findings reviewed above, it is hypothesised that:

1. parental supervision has a mediating effect on the relations between two variables from the exosystem (media and neighbourhood) and peer aggression and victimisation
    a. parental supervision reduces the negative impact of media use, that is, parental supervision makes the association between time spent with media and a) peer aggression and b) peer victimisation weaker
    b. parental supervision reduces the negative impact of neighbourhood

dangerousness, that is, parental supervision makes the association between neighbourhood dangerousness and a) peer aggression and b) peer victimisation weaker

2. school climate has a mediating effect on relations between two variables from the exosystem (media and neighbourhood) and peer aggression and victimisation
   a. a positive school climate reduces the negative impact of media use, that is, a positive school climate makes the association between time spent with media and a) peer aggression and b) peer victimisation weaker
   b. a positive school climate reduces the negative impact of neighbourhood dangerousness, that is, a positive school climate makes the association between neighbourhood dangerousness and a) peer aggression and b) peer victimisation weaker.

Method

Participants

Students from Grades 5 to 8 from six primary schools in the eastern part of Croatia participated in this study with a total number of 880 participants (52% girls). The students' average age was M = 12.8 (SD = 1.15) years, ranging from 10 to 15 years old.

Instruments

Demographic data. A general form was used to gather data about study participants, including basic demographic information (e.g., age, gender and grade level).

Peer Aggression among School Children Questionnaire (UNŠD; Velki, 2012). This self-assessment instrument measures the level of peer aggression and victimisation; it consists of two scales (k = 38). The Scale of Peer Aggression among children measures the frequency of aggression against peers at school, and the Scale of Peer Victimisation measures the frequency of aggression experienced at school. The Scale of Peer Aggression among children consists of the Subscale of Aggression among Children in Schools (13 items further divided into the Verbal Aggression subscale (k = 6; e.g., *I spread gossip about someone*) and Physical Aggression subscale (k = 7; e.g., *I hit or push someone*)) and the Subscale of Electronic

Aggression (k = 6; e.g., *I insult others through social networks, like Facebook, Twitter, etc.*). The Scale of Victimisation among children consists of the Subscale of Victimisation among children in schools (13 items further divided into the Verbal Victimisation subscale (k = 6) and the Physical Victimisation subscale (k = 7)) and the Subscale of Electronic Victimisation (k = 6). For this study, only the results on the Subscale of Aggression among Children in Schools and the Subscale of Peer Victimisation among Children in Schools were used. The participants were invited to respond to the items by indicating frequency for each act of aggression committed and experienced. A five-point Likert scale was used with '1' meaning 'never', '2' 'rare (a few times per year)', 3 'sometimes (once a month)', 4 'frequently (several times per month)', and 5 'always (nearly every day)'. The result on each subscale was computed as the arithmetic mean of responses to the corresponding items and theoretically ranges from 1 to 5. The internal consistency was high for both the Subscale of Aggression among Children in Schools (α = .82) and for the Subscale of Victimisation among Children in Schools (α = .85).

Parental Behaviour Questionnaire (URP29; Keresteš, et al., 2012). The Parental Behaviour Questionnaire examines the most common behaviour towards the child. There are three versions of the questionnaire: the mother's, the father's and the child's version. Only the child's version of the questionnaire was used. It includes two identical questionnaires: one pertaining to the mother's and the other to the father's behaviour. Each questionnaire consists of 29 items. Participants specify their level of agreement with a described parental behaviour on a 4-point Likert scale where '1' stands for 'not true at all', '2' for 'not very true', '3' for 'quite true', and '4' for 'entirely true'. The result for each subscale was computed as an arithmetic mean of the responses to the corresponding items and theoretically ranges from 1 to 4. The questionnaire has a total of seven subscales: Warmth (k = 4), Autonomy (k = 4), Intrusiveness (k = 4), Supervision (k = 4), Permissiveness (k = 3), Inductive Reasoning (k = 5) and Punishment (k = 5). For the purpose of the present study, only the responses obtained on the Supervision subscale (the item example: *He/She knows my friends well)* were used, and the internal consistency was high (α = .84).

Croatian School Climate Survey for students (HUŠK-U, version for students; Velki, et al., 2014). Croatian School Climate Survey for students measures the overall climate of the school. More specifically, it addresses the sense of safety and belonging to the school, the relationship between teachers and students, learning atmosphere, parental involvement in school and predicting the future based on education. HUŠK-U consists of 15 items (e.g., *I enjoy learning in my school*). It is a self-assessment scale of agreement employing a five-point Likert scale (ranging from 'strongly agree' to 'strongly disagree') for the specification of participants' level of agreement with the statements. The total score was computed as an arithmetic mean of responses to all items and theoretically ranges from 1 to 5. Larger values indicate a more positive school climate. The internal consistency of the total scale was high (α = .92).

Exposure to the Media Scale (UM; Velki, 2012). This self-report scale consists of three items related to the amount of time children spend using media (watching TV daily, playing computer games and browsing the Internet weekly). Participants specify the frequency of use of each type of media on a five-point Likert scale (ranging from 'never' to 'more than 10 hours of watching television per day' or 'more than 10 hours of browsing the Internet or playing games per week'). The total score is obtained as an arithmetic mean of answers to all the items and theoretically ranges from 1 to 5. The internal consistency was satisfactory (α = .66).

Scale of Perception of Neighbourhood Dangerousness (POS; Velki, 2012). This self-report scale consists of six items that measure different types of dangerous situations to which children can be potentially exposed in their neighbourhood (e.g., *There are drugs in my neighbourhood*). On a five-point Likert scale (ranging from 'strongly disagree' to 'strongly agree'), the participants indicated their agreement with the statements on the scale. The total score was computed as an arithmetic mean of responses to all items and theoretically ranges from 1 to 5. The internal consistency of the scale was high (α = .81).

Procedure

Parents were asked to give their written consent for the child's participation in the study at PTA meetings. The main researcher explained the purpose and procedure of the study. Data were collected from students during their regular school hours. All students were informed that they could withdraw at any time. Also, it was made clear that all information collected for the study would be treated confidentially. It took the participants about 45 minutes to complete the questionnaires.

Results

All calculations relating to variables were based on arithmetic means of the previously described items on the questionnaires and scales. Criteria for the use of parametric statistics and assumptions for regression analysis were met.

Table 1

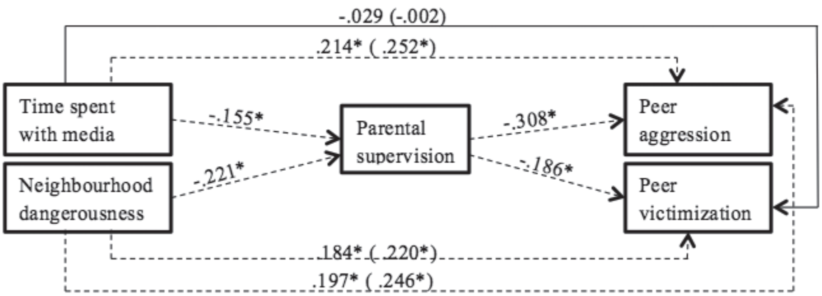
Descriptive statistics

Variable	N	Min	Max	M	SD	Sk	Ku
Peer victimisation	880	1.00	4.54	1.80	.560	1.51	1.62
Peer aggression	880	1.00	4.08	1.40	.381	2.07	3.83
Parental supervision	880	1.00	4.00	3.17	.643	-.73	.10
School climate	880	1.00	5.00	3.67	.782	.52	-.10
Time spent with media	880	1.00	5.00	2.83	.869	.49	-.54
Perceived neighbourhood dangerousness	880	1.00	5.00	1.82	.752	-1.26	1.71

Note. N = number of participants, Min = minimal score, Max = maximal score, M = mean, SD = standard deviation, Sk = skewness, Ku = kurtosis.

A series of regression analyses (Figure 1) was run. First, whether time spent with media and neighbourhood dangerousness (independent variables) significantly affected parental supervision (the mediator) was verified ($F_{(2,878)} = 38.33$, $p < .001$, $R^2 = .082$). Next, whether the independent variables (time spent with media and neighbourhood dangerousness) significantly affect the dependent variables when parental supervision (mediator) is removed, that is, peer aggression ($F_{(2,878)} = 70.83$, $p < .001$, $R^2 = .141$) and peer victimisation was examined ($F_{(2,878)} = 21.82$, $p < .001$, $R^2 = .048$). Finally, whether the effect of the independent variables (time spent with media and neighbourhood dangerousness) on the dependent variables (peer aggression and peer victimisation) changed upon the addition of parental supervision to the model was examined (effect on peer aggression $F_{(3,877)} = 47.21$, p < .001, $\Delta R^2 = .044$; effect on peer victimisation $F_{(3,877)} = 20.17$, $p < .001$, $\Delta R^2 = .022$).

Figure 1
Mediation model: parental supervision as a mediator variable in the relationship between two exosystem variables (time spent with media and perceived neighbourhood dangerousness) and peer aggression and victimisation.



**p< .001.*

Parental supervision had a partially mediating role that resulted in a weakening of the association between peer aggression and a) time spent with media ($\beta_1 = .252$, $p < .001$; $\beta_2 = .214$, $p < .001$) and b) neighbourhood dangerousness ($\beta_1 = .246$, $p < .001$; $\beta_2 = .197$, $p < .001$). Likewise, parental supervision had a partially mediating role that made the association between neighbourhood dangerousness and peer victimisation weaker ($\beta_1 = .220$, $p < .001$; $\beta_2 = .184$, $p < .001$). No statistically significant association between time spent with media and peer victimisation was found; hence, a mediating effect test was not performed. The mediating role of parental supervision was statistically significant for all significant associations (Sobel and Goodman tests, Table 2) and, according to Preacher and Kelley (2011), the effect size of parental supervision was medium.

Table 2
Findings on the mediating role of parental supervision and its effect size

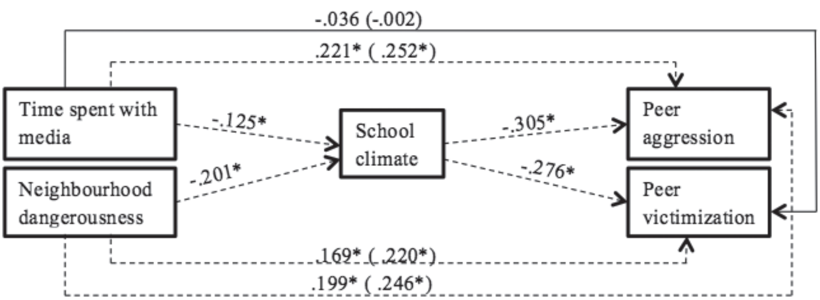
Associations	*Sobel test*	*SE*	*Goodman test*	*SE*	κ^2
peer aggression					
time spent with media	3.92*	.004	3.95*	.004	.035
perceived neighbourhood dangerousness	4.81*	.005	4.83*	.005	.049
peer victimisation					
time spent using media	-	-	-	-	-
perceived neighbourhood dangerousness	3.71*	.007	3.73*	.007	.034

Note. κ^2 = effect size, SE = standard error. *p < .001

The independent variables (time spent with media and neighbourhood dangerousness) significantly affected not only the school climate ($F_{(2,878)} = 28.83$, $p < .001$, $R^2 = .063$) but also the dependent variables, peer aggression ($F_{(2,878)} = 70.83$, $p < .001$, $R^2 = .141$) and peer victimisation ($F_{(2,878)} = 21.82$, $p < .001$, $R^2 = .048$) when the school climate (mediator) was removed from the model. The effect of the independent variables on the dependent variables changed upon the addition of school climate (mediator) to the model (Figure 2), both on peer aggression ($F_{(3,877)} = 52.87$, $p < .001$, $\Delta R^2 = .050$) and peer victimisation ($F_{(3,877)} = 54.94$, $p < .001$, $\Delta R^2 = .057$).

Figure 2

Mediation model: school climate as a mediator variable in the relationship between two exosystem variables (time spent with media and neighbourhood dangerousness) and peer aggression and victimisation.



**p < .001.*

School climate had a partially mediating role that resulted in a weakening of the association between peer aggression and a) time spent with media ($\beta_1 = .252$, $p < .001$; $\beta_2 = .221$, $p < .001$), and b) neighbourhood dangerousness ($\beta_1 = .246$, $p < .001$; $\beta_2 = .199$, $p < .001$). Likewise, school climate had a partially mediating role that made the association between neighbourhood dangerousness and peer victimisation weaker ($\beta_1 = .220$, $p < .001$; $\beta_2 = .169$, $p < .001$). No statistically significant association between time spent with media and peer victimisation was found. The mediating role of school climate was statistically significant for all significant associations and had a medium effect size (Table 3).

Table 3

Findings on the mediating role of school climate and its effect size

Associations	*Sobel test*	*SE*	*Goodman test*	*SE*	κ^2
peer aggression					
time spent with media	3.36*	.004	3.39*	.004	.030
neighbourhood dangerousness	4.66*	.005	4.69*	.005	.047
peer victimisation					
time spent with media	-	-	-	-	-
neighbourhood dangerousness	4.63*	.008	4.66*	.008	.049

Note. κ^2 = effect size, SE = standard error. *p < .001

Discussion

In Bronfenbrenner's ecological theory of peer aggression (Swearer & Doll, 2001; Swearer & Espelage, 2004), distal systems (e.g., exosystem and macrosystem) have an indirect influence on children's behaviour through a closer system (e.g., microsystem). Children's reactions to certain situations are surely dependent not only on what they learn from media or see in their community but also on how the related information and knowledge is processed within their close environments (e.g., family and school). Depending on the feedback received from parents and teachers, everyday events related to media and neighbourhood could be understood and interpreted differently and lead to choices of actions in accordance with these interpretations. It is the children's parents and teachers who emerge as mediators when attempting to explain the indirect influence of community and media on their behaviour.

In line with the first hypothesis, the mediation effects of parental supervision were examined. Regarding the distal influence of media, parental supervision had a partially mediating effect with a medium effect size only on the association between time spent with media and peer aggression, not peer victimisation. The findings are consistent with previous research in which more exposure to violent media was reported to predict peer aggression, but not victimisation (Barboza et al., 2009; Gentile & Walsh, 2002). Continuous exposure to aggressive content in the media could change an individual's attitudes towards aggression and teach aggressive behaviour. Children do not comprehend the negative consequences of violent behaviour from exposure to violence in the media. Instead, from the media, they receive the message that a goal can easily be achieved with aggressive behaviour (even heroes, 'good guys', use extremely violent strategies to achieve positive goals), and, later, transfer such experiences to school situations. A decrease in aggressive behaviour is accomplished by limiting time spent on violent media content and by monitoring and explaining negative effects of media violence (Gentile et al., 2004; Vaala & Bleakley, 2015). This finding is supported by the present study, which points at a significant mediation effect of parental supervision.

Regarding the distal influence of the neighbourhood, a partially mediating role of parental supervision was obtained for associations between neighbourhood dangerousness and both dependent variables (peer aggression and victimisation), again with a medium effect size. Parental supervision buffers the association between neighbourhood dangerousness and peer aggression and victimisation. In line with previous studies, living in dangerous neighbourhoods characterised by high levels of aggression and criminality is a strong predictor of peer aggression and victimisation (Bradshaw et al., 2009; Bowes et al., 2009; Lambert et al., 2005; Tolan et al., 2003). In dangerous communities, children are likely to observe conflicts on a daily basis and learn that violence is a suitable way to deal with problems. Although school is a relatively safe place, according to Bandura's theory of social learning, children transfer these aggressive behaviours to school situations and react excessively violently at the slightest sign of potential danger. Unfortunately, some children become victims of violence in their community and transfer submissive behaviour to school contexts, which makes them easy victims. In these dangerous neighbourhoods, parental supervision can have a major role (Low & Espelage, 2014; Pettit et al., 1999). Children without parental supervision are more likely to engage in violent activities because they might believe that their behaviour will go unnoticed and, therefore, without negative consequences. Also, they seem to be more inclined to engage in other risky activities (e.g., alcohol abuse), probably because of lack of control. Moreover, when parents have no influence on the choice of people their children socialise with, these violent children tend to choose violent and delinquent peer groups, which further supports and encourages aggressive behaviour toward peers (Orpinas & Horne, 2006).

Other than the family environment, the school setting is the place where children spend most of their time. Regarding peer violence, school climate has a similar operating mechanism as the family. The school climate is a reflection of social interactions in the classroom, school canteen, hallways, and elsewhere (Tubbs & Garner, 2008). A positive school climate supports for a child's sense of safety and belonging to the school as well as a good relationship with peers and teachers, and it represents a protective factor against children's unwanted behaviour, especially in cases of children coming from problematic families (Nader, 2008). In line with the second hypothesis, mediation effects of school climate on the association between two variables from the exosystem (media and neighbourhood) and peer aggression and victimisation were examined. The results were identical to those obtained for the mediation effect of parental supervision.

Regarding the distal influence of media, school climate had a partially mediating effect only in association with peer aggression, not peer victimisation. Regarding the distal influence of neighbourhood dangerousness, school climate had a partially mediating effect on associations with peer aggression and peer victimisation. The effect size of school climate was medium. Previous studies have shown that positive school climate can reduce negative impact from violent TV content (Barboza et al., 2009), but results from this study show that school climate can serve as a buffer that mediates the negative impact of excessive use of media on peer aggression. In school environments with a prevailingly positive school climate, children learn about values and ethics, develop empathy and prosocial behaviour, all of which helps in prevention of peer aggression. As exposure to violent contents in the media had no influence on victimisation, no significant mediation of the school climate was expected.

Next, a positive school climate has been reported to have a significant role in dangerous neighbourhoods, decreasing its association with peer aggression (Gaias et al., 2019; Velki, 2012). Moreover, the findings of the current study show that positive school climate has a mediating effect on associations between neighbourhood dangerousness and peer aggression, *and* neighbourhood dangerousness and peer victimisation. When children transfer violent behaviour experienced in neighbourhoods to school situations, it is important to show them that school is a safe place where all children are accepted and welcomed. In schools with a positive school climate, children can relearn that behaving aggressively in order to survive is unnecessary: other words, that aggression is an unacceptable response to problems. Likewise, children who are victimised can ask for help from teachers and educational practitioners in schools where they experience the feeling of emotional and social support (Demaray & Malecki, 2003; Kasen et al., 2004).

There are certain shortcomings to the present study that need to be mentioned. The selection of schools that contributed to the sample was random; however, they were all from one county. The fact that only primary school students participated in the study (Grades 5 to 8) limits attempts to generalise across student populations. In comparison to data from the national sample (Rajhvan-Bulut & Ajduković, 2012), a slightly higher prevalence rate of peer aggression was found, probably a consequence of the sample profile that consisted of participants from the county most affected by the Croatian War of Independence. Further, the data were collected from students only and, at least for some variables, it would be interesting to see if other significant persons in the participants' lives share their opinions on issues at hand. For example, in connection with parental supervision, it could be important to examine how parents self-evaluate their supervision. As for school climate, it could be revealing to investigate teachers' perceptions of it. With reference to peer aggression and victimisation, contributions in the form of peer evaluations would be of extreme importance. Finally, the study was cross-sectional in design.

Conclusion

Once again, family and school-related variables, acting through both direct and mediation mechanisms, emerged as significant protective factors in the prevention and reduction of peer aggression problems, which is of key importance for educational practitioners. As the primary formal educational institution, the school can cater for a wide range of prevention and intervention programmes that take into account media content children are exposed to on a daily basis and features of the community in which they live. Some previous research (Tušak, 2001) confirmed that educational practitioners should devote more time to developing children, not only educating them, in order to prevent peer aggression. For example, watching some educational movies together, talking about superheroes and their aggressive action in movies and cartoons, discussing all possible negative consequences of such behaviour could help students to understand violence better and to learn how to act in a nonviolent way.

Furthermore, educational practitioners should consider the active involvement of parents in these prevention programmes on account of parental supervision's emergence as a buffer that moderates the negative impact of environments, both physical and virtual. Sometimes educational practitioners are not aware of what kind of community the children live in, and parents can give them additional information (Dusi, 2012). During PTA meetings, teachers and parents can discuss potential dangers in their neighbourhood and a positive solution that both of them can offer to a child. Finally, it is of crucial importance that children from troubled families feel accepted, safe, and welcome in their school, that is, in the setting, which can help them to resolve their problems and develop value systems, code of ethics, and rules of behaviour when their parents fail to do so.

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Experiences of Slovenian In-Service Primary School Teachers and Students of Grades 4 and 5 with Outdoor Lessons in the Subject Science and Technology

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The present paper presents the results of a survey on outdoor lessons conducted by teachers of the subject Science and Technology in the 4th and 5th grades of primary school in the school's vicinity. It examines differences between teachers themselves and between teachers and students, as well as the ideas and limitations of outdoor lessons. The study included 70 in-service primary school teachers of the 4th and 5th grades and 154 students of the 4th grade and 151 students of the 5th grade of primary school. The data were obtained with two questionnaires: an e-questionnaire for teachers and a paper-pencil questionnaire for students. The results show that 13 per cent of teaching time in the subject Science and Technology consists of outdoor lessons. Statistically significant differences were found between teachers with different amounts of teaching experience, while differences in the quantity of outdoor lessons did not arise among teachers of different school strata and among teachers who had an early experience with outdoor lessons in the vicinity of school themselves as students compared to teachers who had no such experience. The teachers had several specific and general ideas for outdoor activities for the thematic sets of the Science and Technology curriculum and reported similar difficulties in planning outdoor lessons to those reported in other countries. The results of the research show that the teachers report the use of outdoor lessons in the vicinity of school more often than recalled by the students. The students reported that such activities typically take place about twice a year, mostly in playgrounds, meadows, and forests. The results provide an insight into the state of the teachers' initiatives for outdoor lessons in the subject Science and Technology and indirectly offer opportunities to reflect and act on outdoor lessons from different perspectives.

Keywords: outdoor lesson, primary school, the subject Science and Technology, teachers' experiences, students' experiences

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Izkušnje slovenskih učiteljev in učencev 4. in 5. razreda osnovne šole s poukom na prostem pri predmetu naravoslovje in tehnika

Maruša Novljan in Jerneja Pavlin

Ta prispevek predstavlja izsledke raziskave o pouku na prostem pri predmetu naravoslovje in tehnika v 4. in 5. razredu osnovne šole v okolici šole. Preučuje razlike med učitelji ter med učitelji in učenci pa tudi ideje in omejitve pouka na prostem. V raziskavo je bilo vključenih 70 učiteljev 4. in 5. razreda ter 154 učencev 4. razreda in 151 učencev 5. razreda osnovne šole. Podatki so bili pridobljeni z dvema vprašalnikoma: e-vprašalnikom za učitelje in z vprašalnikom tipa papir – svinčnik za učence. Učitelji navajajo, da 13 odstotkov časa pri predmetu naravoslovje in tehnika predstavlja pouk na prostem. Statistično pomembne razlike so bile ugotovljene med učitelji z različno količino izkušenj s poučevanjem, medtem ko se razlike v količini pouka na prostem niso odrazile med učitelji glede na stratum šole in učitelji, ki so imeli kot učenci zgodnje izkušnje s poukom na prostem v bližini šole, v primerjavi z učitelji, ki niso imeli takšnih izkušenj. Učitelji so imeli več specifičnih in splošnih idej za dejavnosti na prostem za tematske sklope iz učnega načrta za predmet naravoslovje in tehnika in so poročali o podobnih težavah pri načrtovanju pouka na prostem, kot so zaznane v tujih virih. Rezultati raziskave kažejo, da učitelji pogosteje poročajo o uporabi pouka na prostem v bližini šole, kot so ga učenci zaznavajo. Učenci so poročali, da se takšne dejavnosti običajno odvijajo približno dvakrat letno, večinoma na igriščih, travnikih in v gozdovih. Rezultati omogočajo vpogled v stanje pouka na prostem pri predmetu naravoslovje in tehnika ter posredno ponujajo priložnosti za razmislek in ukrepanje glede pouka na prostem z različnih vidikov.

Ključne besede: pouk na prostem, osnovna šola, predmet naravoslovje in tehnika, izkušnje učiteljev, izkušnje učencev

Introduction

Today's children spend too much time sitting (playing computer games, using social networks, online learning, watching TV, etc.) (Bank & Greve, 2013; Štemberger, 2012). School should ensure that students are as active as possible in their classes. Ceciliani and Bortolotti (2013) report that students are mainly active outdoors, engaging in activities, such as walking, socialising, and similar. They move in the manner that the natural environment enables them to move, even if no toys are available. If students do not have the tools to move in a certain way, e.g., by walking, they engage with natural objects (pebbles, sand, small sticks, etc.). Furthermore, it has been demonstrated that outdoor learning positively influences children's wellbeing (health, immune system), creativity and teamwork (DfES, 2006). It is therefore important that students acquire new knowledge and skills not only in the classroom but also outdoors.

The term 'learning outdoors' indicates that learning takes place outside of buildings. Such a definition of outdoor learning is endorsed by many authors, who provide a more precise definition of the term. Outdoor learning is teaching and learning with an emphasis on multisensory experiences (Gilbertson et al., 2006), and it is a concept involving educational activities in a different environment. The English Outdoor Council (n. d.) describes outdoor learning as a physical activity related to the natural environment. Outdoor learning does not include activities such as visiting museums and art galleries or physical education (Rickinson et al., 2004). However, Tuuling et al. (2018) report that outdoor learning is most often perceived by teachers as a free activity or play outdoors rather than as learning. The authors emphasise that outdoor learning can also be a kind of journey involving trying out, smelling and touching objects in the 'home' environment.

Gilbertson et al. (2006) emphasise that the place of outdoor learning is the outside world away from the classroom. Classes are taught outdoors or in an environment that is physically different from the classroom (Peacock & Pratt, 2009). Local spaces such as farms, beehives, hunting lodges, botanical gardens, parks or the schoolyard support teachers by making it easier to organise learning outdoors in locations that are readily accessible (MacQuarrie, 2016). Teachers can organise outdoor learning for only a few minutes (motivation for class), for one school lesson (learning new content), for several school lessons, for one day (outdoor classroom day) or for several days (camp), but the duration of outdoor learning also affects the teacher's organisation: the longer the outdoor learning lasts, the more difficult it is to plan. Research carried out in Scotland (Nicol et al., 2007) reports that students study outdoors for approximately 19 minutes every week. Waite (2011) found that the majority (80%)

of students (6–11 years) perceive that they are physically active outdoors every week or at intervals of several weeks. In organising outdoor learning, teachers can also consider cross-curricular integration of the mother tongue, mathematics, the physical environment, and similar., as Tuuling et al. (2018) found in their research. Šebjanič and Skribe Dimec (2019) point out that outdoor learning makes children curious and does not bore them.

Furthermore, students should be equipped with skills that can be transferred to later life. The effective organisation of outdoor learning is possible if the teacher likes to be outdoors, is creative and interacts with different people (foresters, beekeepers, farmers, etc.). However, Skribe Dimec and Kokalj (2018) suggest involving students in planning activities for outdoor programmes, making them more motivated, creative, and focused.

Outdoor lessons have both positive and negative sides. The positive sides of learning outdoors are reflected in various aspects of the child's development. Physical activity outdoors improves children's manual skills, coordination, balance, and physical activity. Students become more relaxed when learning or playing spontaneously, which leads to better attention, more motivation and faster perception (promotion of a higher level of knowledge), thus improving student success and performance (Fiskum & Jacobsen, 2012). Some researchers (Gill, 2014; Mygind, 2009; Rickinson et al., 2004; Sjöblom & Svens, 2019; Waite, 2010) report that students are more relaxed when playing, resulting in group trust, connection, and participation. Children are dynamic while learning outdoors, constantly changing the environment they explore to satisfy their curiosity (Tovey, 2008). Malone (2008) also argues that outdoor learning has a positive impact on children's learning and supports healthy children's development. Also, outdoor learning is expected to influence cognitive (learning), physical (physical experience), social (social interaction), emotional (emotional wellbeing) and personal (the child's response) development.

Outdoor lessons use individual teaching methods and offer opportunities for rich interdisciplinary connections/learning in the open air in a real environment (Beames et al., 2012; Potočnik & Devetak, 2019; Štemberger, 2012). It is also possible to include ICT, including as apps such as the Woody Species Identification Digital Dichotomous Key (Laganis et al., 2017). Outdoor learning plays a key role in educating young people about our planet and thus in providing environmental education. The English Outdoor Council (n. d.) notes that such learning makes it easier for students to understand the importance of nature conservation better. Outdoor learning develops the students' understanding of the importance of sustainable development (Beames et al., 2012; English Outdoor Council, n. d.; Torkar, 2013; Torkar et al., 2020).

However, Skribe Dimec and Kokalj (2018) emphasise that Slovenian teachers, parents, and students are not sufficiently aware of the positive aspects and opportunities of outdoor learning. The negative aspects of outdoor learning are primarily economic concerns (professionalism of teachers, literature), limited time, and the number of teachers required to ensure the safety and health of children (Jeronen & Jeronen, 2012), especially with regard to weather and the overcrowding of curricula (Rickinson et al., 2004; MacQuarrie, 2016). Numerous authors have proposed strategies to avoid the negative aspects. For example, Barker et al. (2002) suggest that outdoor learning could be conducted at shorter intervals and last for a longer period. Tuuling et al. (2018) found that outdoor learning was not effective when there was noise, traffic, or other children in the playground. They pointed out the following negative factors regarding outdoor teaching: lack of time, lack of outdoor lesson space, lack of equipment for outdoor work, lack of knowledge and experience of teachers, and lack of safety (ticks, stray dogs, etc.). The outdoor groups of students in many Finnish schools are often large; therefore, some students do not feel comfortable due to differences in learning and phobias (Jeronen & Jeronen, 2012). It has been pointed out that teachers prefer traditional teaching and do not consider research showing students' reactions to outdoor learning. However, Walan and Chang Rundgren (2014) noted that curricular changes could lead to teachers becoming aware of science knowledge and the need for further education.

As mentioned above, outdoor lessons have both positive and negative aspects, but there are many projects (e.g., the day out) that help the teacher organise outdoor lessons more quickly and easily. Outdoor Classroom Day is when many teachers in different countries use a tree instead of classical boards, grass instead of chairs, and so on. The project aims to spend at least one hour outdoors with students (Outdoor Classroom Day, 2020). Fägerstam (2013) reports on a project in which teachers were questioned at the beginning of the project and after one year. It was found that the project teachers had more ideas for outdoor lesson activities and that they found learning with all of the senses more effective because the students were enthusiastic about the unusual space during outdoor learning. Policymakers designing curricula must also consider the initiatives of teachers and students in order to provide them with a more focused, playful and natural environment (Gill, 2014). The European Social Fund allows EU Member States to receive funding for such initiatives, so many updates and adaptions in outdoor education are expected (Skribe Dimec & Kokalj, 2018).

The Slovenian school system and the integration of outdoor learning

This study on outdoor lessons refers to the Slovenian school system. Students enter compulsory schooling at the age of six years. Primary school has nine grades, of which five correspond to the primary level (grades 1–5) and four to the lower secondary level (grades 6–9 in other countries). The Council of Experts for General Education in Slovenia approves the national curricula and determines the subjects and the curricula of the subjects, but the choice of teaching methods and textbooks is left to the teachers (MIZŠ, 2018; Taštanoska, 2017). Students start to learn about the world of science, technology, and society through the subject Environmental Studies (Kolar et al., 2011). In the 4th and 5th grades (students aged 9 and 10), they deepen their knowledge in the subject Science and Technology and the subject Society (Vodopivec et al., 2011).

Outdoor lessons are integrated into the national curriculum for primary and secondary schools in Slovenia. On the organisational level, they are mainly implemented through activity days (15 days) and outdoor school (Skribe Dimec, 2019; Dnevi dejavnosti, 1998). Three activity days with science activities are prescribed for grades 4 and 5, each with five school hours. Most schools conduct science activities outside the school area, but activities do not have to take place outside: an activity day can also take place in a museum, gallery, or similar. Outdoor school is also part of the compulsory curriculum. It lasts three or more days in a row and takes place away from the school area. The school must organise outdoor school at least twice in the nine years of primary school (Gros et al., 2001). In Slovenia, outdoor school usually takes place in CŠOD (Centre for School and Outdoor Education) centres, where it is executed by external contractors (Šebjanič & Skribe Dimec, 2019).

As mentioned above, the subject Science and Technology is taught in grades 4 and 5, with a total of 105 school hours per school year (Vodopivec et al., 2011). The general objective of the Science and Technology curriculum emphasises the need for students to gain experience in the field of soft scientific and technical research appropriate for school students. This indicates that, according to the abilities and age of students, outdoor learning is an essential part of the subject.

In the subject Science and Technology, students must have the opportunity to learn experientially. Teachers can provide time in various natural and artificial environments, where students can make observations using simple aids. At the same time, they learn about natural processes and phenomena, asking themselves questions and finding answers through experiments. Students experience some natural (not dependent on human intervention) and artificial (dependent on human intervention) systems by directly observing how they work and how they are composed (Vodopivec et al., 2011).

The Science and Technology curriculum also includes operational learning objectives (compulsory and optional) and content under five thematic sets: Matter, Forces and Motion, Phenomena, Humans, and Living Beings. (Table 1). The large number of operational objectives is not reflected in a large number of content areas. However, the operational learning objectives do not indicate whether the objective can be achieved through outdoor learning. This demonstrates how important it is for the teacher to read the curriculum and know the content of general didactics and special didactics. A well-qualified teacher can find an opportunity to plan outdoor lessons and determine their location (Pečar et al., 2020).

Table 1

Number of operational learning objectives and listed content areas for each specific thematic set of the Science and Technology curriculum

Thematic set	Matter	Forces and Motion	Phenomena	Humans	Living Beings
Number of operational objectives	57 (48*+9**)	34 (29*+5**)	52 (39*+13**)	32 (27*+5**)	40 (28*+12**)
Number of content areas	12 (12*)	16 (13*+3**)	12 (9*+3**)	15 (15*)	(13*)

Note. *Compulsory learning objectives; **Optional learning objectives. Adapted from Vodopivec et al., 2011.

Research problem and research questions

In Slovenia, the study of various aspects of outdoor learning is increasing, especially at the primary level. For example, a search using the COBISS library cataloguing system on 10 September 2020 for the keywords in Slovenian 'outdoor education' (*pouk na prostem*) and 'primary level' (*razredni pouk*) returned 159 graduation theses, 23 master's theses and one doctoral thesis. When the search was narrowed with the addition of the keyword 'science' (*naravoslovje*), four graduation theses and three master's theses were returned. The data search was done by checking documents and not only the keywords listed in the documents.

The 'science' graduation and master's theses describe various aspects of outdoor learning, with a description and evaluation of activities outdoors, learning pathways, CŠOD centre activities, working in combined classes with an emphasis on elementary science, and making use of natural resources in the surroundings of the specific primary school. The authors note that the immediate surroundings of the school play an important role in non-obligatory outdoor lessons. To the best of our knowledge, there is no Slovenian study about the quantity of non-obligatory outdoor lessons in the subject Science and Technology, nor about the perception of outdoor lessons from the perspective of teachers and students. Therefore, we wanted to investigate the extent of outdoor Science and Technology classes in the vicinity of school in the 4th and 5th grades of primary school, focusing on classes that are not compulsory, are not part of outdoor activity days or school outdoors, and are not part of formal out-of-school programmes, which are often led by staff from outdoor centres and can take place in CŠOD centres. In addition, we wanted to investigate the differences between teachers in terms of the location of the school, the teachers' work experience, and their own early experience of outdoor lessons (when they were primary school students) compared to their current teaching. The study also focuses on identifying the status of the organisation of outdoor lessons in the vicinity of school and the identification of outdoor lessons in the school environment by students and their views on such lessons.

As described in *The Slovenian School System and the Integration of Outdoor Learning*, the Science and Technology curriculum does not determine the number of hours spent outdoors (Vodopivec et al., 2011). Therefore, our study aimed to investigate the status of non-compulsory outdoor lessons in the vicinity of the school (which are not part of science day activities or outdoor school) and to compare the results with the location of the school where the teachers teach, as well as the teachers' level of teaching experience and their early experiences with outdoor lessons (during their own schooling). Other aims were to examine the teachers' ideas and identify barriers with content prescribed in the curriculum and evaluate the students' experiences of outdoor lessons in the school environment.

With regard to the research aims, the following research questions (RQs) were addressed:

RQ1: How frequently do teachers teach specific thematic sets from the Science and Technology curriculum as outdoor lessons, and how much time is devoted to such teaching in total?

RQ2: Are there statistically significant differences between teachers in the time devoted to outdoor lessons in the vicinity of the school in Science and Technology regarding: a) the location of the school, b) the teachers' work experience, and c) the teachers' own experience in their early years?

RQ3: Are there differences between the specific and general ideas of teachers regarding conducting outdoor lessons in the vicinity of the school and the thematic sets written in the Science and Technology curriculum?

RQ4: With which thematic sets from the Science and Technology curriculum do teachers have more difficulties in preparing outdoor lessons in the vicinity of school?

RQ5: How often and where do students perceive outdoor lessons in the vicinity of the school in the subject of Science and Technology?

RQ6: Which topics are of interest to students with regard to conducting outdoor lessons in the vicinity of the school in the subject Science and Technology?

Method

The study used a descriptive pedagogical research method and a quantitative research approach.

Sample

An email with a link to an online anonymous questionnaire was sent to the headmasters of all Slovenian primary schools (451) with an email address publicly available on their websites and forwarded to teachers. A total of 70 in-service primary school teachers working with 4^{th} and 5^{th} grade students responded positively and participated in the study. In addition, 21 randomly selected primary schools were requested to gather data about 4^{th} and 5^{th} grade students' experiences with outdoor lessons. Of these, six schools gained the approval of school management and parents, resulting in 26 classrooms responding positively to the request. A total of 305 students from all statistical regions of Slovenia completed a paper-and-pencil questionnaire: 154 fourth-grade students and 151 fifth-grade students. The teachers were not pre-selected.

Data collection

The data were collected using an electronic questionnaire in Slovenian for teachers and a paper-and-pencil questionnaire for students. Both questionnaires were designed for the purpose of the study. The existing literature was first reviewed. Based on the set of research questions, pilot versions of the instruments were prepared and reviewed by two independent science educators. The questions were then modified, pilot-tested (the teacher questionnaire on five fellow teachers and the student questionnaire on five students) and adapted. The instrument used descriptive categories and appropriate Likert scales. Both questionnaires included a definition of the term outdoor lessons as lessons that take place outside the classroom, outdoors, in the natural environment; they take place in the vicinity of the school, and do not include lessons during nature and technology days, visits to the zoo, botanical garden, museums, or similar.

The teacher questionnaire contained 19 questions, divided into three parts. The first part of the questionnaire contained four closed-ended questions about the respondent (gender, class taught, years of teaching experience, and location of the school). The second part included four closed-ended questions, one open-ended question and one semi-open question, all asking about outdoor lessons. In the third part, four closed-ended questions, two open-ended questions and three semi-open questions asked about the content of outdoor lessons in the vicinity of the school, as required by the Science and Technology curriculum.

The student questionnaire was divided into three parts. Three closed-ended questions asked about gender, class, and the location of the school. Two semi-open questions and one open-ended question inquired about outdoor lessons in the school environment, and three open-ended questions were about the students' specific experience of outdoor lessons in the school environment.

Data analysis

The anonymity of the data was guaranteed for research purposes when processing the data. The data was collected in Microsoft Office Excel and statistically processed in SPSS (Statistical Package for the Social Sciences). Basic statistics were used to describe the distribution of the individual variables. Descriptive statistics were used to describe the data: *M* arithmetic mean, *SD* standard deviation, *N* number of teachers/students, and *f* (%) (relative) frequency of occurrence of each answer. The associations between the individual variables were calculated using the nonparametric Mann-Whitney test and the Kullback 2Î-test to explain the relationship between the quantity of outdoor lessons in the vicinity of the school and the school location or the level of work experience or early years' experience with outdoor lessons (Pallant, 2011). Because the Mann-Whitney test works by examining differences in the ranked positions of scores in different groups, the values of mean ranks (*MR*) are added. The statistical hypotheses were tested with an alpha error rate of 5%. To describe whether the effects have a relevant magnitude, the effect size measure eta squared was used to describe the strength of a phenomenon. Benchmarks to define small (.01), medium (.06) and large (.14) effects were provided by Cohen (1988).

The answers to the open-ended questions were analysed qualitatively and quantitatively. Two researchers (i.e., the authors of the present paper) independently read the answers several times, identifying the thoughts with the most important meanings and assigning codes. The coding played a crucial role in analysing the data, as it enabled their organisation and interpretation through the similarities and differences of the answers. The codes were grouped into categories based on the study's research questions and objectives (Vogrinc, 2008). Cross-checking showed a high degree of agreement between the codes assigned by the two researchers: 98% of the codes were the same, and the authors discussed the remaining cases and reached a compromise.

Results and discussion

The results and discussion are presented according to the research questions.

RQ1: How frequently do teachers teach specific thematic sets from the Science and Technology curriculum as outdoor lessons, and how much time is devoted to such teaching in total?

When asked what proportion of Science and Technology hours are taught as outdoor lessons in the vicinity of the school, 39.1% of the 69 (one respondent did not reply to this question) 4^{th}- and 5^{th}-grade teachers answered that they spend 10 out of 105 hours in the subject Science and Technology doing outdoor lessons in the school environment (Table 2). Some 66.5% of the teachers answered that they teach 10% or less of the subject as outdoor lessons, while 33.1% answered that they teach more than 10% as outdoor lessons. If we compare these responses with the hours required by the Science and Technology curriculum (105 hours), we find that teachers spend 10.5 hours doing outdoor lessons. While less than a third (27.4%) of the 4^{th}- and 5^{th}-grade teachers responded that they offer fewer than 10 hours of outdoor lessons in the school environment, 33.1% responded that they offer more than 10 hours of outdoor lessons (Table 2).

Table 2

Teachers' responses to the question about the extent of outdoor school-based lessons within the subject Science and Technology

The quantity of outdoor lessons in the vicinity of school as a percentage of total time for the subject	*f*	*f* %	Hours of curriculum (out of 105)
2	1	1.4	2.1
3	2	2.9	3.2
5	15	21.7	5.3
8	1	1.4	8.4
10	27	39.1	10.5
15	1	1.4	15.8
20	11	15.9	21.0
25	1	1.4	26.3
30	9	13.0	31.5
40	1	1.4	42.0
Total	**69**	**100.0**	

The mean of teachers' responses regarding the extent of outdoor lessons in the subject Science and Technology is 13.49% (*SD* = 9.03%), which means that on average, teachers spend 14 of the 105 total hours required by the Science and Technology curriculum outside. The range was from 4.46% to 22.52%, which means the respondents spent between 5 and 24 hours outdoors in the vicinity of the school.

The teachers' answers refer to the entire school year, which has 35 weeks. If 14 school hours (with a duration of 45 minutes) per school year are spent learning outdoors in the vicinity of the school, this means that outdoor Science and Technology lessons are typically held once or twice a month (18 minutes per week). Research by Waite (2011) shows that students have outdoor physical activities at least once per week, but it should be noted that the research does not specify whether this relates exclusively to the subject Science and Technology or whether it involves another subject (e.g., sport). Nicol et al. (2007) report that students in a study on Scottish primary schools spent a total of 19 minutes per week outdoors in a variety of subjects, which means that the percentage in Slovenia for just one subject is higher. However, the percentage sometimes does not reveal the complete picture, as there are always some teachers who teach outdoors very little, while a few do so very often, as shown in Table 2.

Moreover, from Table 3, it is evident that teachers rarely teach content from the thematic set Living Beings outdoors, while content from the other four thematic sets is sometimes taught as outdoor lessons.

Table 3

Frequency of outdoor lessons for a particular thematic set from the Science and Technology curriculum on a 5-point Likert scale (1 is never and 5 is always)

Thematic Set	Matter	Forces and Motion	Phenomena	Humans	Living Beings
M	2.69	2.94	2.96	2.35	2.97
SD	.75	1.08	.96	.94	1.03

RQ2: Are there statistically significant differences between teachers in the time devoted to outdoor lessons in the vicinity of the school in Science and Technology regarding: a) the location of the school, b) the teachers' work experience, and c) the teachers' own experience in their early years?

The data collected showed that teachers in rural schools devoted more time to outdoor lessons in the vicinity of the school than teachers in urban schools did (Table 4). However, there are no statistically significant differences (U = 446.500, p = .077) between rural and urban schools in the quantity of outdoor lessons taught in the subject Science and Technology. Vidmar (2016) argues that the location of the school in Slovenia does not influence the need for a school garden, which could confirm that the location does not play a role in the quantity of outdoor activities, as shown in our study.

Table 4

The average percentage of time that the subject Science and Technology is taught outdoors in the vicinity of the school according to the location of the school

School Location	*N*	*M* [%]	*SD* [%]	*MR*
Rural school	28	15.54	10.12	40.57
Urban school	42	11.81	8.16	32.12

From the results presented in Table 5, we can conclude that the majority of teachers (81.8%) with up to five years of teaching experience teach 0–10% of the total subject hours as outdoor learning in the vicinity of the school. In contrast, 45.8% of more experienced teachers, who have taught for more than 15 years, report that the proportion of outdoor lessons is greater than 10%. The results of Kullback's 2Î-test revealed statistically significant differences with a high effect size (2Î = 13.550, g = 6, p = .035, =.194) between the years of teaching experience of the 4^{th}- and 5^{th}-grade teachers in terms of the extent of outdoor lessons in the subject Science and Technology: teachers with more teaching experience conduct outdoor lessons more often. The results are consistent with a study by Harlen and Holroyd (1997), who argue that work experience plays an important role in teachers' self-esteem. The more work experience teachers have, the greater their self-confidence (Walan & Chang Rundgren, 2014).

Table 5

The quantity of outdoor lessons expressed as a percentage of time in the Science and technology curriculum according to years of teaching experience

			Quantity of outdoor lessons in the vicinity of the school				
			0–10%	**11–20%**	**21–30%**	**Over 40 %**	**Total**
Years of teaching experience	0–5 years	*f*	9	1	1	0	11
		f %	81.8	9.1	9.1	.0	100.0
	5–15 years	*f*	11	0	0	0	11
		f %	100.0	.0	.0	.0	100.0
	Over 15 years	*f*	26	11	10	1	48
		f %	54.2	22.9	20.8	2.1	100.0
Total		***f***	**46**	**12**	**11**	**1**	**70**
		***f* %**	**65.7**	**17.1**	**15.7**	**1.4**	**100.0**

Table 6 shows that about half of the teachers surveyed experienced outdoor lessons in their early years, meaning that they remember having outdoor lessons themselves as primary school students. Rogoff's theory, which was confirmed in 1990, states that early experiences should influence further belief (Blatt & Patrick, 2014). Hawley and Gunner (2000) argue that knowledge, attitudes, and social skills in later life depend on early childhood experiences. Klofutar et al. (2020) report that direct outdoor experiences lead to a greater increase and persistence of acquired skills. Tomažič and Vidic (2013) also report that early experiences are of great importance in shaping lifelong attitudes. Vidmar (2016) notes that nearly half of classroom teachers (45.8%) had experience gardening before primary education. According to Vidmar (2016), teachers with early experience are more in favour of school gardens than teachers without early gardening experience. A study by Shume and Blatt (2019) also shows the importance of participants' youthful experiences in the outdoors for their positive intentions with regard to taking students outside. In our case, however, the Mann-Whitney U-test did not show statistically significant differences (U = 552.500, p = .878) between teachers with early experiences of outdoor lessons and those without such experiences with regard to the quantity of outdoor lessons.

Table 6
The average percentage of time that the subject Science and Technology is taught outdoors in the vicinity of school according to the teacher's early experience with outdoor lessons

Teacher's early experience	*N*	*M* [%]	*SD* [%]	*MR*
Yes	33	13.97	10.73	32.17
No	31	12.48	7.18	32.85

RQ3: Are there differences between the specific and general ideas of teachers regarding conducting outdoor lessons in the vicinity of school and the thematic sets written in the Science and Technology curriculum?

The teachers' responses to ideas for specific content from the thematic sets in the Science and Technology curriculum were divided into two categories. One category was specific ideas, (i.e., teachers wrote down an activity that they could or could not carry out with students outdoors in the school's vicinity). The second category was general ideas (i.e., ideas that teachers wrote down that could be implemented outdoors in the vicinity of school for specific content). This included all of the answers given by teachers. For example, a suggestion written by one teacher as a general idea was the movement of a car, while another teacher proposed the specific idea of measuring the distance travelled by a car on different surfaces (grass, asphalt, macadam, etc.).

Ideas for outdoor activities (Table 7) that teachers could carry out in the vicinity of the school were provided by the teachers with regard to all of the thematic sets of the Science and Technology curriculum. The teachers provided the most ideas for outdoor lessons, both specific and general, in the thematic set Forces and Motion (54 ideas). They wrote the most specific ideas (29 ideas) in the thematic set Phenomena and the most general ideas in the thematic set Living Beings (32 ideas). Comparing the results from Tables 3 and 7, we find the lowest number of ideas written in the thematic set Humans, which is reflected in the rare implementation of outdoor lessons for this thematic set. For the other four thematic sets, the number of all of the ideas listed and the frequency of outdoor lessons are more similar. However, counting the curriculum items presented in Table 1 shows that a smaller number of operational learning objectives are listed in the thematic set Humans, which is reflected in the lowest number of ideas for outdoor lessons.

Table 7

Teachers' ideas for each specific thematic set of outdoor lessons in the subject Science and Technology

	Matter		Forces and Motion		Phenomena		Humans		Living Beings	
	f	*f* %	*f*	*f* %	*f*	*f* %	*f*	v %	*f*	*f* %
Specific ideas	28	52.8	27	50.0	29	59.2	15	38.5	18	36.0
General ideas	25	47.2	27	50.0	20	40.8	24	61.5	32	64.0
Total	**53**	**100.0**	**54**	**100.0**	**49**	**100.0**	**39**	**100.0**	**50**	**100.0**

When researching outdoor lesson activities, we often find biology-related activities; for example, Blatt and Patrick (2014) describe an outdoor lesson activity that focuses on how photosynthesis can be taught to students through experience rather than through diagrams. When we look at the Internet, we find that many websites (Bocks, 2018; Education.com, 2012; Hamid, 2018; Outdoor Classroom Day, 2020; Teach Junkie, 2017) offer the outdoor lesson activities described above, but most of these activities are biology-related. Outdoor lesson activities include driving a small car faster (Teach Junkie, 2017), making bird food and finding micro-animals (Outdoor Classroom Day, 2020; Education.com, 2012; Hamid, 2018).

The reason that one may find several biology-related ideas for outdoors lessons on the Internet in English, even though the authors' country has a similar curriculum for the subject science, might lie in the fact that primary school teachers have an affinity for biology content. This can also be identified in the more common difficulties that pre-service and in-service primary school teachers face in their knowledge and understanding of chemistry and physics content (Jurišević et al., 2008; Lelliott & Rollnick, 2010; National Curriculum in England, 2013; Pavlin & Čepič, 2015). The other reason for the number of biology ideas for outdoor lessons on the Internet may be that in some countries, science education in the lower grades used to focus on biological sciences, while physics and chemistry were only included in the primary school curriculum to a limited extent (Kinnunen et al., 2016).

RQ4: With which thematic sets from the Science and Technology curriculum do teachers have more difficulties in preparing outdoor lessons in the vicinity of school?

The Slovenian in-service primary school teachers surveyed either reported difficulties with certain content or stated that they had no difficulties with outdoor lessons. Their responses were then coded into individual coding units (guidance, safety, time limit, space, weather conditions, curriculum overload, material limitations, organisation, etc., as well as no problems). The results presented in Table 8 show that teachers reported the most difficulties with the thematic set *Substances* (64 teachers) from the Science and Technology curriculum. Only four teachers reported that they had no problems with the content of the thematic set *Humans*, while 30 teachers reported that they had difficulties with outdoor lessons in the vicinity of the school. The teachers' problems with outdoor lessons were very general, which corresponds to observations found in the literature (Barker et al., 2002; Rickinson et al., 2004; Tuuling et al., 2018; Waite, 2010). For example, Rickinson et al. (2004) and Barker et al. (2002) list accompaniment and limited time as limitations, while Tuuling et al. (2018) also list safety, material limitations, and space.

Table 8

Teachers' difficulties according to the thematic sets of the Science and Technology curriculum in the field of outdoor lessons in the vicinity of the school

Thematic Set	Substances		Force and Motion		Phenomena		Humans		Living Beings		Total	
Teacher's difficulty	*f*	*f* %	*f*	*f* %	*f*	*f* %	*f*	*f* %	*f*	*f* %	*f*	*f* %
Guidance	12	18.5	8	16.3	6	12.8	6	20.1	9	21.4	41	100.0
Safety	6	9.3	8	16.3	5	10.6	5	16.7	7	16.7	31	100.0
Time limit	7	10.7	6	12.2	10	21.3	3	10.0	5	11.9	31	100.0
Space	11	16.9	5	10.2	4	8.5	4	13.3	6	14.3	30	100.0
Weather conditions	2	3.1	2	4.1	6	12.8	1	3.3	1	2.4	12	100.0
Curriculum overload	2	3.1	0	0.0	1	2.2	0	.0	0	.0	3	100.0
Material limitations	12	18.5	10	20.4	6	12.8	0	.0	4	9.5	32	100.0
Organisation	5	7.7	2	4.1	5	10.6	4	13.3	4	9.5	20	100.0
Other	7	10.7	5	10.2	2	4.2	3	10.0	4	9.5	21	100.0
No problems	1	1.5	3	6.2	2	4.2	4	13.3	2	4.8	12	100.0
Total	**65**	**100.0**	**49**	**100.0**	**47**	**100.0**	**30**	**100.0**	**42**	**100.0**	**233**	**100.0**

RQ5: How often and where do students perceive outdoor lessons in the vicinity of the school in the subject of Science and Technology?

The participating students recall having rarely experienced outdoor lessons in science and technology in the vicinity of their school, with 57.7% of the 305 students selecting the response once or twice per school year (Table 9). Some of the students (11.1%) reported that they had either never studied outdoors, or had studied outdoors three times a week or three times every four months. We believe that students do not perceive outdoor activities as a lesson if the activities are playful, instead perceiving such activities as play, as reported by Tuuling et al. (2018). It should be mentioned once again that the students who completed the questionnaire were not necessarily those of the participating teachers, or even the participating schools. Nonetheless, the results still provide some insight into outdoor lessons, as the students were taught by 26 different teachers. There may, however, be a discrepancy between the teachers' answers (once or twice a month) and students' answers in Table 9, as the students perceived outdoor lessons in the vicinity of the school to a lesser extent than the adults did. The reason for the poor perception of outdoor lessons might be that students are often not aware that they are learning, even if the lessons take place in the schoolyard with various activities (Ginnis, 2002; Ross et al., 2007).

Table 9
The quantity of outdoor lessons in the subject Science and Technology in the vicinity of school, as reported by students

The quantity of outdoor lessons in the vicinity of school	f%
Once or twice per school year	57.7
Once per month	22.6
Twice per month	6.6
Every week, once	1.0
Every week, twice	1.0
Other	11.1
Total	**100.0**

We were interested in the location of Science and Technology classes that students remembered as they experienced outdoor lessons. The students' open-ended answers were coded into the following coding units: forest, field, meadow, river, schoolyard, school garden, school environment, park, institution, nature school, elsewhere, nowhere, and meaningless answer. The students most commonly mentioned the schoolyard (56.5%). Slightly more than a third of the students (37.8%) replied that they had lessons in a forest near the school grounds, and slightly less than a quarter (23.7%) wrote that the outdoor lessons took place on the school meadow or another nearby meadow (Table 10). Some 7.6% of the students wrote outdoor institutions (e.g., museum, zoo, botanic garden) at which they had studied, but that are not within 500 m of the school.

When we compare the research results with the literature, we find that a hard surface, e.g., the schoolyard, often serves as an environment for outdoor lessons (Nicol et al., 2007). Blatt and Patrick (2014) report on the lifestyle of children in urban and rural areas. They claim that children mostly use asphalt surfaces for outdoor physical activities, whereas children in rural areas tend to use natural areas (e.g., garden, meadow, forest, etc). In our study, the students from urban schools most often listed the schoolyard and forests as the location of outdoor lessons, while students in rural schools listed meadows, forest, rivers and fields (Table 10).

Although it is indicated that the students remembered different experiences of outdoor lessons outside the vicinity of the school, teachers probably took advantage of various locations within 500 m of the school for outdoor lesson activities. The students might have reported the forest so often because the curriculum lists content that could be presented in it: moss, ferns and seed plants; flowering and non-flowering plants; tree and shrub species in the immediate environment; invertebrates; or vertebrates (Verovnik et al., 2011).

Table 10

Locations of outdoor lessons around the school given by 262 students (88 from rural schools and 174 from urban schools) out of a total sample of 305 students. Some students provided more than one location of outdoor lessons.

		Students giving the answer	
Location of outdoor lessons	**Attending**	***f***	***f* %**
Forest	Rural school	44	50.0
	Urban school	55	31.6
	Total	**99**	**37.8**
Field	Rural school	16	18.2
	Urban school	0	.0
	Total	**16**	**6.1**

		Students giving the answer	
Location of outdoor lessons	Attending	*f*	*f* %
Meadow	Rural school	49	55.7
	Urban school	13	7.5
	Total	**62**	**23.7**
River	Rural school	22	25.0
	Urban school	18	10.3
	Total	**40**	**15.3**
Schoolyard	Rural school	57	64.8
	Urban school	107	61.5
	Total	**148**	**56.5**
School garden	Rural school	8	9.1
	Urban school	0	.0
	Total	**8**	**3.1**
School surroundings	Rural school	17	19.3
	Urban school	15	8.6
	Total	**32**	**12.2**
Park	Rural school	2	2.3
	Urban school	0	.0
	Total	**2**	**.8**

RQ6: Which topics are of interest to students with regard to conducting outdoor lessons in the vicinity of the school in the subject Science and Technology?

The present study also focused on students' interest in learning outdoors in the vicinity of their schools. They expressed their ideas on the topics (content) they wanted to learn. Their answers were coded and divided into categories similar to the curriculum content, in order to examine whether it would be possible to implement the students' topics of interest in the existing curriculum (Table 11). The most common responses were that the students wanted to learn about animals (43.5%), plants (25.7%) and other topics (22.6%). Under the heading *Other*, we mainly listed general answers (e.g., about nature, about other things, how to build a house, etc). However, some students also expressed interest in outdoor lessons about water, soil, weather, air, space and the stars, electricity, matter, fire, pollution and gravity. In the vast majority of cases, students' answers corresponded with the content of the curriculum (Vodopivec et al., 2011). Based on the students' short answers, it is not possible to deduce the extent to which they would like to continue with the given content, but in most cases it would be possible to incorporate the content into the existing curriculum and implement it outdoors. One exception is content on astronomy, especially on space and the stars, which always sparks the curiosity and interest of students of different ages (Susman & Pavlin, 2020). The students' responses do, however, show their interest in outdoor lessons. Šebjanič and Skribe Dimec (2019) found that when teachers give outdoor lessons that are not boring for students and that engage their interest, they equip students with life skills.

Table 11
Content that the students would like to learn in the subject Science and Technology during outdoor lessons, provided by 230 students out of 305

Content	Students giving the answer	
Animals	*f*	100
	f %	43.5
Plants	*f*	59
	f %	25.7
Other	*f*	52
	f %	22.6
Water	*f*	17
	f %	7.4
Soil	*f*	5
	f %	2.2
Weather	*f*	9
	f %	3.9
Air	*f*	8
	f %	3.5
Space and stars	*f*	8
	f %	3.5
Electricity	*f*	6
	f %	2.6
Matter	*f*	5
	f %	2.2
Fire	*f*	4
	f %	1.7
Pollution	*f*	4
	f %	1.4
Gravity	*f*	3
	f %	1.3

Conclusion

This paper presents a Slovenian study on outdoor lessons in the vicinity of schools. It is based on a sample of 4^{th}- and 5^{th}-grade primary school teachers and students and focuses on lessons outside the prescribed framework of activity days and outdoor school. More specifically, the aim was to investigate the extent to which outdoor lessons in the vicinity of school are carried out by teachers within the subject Science and Technology, and whether this differs according to years of teaching experience, school location and early experiences of teachers. The content taught and how outdoor lessons are perceived by students was also investigated. The participating teachers reported that they teach an average of 18 minutes per week outdoors in the vicinity of their schools. It is clear that the location of the school and early experience have no statistically significant influence on the duration of outdoor lessons in the vicinity of the school, while statistically significant differences in the extent of outdoor learning near the school are found among teachers with different levels of experience. Slovenian teachers have ideas for outdoor lessons close to the school but also report difficulties in implementing such lessons. However, most of their ideas for outdoor activities fall within the thematic set Forces and Motion, while fewer ideas exist in terms of Humans. Students recalled that they had experienced outdoor lessons once or twice per school year and mostly remembered schoolyards, forests, and meadows as locations of their school-related outdoor lesson. They had ideas for the content of the outdoor lessons that could largely be implemented directly in Science and Technology lessons within the existing curriculum.

Due to the sampling procedure, generalisation of the research results regarding teachers is only possible to a limited extent. Although the questionnaire accompanied by a request to participate was sent to 451 of 455 Slovenian primary schools, only 70 teachers answered the questions of the e-questionnaire. Another limitation is that the students who responded were taught by 26 teachers not necessarily included in the research and from only six different primary schools. The location in which they could experience outdoor lessons may therefore be similar. The choice of instrument also partially limits the conclusions, partly because the questions were mostly closed-ended and that the open-ended questions were sometimes poorly answered.

The presented results of the study can be used by teachers to raise awareness of their own outdoor Science and Technology lessons, and to plan outdoor lessons, some of which could be designed together with students, thus strengthening their personal responsibility for the importance of their progress. The results indirectly indicate guidelines for the preparation of materials, training and the encouragement of teachers to teach outdoors near their schools, taking into account all subjects in the educational process.

The responses to the research questions gave rise to many ideas for further research. It would be interesting to design specific teaching materials based on content for which teachers listed fewer ideas for outdoor lessons, and to include and evaluate the students' ideas. The comparison of activities and the organisation of outdoor lessons in urban or rural environments, research on the level of motivation, achievements and the attitudes of students towards outdoor lessons in the vicinity of the school, the extent of outdoor lessons through the stages of education in Slovenia, and similar are all topics that would be relevant for future qualitative and quantitative research and would provide specific guidance for improving outdoor lessons.

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Biographical note

Maruša Novljan is the second teacher of the first grade in Nove Jarše Elementary School. Aware that outdoor lessons have a positive effect on students and their holistic development, she has prepared a master's thesis in this field. Even as a class teacher, she strives to conduct outdoor lessons in the school's surroundings as often as possible.

Jerneja Pavlin, PhD, is an Associate Professor of Physics Education at University of Ljubljana, Faculty of Education, Slovenia. Her research focuses on the investigation of different approaches to physics and science teaching (e.g. outdoor teaching and learning, using 3D sub-microscopic representations, didactic games, learning by inquiry, peer instruction etc.), aspects of scientific literacy and the introduction of contemporary science into physics teaching. She is involved in several research projects in the field of science and physics education (e.g. NA-MA POTI, AR Physics made for students, Movement-dance stories in connection with science and social sciences in kindergarten and school, Connectedness with nature, the organization of early school-aged students' free time and digital technologies).

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Examining the Mediating Role of Altruism in the Relationship between Empathic Tendencies, the Nature Relatedness, and Environmental Consciousness[1]

Nudar Yurtsever[2] Duriye Esra Angin*[3]

A study was conducted using a correlational screening model to determine the mediating effect of altruism in the relationship between empathic tendencies, the nature relatedness and environmental consciousness. The participants of the study, selected via random cluster sampling design, are composed of 305 pre-school teachers working in pre-schools and kindergartens in a city located in Turkey's Aegean region. The 'Empathic Tendency Scale', 'Altruism Scale', 'Nature Relatedness Scale', and 'Environmental Consciousness Scale' were used as data collection tools. The analyses of the sub-purposes were carried out using the PROCESS macro (Model 4) developed by Andrew Hayes using the SPSS infrastructure. When the study results were examined, the indirect effects of the empathic tendency on nature relatedness and environmental consciousness were found to be significant. Thus, altruism was the mediator for the relationship between the empathic tendency and nature relatedness (β=.13, 95% BCA CI [.08; .19]) and for the relationship between emphatic tendency and environmental consciousness (β=.36, %95 BCA CI [.18; .57]).

Keywords: altruism, empathic tendency, environmental consciousness, nature relatedness, pre-school teacher

1 This article was developed from the master's thesis of the first author titled as "The mediating role of altruism in the relationship between the empathic tendencies of preschool teachers and the nature relatedness and environmental consciousness".

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Preučevanje posredniške vloge altruizma glede na empatična nagnjenja, povezanost z naravo in okoljsko zavedanje

Nudar Yurtsever in Duriye Esra Angin

Da bi ugotovili mediacijski učinek altruizma glede na empatična nagnjenja, povezanost z naravo in okoljsko zavedanje, smo izvedli raziskavo s korelacijskim modelom preverjanja. Udeležence raziskave, izbrane z naključnim vzorčenjem v obliki grozda, predstavlja 305 vzgojiteljev predšolskih otrok, ki delajo v vrtcih in šolah v mestu v egejski regiji v Turčiji. Kot orodja za zbiranje podatkov so bile uporabljene lestvice empatičnosti, altruizma, povezanosti z naravo in okoljskega zavedanja. Analize podciljev so bile izvedene z uporabo makroprograma PROCESS (model 4), ki ga je z uporabo infrastrukture SPSS razvil Andrew Hayes. Pri pregledu rezultatov je bilo ugotovljeno, da so posredni učinki empatične nagnjenosti na povezanost z naravo in okoljskim zavedanjem statistično pomembni. Tako je bil altruizem mediator v razmerju med empatično nagnjenostjo in povezanostjo z naravo (β = ,13, 95 % BCA CI [,08; ,19]) ter v razmerju med empatično nagnjenostjo in okoljskim zavedanjem (β = ,36, 95 % BCA CI [,18; ,57]).

Ključne besede: altruizem, empatična nagnjenost, okoljsko zavedanje, povezanost z naravo, vzgojitelj

Introduction

Humanity has always had a relationship with nature. Although this relationship is based on the principle of equality in hunter-gatherer societies, it appears that this equality is gradually disappearing in agricultural societies, while the idea that natural resources can be used indefinitely for human well-being appears to be dominant in industrial societies (Harper & Snowden, 2017; Lenski, 1966). During this process, the increase in human population and technological advances have caused severe consumption of natural resources such as land and energy. Also, environmental problems, such as ozone depletion, acid rain, and global warming, have acquired a global dimension (Steg & Vlek, 2009). The fact that critical environmental problems facing the earth have reached global dimensions has prompted people to find solutions on both an individual and social scale. For example, an action plan was implemented in 1992 globally, nationally, and locally by organisations of the United Nations, governments, and major groups in every area where people impact the environment (United Nations Sustainable Development, 1992). Also, the United Nations (2015) made an urgent call for action by all countries in a global partnership to overcome climate change and work to preserve oceans and forests within the 17 Sustainable Development Goals.

In many developing countries, people are beginning to realise that the long-term costs of ignoring environmental protection are high, and environmental concerns are no longer a luxury that only some nations can afford (Economic Commission for Latin America and the Caribbean, 2000), and they searched for solutions. In their research, Steg and Vlek (2009), Kurisu (2005), and Vlek and Steg (2007) stated that environmental problems can change through behavioural change. For this reason, environmental training is planned and implemented. However, some studies have stated that although it is important to provide environmental training, attitudes towards the environment and information about the environment are insufficient to explain environmental behaviour (Erten, 2005; Kollmuss & Agyeman, 2002). In parallel with personality development, it appears that the concepts of empathy and altruism, which are personality traits, also influence this process (Schultz, 2000; Schultz et al., 2005; Tam, 2013). It is believed that the individual's correct communication and the realisation that the existence of all living creatures in nature is just as important as his/her existence is due to empathy (Mayer & Frantz, 2004; Tam, 2013). Many studies have also shown that empathy can be established with all other living creatures and that we can better understand nature (Berenguer, 2007; Schultz, 2000; Sevillano et al., 2007). Empathy has a positive effect on environmental problem solving, as well as an individual's altruistic values about the environment (Stern et al., 1995; Stern et al., 1999); thus, people's empathetic tendencies have a positive relationship with their altruistic behaviours (Andreoni & Rao, 2011; Batson et al., 1981; Burks et al., 2012; Cialdini et al., 1997; Rushton et al., 1981). Berenguer (2010) stated that altruism and empathy play an important role in explaining behaviour, attitudes, and personal norms about the environment. It is believed that people who exhibit altruistic behaviour participate more actively in environmental issues (Ghazali et al., 2019; Stern et al., 1995; Stern et al., 1999).

The role of the pre-school teacher

According to Keenan and Evans (2009), in early childhood, individuals develop their basic values, attitudes, skills, behaviours and habits; thus, it is a receptive period for the development of the child's personality. The early childhood period also has enormous potential for the creation of environmental attitudes (Samuelsson & Kaga, 2008). Therefore, the period that includes the pre-school education process will be the most important and ideal time for children to be aware of the effects of environmental problems, which have serious consequences in their own and others' lives, on nature and social life, and about the precautions to be taken against them.

For this reason, in this process, children should have learning and life experiences that allow them to become acquainted with nature, strengthen their bonds, and make them interested in nature in order to cope with environmental problems (Kidd & Kidd, 1997; Selby, 2017) because it is thought that children who can spend time in nature will be more sensitive to environmental problems and will be more likely to maintain this sensitivity in adulthood (Tanja Dijkstra et al., 2019). Studies have demonstrated that the acquisition of basic environmental behaviours in early childhood positively affects the individual's attitudes and behaviours towards the environment in later life stages (Basile, 2000; Chawla, 1999; Lohr & Pearson Mims, 2005). However, factors such as the density of people living in urban centres, children's encounters with a limited environment, and the increase in the time spent in front of screens negatively affect their interaction with nature (Kernan, 2010; Tanja Dijkstra et al., 2019). Children grow up by normalising living their lives, regardless of the connection between themselves and the ecological system (Louv, 2008).

In addition, role models interested in nature, sensitive to environmental problems, and displaying environmental protection behaviour are as important as forming environmental awareness during early childhood (Darling

Hammond, 2000; Palmer et al., 1998; Palmer et al., 1999). In this period, pre-school teachers are among the important role models first in terms of their physical, cognitive, and affective connectedness with nature; they also play a critical role in solving environmental problems (Landy, 2018; Meier & Sisk Hilton, 2017; Orbanić & Kovač, 2021; Sirivianou & Papadimitriou, 2018). For this reason, pre-school teachers should have a positive attitude towards the environment to develop children's relationships with nature and raise environmental awareness. Depending on the 'value basis of environmental concern' theory of Stern and Dietz (1994), Schultz (2001) suggested three factors for environmental concern: egoistic, altruistic, and biospheric. Considering that children learn by observing the model, the pre-school teacher should also be altruistic (Nyaga, 2015; Robinson & Curry, 2005), because altruism, which emerges in early childhood, is one of the concepts used to explain environmental behaviours (Bruni et al., 2012; Kawakami & Takai Kwakami, 2015; Lam, 2012; Snelgar, 2006).

Studies have shown that empathy is necessary for the emergence of altruism and that it affects environmental awareness by strengthening the individual's bond with nature. When the relevant literature was examined, many studies on empathic tendency, altruism, nature relatedness, and environmental consciousness were found. In some of these studies, empathy has been associated with altruism (Andreoni & Rao, 2011; Batson et al., 1981; Burks et al., 2012; Cialdini et al., 1997; Rushton et al., 1981) and with nature and environmental consciousness (Berenguer, 2010; Dökmen, 2017; Schultz, 2000; Sevillano et al., 2007). Studies have been found in which altruism is associated with nature and environmental consciousness (Ghazali et al., 2019; Stern et al., 1995; Stern et al., 1999). Thus, in this study, altruism was accepted as the mediating variable. Considering the previous studies, it raised new questions; in the current study, it is thought that determining the mediating role of altruism in the relationship between empathic tendency and nature relatedness and environmental consciousness will provide a different perspective.

Nonetheless, based on the statement set out in United Nations Sustainable Development, Agenda 21 (1992) subclause B, item 25.12:

> Children are also highly aware supporters of environmental thinking. The specific interests of children need to be taken fully into account in the participatory process on environment and development in order to safeguard the future sustainability of any actions taken to improve the environment (p. 277).

It is thought that working with the pre-school teachers as the most important actors in this regard will contribute to the relevant field.

Aims of the Study

The study aims to determine whether altruism plays a mediating role in the relationship between empathic tendencies, the nature relatedness, and environmental consciousness. In the light of this general aim, the sub-purposes have been determined as follows:

1. Is there a mediating effect of altruism in the relationship between the empathic tendency and nature relatedness?
2. Is there a mediating effect of altruism in the relationship between the empathic tendency and environmental consciousness?

Figure 1 and 2 shows the models for the first and second sub-purposes.

Figure 1

Model showing the mediating role of altruism on the relationship between empathic tendency and nature relatedness

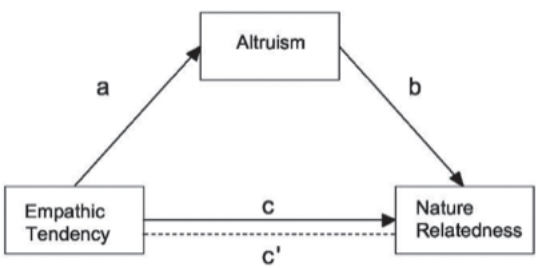
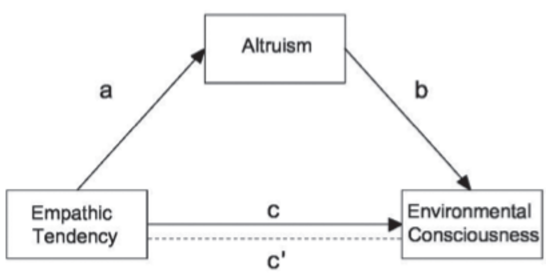


Figure 2

Model showing the mediating role of altruism on the relationship between empathic tendency and environmental consciousness



The empathic tendency prediction variable, altruism mediator variable, and nature relatedness outcome variable are included in the model related to the first sub-purpose. As seen in the model, the study examines whether altruism has a mediating role in the relationship between empathic tendency and nature relatedness. The effect of the empathic tendency on the altruism mediating variable is symbolised by *path a* and the effect of altruism on nature relatedness *path b*. When altruism and the empathic tendency, which is the mediator variable, are included in the model simultaneously, the effect of empathic tendency on nature relatedness is shown with *path c'*, and this is the direct effect. Also, *path c* shows the effect in the absence of altruism, that is, the total effect, on the nature relatedness of empathic tendency. The effect obtained by multiplying the paths a and b in the model is the indirect effect, and it shows the mediating role of altruism. The paths symbolised by the aforementioned letters a, b, c, and c' are non-standardised regression coefficients.

In the second sub-purpose model, the empathic tendency prediction variable, altruism mediation variable, and environmental consciousness are outcome variables. Paths a, b, c, and c' present the same effects, as explained in Model 1.

Method

The study was conducted in correlational design among quantitative research methods to collect various information from many subjects at once and allow the researcher to work with a wide range of variables and these variables' interrelations as cited in Creswell (2015).

Participants

The study participants are composed of pre-school teachers working in public and private pre-schools and kindergartens of primary schools in a city (five districts of the province) located in Turkey's Aegean region in the 2018/19 academic year. A total of 323 pre-school teachers were reached in the study, and the study was carried out with 305 teachers after removing the extreme values that were answered incompletely and affecting the normal distribution.

Instruments

The Empathic Tendency Scale, Altruism Scale, Nature Relatedness Scale, and Environmental Consciousness Scale were used as data collection tools to address the research questions in the study. While determining the data collection tools, attention was also paid to whether they were adapted to the Turkish culture or developed in the Turkish language or not.

Empathic Tendency Scale

The Empathic Tendency Scale was developed by Dökmen (1988) and is comprised of 20 items with a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). Negative items in this scale are reverse coded. The Cronbach alpha reliability coefficient of the Empathic Tendency Scale was found as .82. For this study, Cronbach's alpha coefficient value (α) of the scale was found to be .71.

Altruism Scale

The original Altruism Scale was developed by Ruston et al. (1981) and adapted to Turkish by Tekeş ve Hasta (2015). The scale was comprised of 20 items. The scale is a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). No reverse coding/scoring was done for this scale since there were no negative items included. The internal reliability (α) co-efficient of the Altruism Scale was computed as .85. For this study, (α) of the scale was found to be .86.

Nature Relatedness Scale

The original nature relatedness scale was developed by Nisbet et al. (2009) and adapted to Turkish by Çakır et al. (2015). The scale is composed of 21 items; it is a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). Negative items in this scale were reverse coded. The internal reliability (α) co-efficient of the Nature Relatedness Scale was computed as .88. For this study, (α) of the scale was found to be .78.

Environmental Consciousness Scale

The original Environmental Consciousness Scale was developed by Milfont and Duckitt (2006), and the Turkish adaptation was made by Ak (2008). The scale comprises 53 items; it is a seven-point Likert scale ranging from 1 (strongly disagree) to 7 (strongly agree). Negative items in this scale were reverse coded. (α) reliability coefficient of the Environmental Consciousness Scale was computed as .87. For this study, (α) of the scale was found to be .79.

Data Collect Procedures

As a first step, in order to use the scales, all necessary permissions were obtained from the researchers who developed the Empathic Tendency Scale, Altruism Scale, Nature Relatedness Scale and Environmental Consciousness Scale via e-mail. Secondly, after obtaining the necessary permissions from the Provincial Directorate of National Education, meetings were arranged with school principals and vice-principals. Following these meetings, scales were delivered to the pre-school teachers who wanted to participate in the study. After the scales were delivered, the teachers were asked to fill in the scales within one to ten days.

Data Analysis

Before starting data analysis, the normality analysis of scales, descriptive statistics and correlation calculations between variables were performed using the SPSS 21 software package. The sub-purposes model's of the study was tested using PROCESS 3.4.1 (Model 4), developed by Andrew Hayes and runs within the SPSS infrastructure.

Table 1

Descriptive statistics and normality analysis values regarding the scores the participants obtain from the scales

	Skewness	Kurtosis	Min	Max	x	sd
Empathic Tendency Scale	-.025	-.484	56.00	95.00	75.96	7.93
Altruism Scale	.132	-.140	39.00	100.00	69.39	11.91
Nature Relatedness Scale	-.011	-.436	59.87	105.00	83.49	8.98
Environmental Consciousness Scale	-.151	-.310	204.10	356.00	282.76	30.34

Note. n = 305.

As seen from Table 1, the skewness and kurtosis values of the scales are between -1 and +1. According to these values, it is accepted that the data show a normal distribution (Morgan et al., 2004, p.57).

Table 2

Correlation values between variables

Variables	1	2	3	4
Empathic Tendency Scale	-			
Altruism Scale	.35**	-		
Nature Relatedness Scale	.33**	.40**	-	
Environmental Consciousness Scale	.27**	.33**	.53**	-

Note. n = 305, *p < .01, **p < .001.

As Table 2 presents, there is a significant positive relationship between altruism, empathic tendency, nature relatedness and environmental consciousness.

Results

To answer the research questions, the results are provided as tables and comments. The results of the study include *paths a*, *b*, *c*, and *c*'. Table 3, in which the results of the first sub-purpose of the study are shown, shows the *paths a*, *b*, *c*, and *c*' while Table 4, in which the results of the second sub-purpose are shown, shows the *paths b*, *c*, and *c*'. Since *path a* contains the same results for both research questions, the relationship between altruism and the empathic tendency was shown only in Table 3.

Table 3

Results of the analysis regarding the mediating effect of altruism in the relationship between empathic tendency and nature relatedness

Dependent Variable: Altruism						
Model Summary						
R	**R-sq**	**MSE**	**F**	**df1**	**df2**	**p**
.35	.12	125.20	41.25	1.00	303.00	.00
Modal						
	coeff	**se**	**t**	**p**	**LLCI**	**ULCI**
Constant	29.91	6.18	4.84	.00	17.74	42.07
Empathic Tendency	.52	.08	6.42	.00	.36	.68
Standardised coefficients						
	coeff					
Empathic Tendency	.35					
Dependent Variable: Nature Relatedness						
Model Summary						
R	**R-sq**	**MSE**	**F**	**df1**	**df2**	**p**
.45	.20	64.73	38.44	2.00	302.00	.00
Model						
	coeff	**se**	**t**	**p**	**LLCI**	**ULCI**
Constant	47.89	4.61	10.38	.00	38.81	56.97
Empathic Tendency	.24	.08	3.89	.00	.12	.36
Altruism	.25	.04	6.03	.00	.17	.33
Standardised coefficients						
	coeff					
Empathic Tendency	.21					
Altruism	.33					

Total Effect Model						
Dependent Variable: Nature Relatedness						
Model Summary						
R	**R-sq**	**MSE**	**F**	**df1**	**df2**	**p**
.33	.12	72.28	36.33	1.00	303.00	.00
Model						
	coeff	**se**	**t**	**p**	**LLCI**	**ULCI**
Constant	55.34	4.70	11.78	.00	46.10	64.58
Empathic Tendency	.37	.06	6.03	.00	.25	.49
Standardised coefficients						
	coeff					
Empathic Tendency	.33					

Indirect Effect of Empathic Tendency on Nature Relatedness (Mediator Effect)				
	Effect	BootSE	BootLLCI	BootULCI
Altruism	.13	.03	.08	.19

In Table 3, we can first see *path a*, the results of the regression analysis showing the effect of the independent variable, which is the empathic tendency, and on altruism, which is the mediator variable, can be seen. Accordingly, it is seen that empathic tendency affects altruism significantly and positively. The table shows the non-standardized β value as β= .52, %95 CI [.36; .68], t=6.42 and its corresponding p-value as .00 < .01. It is understood that the value of β (non-standardised β) is significant, both because the p-value in the table is less than .01 and the values belonging to the confidential interval do not include 0 (zero) value (Gürbüz, 2019). Also, empathic tendency explains about 12% (R^2 = .12) of the change in altruism.

Secondly, *path b* and *path c'* can be seen on Table 3, showing the effects of altruism (*path b*) and empathic tendency variable (*path c'*), which are mediating variables, on the nature relatedness, which is the dependent variable. According to this, it is seen that altruism affects nature relatedness significantly and positively (β = .25, %95 CI [.17; .33], t = 6.02, p = .00 < .01). It is also seen that empathic tendency has a significant and positive effect on nature relatedness (β = .24, %95 CI [.12; .36], t = 3.89, p = .00 < .01). The empathic tendency and altruism explain about 20% (R^2 = .20) of the change in nature relatedness.

Thirdly, *path c*, the effect of empathic tendency on nature relatedness, is seen in a model without altruism, which is the mediator variable. According to this, in the absence of altruism, it is seen that empathic tendency affects nature relatedness positively (β = .37, %95 CI [.25; .49], t = 6.03, p = .00 < .01).

Lastly, the indirect effect value, which indicates whether the empathic tendency has an indirect effect on the behaviour in nature relatedness, is reported with the confidence intervals obtained by the 'bootstrap technique'. It has been determined that the indirect effect of empathic tendency on nature relatedness is significant, so altruism mediates the relationship between empathic tendency and nature relatedness (β = .13, %95 BCA CI [.08; .19]).

Table 4

Results of the analysis regarding the mediating effect of altruism in the relationship between empathic tendency and environmental consciousness

Dependent Variable: Environmental Consciousness						
Model Summary						
R	**R-sq**	**MSE**	**F**	**df1**	**df2**	**p**
.37	.14	789.40	24.27	2,00	302.00	.00
Model						
	coeff	**se**	**t**	**p**	**LLCI**	**ULCI**
Constant	183.26	16.20	11.31	.00	151.38	215.13
Empathic Tendency	.69	.22	3.11	.00	.25	1.11
Altruism	.70	.15	4.77	.00	.41	.98
Standardised coefficients						
	coeff					
Empathic Tendency	.18					
Altruism	.27					
Total Effect Model						
Dependent Variable: Environmental Consciousness						
Model Summary						
R	**R-sq**	**MSE**	**F**	**df1**	**df2**	**p**
.27	.07	855.71	24.05	1.00	303.00	.00
Model						
	coeff	**se**	**t**	**p**	**LLCI**	**ULCI**
Constant	203.95	16.16	12.62	.00	172.16	235.75
Empathic Tendency	1.04	.21	4.90	.00	.62	1.45
Standardised coefficients						
	coeff					
Empathic Tendency	.27					
Indirect Effect of Empathic Tendency on Environmental Consciousness (Mediator Effect)						
	Effect		BootSE		BootLLCI	BootULCI
Altruism	.36		.11		.18	.57

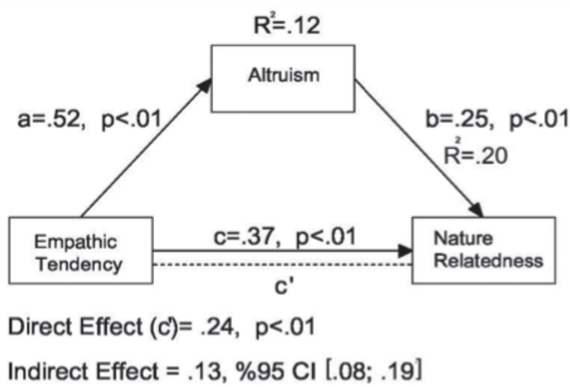
Firstly *path b* and *path c'* on Table 4, showing the effects of the altruism (*path b*) and empathic tendency variables (*path c'*) on the environmental consciousness, which is the dependent variable, are seen. Accordingly, altruism is seen to have a significant and positive impact on environmental consciousness (β = .70, %95 CI [.41; .98], t = 4.77, p = .00 < .01). In addition, it is also seen that empathic tendency significantly and positively affects environmental consciousness (β = .69, %95 CI [.25; 1.11], t = 3.11, p = .00 < .01). The empathic tendency and altruism explain about 14% (R^2 = .14) of the change in environmental consciousness.

Secondly, *path c*, in a model without altruism, the effect (*path c*) of the empathic tendency on environmental consciousness is seen. According to this, in the absence of altruism, the empathic tendency significantly affects environmental consciousness in the positive direction (β = 1.04, %95 CI [.62; 1.45], t = 4.90, p = .00 < .01).

Lastly, it has been determined that the indirect effect of empathic tendency on environmental consciousness is significant, so altruism mediates the relationship between empathic tendency and environmental consciousness (β = .36, %95 BCA CI [.18; .57]. The results of the study are summarised in Figures 3 and 4.

Figure 3

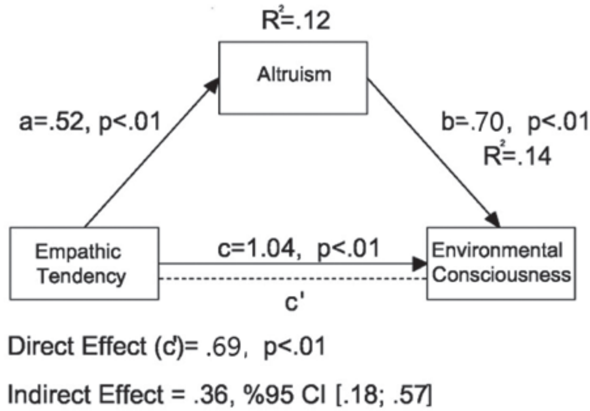
Report of analysis results on the mediating role of altruism in the relationship between empathic tendency and nature relatedness



Note: Unstandardised beta coefficients are not reported. R^2 values show the variance explained. N = 305.

Figure 4

Report of analysis results on the mediating role of altruism in the relationship between empathic tendency and environmental consciousness



Note: Unstandardised beta coefficients are not reported. R^2 values show the variance explained. *N* = 305.

Discussion

According to the findings of the analyses conducted for both the first and second sub-purposes, the empathic disposition significantly and positively affected altruism (*path a*). When previous studies were examined, a relationship between empathic tendency and altruism was found (Andreoni & Rao, 2011; Batson et al., 1981; Burks et al., 2012; Cialdini et al., 1997; Rushton et al., 1981). Mehrabian and Epstein (1972) have stated that people with a high empathic tendency tend to display helping behaviour, while Batson et al. (1981) stated that helping behaviour can be selfish or altruistic. It has been found that if someone's thinking is selfish, in other words, if his/her own interests are more in the foreground, then he/she will not help if it is easier to avoid doing so. While altruistic behaviour, which is defined as anti-selfish behaviour (Scott & Seglow, 2007), is a non-selfish behaviour without expecting any benefit by trying to be useful to people and by considering the benefits of others as his/her own (Gintis et al., 2003; Khalil, 2004), he/she will display this behaviour, whether it is easy to help or difficult (Batson et al., 1981). Therefore, empathy will stimulate altruism (Barr & Higgins D'Alessandro, 2007; Batson, 1991; Batson & Ahmad, 2001; Coke et al., 1978; Krebs, 1975). From these points of view, it is seen that this finding of the study is consistent with the studies in the literature.

Findings about the relationship between altruism belonging to the first sub-purpose and nature relatedness (*path b*) show that altruism significantly and positively affects nature relatedness. When previous studies are examined, it is stated that altruism will raise awareness in terms of the consequences of environmental problems (Ghazali et al., 2019; Stern et al., 1999). Therefore, it can be said that awareness in terms of environmental consequences will strengthen individuals' nature relatedness. When the literature is examined; it has been stated that people with a high nature relatedness regard themselves as a part of nature and feel a responsibility towards other living creatures and that they are more concerned about the damage caused by environmental problems and engage in environmental behaviours compared to people who have a weak nature relatedness (Dutcher et al., 2007; Schultz et al., 2004). Since people who have a high nature relatedness are aware of environmental problems, care about the suffering of animals, think that what they do about the environment will provide solutions to problems in different parts of the world, the individual must have a high nature relatedness in order to achieve success in solving environmental problems (Baylan, 2009; Çakır et al., 2015). It is also stated that altruistic people act more voluntarily (Eisenberg, 2015; Leeds, 1963). It has been stated that altruistic people act against selfishness and individualism, are people who are also observing the concerns and interests of others, behave against selfishness and individualism (Boudon & Bourricaud, 2003) and sometimes even pay a personal price to consider the interests and well-being of others (Budak, 2009). From this point of view, it is thought that the relationship between altruism and nature is because altruists can volunteer and take responsibility in the process of helping not only humans but also living creatures whose existence is endangered or in the process of helping the global environmental problems created by the industrial revolution.

According to the findings of the analysis for the relationship between the empathic tendency of the first sub-purpose and the nature relatedness (*path c*), it is seen that empathic tendency affects nature relatedness positively, even in the absence of altruism. When previous studies are examined, it is seen that empathy can be established with all other living creatures and, in this way, nature will be more understandable (Berenguer, 2007; Schultz, 2000; Sevillano et al., 2007). It has been stated that empathising with creatures such as birds and trees in nature displays behaviours that protect the environment (Berenguer, 2007). The reason for the relationship between empathic tendency and nature; can be explained by the statement that a person who can empathise enough with one of the elements that make up the environment, such as nature, people and man-made products, can also empathise with other elements (Dökmen, 2017).

As a result of the analyses belonging to the first sub-purpose, it has been determined that the indirect effect of empathic tendency on nature relatedness is significant, so altruism mediates the relationship between empathic tendency and nature relatedness.

According to the findings for the relationship between altruism and environmental consciousness (*path b*) belonging to the second sub-purpose, it was determined that altruism significantly and positively affected environmental awareness. Altruism has positively affected the environmental consciousness as well as in the relationship with nature. Batson (1991) and Krebs and Van Hesteren (1992) stated that altruism is an effective factor in creating environmental behaviour changes, and motivation is emphasised when defining altruism. Studies to explain altruism are based on social dilemmas in which people have to choose between acting in their self-interest or helping others. Although it has been revealed that people mostly display self-sacrificing attitudes in social dilemmas, social dilemmas are of great importance in terms of being within life and affecting life in the economic and cultural areas (Üzümçeker et al., 2019). The fact that this is continuing, although social dilemmas are known to have limited natural energy resources, it is also thought to be the case for environmental problems, such as the killing of endangered animals for various purposes. In this dilemma, it will be more profitable for everyone to cooperate, that is, to act altruistically, even though people can gain the highest interest just by thinking about themselves. Therefore, the relationship between environmental awareness, which means that the individual realises the environment by making sense of his/her perspective (Ak, 2008) and altruism, can be explained by the fact that altruistic people can also show their motivation to help in social dilemmas such as environmental problems.

Findings of the analysis for the relationship between the empathic tendency of the second sub-purpose and environmental consciousness (*path c*) show that empathic tendency affects environmental consciousness significantly and positively, even in the absence of altruism. A person can develop positive attitudes, consciousness and emotions about other living creatures based on his/her own experiences through empathy (Dökmen, 2017). The person who empathises will play an important role in reducing social problems by trying to understand the other, understanding social problems, and seeking solutions (Genç & Kalafat, 2010). Their interaction with other living creatures affects the individual's conscious behaviour towards environmental problems (Erten, 2012; Karaismailoğlu, 2007). The relationship between empathic tendency and environmental awareness; can be explained by the fact that people with high empathic tendencies are more sensitive to environmental problems, which is a social problem.

As a result of the analysis belonging to the second sub-purpose, it was determined that the indirect effect of empathic tendency on environmental consciousness was significant, so altruism mediated the relationship between empathic tendency and environmental consciousness. Based on the results obtained from the study, considering the mediating effect of altruism, the importance of increasing the empathic tendency and altruism of individuals is better understood in order to increase the awareness of nature and the environment.

Limitations

Based on the research design, the following limitations can be identified:

1. This study is limited to the pre-school teachers in Turkey.
2. The Empathic Tendency is limited to the qualities measured by the Empathic Tendency Scale; Altruism is limited to the qualities measured by the Altruism Scale; Nature relatedness is limited to the qualities measured by the Nature Relatedness Scale; Environmental Consciousness is limited to the qualities measured by the Environmental Consciousness Scale.
3. Since the scales used in the study are based on self-reporting, the data obtained are limited to the participants' perceptions of themselves and the concepts evaluated in the study.

Conclusions

As a result of the study, it was seen that altruism had a mediating role between empathic tendency and nature relatedness and environmental consciousness. Taking into account the limitations of the study, future researchers can focus on the effect of demographic characteristics on the relationship between pre-school teachers' empathic tendencies, altruism, nature relatedness, and environmental consciousness. This study can be repeated with teachers in different branches. Also, to understand the mediating role of altruism, more detailed information can be obtained by using different measurement tools or by making detailed interviews. Based on the results obtained from the study, developing empathic tendency and altruism can also be included in activities/seminars to develop nature relatedness or environmental consciousness. In this way, an important step will be taken in solving environmental problems.

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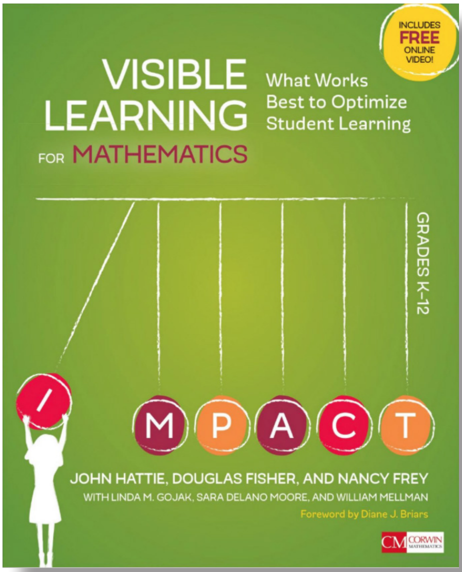
John Hattie, Douglas Fisher and Nancy Frey, *Visible Learning for Mathematics: Grades K-12: What Works Best to Optimize Student Learning*, Corwin Mathematics: 2017; 269 pp.: ISBN: 9781506362946

Reviewed by Monika Zupančič[1]



This book is devoted to the question of how mathematics teachers can make learning visible (evident) to students and to themselves. The content is divided into seven chapters and is supported throughout the book by practical examples that can be applied to mathematics teaching. In addition, each chapter includes QR codes that link to online videos (23 in total, throughout the book). The videos are a mixture of clips from real mathematics lessons and comments from teachers about the structure of the lessons or the methods used. In the videos, we meet seven teachers who teach mathematics from kindergarten through twelfth grade. In addition to the videos, readers have access to some online materials ('reproducibles') consisting of templates, checklists, rubrics, and similar, that teachers can use in their lessons. Each of the seven chapters concludes with a summary, while questions for reflection and discussion guide readers to focus on the key concepts described in each chapter for their teaching.

The first chapter forms the basis for all the following chapters. One of the main themes in this book is 'knowing *what* strategies to implement *when* for maximum impact' (p. 26). The authors call this *precision teaching*. Based on 800 meta-analyses involving more than 250 million students, the co-author of this book John Hattie in his books *Visible Learning* (2009) and *Visible Learning*

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for Teachers (2012) studied the effect size of 150 'influences (instructional strategies, ideas, or tools) that we use in school' (p. 20). He found that most (95%) of the influences studied have a positive effect. He defines an effect value of .40 as a key point: the influences studied that have a value greater than .40 fall into the so-called *zone of desired effects*, and it is these on which this book focuses. The chapter concludes with a categorisation of three types of learning: surface, deep, and transfer, and with a brief description of each type and an explanation of the relationship between them. Claiming that *surface learning* (understanding of concepts, skills, and vocabulary) builds on prior knowledge, the linking of multiple concepts, and the application of these links to procedural skills leads to the level of *deep learning*, which serves as an important foundation for *transfer learning* when understanding of concepts and acquired procedural skills are applied to new contexts. Each of the phases of learning is discussed later in separate chapters. In reading this book, the reader may recall a similar (but in some respects different) classification by Brown (1978).

In the second chapter, the authors assert that visible learning begins with *teacher clarity*. They spend most of their time explaining how teachers can achieve this by setting learning intentions ('what students are expected to learn') and success criteria ('what success looks like when the learning goal is reached') (p. 39). When explaining why learning intentions are important and how to set them, they mention that it is also important to communicate them to students. Then they highlight an interesting idea: teachers take the part about communicating learning intentions more seriously when it is presented as 'students have a right to know what they're supposed to learn, and why they're supposed to learn it' (p. 43). This is an interesting and potential pointer to further debate. The chapter continues with a section on how learning intentions should be linked to prior knowledge and how they can be formulated to invite students to engage in the learning process. Special attention is given to two types of learning intentions: language and social. The reader will later notice that these two topics are well represented in this book as the authors write about communication, mathematical language, collaboration, and related topics. They point out that there are fewer and more appropriate ways to frame the statements for the success criteria. Success criteria that are too general, such as 'I work hard' or 'I can do math', do not have much impact. Instead, the authors suggest that statements should be specific and focus on an achievement in the near future. To this end, they offer some advice and examples of good success criteria and suggest that teachers involve students in the process of establishing these criteria.

The third chapter focuses on guiding the learning process through a mathematical task and a conversation. These two themes are also found in

Chapters Four through Six when applied specifically to each phase of learning. In the first part of the chapter, the authors emphasise that teachers need to be careful about what kind of tasks they choose and when they present them to students to help them move from surface to deep learning and then to transfer learning. They distinguish between exercises and problems, advocate spaced practice over massed practice, and stress the importance of introducing real-world problems at the beginning of the learning process rather than at the end. Believing that increasing the quantity of student work cannot be as beneficial as preparing high-quality and varied tasks, they help teachers distinguish between complexity and difficulty. Thus, they present task types that address student fluency (low-difficulty, low-complexity tasks), strategic thinking (low-difficulty, high-complexity tasks), stamina (high-difficulty, low-complexity tasks), and expertise (high-difficulty, high-complexity tasks). The second part, devoted to mathematical talk, begins with a discussion of classroom discourse and how it can be supported. The authors devote considerable attention to asking questions, noting the importance of framing questions in a way that leaves room for students' thinking and allows them to do the cognitive work. They conclude the subchapter on questions with the suggestion to help students ask themselves questions when they get stuck, as this helps them become more independent.

The fourth chapter is devoted to equipping teachers with information, techniques, and examples to promote *surface learning* in their mathematics classrooms. They point out that the word surface might evoke a negative connotation in some people, and attempt to distinguish this from the meaning of *surface learning*, which is a fundamental part of learning as it equips learners with tools for future action and learning. The rest of the chapter is divided into two larger parts. In the first part, the authors introduce four different types of mathematical talk (number talk, guided questions, worked examples, and direct instruction) that are helpful for students in the *surface learning* phase. All sections presenting the four types of mathematical talk include an example of an activity or a QR code to an online video on the topic (sometimes both). This adds immeasurably to the value of this book by giving readers the knowledge, illustration, and motivation to apply the practices presented in their classrooms. In the second part of the chapter, they describe several effective teaching practices that promote *surface learning*, such as vocabulary instruction, manipulatives for *surface learning*, spaced practice with feedback, and mnemonics. The authors devote most of their writing to teaching practices, which include vocabulary instruction. They encourage readers to be intentional about helping their students build their academic language.

Chapter Five is, in our opinion, the most comprehensive of all. Its content relates to the *deep learning* phase. The authors write about the nature of *deep learning* and highlight the core meaning of *deep learning* – making connections between ideas. They emphasise the importance of discourse (which is more than just discussion) as it provides an opportunity to express agreement or disagreement in different ways. As in Chapter Four, the authors give advice on choosing a mathematical task that will challenge students in the *deep learning* phase. They emphasise the importance of students' *surface learning* to the success of this learning phase, but they do not stop there. They argue that successful *deep learning* requires more than just offering students the right task. One of the ways to support student learning in the *deep learning* phase is through accountable talk – 'a set of expectations for students that is supported through the use of language frames that scaffold the use of language to explore a topic' (p. 144). They provide insight into how accountable talk is achieved by using pre-established language frames that teachers and students can use to ask for clarification, request justification, challenge misconceptions, and similar The authors provide advice on how teachers can motivate their students to use accountable talk in their communication in mathematics classrooms but point out that the responsibility lies first with the teachers themselves, who must practice accountable talk in their communication with students. Next, the chapter looks at the use of accountable talk in small groups and in the whole class. The chapter continues with a debate about when to put students in a small group to organise their work together, how to group them, how to support collaborative learning in groups on the one hand, and how to promote student accountability on the other. It continues with a section on whole-class collaboration that focuses on whole-class discourse when to use it and how to support it.

The sixth chapter is devoted to *transfer learning*. The authors point out that the goal of teaching should be that students can transfer their knowledge to new contexts and eventually become independent learners. They further argue that if teachers are to achieve this goal, they must provide students with tools to assist them in this process. They suggest that helping students develop their metacognitive skills would impact their independence as they begin to monitor and regulate their learning process. They develop this idea further in two sub-chapters, discussing why self-questioning and self-reflection are critical metacognitive techniques and how to help students develop them. The chapter concludes with a description of how to help students transfer and connect their mathematical understanding. One suggested way for students to develop connections in their understanding is to tutor younger students. They argue that tutors need to organise and understand their own thinking when explaining, which helps them improve as learners.

The final chapter focuses on properly using formative and summative evaluation to provide feedback to students and teachers. The authors present several ways teachers can check students' understanding and provide feedback that can vary in timing, amount, type, and audience, and encourage teachers to cultivate a positive attitude toward student errors. Next, the authors highlight the importance of differentiation as a way teachers can address the needs of individuals. They describe three ways teachers can differentiate instruction (adjusting content, process, or product) but advise adjusting only one area at a time. They also present the RTI (Response to Intervention) model in detail, as it has been shown to have one of the most significant effect sizes (1.07). It ranks third out of 150 influences measured. One of the interesting emphases in this section is that teachers should provide interventions for high-achieving students because they sometimes have difficulty justifying why their answer is correct or because they tend to get bored. Although the authors highlight influences throughout the book that have a positive impact on student learning, they do not disappoint and also present some that do not work as well, such as grouping students by their ability or teaching students how to take a test. The book concludes with 'ten teacher mind frames that together summarize a great deal of the "what works" literature' (p. 199) which encourage teachers to collaborate with their colleagues and acknowledge learning as hard work (Hattie, 2012).

This book is relevant to target readers (mathematics teachers) both in terms of the topic chosen and the way it is written. It contains some very interesting ideas and themes (e.g., categorising learning into three phases, effect sizes for 150 influences, highlighting the importance of communication and mathematical talk) that offer the possibility to be elaborated upon at the college level, as professors of didactics could include them in their didactics courses for pre-service teachers.

We conclude that the book covers many different topics, supported with practical examples in the text or content from online videos that encourage the reader to reflect on them and put them into practice. There is a common theme throughout the book: how making learning visible can help students transfer their mathematical knowledge to new contexts and help them become independent as learners. One theme that we believe makes an essential contribution to the value of this book is the importance of using mathematical language in the process of teaching and learning mathematics. The content of this book, through teachers' reflection, represents a great opportunity to strengthen teachers' didactic knowledge and contribute to the quality of their teaching.

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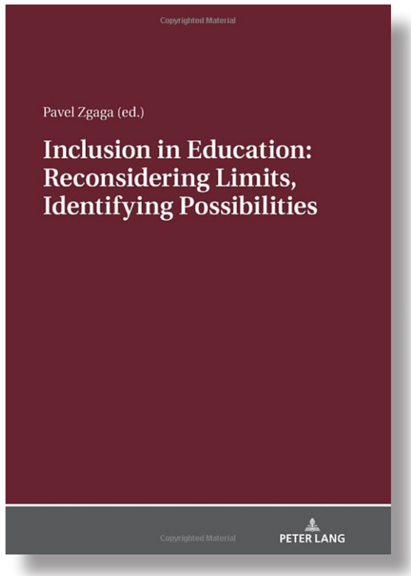
Pavel Zgaga (ed.), *Inclusion in Education: Reconsidering Limits, Identifying Possibilities*, Peter Lang: 2019; 271 pp.: ISBN 978-3-631-77859-3

Reviewed by Melina Tinnacher[1]



Edited by Pavel Zgaga and published by Peter Lang, this book on inclusion represents an excellent contribution to the ongoing debate on educational and social inclusion. The book originated from an interdisciplinary research group at the University of Ljubljana (Slovenia) and is published as an edited volume in collaboration with international scholars working on inclusion. It is addressed to anyone interested in inclusion, be it in a social, educational, or theoretical context.

The volume comprises 10 contributions divided into three parts, which discuss inclusion from a multi- and inter-disciplinary perspective. Common to all chapters is the idea of inclusion as fundamental to the challenges and changing perspectives of the 21st century. However, what distinguishes it from other volumes on the subject is the broader context in which it situates its discussion, namely society in general rather than the more restricted frame of reference of the school system. It also provides an unprecedented insight into the inclusion debate in Slovenia, comparing it with other countries.

The book is divided into three sections. Part 1 consists of four texts on educational policy, pedagogical discussions around inclusion, and educational inclusion as it relates to schools in Slovenia and Italy. Part 2, which consists of three articles, extends the perspective to the broader concept of inclusion at the societal level, dealing with migration, housing, and intersectionality. Part 3

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concludes the analysis with three articles providing a theoretical discussion of concepts of inclusion, integration, and exclusion.

Many of the contributions provide new ideas and prompt further reflection on the concept of inclusion. In the first part of the book, dealing with pedagogical and school-specific aspects of inclusion, Lani Florian highlights how teaching and differentiation techniques are changing because of the lowered expectations accompanying any official diagnosis of a special need. Her article takes a novel approach to a cornerstone of inclusive education, providing a new perspective on differentiation in the classroom. Doubts may arise while reading her argument that differentiation leads to increased exclusion in the classroom and a more significant burden in terms of marking, but the sociocultural perspective brought by the chapter is of great relevance. The need for changes in pedagogical thinking is unquestionable, and Florian's criticisms relate to the bell curve that has been an established feature of education and student achievement for far too long. She encourages readers to rethink and question approaches to inclusive education, both in their classroom practice and in research.

Peček and Macura argue that approaches should focus on the principle of need rather than the principle of equity and that it is thus vital to consider disadvantage. Their use of examples in their discussion of diversity encourages the reader to rethink stereotypes relating to teachers. Lesar and Žveglič Mihelič do not explicitly pose questions, instead offering a text on how teachers think about inclusion and a highly effective presentation of the importance of the microenvironment.

A Comparative Analysis of Inclusion in Slovenia and Italy by Zorc-Maver, Morganti, and Vogrinc takes a transnational look at inclusion, first presenting a historical background of inclusive education in both countries and then comparative research looking at teachers' perspectives and skills in Italy and Slovenia. This is followed by a discussion of the significant differences between the countries' perceptions and the development of inclusive education. *Intersectionality as a Tool for Overcoming Barriers to Inclusion* by Špela Razpotnik deals with an approach based on empirical research into families at risk. The analysis here is clear and specific as the author reflects on binary approaches and categorisations, which trap us as human beings and only allow us to actively deviate and question why we persist in certain types of discourses in the context of our nature as relational beings.

Bojan Dekleva focuses on exclusion based on residence and housing rights and references a more extensive European framework. His chapter is followed by Nina Marin's consideration of refugees, which focuses more on an inclusive approach to migration than on integrating refugees into society. Discussion of pedagogical and sociological issues is followed by a chapter on locating the phenomenon of inclusion in our thinking. Nika Šušterič situates inclusion in history and points out how the concept, and thinking related to it, may have been diluted. Kordeš and Klauser, in turn, attempt to address the question we may have been pondering throughout the previous chapters, namely the place of data when considering the phenomenon of inclusion.

In the final chapter, Krek rhetorically discusses subjective inclusion on human rights, universal historical, and epistemological perspectives. Differences are discussed throughout the chapter and form a thread throughout a self-reflection on subjective inclusion. The author presents the topic of subjective inclusion rhetorically, almost philosophically, in such an interesting way and relates it to traditional concepts of inclusion that it is absolutely stimulating to read his contribution to this extraordinary book.

In the canon of books on inclusion, this book adds value by taking the concept further and extending it to all socially disadvantaged groups. The personal reflection that it prompts makes it difficult to put the book down. It should be incorporated into scholarly discourse and research in an equally interdisciplinary way and used to encourage social science and educational science researchers to acquire new perspectives on inclusion.

The book brings expertise, discourses, and perspectives from Slovenia to the international discourse on inclusion. Although the definition of inclusion has broadened from being a purely disability-related term to cover all marginalised groups, publications still tend to refer to disadvantage as affecting one single group of people or one specific social or educational setting. This book attempts to map the full range of inclusion.

Inclusion is explored not only in an educational context but also in a variety of social contexts, which is very much one of the book's strengths. From a personal perspective, I would recommend this book to anyone with an interest in the topic. Its interdisciplinary nature makes it a fascinating read, and its thought-provoking and varied selection of articles prompt profound reflection.

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