

Digraphs with small automorphism groups that are Cayley on two nonisomorphic groups*

Luke Morgan[†] 

*Centre for the Mathematics of Symmetry and Computation,
Department of Mathematics and Statistics (M019), The University of Western Australia,
35 Stirling Highway, Crawley, 6009, Australia*

*Current address: University of Primorska, FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia,
and University of Primorska, IAM, Muzejski trg 2, 6000 Koper, Slovenia*

Joy Morris[‡] 

*Department of Mathematics and Computer Science, University of Lethbridge,
Lethbridge, AB T1K 3M4, Canada*

Gabriel Verret 

*Department of Mathematics, The University of Auckland,
Private Bag 92019, Auckland 1142, New Zealand*

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Abstract

Let $\Gamma = \text{Cay}(G, S)$ be a Cayley digraph on a group G and let $A = \text{Aut}(\Gamma)$. The Cayley index of Γ is $|A : G|$. It has previously been shown that, if p is a prime, G is a cyclic p -group and A contains a noncyclic regular subgroup, then the Cayley index of Γ is superexponential in p .

We present evidence suggesting that cyclic groups are exceptional in this respect. Specifically, we establish the contrasting result that, if p is an odd prime and G is abelian but not cyclic, and has order a power of p at least p^3 , then there is a Cayley digraph Γ on G whose Cayley index is just p , and whose automorphism group contains a nonabelian regular subgroup.

Keywords: Cayley digraphs, Cayley index.

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E-mail addresses: luke.morgan@famnit.upr.si (Luke Morgan), joy.morris@uleth.ca (Joy Morris),



Digrafi z majhnimi grupami avtomorfizmov, ki so Cayleyjevi na dveh neizomorfnih grupah*

Luke Morgan[†] 

Centre for the Mathematics of Symmetry and Computation,

Department of Mathematics and Statistics (M019), The University of Western Australia,

35 Stirling Highway, Crawley, 6009, Australia

Trenutni naslov: *University of Primorska, FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia,*

in University of Primorska, IAM, Muzejski trg 2, 6000 Koper, Slovenia

Joy Morris[‡]

Department of Mathematics and Computer Science, University of Lethbridge,

Lethbridge, AB T1K 3M4, Canada

Gabriel Verret 

Department of Mathematics, The University of Auckland,

Private Bag 92019, Auckland 1142, New Zealand

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Povzetek

Naj bo $\Gamma = \text{Cay}(G, S)$ Cayleyjev digraf na grupi G in naj bo $A = \text{Aut}(\Gamma)$. Cayleyjev indeks grafa Γ je $|A : G|$. Dokazano je že bilo, da če je p praštevilo, G ciklična p -grupa, A pa vsebuje neciklično regularno podgrupo, potem je Cayleyjev indeks grafa Γ nadeksponenten v p .

Predstavimo rezultate, ki kažejo, da so ciklične grupe izjemne v tem pogledu. Konkretno, pokažemo nasprotni rezultat: če je p liho praštevilo in G abelska, ne pa ciklična, in ima red potence števila p najmanj p^3 , potem obstaja Cayleyjev digraf Γ na G , katerega Cayleyjev indeks je samo p , in katerega grupa avtomorfizmov vsebuje neabelsko regularno podgrupo.

Ključne besede: Cayleyjevi digrafi, Cayleyjev indeks.

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g.verret@auckland.ac.nz (Gabriel Verret)

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E-poštni naslovi: luke.morgan@famnit.upr.si (Luke Morgan), joy.morris@uleth.ca (Joy Morris),

