

Products of subgroups, subnormality, and relative orders of elements

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Abstract

Let G be a group. We give an explicit description of the set of elements $x \in G$ such that $x^{|G:H|} \in H$ for every subgroup of finite index $H \leq G$. This is related to the following problem: given two subgroups H and K , with H of finite index, when does $|HK : H|$ divide $|G : H|$?

Keywords: Relative order, product of subgroups, subnormal subgroup.

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Produkti podgrup, subnormalnost in relativni vrstni red elementov

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Povzetek

Naj bo G grupa. Podamo ekspliciten opis množice elementov $x \in G$, za katere je $x^{|G:H|} \in H$ za vsako podgrupo končnega indeksa $H \leq G$. To je povezano z naslednjim problemom: če sta dani dve podgrupi H in K , in ima H končen indeks, kdaj $|HK : H|$ deli $|G : H|$?

Ključne besede: Relativni vrstni red, produkt podgrup, podnormalna podgrupa.

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