A realistic estimate of the accuracy of position measurements of characteristic terrain points via the RTK-GPS method

Realna ocena natančnosti določanja karakterističnih točk terena po metodi RTK-GPS

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Abstract: Here at the Faculty of Natural Sciences and Engineering, we are developing our very own method for determining said volumes; a method that has thus far been lacking in an estimation of the accuracy of determining the position of characteristic points of the plane via the RTK-GPS method. With the aid of measurements of three different types of points, i.e. points with three different point placements, we have managed to engulf the measurements, which we then use to shape an estimation regarding the accuracy of characteristic terrain points' measurements when it comes to those points which are not materialised i.e. labelled - the third type. When measuring the positions of characteristic terrain points, what is encompassed in the procedure is both the accuracy of the GPS measuring aid as well as the accuracy regarding of the positioning of the characteristic points themselves with an average operator in mind. For the determination of the characteristic points of the plane, and with the aid of an overviewing computer programme (Microsoft Excel), a model intended for the levelling measurements of the positions of characteristic plane points and the determination of accuracy when it comes to the determination of said point positions has now been designed. The model represents an automatised process of point position measurements' levelling, and allows one to enter measuring data – the coordinates of a given point, the *a priori* standard deviation $\hat{\sigma}_0$ and the correlation matrix **Q** as the incoming data. The outgoing data is then obtained in the form of already levelled coordinates of characteristic points, the standard deviation of the weighted standard deviation $\hat{\sigma}_0$ *a posteriori*, deviations in the directions of coordinate axes $\hat{\sigma}_Y$, $\hat{\sigma}_X$, $\hat{\sigma}_z$, the main standard deviations i.e. the attributed values of the tensor of deviations of point measurements and deviation pedaloid itself. In the computer programme MS Excel, a model aimed at the calculation of main deviations i.e. attributed

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values of the points' deviation tensor as well as the determination of pedalloid (ellipsoid) deviations is given.

Izvleček: Na Naravoslovnotehniški fakulteti se razvija lastna metoda za določanje prostornin, kateri je do sedaj manjkala ocena natančnosti določanja položaja karakterističnih točk ploskve po metodi RTK-GPS. Z meritvami točk s tremi različnimi postavitvami na točkah, in sicer s prisilnim centriranjem, s togim grezilom na znanih točkah in s togim grezilom s simuliranjem postavitve na karakt. točkah smo zajeli meritve, s katerimi smo podali oceno natančnosti merjenj karakt. točk terena, ki niso materializirane oziroma označene. Tako je zajeta natančnost GPS izmere kot tudi natančnost postavitve na karakt. točkah terena pri povprečnem operaterju. Za določitev natančnosti karakt. točk ploskve je sedaj, s pomočjo računalniškega programa za preglednice, izdelan model za izravnavo meritev položaja karakt. točk ploskve ter določitev natančnosti določanja položaja karakt. točk ploskve. Model predstavlja avtomatiziran proces izravnave merjenj položaja točke in omogoča, da se kot vhodni podatki vstavijo podatki merjenj, in sicer koordinate točke, *a priori* srednji pogrešek $\hat{\sigma}_0$ in korelacijska matrika **Q**. Izhodni podatki pa se dobijo kot izravnane koordinate karakt. točk, srednji pogrešek utežne enote $\hat{\sigma}_0$ *a posteriori*, pogreški v smereh koordinatnih osi $\hat{\sigma}_Y$, $\hat{\sigma}_X$, $\hat{\sigma}_z$, glavni srednji pogreški oziroma lastne vrednosti tenzorja pogreškov merjenja točke in pedaloid pogreškov. V računalniškem programu MS Excel je podan tudi model za izračun glavnih pogreškov oziroma lastnih vrednosti tenzorja pogreškov točke in določitev pedaloidov (elipsoidov) pogreškov.

Key words: RTK-GPS, RTK-DGPS, main deviations, accuracy
Ključne besede: RTK-GPS, RTK-DGPS, glavni pogreški, natančnost

INTRODUCTION

Focusing on realistic, on-location measurements of characteristic terrain points and their belonging errors, rather than only those allotted by whatever programme we happen to be using to later on process said measured data, we are able to get a truer-to-life assessment of the accuracy of our measurements. The RTK-GPS (Real Time Kinematic - Global Positioning System) is one of the tools we may use to aid us in the accomplishment of this. It may actually be used for both determining the shape of our measured terrain as well as its belonging surface and volume, which proves useful in engulfing the measurements of points of three different point placements, further categorised as we elaborate on the procedure in pages to come. Thus, we are one step closer to shaping an estimation regarding the accuracy of characteristic terrain points' measurements.

What is important is that we keep in mind both the accuracy of the GPS measuring aid as well as the accuracy of an average human operator dealing with it when it comes to the positioning of the the characteristic points. As for the actual determination of the latter, what has now been designed with the aid of Microsoft Excel, our chosen overwieving computer tool, is a model intended for the levelling measurements of the positions of characteristic plane points and the determination of accuracy when it comes to said point positions. Here at the Faculty of Natural Sciences and Engineering, the design of such a simple and financially undemanding model - oriented at the levelling that comes with characteristic points' accuracy measurements - has still managed to enhance the accuracy estimations associated with the measurements of characteristic terrain points, as we will now proceed to make evident.

The levelling measurement of points aimed at determining the accuracy of the characteristic points of a plane

When it comes to a derisive levelling of corellated measurements, it is necessary to determine the most probably value of the unknown vector $\underset{u\times1}{\mathbf{x}}$ with the aid of the vector of measured quantities $\underset{nu\times1}{\mathbf{l}}$ with the aid of *a priori* known corellations $\underset{nu\times nu}{\Sigma}$, and give an estimate i.e. evaluation of the accuracy of all these measuremants i.e. the sought-for quantities. When it comes to the latter, the number $\boldsymbol{n}$ is the number of measured quantities i.e. measurements, and $\boldsymbol{u}$ is the number of sought-for (unknown) quantities.

With one derisive levelling of corellated measurements, the sought-for (unknown) quantities are determined via a string of measurements, but all this under the condition that the sum of squares of their correlated residuals is smallest:

$$\underset{1\times nu}{\mathbf{v}^{\mathrm{T}}}\ \underset{nu\times nu}{\mathbf{\Sigma}_{ll}^{-1}}\ \underset{nu\times1}{\mathbf{v}} = \min \tag{1}$$

where:

- $\underset{nu\times nu}{\mathbf{\Sigma}_{ll}^{-1}}$, the correlation matrix of measured quantities is the matrix of cofactors of said measured quantities
- $\underset{nu\times1}{\mathbf{v}}$ is the vector of residuals of the measured quantities.

When it comes to measuring sciences, the determination of a uniform solution is obtained through the least squares method (LS).

In the case of a derisive levelling procedure, what is determined first is the residuals' vector $\underset{u\times1}{\xi}$ of approximate values of the unknown values $\underset{u\times1}{\mathbf{x}}$, and then the residuals of measured quantities $\underset{nu\times1}{\mathbf{v}}$ may be dealt with. For n derisive measurements, the vector of residuals is labelled as $\underset{nu\times1}{\mathbf{v}}$, the matrix of coefficients of equations of the residuals – the design matrix - as $\underset{nu\times u}{\mathbf{A}}$, the vector of derivatives of unknowns as $\underset{u\times1}{\xi}$, and the vector of free terms with $\underset{n\times1}{\mathbf{f}}$, which would make it:

$$\underset{nu\times1}{\mathbf{v}}=\begin{bmatrix}\underset{u\times1}{\mathbf{v}_1}\\ \underset{u\times1}{\mathbf{v}_2}\\ \underset{u\times1}{\mathbf{v}_3}\\ \vdots\\ \underset{u\times1}{\mathbf{v}_n}\end{bmatrix}_{nu\times1},\ \underset{nu\times u}{\mathbf{A}}=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1u}\\ a_{21} & a_{22} & \cdots & a_{2u}\\ \vdots & \vdots & \ddots & \vdots\\ a_{n1} & a_{n2} & \cdots & a_{nu}\end{bmatrix},\ \underset{u\times1}{\xi}=\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\\ \vdots\\ \xi_u\end{bmatrix}_{u\times1},\ \underset{nu\times1}{\mathbf{f}}=\begin{bmatrix}\underset{u\times1}{\mathbf{f}_1}\\ \underset{u\times1}{\mathbf{f}_2}\\ \underset{u\times1}{\mathbf{f}_3}\\ \vdots\\ \underset{u\times1}{\mathbf{f}_n}\end{bmatrix}_{nu\times1} \tag{2}$$

That is how, for n derisive measurments, with the aid of with the u of unknowns is to be determined, the system of residual equations of measured quantities equals:

$$\underset{nu\times1}{\mathbf{v}} = \underset{nu\times u}{\mathbf{A}} \cdot \underset{u\times1}{\xi} + \underset{nu\times1}{\mathbf{f}} \tag{3}$$

The vector of free terms $\underset{nux1}{\mathbf{f}}$ can symbolically be denoted as:

f = approximate value – measured value

$$\mathbf{f} = \mathbf{f}_{pribl} - \mathbf{1}$$
$$\mathbf{f} = \mathbf{f}\,(\xi_1\xi_2\xi_3...\xi_u) - \mathbf{1} \qquad (4)$$

For *u* unknowns, equations of free terms for an *n* number of measurements may be written as:

$$\begin{aligned} \underset{ux1}{\mathbf{f}_1} &= \mathbf{f}_1\,(\xi_1\xi_2\xi_3...\xi_u) - \underset{ux1}{\mathbf{1}_1} \\ \underset{ux1}{\mathbf{f}_2} &= \mathbf{f}_2\,(\xi_1\xi_2\xi_3...\xi_u) - \underset{ux1}{\mathbf{1}_2} \\ \underset{ux1}{\mathbf{f}_3} &= \mathbf{f}_3\,(\xi_1\xi_2\xi_3...\xi_u) - \underset{ux1}{\mathbf{1}_3} \\ &\vdots \\ \underset{ux1}{\mathbf{f}_n} &= \mathbf{f}_n\,(\xi_1\xi_2\xi_3...\xi_u) - \underset{ux1}{\mathbf{1}_n} \end{aligned} \qquad (5)$$

The determination of a vector of unknowns $\underset{ux1}{\xi}$[1], the calculation of a matrix of coefficients of normal equations $\underset{uxu}{\mathbf{N}}$ and the determination of the vector of free terms $\underset{ux1}{\mathbf{n}}$ (FEIL, 1990)

The vector of unknowns $\underset{ux1}{\xi}$ is begotten:

$$\underset{ux1}{\xi} = -\underset{uxu}{\mathbf{N}^{-1}}\,\underset{ux1}{\mathbf{n}} \qquad (6)$$

where:

- $\underset{uxu}{\mathbf{N}}$ is the matrix of the coefficients of normal equations:

$$\underset{uxu}{\mathbf{N}} = \underset{uxnu}{\mathbf{A}^T}\,\underset{nuxnu}{\Sigma_{\hat{l}\hat{l}}^{-1}}\,\underset{nuxu}{\mathbf{A}} \qquad (7)$$

- $\underset{ux1}{\mathbf{n}}$ is the vector of free terms:

$$\underset{ux1}{\mathbf{n}} = \underset{uxnu}{\mathbf{A}^T}\,\underset{nuxnu}{\Sigma_{\hat{l}\hat{l}}^{-1}}\,\underset{unx1}{\mathbf{A}} \qquad (8)$$

- $\underset{nuxu}{\mathbf{A}}$ is the matrix of coefficients of residual equations - the so-called design matrix
- $\underset{nuxnu}{\Sigma_{\hat{l}\hat{l}}^{-1}}$ is the corellation matrix of measured quantities

Considering that:

$$\underset{uxu}{\mathbf{Q}_{\xi\xi}} = \underset{uxu}{\mathbf{N}^{-1}} \qquad (9)$$

- $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$ is the matrix of covariances and the vector of unknowns denoted as $\underset{ux1}{\xi}$ may also be written down as:

$$\underset{ux1}{\xi} = -\underset{uxu}{\mathbf{Q}_{\xi\xi}}\,\underset{ux1}{\mathbf{n}} \qquad (10)$$

The correlation matrix of measured quantities $\underset{nuxnu}{\Sigma_{\hat{l}\hat{l}}}$ (FEIL, 1990)

For each correlated measurement l_i there is a belonging – on the basis of calculated variances and covariances pertaining to prior variantly known measurements – covariance matrix i.e. correlation between measured quantities $\underset{uxu}{\Sigma}$ and the *a priori* standard deviation of the (prior) levelling σ_0. For murtually dependent measurements of varying accuracies, the matrix of cofactors of each measurement by itself is a symmetrical matrix, as seen below:

$$\underset{uxu}{\Sigma_{\hat{l}\hat{l}}} = \hat{\sigma}_{0i}^2\,\underset{uxu}{\mathrm{Q}_i} \qquad (11)$$

$$\underset{u\times u}{\Sigma_{\hat{l}\hat{l}i}} = \hat{\sigma}_{0i}^2 \cdot \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} & \cdots & q_{xu} \\ q_{xy} & q_{yy} & q_{yz} & \cdots & q_{yu} \\ q_{xz} & q_{yz} & q_{zz} & \cdots & q_{zu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{xu} & q_{yu} & q_{zu} & \cdots & q_{uu} \end{bmatrix}_{u\times u} \qquad (12)$$

[1] In an ample amount of books pertaining to this area, the vector of unknowns is denoted also as **x**. Due to practical reasons, however, the vector of unknowns may also be denoted with the Greek letter ξ, thus making matters more concise when it is to be dealt with in actual equations.

$$\underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}_i}} = \begin{bmatrix} q_{xx}\hat{\sigma}_{0i}^2 & q_{xy}\hat{\sigma}_{0i}^2 & q_{xz}\hat{\sigma}_{0i}^2 & \cdots & q_{xu}\hat{\sigma}_{0i}^2 \\ q_{xy}\hat{\sigma}_{0i}^2 & q_{yy}\hat{\sigma}_{0i}^2 & q_{yz}\hat{\sigma}_{0i}^2 & \cdots & q_{yu}\hat{\sigma}_{0i}^2 \\ q_{xz}\hat{\sigma}_{0i}^2 & q_{yz}\hat{\sigma}_{0i}^2 & q_{zz}\hat{\sigma}_{0i}^2 & \cdots & q_{zu}\hat{\sigma}_{0i}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{xu}\hat{\sigma}_{0i}^2 & q_{yu}\hat{\sigma}_{0i}^2 & q_{zu}\hat{\sigma}_{0i}^2 & \cdots & q_{uu}\hat{\sigma}_{0i}^2 \end{bmatrix}_{u\times u} = \hat{\sigma}_{0i}^2 \underset{u\times u}{\mathbf{Q}_{lli}} \qquad (13)$$

The correlatory matrix of measured quantities $\underset{nu\times nu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}}}$ for u unknowns and n measurements is made up of the following sub-matrices:

$$\underset{nu\times nu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}ij}} = \begin{bmatrix} \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}11}} & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}12}} & \cdots & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}1n}} \\ \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}12}} & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}22}} & \cdots & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}2n}} \\ \vdots & \vdots & \ddots & \vdots \\ \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}1n}} & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}2n}} & \cdots & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}nn}} \end{bmatrix}_{nu\times nu} \qquad (14)$$

Sub-matrices are of the shape depicted below:

$$\underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}ii}} = \begin{bmatrix} \Sigma_{xxi} & \Sigma_{xyi} & \Sigma_{xzi} & \cdots & \Sigma_{xui} \\ \Sigma_{xyi} & \Sigma_{yyi} & \Sigma_{yzi} & \cdots & \Sigma_{yui} \\ \Sigma_{xzi} & \Sigma_{yzi} & \Sigma_{zzi} & \cdots & \Sigma_{zui} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{xui} & \Sigma_{yui} & \Sigma_{zui} & \cdots & \Sigma_{uui} \end{bmatrix}_{u\times u} \qquad (15)$$

$$\underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}ij}} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{u\times u} \qquad (16)$$

That is how the correlation matrix of measured values is composed of the following sub-matrices:

$$\underset{nu\times nu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}}} = \begin{bmatrix} \begin{bmatrix} \Sigma_{xx1} & \Sigma_{xy1} & \Sigma_{xz1} & \cdots & \Sigma_{xu1} \\ \Sigma_{xy1} & \Sigma_{yy1} & \Sigma_{yz1} & \cdots & \Sigma_{yu1} \\ \Sigma_{xz1} & \Sigma_{yz1} & \Sigma_{zz1} & \cdots & \Sigma_{zu1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{xu1} & \Sigma_{yu1} & \Sigma_{zu1} & \cdots & \Sigma_{uu1} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} & \cdots & \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} & \begin{bmatrix} \Sigma_{xx2} & \Sigma_{xy2} & \Sigma_{xz2} & \cdots & \Sigma_{xu2} \\ \Sigma_{xy2} & \Sigma_{yy2} & \Sigma_{yz2} & \cdots & \Sigma_{yu2} \\ \Sigma_{xz2} & \Sigma_{yz2} & \Sigma_{zz2} & \cdots & \Sigma_{zu2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{xu2} & \Sigma_{yu2} & \Sigma_{zu2} & \cdots & \Sigma_{uu2} \end{bmatrix} & \cdots & \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \\ \vdots & \vdots & \vdots & \vdots \\ \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} & \cdots & \begin{bmatrix} \Sigma_{xxn} & \Sigma_{xyn} & \Sigma_{xzn} & \cdots & \Sigma_{xun} \\ \Sigma_{xyn} & \Sigma_{yyn} & \Sigma_{yzn} & \cdots & \Sigma_{yun} \\ \Sigma_{xzn} & \Sigma_{yzn} & \Sigma_{zzn} & \cdots & \Sigma_{zun} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Sigma_{xun} & \Sigma_{yun} & \Sigma_{zun} & \cdots & \Sigma_{uun} \end{bmatrix} \end{bmatrix}_{nu\times nu} \qquad (17)$$

which can, in short, be denoted as:

$$\underset{nu\times nu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}}} = \underset{nu\times nu}{\begin{bmatrix} \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}11}} & \underset{u\times u}{\mathbf{0}} & \cdots & \underset{u\times u}{\mathbf{0}} \\ \underset{u\times u}{\mathbf{0}} & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}22}} & \cdots & \underset{u\times u}{\mathbf{0}} \\ \vdots & \vdots & \ddots & \vdots \\ \underset{u\times u}{\mathbf{0}} & \underset{u\times u}{\mathbf{0}} & \cdots & \underset{u\times u}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}nn}} \end{bmatrix}} \tag{18}$$

The inverse (matrix) of the matrix of measured quantities is thus denoted as:

$$\underset{nu\times nu}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}}} = \underset{nu\times nu}{\begin{bmatrix} \underset{u\times u}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}11}} & \underset{u\times u}{\mathbf{0}} & \cdots & \underset{u\times u}{\mathbf{0}} \\ \underset{u\times u}{\mathbf{0}} & \underset{u\times u}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}22}} & \cdots & \underset{u\times u}{\mathbf{0}} \\ \vdots & \vdots & \vdots & \vdots \\ \underset{u\times u}{\mathbf{0}} & \underset{u\times u}{\mathbf{0}} & \cdots & \underset{u\times u}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}nn}} \end{bmatrix}} \tag{19}$$

which may, again, be shortened to:

$$\underset{nu\times nu}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}}} = \mathrm{diag}\begin{bmatrix} \underset{u\times u}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}11}} & \underset{u\times u}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}22}} & \cdots & \underset{u\times u}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}nn}} \end{bmatrix} \tag{20}$$

and is the row-specific matrix of the inverse correlation sub-matrices $\underset{uxu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}ii}}$ of individual measured unknowns $\xi_1, \xi_2, \xi_3, ..., \xi_u$.

When we are dealing with the measurements of point positions via the RTK-GPS method, each measurement in itself may beget its own varance-covariance matrix $\underset{nuxnu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}}}$. The variance-covariance matrix $\underset{nuxnu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}}}$ is determined on the basis of different measurement influences of the time. Sad »influences« are, in fact, the disparations of the receiver's an the satellite's timers, the atmosphere's influence upon wave expansion, a mistake pertaining to the determination of the hight of the antennae etc.

For an individual point measurement, we are thus able to obtain the coordinates of the measured point, as well as the sub-matrix $\underset{3x3}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}i}}$.

As has already been depicted in equations (14) to (17), for the determination of a variance-covariance matrix $\underset{nuxnu}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}}}$, it is already enough when we are acquainted with the correlation matrices of an individual measurement $\underset{3x3}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}i}}$. The sub-matrices $\underset{3x3}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}i}}$ are symmetrical matrices, and can therefore be written as below for each of the individual measurements:

$$\underset{3\times 3}{\boldsymbol{\Sigma}_{\hat{l}\hat{l}i}} = \hat{\sigma}_0^2 \begin{bmatrix} Q11 & Q12 & Q13 \\ & Q22 & Q23 \\ & & Q33 \end{bmatrix} = \hat{\sigma}_0^2 \begin{bmatrix} Q11 & Q12 & Q13 \\ Q12 & Q22 & Q23 \\ Q13 & Q23 & Q33 \end{bmatrix} = \hat{\sigma}_0^2 \underset{3\times 3}{\mathbf{Q}_{lli}} \tag{21}$$

- i , the number of the measurement

THE EVALUATIONS OF ACCURACY OF THE LEVELLING OF AN INDIVIDUAL QUANTITY

The standard deviation of a given levelling procedure - $\hat{\sigma}_0$

$$\hat{\sigma} = \sqrt{\frac{\underset{1\times un}{\mathbf{v}^T} \underset{nu\times nu}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}}} \underset{un\times 1}{\mathbf{v}}}{nu - u}} \tag{22}$$

- $r = nu - u$, the number of meaurements above the required number
- nu, the number of measured quantities i.e .the number of measurements
- u, the number of sought-for (unknown) quantities

- $\underset{nuxnu}{\boldsymbol{\Sigma}^{-1}_{\hat{l}\hat{l}}}$, the correlation matrix of measured quantities, the matrix of cofactors of measured quantities
- $\underset{unx1}{\mathbf{v}}$, the vector of residuals of measured quantities

The standard deviation of unknown quantities - $\hat{\sigma}_{\xi}$

The medium residuals of unknowns are the functions of standard deviations of meaurements. Unknowns can thus be denoted as the functions of measurements bearing in mind that we must take into consideration the Law of Cofactor Increment. The matrix of covariances $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$ is equal to the inverse matrix of coefficients of normal equations $\underset{uxu}{\mathbf{N}^{-1}}$.

For three unknowns, the matrix of covariances $\underset{3x3}{\mathbf{Q}_{\xi\xi}}$ may be written in the form of:

$$\underset{3\times3}{\mathbf{Q}_{\xi\xi}} = \underset{3\times3}{\mathbf{N}^{-1}} = \underset{3\times3}{\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xz} \\ Q_{xy} & Q_{yy} & Q_{yz} \\ Q_{xz} & Q_{yz} & Q_{zz} \end{bmatrix}} \quad (23)$$

The necessary cofactors, intended for the estimation of the accuracy of the unknowns, are on the main diagonal of the covariance matrix (the matrix of the cofactors of the unknowns) $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$. The standard deviations of the unknown quantities are therefore:

$$\begin{aligned} \hat{\sigma}_{\xi_1} &= \hat{\sigma}_0 \sqrt{Q_{\xi_1\xi_1}} \\ \hat{\sigma}_{\xi_2} &= \hat{\sigma}_0 \sqrt{Q_{\xi_2\xi_2}} \\ \hat{\sigma}_{\xi_3} &= \hat{\sigma}_0 \sqrt{Q_{\xi_3\xi_3}} \\ &\vdots \\ \hat{\sigma}_{\xi_u} &= \hat{\sigma}_0 \sqrt{Q_{\xi_u\xi_u}} \end{aligned} \quad (24)$$

or even:

$$\hat{\sigma}_{\xi} = \hat{\sigma}_0 \sqrt{Q_{\xi\xi}} \quad (25)$$

The standard deviation of levelled quantities - $\hat{\sigma}_{l'l'}$

$$\hat{\sigma}_{l'l'i} = \hat{\sigma}_0 \sqrt{K_{l'l'ii}} \quad (26)$$

The necessary cofactors for an evaluation of accuracy of said levelled quantities are on the main diagonal of the matrix of cofactors of levelled quantities, $\underset{nuxnu}{\boldsymbol{\Sigma}_{\hat{l}'\hat{l}'}}$:

$$\underset{nuxnu}{\mathbf{Q}_{l'l'}} = \underset{nuxu}{\mathbf{A}} \; \underset{uxu}{\mathbf{Q}_{\xi\xi}} \; \underset{uxnu}{\mathbf{A}^T} \quad (27)$$

The standard deviation of residuals of measured quantities - $\hat{\sigma}_{vv}$

The standard deviation of residuals of measured quantities may be gotten through the following:

$$\hat{\sigma}_{vvi} = \sqrt{\Sigma_{vvii}} = \sqrt{\Sigma_{llii} - \Sigma_{l'l'ii}} \quad (28)$$

The cofactors necessary to obtain an evaluation of the accuracy of the residuals of our measured values $\boldsymbol{\Sigma}_{vvii}$, can be obtained as the difference between the cofactors of measured quantities $\boldsymbol{\Sigma}_{llii}$ and the cofactors of levelled quantities $\boldsymbol{\Sigma}_{l'l'ii}$.

Determination of the main standard deviations of unknowns:

During a given levelling of derisive measurements, the position of a point may be determined on the basis of certain previously measured quantities. That is how, besides levelled quantities of unknowns, ξ_1, ξ_2, ξ_3, ..., ξ_u $\underset{ux1}{\boldsymbol{\xi}}$, one also obtains the covariance matrix, $\boldsymbol{\Sigma}_{\xi\xi}$:

$$\underset{u\times 1}{\boldsymbol{\xi}} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \vdots \\ \xi_u \end{bmatrix}_{u\times 1} \qquad (29)$$

$$\underset{u\times u}{\boldsymbol{\Sigma}_{\xi\xi}} = \begin{bmatrix} \hat{\sigma}_x^2 & \hat{\sigma}_{xy} & \hat{\sigma}_{xz} & \cdots & \hat{\sigma}_{xu} \\ \hat{\sigma}_{xy} & \hat{\sigma}_y^2 & \hat{\sigma}_{yz} & \cdots & \hat{\sigma}_{yu} \\ \hat{\sigma}_{xz} & \hat{\sigma}_{yz} & \hat{\sigma}_z^2 & \cdots & \hat{\sigma}_{zu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{xu} & \hat{\sigma}_{yu} & \hat{\sigma}_{zu} & \cdots & \hat{\sigma}_u^2 \end{bmatrix} = \qquad (30)$$

$$= \hat{\sigma}_0^2 \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} & \cdots & q_{xu} \\ q_{xy} & q_{yy} & q_{yz} & \cdots & q_{yu} \\ q_{xz} & q_{yz} & q_{zz} & \cdots & q_{zu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{xu} & q_{yu} & q_{zu} & \cdots & q_{uu} \end{bmatrix} = \hat{\sigma}_0^2 \underset{u\times u}{\mathbf{Q}_{\xi\xi}}$$

The elements on the main diagonal of the covariance matrix, $\underset{uxu}{\boldsymbol{\Sigma}_{\xi\xi}}$ are squares of standard deviations of the determination of the position of our given point in the direction of the coordinate axis. Many times, however, it is also necessary to determine the maximum - or minimum, depending on the case - values of said deviations.

The values of our main deviations (the maximum, minimum and binormal standard deviations) are determined with the aid of the selves' values of λ_i and their belonging selves' vectors $\mathbf{s}_i$ of the covariance matrix $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$. The »self values« of the covariance matrix, $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$, are obtained according to the equation:

$$\det(\underset{uxu}{\mathbf{Q}_{\xi\xi}} - \lambda \mathbf{I}) = 0 \qquad (31)$$

and the equation for the determination of self values λ_i of the covariance matrix $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$ is obviously not much different:

$$\begin{bmatrix} q_{xx}-\lambda & q_{xy} & q_{xz} & \cdots & q_{xu} \\ q_{xy} & q_{yy}-\lambda & q_{yz} & \cdots & q_{yu} \\ q_{xz} & q_{yz} & q_{zz}-\lambda & \cdots & q_{zu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{xu} & q_{yu} & q_{zu} & \cdots & q_{uu}-\lambda \end{bmatrix} = 0 \quad (32)$$

The characteristic equation then goes like this:

$$P_u(\lambda)=(-1)^u \lambda^u+a_{u-1}\lambda^{u-1}+...+a_1\lambda+a_0 \qquad (33)$$

- $P_u(\lambda)$ being the characteristic polynomial

The »zeroes« of the characteristic polynomial $P_u(\lambda)$ are the self values of the covariance matrix $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$.

The self values λ_i, of course, have belonging self vectors denoted as $\mathbf{s}_i$, of the covariance matrix $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$, and the latter are defined as:

$$\underset{uxu}{\mathbf{Q}_{\xi\xi}} \cdot \mathbf{s} = \lambda \cdot \mathbf{s} \qquad (34)$$

Each self value of λ_i provides one witha system of equations with the aid of which the afore-mentioned $\mathbf{s}_i$ vectors may be determined.

Determining the solutions of higher-degree polynomials is a very expansive and demanding practice, which is why various computer programmes have been enlisted with this very purpose in mind, aiding us in this quest to determine the previously mentioned self values and vectors.

The self values of λ_i and their belonging self vectors $\mathbf{s}_i$ of the covariance matrix $\underset{uxu}{\mathbf{Q}_{\xi\xi}}$ determine the quantitative values and directions of our main deviations. The quantities of main standard deviations of unknowns are thus determined with the aid of the following equations:

$$\begin{aligned}
\hat{\sigma}_{1_eigen} &= \hat{\sigma}_0 \sqrt{\lambda_1} \\
\hat{\sigma}_{2_eigen} &= \hat{\sigma}_0 \sqrt{\lambda_2} \\
\hat{\sigma}_{3_eigen} &= \hat{\sigma}_0 \sqrt{\lambda_3} \\
&\vdots \\
\hat{\sigma}_{u_eigen} &= \hat{\sigma}_0 \sqrt{\lambda_u}
\end{aligned} \tag{35}$$

- $\hat{\sigma}_0$, the standard deviation of the levellin of the measured quantities
- λ_1, λ_2, λ_3, ..., λ_n the self values of the covariance matrix $\mathbf{Q}_{uxu\,\zeta\zeta}$
- $\hat{\sigma}_{u\_eigen}$, the main deviation of unknowns in the direction of itself's coordinate system plane (of a pedalloid as well as ellipoid)

The main standard deviations and the pedalloid (ellipsoid) of deviations

The directions of mai deviation unknowns $\hat{\sigma}_{1\_eigen}$, $\hat{\sigma}_{2\_eigen}$, $\hat{\sigma}_{3\_eigen}$, ..., $\hat{\sigma}_{u\_eigen}$ are determined by the self vectors $\mathbf{s}_i$. That is how the $s_{\hat{\sigma}i}$ vectors, which obviously determine the directions and quantities of main standard deviations, $\hat{\sigma}_{1\_eigen}$, $\hat{\sigma}_{2\_eigen}$, $\hat{\sigma}_{3\_eigen}$, ..., $\hat{\sigma}_{u\_eigen}$, of unknowns, may be written down as:

$$\begin{aligned}
\mathbf{s}_{\hat{\sigma}1} &= \hat{\sigma}_0 \sqrt{\lambda_1} \cdot \mathbf{s}_1 \\
\mathbf{s}_{\hat{\sigma}2} &= \hat{\sigma}_0 \sqrt{\lambda_2} \cdot \mathbf{s}_2 \\
\mathbf{s}_{\hat{\sigma}3} &= \hat{\sigma}_0 \sqrt{\lambda_3} \cdot \mathbf{s}_3 \\
&\vdots \\
\mathbf{s}_{\hat{\sigma}u} &= \hat{\sigma}_0 \sqrt{\lambda_u} \cdot \mathbf{s}_u
\end{aligned} \tag{36}$$

If the main standard deviations are written down for three particular coordinates *X, Y, Z*, then:

$$\begin{aligned}
\hat{\sigma}_{\text{binor}} &= \hat{\sigma}_0 \sqrt{\lambda_1} = a \\
\hat{\sigma}_{\text{minor}} &= \hat{\sigma}_0 \sqrt{\lambda_2} = b \\
\hat{\sigma}_{\text{major}} &= \hat{\sigma}_0 \sqrt{\lambda_3} = c
\end{aligned} \tag{37}$$

are the equations for the determination of main deviations of the position of the point. The main standard deviations $\hat{\sigma}_{\text{binor}}$, $\hat{\sigma}_{\text{minor}}$, $\hat{\sigma}_{\text{major}}$ represent the values of the half-axes *a, b, c* of the deviations' ellipsoid. The directions and quantitative values of the half-axes *a, b, c* of the deviations' ellipsoid may be obtained with the aid of vectors $s_{\hat{\sigma}x}$, $s_{\hat{\sigma}y}$, $s_{\hat{\sigma}z}$, as in:

$$\begin{aligned}
\mathbf{s}_{\hat{\sigma}x} &= \hat{\sigma}_0 \sqrt{\lambda_1} \cdot \mathbf{s}_1 \\
\mathbf{s}_{\hat{\sigma}y} &= \hat{\sigma}_0 \sqrt{\lambda_2} \cdot \mathbf{s}_2 \\
\mathbf{s}_{\hat{\sigma}z} &= \hat{\sigma}_0 \sqrt{\lambda_3} \cdot \mathbf{s}_3
\end{aligned} \tag{38}$$

A realistic evaluation of the accuracy of the measurements of the position of a plane's characteristic points

The characteristic points of a plane

The data in connection to the positions of characteristic points of a given plane have been obtained on-terrain, via the methods of RTK-GPS for three different types of point placement, namely via forced centering (type 01), a lame plumb line upon the known points (type 02), or a lame plumb line and a simulated placement upon the characteristic points (type 03). Even the levelling and evaluation of measurements according to the RTK-GPS (Real Time Kinematic - Global Positioning System) method has been developed in accordance with all three types of point placement, which enables a comparison of the measurement results and their accuracy for all three types. The points are sorted as the figure directly below depicts.

For each type of point (type 01 – type 03), four points have been measured with 25 separate measurements, and four points with five measurements per point. That is how a number of measurements above the required number has been secured, and the procedure of levelling and accuracy evaluation may thus commence. The label of a given point represents its type and how many times exactly it has been measured. Points with an odd first digit have been measured 25 times, and those with an even first digit have had their positions measured five times. The second digit pertaining to the label on each individual point, on the other hand, denotes the type of positioning upon the point, i.e. the numbers 1, 2 and 3 inform us whether is is a (type 01), (type 02) or (type 03), respectively. The palcement upon a point which is labelled as belonging to (type 01) is a placement upon a known point on the ground, with forced centering via a tripod and antenna latched onto said tripod, whereas a (type 02) informs us that teh positioning has been done with a lame plumb line upon a known point on the ground and (type 03) with the same sort of lame plumb line upon a point, except that here it is on the ground within the area which has been labelled via circularly distributed spray paint with a circumference of approximately 1m. These circles have not been drawn with the utmost precision, but via free-hand, and

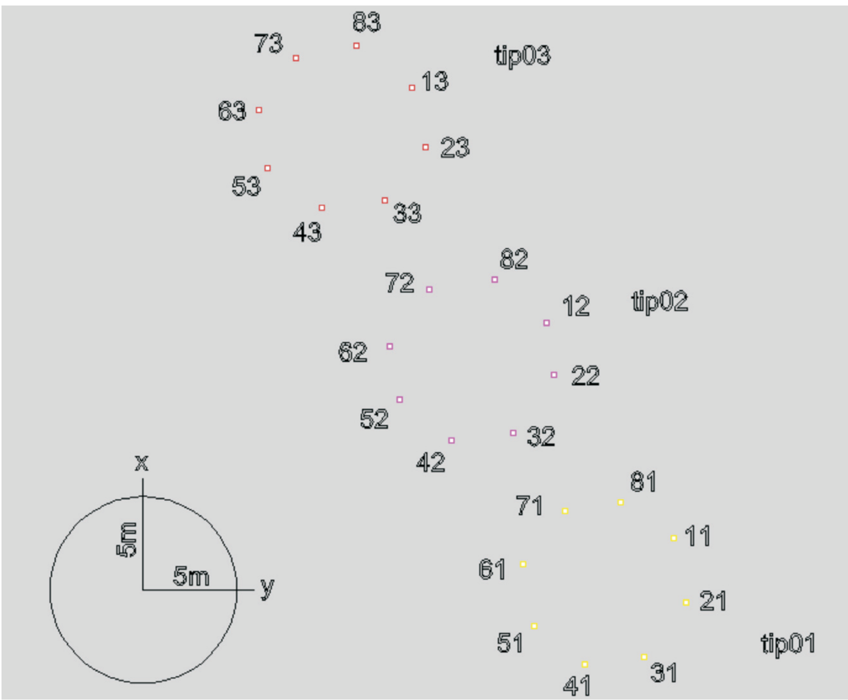

Figure 1. The point arrangement and labels
Slika 1. Razporeditev in oznake točk

Table 1. Maximum deviations in the direction of coordinate axes and main deviations (type 01)
Tabela 1. Maksimalni pogreški v smeri koordinatnih osi in glavni pogreški (tip 01)

Tripod points (type 01):

			$\hat{\sigma}_{x1}$	
		Maximum deviation of the coordinate axes		
No. of measurements	Point	Z	X	Y
25	11	0,0020	0,0010	0,0008
25	31	0,0017	0,0008	0,0006
25	51	0,0016	0,0007	0,0006
25	71	0,0019	0,0009	0,0007
5	21	0,0037	0,0018	0,0014
5	41	0,0036	0,0017	0,0013
5	61	0,0040	0,0019	0,0015
5	81	0,0045	0,0021	0,0017

		$\hat{\sigma}_{max1}$	
	Main deviations		
Point	ζ	ξ	ψ
11	0,0020	0,0010	0,0008
31	0,0017	0,0008	0,0006
51	0,0016	0,0007	0,0006
71	0,0019	0,0009	0,0007
21	0,0037	0,0018	0,0014
41	0,0036	0,0017	0,0013
61	0,0041	0,0019	0,0015
81	0,0045	0,0021	0,0017

were consequently less like circles than ellipses. The figurant had tried to place the plumb line on the same place every time, but only with the aid of a visual evaluation of where the pervieved centre of the circle is.

The results of the levelling of characteristic point measurements

In the comparative tables below, the results of said levelling of characteristic point measurements are presented.

The said results have shown that the folowing for point of forced centering (type 01):

- Deviations in the direction of coordinate axis *Z* do not exceed 5 mm (the largest one would be 4.5 mm), even for the smallest number of measurements (5).
- Deviations in the direction of coordinate axis *Y* do not surpass 2 mm (the largest one would be 1.7 mm).
- Deviations in the direction of coordinate axis *X* do not surpass 2.5 mm (the largest one would be 2.1 mm).

This is all depicted in the table 1.

Results of the levelling of characteristic point measurements with a lame plumb line upon known points show that:

- Deviations in the direction of coordinate axis *Z* do not exceed 8 mm (the largest one would be 7.4 mm), even when it comes to the smallest number of measurements (3).
- Deviations in the direction of coordinate axis *Y* do not exceed 3 mm (the largest one would be 2.5 mm).
- Deviations in the direction of coordinate axis *X* do not exceed 3 mm (the largest one would be 3.0 mm).

Again, this is all depicted in the table 2.

Table 2. Maximum deviations in the direction of coordinate axes and main deviations (type 02)
Tabela 2. Maksimalni pogreški v smeri koordinatnih osi in glavni pogreški (tip 02)

Lame plumb line points – determined in the ground (type 02):

		$\hat{\sigma}_{x2}$				$\hat{\sigma}_{max2}$		
		Maximum deviation of the coordinate axes				Main deviations		
No. of measurements	Point	Z	X	Y	Point	Z	X	Y
19	12	0,0026	0,0012	0,0010	12	0,0027	0,0012	0,0010
19	32	0,0027	0,0013	0,0010	32	0,0027	0,0013	0,0010
19	52	0,0025	0,0011	0,0009	52	0,0025	0,0011	0,0009
21	72	0,0022	0,0010	0,0008	72	0,0022	0,0010	0,0008
3	22	0,0037	0,0016	0,0013	22	0,0037	0,0016	0,0013
3	42	0,0074	0,0030	0,0025	42	0,0074	0,0030	0,0025
4	62	0,0053	0,0022	0,0018	62	0,0053	0,0023	0,0018
4	82	0,0049	0,0023	0,0018	82	0,0049	0,0023	0,0018

Table 3. Maximum deviations in the direction of coordinate axes and main deviations (type 03)
Tabela 3. Maksimalni pogreški v smeri koordinatnih osi in glavni pogreški (tip 03)

Lame plumb line points – not determined in the ground (type 03):

		$\hat{\sigma}_{x3}$				$\hat{\sigma}_{max3}$		
		Maximum deviation of the coordinate axes				Main deviations		
No. of measurements	Point	Z	X	Y	Point	Z	X	Y
20	13	0,0045	0,0022	0,0017	13	0,0045	0,0022	0,0017
20	33	0,0036	0,0017	0,0014	33	0,0036	0,0017	0,0014
20	53	0,0038	0,0018	0,0014	53	0,0038	0,0018	0,0014
20	73	0,0039	0,0018	0,0014	73	0,0039	0,0018	0,0014
5	23	0,0072	0,0035	0,0028	23	0,0072	0,0035	0,0027
5	43	0,0037	0,0020	0,0016	43	0,0038	0,0020	0,0015
5	63	0,0099	0,0047	0,0039	63	0,0099	0,0047	0,0038
5	83	0,0099	0,0045	0,0037	83	0,0099	0,0045	0,0037

Results of the levelling of characteristic point measurements with a lame plumb line via simulating the approximate positioning upon the characteristic points (type 03) have shown that:

- Deviations in the direction of coordinate axis *Z* do not exceed 1 cm (the largest one would be 9.9 mm), even when it comes to the smallest number of measurements (5).
- Deviations in the direction of coordinate axis *Y* do not exceed 4 mm (the largest one would be 3.9 mm).
- Deviations in the direction of coordinate axis *X* do not exceed 5 mm (the largest one would be 4.7 mm).

Once again, this is all depicted in the table 3.

On the following picture, a pedalloid and ellipsoid of deviations of measurement point 13 are shown:

Conclusions

With the on-terrain measurements, we have managed to obtain a realistic estimate i.e. evaluation of the accuracy of the characteristic point meaurements on our terrain. With the measurements of three different types of points, i.e. points with three different varieaties of on-point placement – namely with forced centering (type 01), a lame plumb line on known points (type 02) and a lame plumb line and simulated placement on characteristic points (type 03) – we have obtained the measurements necessary for us to provide an evaluation of the accuracy of said characteristic point measurements, which have not been materialised i.e. labelled (type 03). When measuring these characteristic point upon our terrain, the accuracy of GPS measurements has thus also been captured, as well as the accuracy of average operator placement of said characteristic terrain points.

With the measurements of said terrain points, the designing of a simple and financially undemanding model intended for the levelling of the afore-mentioned measurements, we have, at the Faculty of Natural Sciences and Engineering, managed to complement the accuracy estimations pertaining to the measurement of characteristic terrain points.

The characteristic points of the terrain which have not been materialised i.e. labelled (type 03) can be detected and measured with an accuracy where the standard deviation in the direction of coordinate axis *Z* does not, even with the smallest given number of measurements (5) surpass 1 cm (the largest having been 9.9 mm), the standard deviations in the direction of coordinate axis *Y* do not surpass 4 mm (the largest having been 3.9 mm), and the deviations in the direction of coordinate axis *X* do not surpass 5 mm (the largest having been 4.7 mm).

The comparison of measurements of all three types of points that have been measured has shown us that the designed model has provided us with realistic results. We are thus able to claim that the accuracy of the determination of characteristic terrain points via the RTK-GPS method is 1 cm or less.

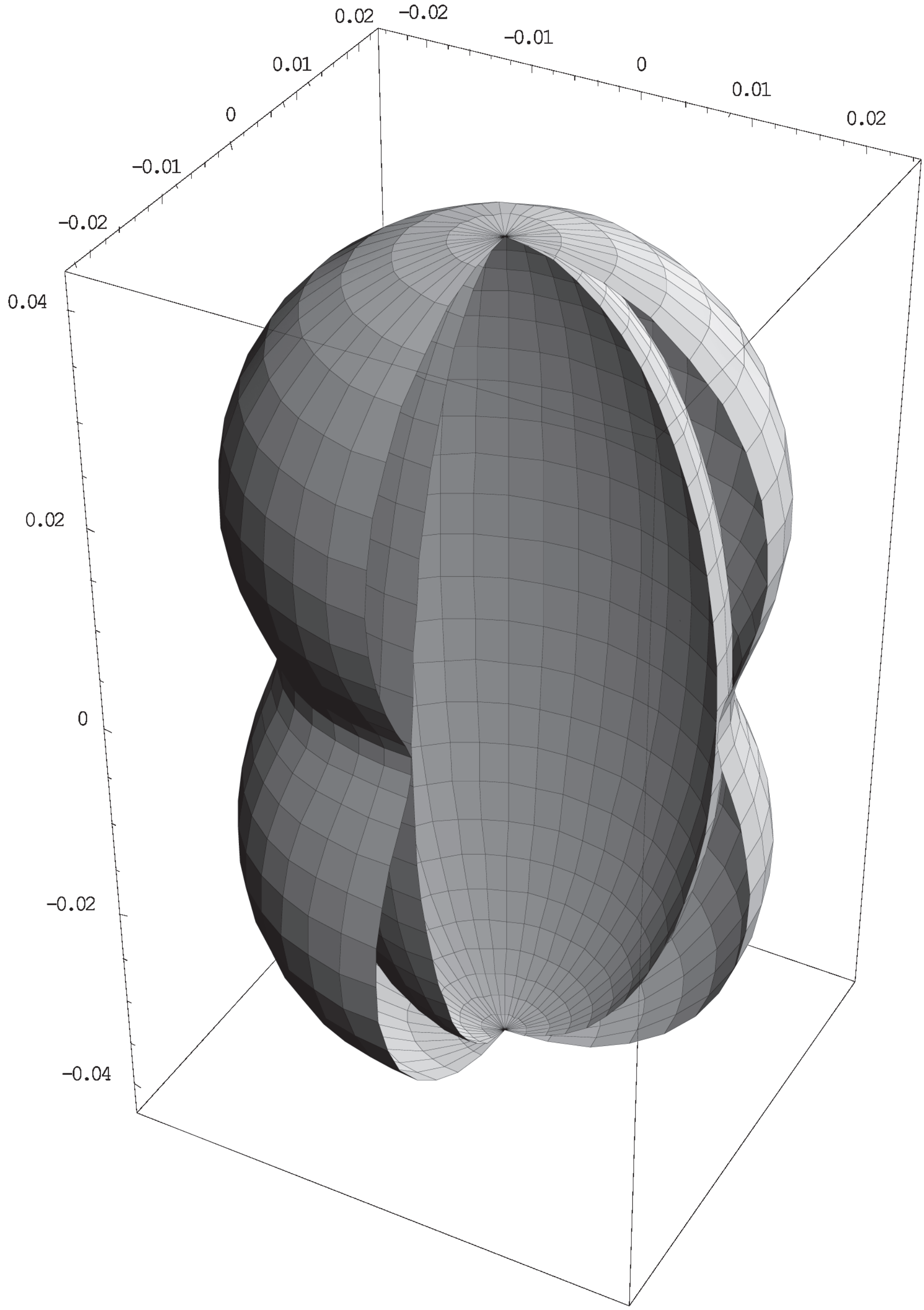

Figure 2. The pedalloid and ellipsoid of deviations of point 13
Slika 2. Pedaloid in elipsoid pogreškov točke 13

Povzetek

Realna ocena natančnosti določanja karakterističnih točk terena po metodi RTK-GPS

Z meritvami na terenu smo dobili realno oceno natančnosti meritev položaja karakterističnih točk terena. Z meritvami treh različnih tipov točk oz. točk s tremi različnimi postavitvami na točkah, in sicer s prisilnim centriranjem (tip01), s togim grezilom na znanih točkah (tip02) in s togim grezilom s simuliranjem postavitve na karakterističnih točkah (tip03) smo zajeli meritve, s katerimi smo podali oceno natančnosti merjenj karakterističnih točk terena, ki niso materializirane oziroma označene (tip03). Pri merjenju položaja karakterističnih točk terena je tako zajeta natančnost GPS izmere kot tudi natančnost postavitve na karakterističnih točkah terena pri povprečnem operaterju.

Z meritvami položaja karakterističnih točk terena, izdelavo enostavnega in cenovno ugodnega modela za izravnavo merjenj in podanimi ocenami natančnosti merjenj karakterističnih točk, pri povprečnem operaterju smo lastno metodo za določanje prostornin, ki se razvija na NTF, dopolnili z oceno natančnosti merjenj karakterističnih točk terena.

Karakteristične točke terena, ki niso materializirane oziroma označene (tip03) lahko zaznamo in izmerimo z natančnostjo kjer srednji pogreški v smeri koordinatne osi *Z* tudi pri manjšem številu merjenj (5) ne presegajo 1 cm (največji je 9,9 mm), pogreški v smeri koordinatne osi *Y* ne presegajo 4 mm (največji je 3,9 mm) in pogreški v smeri koordinatne osi *X* ne presegajo 5 mm (največji je 4,7 mm).

Primerjava meritev vseh treh omenjenih tipov točk je pokazala, da je izdelani model podal realne rezultate. Trdimo lahko, da je natančnost določitve karakterističnih točk terena po metodi RTK-GPS 1cm ali manjša.

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