Quasilinearization method and its application to physical problems

R. Krivec[1] and V. B. Mandelzweig[2]

[1] *J. Stefan Institute, P.O. Box 3000, 1001 Ljubljana, Slovenia*

[2] *Racah Institute of Physics, Hebrew University, Jerusalem 91904, Israel*

The general properties of the quasilinearization method [1–3], particularly its fast convergence, monotonicity and numerical stability are analyzed and illustrated on different physical problems. The method approximates a solution of a nonlinear differential equation by treating nonlinear terms as a perturbation about linear ones, and is not based, unlike perturbation theories, on the existence of some kind of a small parameter. Each approximation of the method sums many orders of the perturbation theory. The method provides accurate and stable answers for any coupling strengths, including for super singular potentials for which each term of the perturbation theory diverges and the perturbation expansion does not exist even for a very small couplings.

In order to further analyze and highlight the power and features of the quasilinearization method (QLM), we have made [2] numerical computations on the nonlinear ordinary first order differential equations for the S-wave scattering length $a_0 = a(\infty)$ and phase shifts δ_0, respectively, obtained in the variable phase approach [4]. We have considered different singular and nonsingular, attractive and repulsive potentials, namely Yukawa, Pöschl-Teller and Newton potentials, and have compared the results obtained by the quasilinearization method with the exact solutions.

It is shown also [3] that the quasilinearization method gives excellent results when applied to different nonlinear ordinary differential equations in physics, such as the Blasius, Duffing, Lane-Emden and Thomas-Fermi equations. The first few quasilinear iterations already provide extremely accurate and numerically stable answers.

Our conclusions can be formulated as follows:

i) The QLM treats the nonlinear terms as a perturbation about the linear ones [1] and is not based, unlike perturbation theories, on the existence of some kind of small parameter. As a result, as we see on our examples, it is able to handle, unlike the perturbation theory, large values of the coupling constant.

ii) The method provides very accurate and numerically stable and fast convergent answers for any values of the coupling constant giving the accuracy of at least five significant figures required in this work. Already the first few iterations provide precise answers for small and intermediate values of the coupling constant. The number of iterations necessary to reach a given precision only moderately increases for larger values of the coupling constants.

iii) The method provides very accurate and numerically stable answers also for any potential strength in the case of super singular potentials for which each term of the perturbation theory is infinite and the perturbation treatment is not possible even for a very small coupling.

In view of all this, since most equations of physics, from classical mechanics to quantum field theory, are either not linear or could be transformed to a nonlinear form, the quasilinearization method may turn out to be extremely useful and in many cases more advantageous than the perturbation theory or its different modifications, like expansion in inverse powers of the coupling constant, the $1/N$ expansion, etc.

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