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Coincident-point rigidity in normed planes

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Abstract

A bar-joint framework (G, p) is the combination of a graph G and a map p assigning positions, in some space, to the vertices of G. The framework is rigid if every edge-length-preserving continuous motion of the vertices arises from an isometry of the space. We will analyse rigidity when the space is a (non-Euclidean) normed plane and two designated vertices are mapped to the same position. This non-genericity assumption leads us to a count matroid first introduced by Jackson, Kaszanitsky and the third author. We show that independence in this matroid is equivalent to independence as a suitably regular bar-joint framework in a non-Euclidean normed plane with two coincident points; this characterises when a regular non-Euclidean normed plane coincident-point framework is rigid and allows us to deduce a delete-contract characterisation. We then apply this result to show that an important construction operation (generalised vertex splitting) preserves the stronger property of global rigidity in non-Euclidean normed planes and use this to construct rich families of globally rigid graphs when the non-Euclidean normed plane is analytic.

Keywords: Bar-joint framework, global rigidity, non-Euclidean framework, count matroid, recursive construction, normed spaces, analytic norm.

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Togost koincidenčnih točk v normirani ravnini

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Povzetek

Ogrodje paličnega sklepa (G, p) tvorita graf G in preslikava p, ki točkam grafa G določi položaje v nekem prostoru. Ogrodje je togo, če vsako zvezno gibanje točk, ki ohranja dolžine povezav, izhaja iz izometrije prostora. Analiziramo togost v primeru, ko je prostor (neevklidska) normirana ravnina, dve izbrani točki pa se preslikata v isti položaj. Ta predpostavka negeneričnosti nas vodi v števni matroid, ki so ga prvi vpeljali Jackson, Kaszanitsky in tretji avtor. Pokažemo, da je neodvisnost v tem matroidu ekvivalentna neodvisnosti primernega regularnega ogrodja paličnega sklepa v neevklidski normirani ravnini z dvema koincidenčnima točkama; to omogoča karakterizacijo, kdaj je v pravilni neevklidski normirani ravnini ogrodje koincidenčnih točk togo, in nam omogoča izpeljati karakterizacijo s pomočjo izbrisov in skrčitev. Nato z uporabo tega rezultata pokažemo, da pomembna konstrukcijska operacija (posplošeni razcep točke) ohranja močnejšo lastnost globalne togosti v neevklidskih normiranih ravninah, potem pa to uporabimo za konstrukcijo velikih družin globalno togih grafov v primeru, ko je neevklidska normirana ravnina analitična.

Ključne besede: Ogrodje paličnega sklepa, globalna togost, neevklidsko ogrodje, števni matroid, rekurzivna konstrukcija, normirani prostori, analitična norma.

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