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Analytic structure of nonperturbative quark propagators and meson processes*

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Abstract. The analytic structure of certain *Ansätze* for quark propagators in the nonperturbative regime of QCD is investigated. When choosing physically motivated parameterization of the momentum-dependent dressed quark mass function $M(p^2)$, with definite analytic structure, it is highly nontrivial to predict and control the analytic structure of the corresponding nonperturbative quark propagator. The issue of the Wick rotation relating the Minkowski-space and Euclidean-space formulations is also highly nontrivial in the nonperturbative case. A propagator form allowing the Wick rotation and enabling equivalent calculations in Minkowski and Euclidean spaces is achieved. In spite of its simplicity, this model yields good qualitative and semi-quantitative description of some pseudoscalar meson processes.

Lattice studies of QCD are complemented by the continuum QCD studies utilizing Dyson–Schwinger equations (DSE). Both *ab initio* DSE studies and DSE studies for models of QCD provide an important approach for the study of phenomena in hadronic physics both at zero and finite temperatures and densities – see, for example, Refs. [1, 2]. Just like lattice QCD studies, the large majority of DSE calculations (including those of our group, *e.g.*, [3]) are implemented in the Euclidean metric.

Nevertheless, solutions of the Bethe–Salpeter equation require analytic continuation of DSE solutions for dressed quark propagators $S_q(p)$, into the complex p^2–plane. Similar situation is with the processes that involve quark propagators (QP) and Bethe–Salpeter amplitudes: it is not enough to know propagators and the Bethe–Salpeter amplitude only in the spacelike region, for real and positive p^2. It is important to know the analytic properties in the whole p^2 complex plane.

Alkofer *et al.* [4] have explored the analytic structure of the Landau gauge gluon and quark propagators. They have proposed some simple analytic *Ansätze* for these propagators. Based on their *Ansätze*, Jiang *et al.* [5] provide an analytical approach to calculating the pion decay constant f_π and and the pion mass M_π at finite density.

We want to investigate and further improve the analytic structure of the quark propagator $S(p)$. It can be conventionally parameterized (in Minkowski

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space) as

$$S(p) = -\sigma_v(-p^2)\,\not{p} + \sigma_s(-p^2) = Z(-p^2)\frac{\not{p} + M(-p^2)}{p^2 - M^2(-p^2)}\ ,$$

and correspondingly in Euclidean space as

$$S(p) = i\not{p}\,\sigma_v(p^2) + \sigma_s(p^2) = \frac{Z(p^2)}{-i\not{p} + M(p^2)} = Z(p^2)\frac{i\not{p} + M(p^2)}{p^2 + M^2(p^2)}\ ,$$

where $M(x)$ is the dressed quark mass function and $Z(x)$ is the wave function renormalization. Alkofer et al. [4] have explored the analytic structure of the quark (and gluon) propagator in the Landau gauge, using numerical solutions of the pertinent Dyson-Schwinger equations and fits to lattice data as inputs. Their *Ansätze* for Z and M (or σ_v and σ_s) include meromorphic functions (poles on the real axes or/and pairs of the complex conjugate poles) and functions with branch cut structures. Positivity violation in the spectral representation of the propagator shows the presence of the negative–norm contributions to the spectral function, i.e., the absence of asymptotic states from the physical part of the state space, which is sufficient (but not necessary) criterion for the confinement. While in the gluon propagator a clear evidence for positivity violation is found, the similar analysis shows that there is probably no such violation in the quark propagator [4]. The propagator with pairs of complex conjugate poles violates causality. It has been argued [6,7] that the corresponding S-matrix remains both causal and unitary (see also Ref. [1]).

Furthermore, complex conjugate poles can pose a problem for the analytic continuation from Minkowski to Euclidean space (Wick rotation) used by lattice gauge theory and functional methods. It has been also shown that complex conjugate poles in $S(p)$ cause thermodynamical instabilities at nonvanishing temperature and density [8].

Of crucial importance is the following question: Is it possible to find an analytic *Ansatz* for the quark propagator solely with branching cut (or cuts) on the real timelike axes, with no additional structures (isolated singularities or cuts) in the complex plane? Such an *Ansatz* could be used for practical calculation of the processes involving quark loops.

Because of a complicated interplay between analytic structure of the functions Z and M on one side, and σ_s and σ_v on the other side, the approximation $A = 1$ has been applied. Then, the problem reduces to finding of appropriate functions $M(x)$ and $\sigma(x) = 1/(x + M^2(x))$. The most rigorous constraint is that the propagator $S(p) \to 0$ for all directions $|p^2| \to \infty$ in the complex p^2 plane [9]. Furthermore, for large and positive values of $x = p^2$ (spacelike momenta), function $M(x)$ must be positive and approach to zero from above [4]. In the Euclidean regime, for real and positive values of x, the mass function should be fitted to match the form known from lattice and Dyson-Schwinger calculations.

Number of *Ansätze* for the quark mass function has been investigated. When choosing certain parametrization of the function $M(x)$, with definite analytic structure, it is highly nontrivial to predict and control the analytic structure of the accompanying $\sigma(x)$ function. Relevant mathematical theory and possibly related theorems (like Rouches theorem) are hardly applicable for this concrete problem.

The best results were achieved with the *Ansatz* of the form $M(x) = \log(R(x))$, where R is a rational function with certain good properties. The function $M(x)$ has a few cuts on the real timelike axes, while the propagator dressing function $\sigma(x)$ has both branch cuts and poles on the real timelike axes. No additional structure are present in the complex momentum plane. The quark propagator based on this *Ansatz* should allow for the Wick rotation and equivalent calculation in Minkowski and Euclidean spacetime.

Future work will include an improved fitting of the mass function $M(x)$ and refinement of calculation with $Z(x) = 1$. Furthermore, we are planning to check whether our *Ansatz* satisfies the requirements of positivity violation.

The quark propagator obtained in this way, endowed with good analytic properties, should then be tried and adjusted so that it gives good results in various applications: the $\gamma\gamma$-transition and charge form factors of pions, σ and ρ form factors and decays, are just some of the interesting potential applications of the quark propagator *Ansatz* with good analytic structure. It is also necessary to investigate the related issue of the Bethe-Salpeter equation in Minkowski space. The quark loop contribution to various processes should also be studied using these improved quark propagators. Besides the processes like $\pi, \eta, \eta' \to \gamma\gamma$ that are described by an anomalous triangle diagram, there are interesting anomalous processes based on the pentagon diagram, like $\eta, \eta' \to 4\pi$. (We could expect new results from high-statistics η' experiments like BES-III, ELSA, CB-at-MAMIC, CLAS at Jefferson Lab.) The non–anomalous processes $\eta \to 3\pi$ is especially interesting because it is sensitive to the isospin violation. While the average u and d-quark mass, $(m_u + m_d)/2$, is well known, there exists significant uncertainty in their mass difference, $m_d - m_u$. The $\eta \to 3\pi$ decay is particularly suitable for $m_d - m_u$ difference determination because of the suppressed electromagnetic contributions [10, 11].

Since the microscopic understanding of strongly interacting matter (both in hadronic phase and in quark-gluon phase) is of great importance also for the physics of heavy ion collisions and compact stars, extending the quark propagator *Ansatz* with good analytic structure to finite densities and temperatures should also be investigated. This is necessary, for example (to name one concrete task), for extending our analyses of the η-η' complex [12,13] to finite densities and temperatures. Of particular interest is extending to finite density our analysis of the possible $U_A(1)$ symmetry restoration [14] in the η-η' complex.

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Resonances in the Nambu–Jona-Lasinio model

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Abstract. We have designed a soluble model similar to the Nambu–Jona-Lasino model, regularized in a box with periodic boundary conditions, in order to explore the properties of resonances when only discrete eigenvalues are available. The study might give a lesson to similar problems in Lattice QCD.

1 The quasispin NJL-like model

It is very instructive to understand the key features of a simplified model containing the spontaneous chiral symmetry breaking. Some time ago we have constructed a soluble version of the Nambu–Jona-Lasino model [1, 2]. Now we explore what it tells about the sigma meson.

We make the following simplifications:

1. We assume a sharp 3-momentum cutoff $0 \le |\mathbf{p}_i| \le \Lambda$;
2. The space is restricted to a box of volume $\mathcal{V}$ with periodic boundary conditions. This gives a finite number of discrete momentum states, $\mathcal{N} = N_h N_c N_f \mathcal{V}\Lambda^3/6\pi^2$ occupied by N quarks. (N_h, N_c and N_f are the number of quark helicities, colours and flavours.)
3. We take an average value of kinetic energy for all momentum states: $|\mathbf{p}_i| \to P = \frac{3}{4}\Lambda$.
4. While in the NJL model the interaction conserves the sum of momenta of both quarks we assume that each quark conserves its momentum and only switches from the Dirac level to Fermi level.
5. Temporarily, we restrict to one flavour of quarks, $N_f = 1$.

Let us repeat the "Quasispin Hamiltonian" [1,2].

$$H = \sum_{k=1}^{N} \Big(\gamma_5(k) h(k) P + m_0 \beta(k) \Big) + \\ - \frac{g}{2} \Big(\sum_{k=1}^{N} \beta(k) \sum_{l=1}^{N} \beta(l) + \sum_{k=1}^{N} i\beta(k)\gamma_5(k) \sum_{l=1}^{N} i\beta(l)\gamma_5(l) \Big) \quad .$$

Here γ_5 and β are Dirac matrices, m_0 is the bare quark mass and $g = 4G/\mathcal{V}$ where G is the interaction strength in the original (continuum) NJL.

We introduce the quasispin operators which obey the spin commutation relations

$$j_x = \frac{1}{2}\beta\,, \quad j_y = \frac{1}{2}i\beta\gamma_5\,, \quad j_z = \frac{1}{2}\gamma_5\,,$$

$$R_\alpha = \sum_{k=1}^{N}\frac{1+h(k)}{2}j_\alpha(k)\,,\; L_\alpha = \sum_{k=1}^{N}\frac{1-h(k)}{2}j_\alpha(k)\,,\; J_\alpha = R_\alpha + L_\alpha = \sum_{k=1}^{N} j_\alpha(k)\,.$$

The model Hamiltonian can then be written as

$$H = 2P(R_z - L_z) + 2m_0 J_x - 2g(J_x^2 + J_y^2) \quad .$$

The three model parameters

$$\Lambda = 648\,\text{MeV}, \quad G = 40.6\,\text{MeV fm}^3, \quad m_0 = 4.58\,\text{MeV}$$

have been fitted (in a Hartree-Fock + RPA approximation) to the observables

$$M = \sqrt{\Big(E_g(N) - E_g(N-1)\Big)^2 - P^2} = 335\,\text{MeV}$$

$$Q = \langle g|\bar\psi\psi|g\rangle = \frac{1}{\mathcal{V}}\langle g|\sum_i \beta(i)|g\rangle = \frac{1}{\mathcal{V}}\langle g|J_x|g\rangle = 250^3\,\text{MeV}^3$$

$$m_\pi = E_1(N) - E_g(N) = 138\,\text{MeV}\,.$$

The values of our model parameters are very close to those of the full Nambu-Jona Lasinio model used by the Coimbra group [3] and by Buballa [4].

2 The spectrum of 0^- and 0^+ excitations – Emergence of the σ meson

It is easy to evaluate the matrix elements of the quasispin Hamiltonian using the angular momentum algebra. If N is not too large the corresponding sparse matrix can be diagonalized using *Mathematica*.

Excited levels of the ground state band (R=L=N/4) in Fig. 1 are almost equidistant and are suggestive of n-pion states (in s-state). The level spacings ΔE are slightly decreasing with the assumed number of pions n_π due to the attractive interaction between pions. Inbetween appear also "intruder states" which can be interpreted as sigma excitations. The interpretation as σ meson is further supported by the large value of the matrix element of J_x between the ground state and the "intruder state". (Odd "multipion states" have zero value and even ones have a rather small value.)

n_π	parity	E [MeV]	ΔE [MeV]	Intruder
8	+	866	63	
	-	816		$\sigma(667)+\pi(136)+13$ MeV
7	-	803	93	
6	+	710	99	
	+	667		$\sigma(667)$
5	-	611	108	
4	+	503	115	
3	-	388	123	
2	+	265	129	
1	-	136	136	
0	+	0	0	

Fig. 1. Levels of the ground state band (R=L=N/4), level spacing between opposite parity states, and the assumed number of pions n_π pions

3 The width of the σ meson

In the attempt to describe resonances when only discrete eigenvalues are available we get a discrete sigma resonance energy, but not its width. We are trying to get the complex pole. For that purpose, we explore the method of analytic continuation from the bound state [5]. For this purpose, we vary one of the model parameters, the bare quark mass m from the region where the σ meson would be bound ($E_\sigma < E_{2\pi}$) down to the physical value of $m \to m_0$ (where $E_\sigma \gg E_{2\pi}$).

At $m > 64$ MeV there are two positive parity states between the first and second negative parity states (the one-pion and three-pion excitations); the lower one is the intruder (σ meson) and the upper one is the correlated two-pion state. At $m = 64$ MeV both positive parity states coincide – the threshold for $\sigma \to 2\pi$. When we decrease m further, the energy of the σ meson decreases slower than the 2π energy and it appears at higher multipion states. For the physical value $m = m_0 = 4.58$ MeV σ is already the sixth excited state, next to the six-pion state. It is obviously in the continuum, prompt to decay into 2π, in a more complete choice of interaction.

The method consists of the following steps:

- Determine the threshold value m_{th} and calculate $\epsilon = E_\sigma - E_{2\pi}$ as a function of m for $m > m_{th}$.
- Introduce a variable $x = \sqrt{m - m_{th}}$; calculate $k(x) = i\sqrt{-\epsilon}$ in the bound state region (Fig. 2).
- Fit $k(x)$ by a polynomial $k(x) = i(c_0 + c_1 x + c_2 x^2 + \ldots + c_{2M} x^{2M})$.

- Construct a Padé approximant:

$$k(x) = i\,\frac{a_0 + a_1 x + \ldots + a_M x^M}{1 + b_1 x + \ldots + b_M x^M}\ .$$

- Analytically continue $k(x)$ to the region $m < m_{th}$ (i.e. to imaginary x) where $k(x)$ becomes complex.
- Determine the position and the width of the resonance as analytic continuation in m (Fig. 3 and Fig. 4):

$$E_{res} = \mathrm{Re}\,(\mathrm{cont}_{m\to m_0} k^2)\,, \qquad \Gamma = -2\,\mathrm{Im}\,(\mathrm{cont}_{m\to m_0} k^2)\ .$$

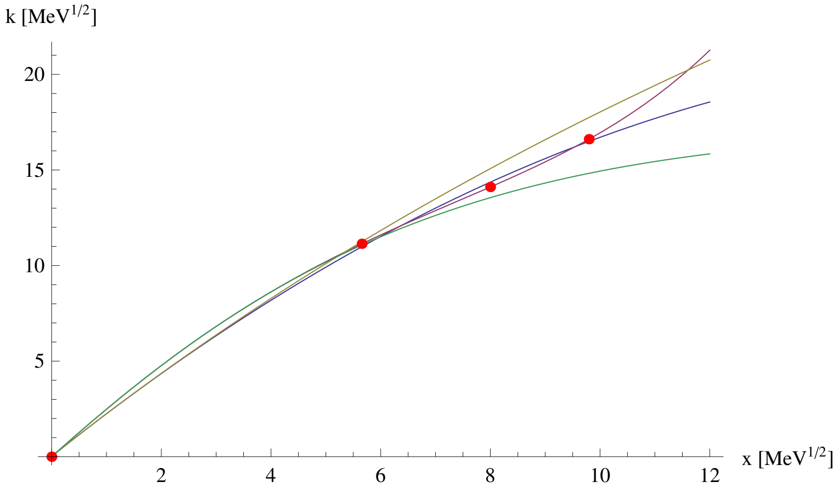

Fig. 2. The fit of $k(x)$ with quadratic(lower middle) and quartic polynomial (upper middle) and with Padé approximants of order 1 (below) and 2 (above)

We notice that the results for E_{res} and Γ in Fig. 3 and 4 deviate strongly for first and second order Padé approximants. This is due to the large stretch for the analytic continuation so that convergence at higher orders cannot be expected. Nevertheless, it is rewarding that the physical values for E_{res} and Γ lie somewhere in the middle between both curves.

To conclude, the method of analytic continuation in this case is just a game, but it is instructive. Intentionally, we have plotted the energy and width of the σ meson as a function of the corresponding pion mass rather than as a function of the model parameter m. This is reminiscent of the extrapolation of pion mass from about 500 Mev towards its physical value the way tha lattice people have to struggle.

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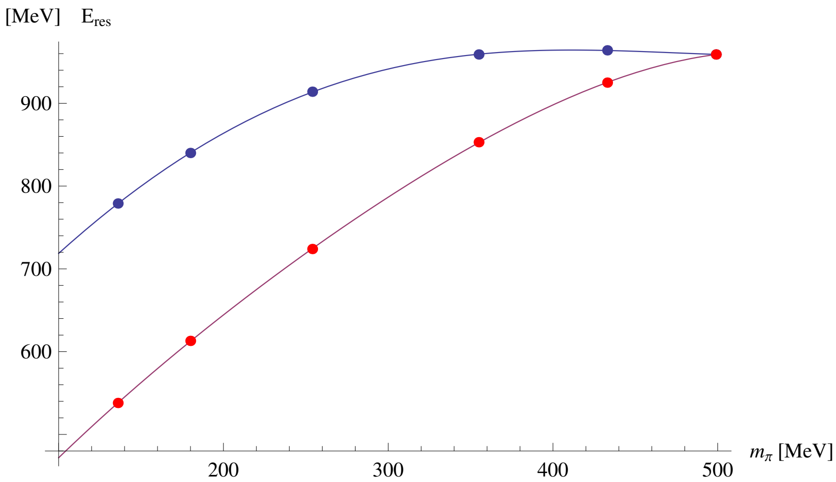

Fig. 3. The resonance energy E_{res} of the σ meson as a function of the pion mass – extrapolation using Padé approximants of order 1 (below) and 2 (above)

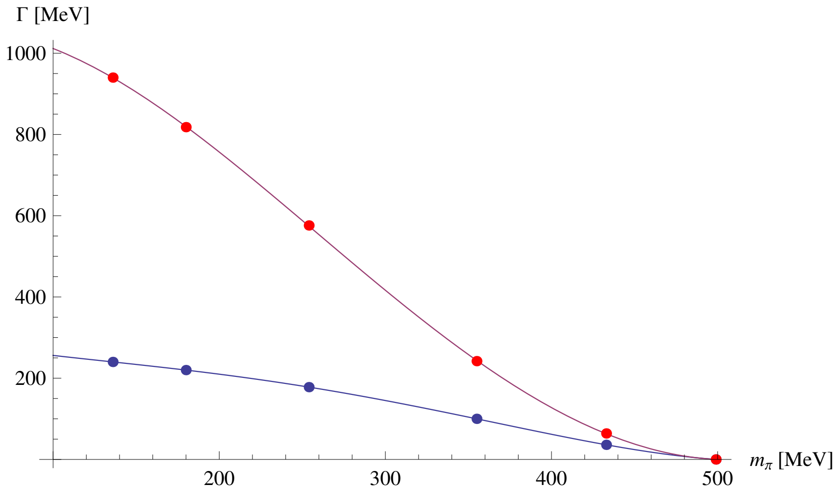

Fig. 4. The width Γ of the σ meson as a function of the pion mass – extrapolation using Padé approximants of order 1 (below) and 2 (above)

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Analitična zgradba neperturbativnih kvarkovih propagatorjev in mezonskih procesov

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Raziskujemo analitično zgradbo nekaterih nastavkov za kvarkove propagatorje v neperturbativnem področju kromodinamike. Če izberemo fizikalno motivirano parametrizacijo masne funkcije $M(p^2)$ oblečenih kvarkov, odvisne od gibalne količine in z določeno analitično zgradbo, je skrajno težavno napovedati in obvladati analitično zgradbo ustreznega neperturbativnega kvarkovega propagatorja. Tudi problem Wickove rotacije, ki povezuje izražavo v prostoru Minkowskega in Evklida, je skrajno težaven v neperturbativnem območju. Izpeljemo obliko propagatorja, ki omogoča Wickovo rotacijo in dopušča enakovredne račune v prostoru Minkowskega in Evklida. Kljub preprostosti nudi ta model dober kvalitativen in semikvantitativen opis nekaterih procesov z psevdoskalarnimi mezoni.

Primerjava med mezoni in resonancami $W_L W_L$ pri energijah več TeV

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Mikavni signali z Velikega hadronskega trkalnika (LHC) namigujejo, da morda obstajajo v področju zloma elektro-šibke simetrije resonance v območju več TeV. Spomnimo na nekaj ključnih resonanc mezon-mezon v območju GeV, ki bi utegnile imeti analogne resonance pri visokih energijah in nam služijo za primerjavo, hkrati z odgovarjujočo unitarizirano efektivno teorijo. Čeprav je podrobna dinamika lahko različna, pa zahteve po unitarnosti, kavzalnosti in globalnem zlomu simetrije (z uporabo metode inverzne amplitude) dovoljujejo prenos intuicije v večinoma neizmerjeno območje visokih energij. Če bo povečano število dogodkov na ATLASU okrog 2 TeV podprlo tako novo resonanco, to lahko pomeni anomalno sklopitev $q\bar{q}W$.

Resonance v konstituentnem kvarkovem modelu.

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Na kratko poročamo o današnjem opisu barionskih resonanc v realističnem modelu s konstituentnimi kvarki, v katerem običajno obravnavamo resonance kot